$$\text{أشكال بالتجزئة}$$

$$\int u \, dv = uv - \int v \, du$$

اوجد:

$$\int x \cos x \, dx \qquad\qquad u = x \qquad dv = \cos x \, dx$$
$$du = dx \qquad v = \sin x$$

$$= x \sin x - \int \sin x \, dx$$
$$= x \sin x + \cos x + C$$

ⓐ $\int (x-3) \, e^{x-3} \, dx$

$$u = x - 3 \qquad dv = e^{x-3} \, dx$$
$$du = dx \qquad v = e^{x-3}$$

$$= (x-3) \, e^{x-3} - \int e^{x-3} \, dx$$

$$= (x-3) \, e^{x-3} - e^{x-3} + C$$

ⓑ $\int 4x \, e^{-5x} \, dx = \dfrac{-1}{5} \int (4x)(-5) \, e^{-5x} \, dx$

$$u = 4x \qquad dv = -5 \, e^{-5x} \, dx$$
$$du = 4 \, dx \qquad v = e^{-5x}$$

$$= \frac{-1}{5} \left[4x \, e^{-5x} - \int 4 \, e^{-5x} \, dx \right]$$

$$= \frac{-1}{5} \left[4x \, e^{-5x} - \int \frac{-1}{5} \, 4 \, (-5) \, e^{-5x} \, dx \right]$$

$$= \frac{-1}{5} \left[4x \, e^{-5x} + \frac{4}{5} \, e^{-5x} \right] + C$$

$$= \frac{-4}{5} \, x \, e^{-5x} - 4 \, e^{-5x} + C$$

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$$\int \ln x \, dx \qquad \qquad ③ \; P.38$$

$$u = \ln x \qquad dv = dx$$
$$du = \frac{1}{x} dx \qquad v = x$$

$$= x \ln x - \int \frac{1}{x} \cdot x \, dx$$

$$= x \ln x - \int 1 \, dx \quad = x \ln x - x + c$$

$$\int (x+1) \ln(x+1) \, dx \qquad ④ \; P.39$$

$$= \int t \ln t \, dt \qquad \qquad t = x+1$$
$$\qquad \qquad dt = dx$$

$$= \frac{t^2}{2} \cdot \ln t - \int \frac{t^2}{2} \cdot \frac{dt}{t} \qquad u = \ln t \quad dv = t \, dt$$
$$\qquad \qquad du = \frac{dt}{t} \qquad v = \frac{t^2}{2}$$

$$= \frac{t^2}{2} \cdot \ln |t| + \frac{1}{4} t^2 + c$$

$$= \frac{t^2}{2} \ln |x+1| + \frac{1}{4} (x+1)^2 + c$$

$$\int x^2 \sin x \, dx \qquad \qquad ⑤ \; P.39$$

$$u = x^2 \qquad dv = \sin x \, dx$$
$$du = 2x \, dx \qquad v = -\cos x$$

$$= -x^2 \cos x + \int 2x \cos x \, dx$$

$$u = 2x \qquad dv = \cos x \, dx$$
$$du = 2 \, dx \qquad v = \sin x$$

$$= -x^2 \cos x + 2x \sin x - \int 2 \sin x \, dx$$

$$= -x^2 \cos x + 2x \sin x + 2 \cos x + c$$

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$I = \int x^2 \, e^{x+2} \, dx$ ⑥ P. 40

$u = x^2 \qquad dv = e^{x+2} \, dx$

$du = 2x \, dx \qquad v = e^{x+2}$

$I = x^2 (e^{x+2}) - \int 2x \, e^{x+2} \, dx$

$u = 2x \qquad dv = e^{x+2} \, dx$

$du = 2 \, dx \qquad v = e^{x+2}$

$I = x^2 e^{x+2} - \left[2x \, e^{x+2} - \int 2 \, e^{x+2} \, dx \right]$

$= x^2 e^{x+2} - 2x e^{x+2} + 2 e^{x+2} + C$

$I = \int e^x \cos x \, dx$ ⑦ P. 41

$u = e^x \qquad dv = \cos x \, dx$

$du = e^x \, dx \qquad v = \sin x$

$= e^x \sin x - \int e^x \sin x \, dx$

$u = e^x \qquad dv = \sin x \, dx$

$du = e^x \, dx \qquad v = -\cos x$

$I = e^x \sin x - \left[-e^x \cos x + \int e^x \cos x \, dx \right]$

$= e^x \sin x + e^x \cos x - \int e^x \cos x \, dx$

$I = e^x \sin x + e^x \cos x - I$

$2I = e^x \sin x + e^x \cos x$

$I = \frac{1}{2} \left[e^x \sin x + e^x \cos x \right] + C$

3

$$f(x) = \frac{2x-1}{x^2-4x+3}$$

@ الكسور الجزئيه

$$\frac{2x-1}{x^2-4x+3} = \frac{2x-1}{(x-1)(x-3)} = \frac{A_1}{x-1} + \frac{A_2}{x-3}$$

$$= \frac{A_1(x-3) + A_2(x-1)}{(x-1)(x-3)}$$

$$2x-1 = A_1(x-3) + A_2(x-1)$$

بعوض عن $x = 3$

$$2(3)-1 = A_1(0) + A_2(3-1)$$

$$5 = 2A_2 \Rightarrow \boxed{A_2 = \frac{5}{2}}$$

بعوض عن $x = 1$

$$2(1)-1 = A_1(1-3) + A_2(0)$$

$$1 = -2A_1 \Rightarrow \boxed{A_1 = \frac{-1}{2}}$$

$$\therefore \frac{2x-1}{x^2-4x+3} = \frac{-1}{2(x-1)} + \frac{5}{2(x-3)}$$

ⓑ $$\int f(x)\, dx = \int \frac{-1}{2(x-1)}\, dx + \int \frac{5}{2(x-3)}\, dx$$

$$= -\frac{1}{2} \ln|x-1| + \frac{5}{2} \ln|x-3| + c$$

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$$\int \frac{x^2-2}{2x^3-5x^2-3x}\,dx$$

$$\frac{x^2-2}{2x^3-5x^2-3x} = \frac{x^2-2}{x(2x+1)(x-3)}$$

$$= \frac{A_1}{x} + \frac{A_2}{2x+1} + \frac{A_3}{x-3}$$

$$= \frac{A_1(2x+1)(x-3) + A_2\,x(x-3) + A_3\,x(2x+1)}{x(2x+1)(x-3)}$$

$$\therefore\ x^2-2 = A_1(2x+1)(x-3) + A_2(x)(x-3) + A_3(x)(2x+1)$$

$x=0$ عند:

$$-2 = A_1(1)(-3) + A_2(0) + A_3(0)$$
$$-2 = -3A_1 \implies \boxed{A_1 = \frac{2}{3}}$$

$x=3$ عند:

$$3^2-2 = A_1(7)(0) + A_2(3)(0) + A_3(3)(7)$$
$$7 = A_3(3)(7) \implies \boxed{A_3 = \frac{1}{3}}$$

$x=\frac{-1}{2}$ عند:

$$\frac{1}{4}-2 = A_1(0)\left(-\frac{1}{2}-3\right) + A_2\left(\frac{-1}{2}\right)\left(\frac{-1}{2}-3\right) + A_3\left(-\frac{1}{2}\right)(0)$$
$$\frac{1}{4}-2 = A_2\left(-\frac{1}{2}\right)\left(-\frac{7}{2}\right)$$
$$\frac{-7}{4} = A_2\left(\frac{7}{4}\right) \implies \boxed{A_2 = -1}$$

$$I = \int \frac{\frac{2}{3}}{x}\,dx + \int \frac{-1}{2x+1}\,dx + \int \frac{\frac{1}{3}}{x-3}\,dx$$

$$= \frac{2}{3}\int \frac{1}{x}\,dx - \frac{1}{2}\int \frac{2}{2x+1}\,dx + \frac{1}{3}\int \frac{1}{x-3}\,dx$$

$$= \frac{2}{3}\ln|x| - \frac{1}{2}\ln|2x+1| + \frac{1}{3}\ln|x-3| + c$$

$$\int \frac{4x^2 - 4x + 1}{x^3 - 2x^2 + x} \, dx$$

$$\frac{4x^2 - 4x + 1}{x^3 - 2x^2 + x} = \frac{4x^2 - 4x + 1}{x(x^2 - 2x + 1)} = \frac{4x^2 - 4x + 1}{x(x-1)^2}$$

$$= \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$$= \frac{A(x-1)^2 + Bx(x-1) + Cx}{x(x-1)^2}$$

$$\therefore \quad 4x^2 - 4x + 1 = A(x-1)^2 + Bx(x-1) + Cx$$

$$x = 0 \ ضع$$

$$1 = A(1) + B(0)(-1) + C(0)$$

$$\boxed{A = 1}$$

$$x = 1 \quad ضع$$

$$4(1)^2 - 4(1) + 1 = A(0) + B(1)(0) + C(1)$$

$$\boxed{1 = C}$$

$$x = 2 \quad ضع$$

$$4(2)^2 - 4(2) + 1 = 1(2-1)^2 + B(2)(2-1) + 1(2)$$

$$9 = 1 + 2B + 2$$

$$9 - 3 = 2B \implies \boxed{B = 3}$$

$$\therefore \quad I = \int \frac{1}{x} \, dx + \int \frac{3}{x-1} \, dx + \int \frac{1}{(x-1)^2} \, dx$$

$$= \ln|x| + 3 \ln|x-1| - \frac{1}{x-1} + C$$

$$\int \frac{x^2+1}{x^3+4x^2} \, dx$$

$$\frac{x^2+1}{x^3+4x^2} = \frac{x^2+1}{x^2(x+4)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+4}$$

$$= \frac{Ax(x+4) + B(x+4) + Cx^2}{x^2(x+4)}$$

$$x^2+1 = Ax(x+4) + B(x+4) + Cx^2$$

$x=0$ если

$$1 = A(0)(0+4) + B(0+4) + C(0)^2$$

$$1 = 4B \implies \boxed{B = \frac{1}{4}}$$

$x=-4$ если

$$(-4)^2+1 = A(-4)(0) + B(0) + C(-4)^2$$

$$17 = C(16) \implies \boxed{C = \frac{17}{16}}$$

$x=1$ если

$$1^2+1 = A(1)(1+4) + \frac{1}{4}(1+4) + \frac{17}{16}$$

$$2 = 5A + \frac{5}{4} + \frac{17}{16}$$

$$5A = 2 - \frac{5}{4} - \frac{17}{16} \implies \boxed{A = \frac{-1}{16}}$$

$$I = \frac{-1}{16} \int \frac{1}{x} \, dx + \frac{1}{4} \int x^{-2} \, dx + \frac{17}{16} \int \frac{1}{x+4} \, dx$$

$$= \frac{-1}{16} \ln|x| + \frac{1}{4x} + \frac{17}{16} \ln|x+4| + c$$

$$\int \frac{x^3 - 2x^2 - 4}{x^3 - 2x^2} \, dx$$

$$
\begin{array}{r}
1 \\
x^3 - 2x^2 \overline{\big)\, x^3 - 2x^2 - 4} \\
-x^3 \pm 2x^2 \\
\hline
-4
\end{array}
$$

$$I = \int 1 \, dx + \int \frac{-4}{x^3 - 2x^2} \, dx$$

$$\frac{-4}{x^3 - 2x^2} = \frac{-4}{x^2(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2}$$

$$= \frac{Ax(x-2) + B(x-2) + Cx^2}{x^2(x-2)}$$

$$-4 = Ax(x-2) + B(x-2) + Cx^2$$

$x = 0$ için

$$-4 = A(0)(0-2) + B(0-2) + C(0)^2$$

$$-4 = -2B \implies \boxed{B = 2}$$

$x = 2$ için

$$-4 = A(2)(2-2) + B(2-2) + C(2)^2$$

$$-4 = 4C \implies \boxed{C = -1}$$

$x = 1$ için

$$-4 = A(1)(1-2) + 2(1-2) + -1(1)^2$$

$$-4 = -A - 2 - 1 \implies \boxed{A = 1}$$

$$I = \int 1 \, dx + \int \frac{dx}{x} + \int 2x^{-2} \, dx - \int \frac{1}{x-2} \, dx$$

$$= x + \ln|x| - \frac{2}{x} - \ln|x-2| + C$$

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$$\int \frac{x^3 - 7x + 9}{x^2 - 3x + 2} \, dx$$

$$x^2 - 3x + 2 \, \overline{\big)\, x^3 \qquad - 7x + 9} \quad \underline{x + 3}$$

$$\underline{-x^3 \overset{+}{\,} 3x^2 \overset{-}{\,} 2x}$$

$$3x^2 - 9x + 9$$

$$\underline{-3x^2 \overset{+}{\,} 9x \overset{-}{\,} 6}$$

$$3$$

$$= \int (x+3) \, dx + \int \frac{3 \, dx}{x^2 - 3x + 2}$$

$$\frac{3}{x^2 - 3x + 2} = \frac{3}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$$

$$= \frac{A(x-2) + B(x-1)}{(x-1)(x-2)}$$

$$\therefore \, 3 = A(x-2) + B(x-1)$$

$$x = 1 \text{ ضع}$$

$$3 = A(1-2) + B(0) \implies \boxed{A = -3}$$

$$x = 2 \text{ ضع}$$

$$3 = A(0) + B(2-1) \implies \boxed{B = 3}$$

$$I = \int (x+3) \, dx + (-3) \int \frac{1}{x-1} \, dx + 3 \int \frac{1}{x-2} \, dx$$

$$= \frac{1}{2} x^2 + 3x - 3 \ln|x-1| + 3 \ln|x-2| + c$$

$$\int \frac{2x^4 + 3x^2 - 7}{x^3 - 6x^2 + 9x} \, dx =$$

$$
\begin{array}{r|l}
 & \underline{2x + 12} \\
x^3 - 6x^2 + 9x & 2x^4 \quad\quad + 3x^2 \quad\quad -7 \\
 & \underline{-2x^4 + 12x^3 - 18x^2} \\
 & \quad\quad 12x^3 - 15x^2 \quad -7 \\
 & \quad\quad \underline{-12x^3 \pm 72x^2 \mp 108x} \\
 & \quad\quad\quad 57x^2 - 108x - 7
\end{array}
$$

$$= \int (2x+12)\,dx + \int \frac{57x^2 - 108x - 7}{x^3 - 6x^2 + 9x} \, dx$$

$$\frac{57x^2 - 108x - 7}{x^3 - 6x^2 + 9x} = \frac{57x^2 - 108x - 7}{x(x^2 - 6x + 9)} = \frac{57x^2 - 108x - 7}{x(x-3)^2}$$

$$= \frac{A}{x} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$$

$$= \frac{A(x-3)^2 + Bx(x-3) + Cx}{x(x-3)^2}$$

$$\therefore 57x^2 - 108x - 7 = A(x-3)^2 + Bx(x-3) + Cx$$

$x = 0$ يضع ر.

$$-7 = 9A \implies A = \frac{-7}{9}$$

$x = 3$ يضع ر.

$$513 - 324 - 7 = 3C \implies C = \frac{182}{3}$$

$x = 1$ يضع ر.

$$57 - 108 - 7 = 4\left(-\frac{7}{9}\right) + (-2B) + \frac{182}{3}$$

$$\implies B = \frac{520}{9}$$

$$I = \int (2x+12)\,dx + \left(\frac{-7}{9}\right)\int \frac{1}{x}\,dx + \frac{520}{9}\int \frac{dx}{x-3} + \frac{182}{3}\int \frac{dx}{(x-3)^2}$$

$$= x^2 + 12x - \frac{7}{9}\ln|x| + \frac{520}{9}\ln|x-3| + \frac{182}{3(x-3)} + C$$