	Funtion	x Domain	y Range
(a) $y = x + 4$	Yes	$(-\infty, \infty)$	$(-\infty, \infty)$
(b) $y = \sqrt{2x-1}$	Yes	$2x - 1 \geq 0$ $x \geq \frac{1}{2}$ $[\frac{1}{2}, \infty)$	$[0, \infty)$
(c) $y^2 = x$ $y = \pm\sqrt{x}$	No	$x \geq 0$ $[0, \infty)$	$(-\infty, \infty)$
(d) $y \leq x - 1$	No	$(-\infty, \infty)$	$(-\infty, \infty)$
(e) $y = \frac{5}{x-1}$	Yes	$x \neq 1$ $\mathbb{R} \setminus \{1\}$ $-\infty$ 1 ∞ $(-\infty, 1) \cup (1, \infty)$	$y \neq 0$ $\mathbb{R} \setminus \{0\}$ $(-\infty, 0) \cup (0, \infty)$
$y = \sqrt[3]{x+1}$	Yes	$(-\infty, \infty)$	$(-\infty, \infty)$

2.5 Absolute Value Equation

① $|\blacksquare| = k \;,\; k > 0 \Rightarrow \blacksquare = \pm k$ (صاحب اهل القيمة)

② $|\blacksquare| = 0 \Rightarrow \blacksquare = 0$ (صاحب اهل القيمة)

③ $|\blacksquare| = k,\; k < 0 \Rightarrow S.S\ \emptyset$

④ $|a| = |b| \Rightarrow a = \pm b$

⑤ $|a-b| = |b-a| \Rightarrow \mathbb{R}$

لما نطلع قيمة المعادلة نعوض ونشوف اذا تحقق absolute value

e.g $|x-5| = x$

راح اشيل دبل تاتوتا $\quad x - 5 = \pm x \quad$. وبحل وأشوف

ons

Rec
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Distan
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-3

LOOKING AHEAD TO CALCULUS

The precise definition of a **limit** in calculus requires writing absolute value inequalities.

A standard problem in calculus is to find the "interval of convergence" of something called a **power series**, by solving an inequality of the form

$$|x-a| < r.$$

This inequality says that x can be any number within r units of a on the number line, so its solution set is indeed an interval—namely the interval $(a-r, a+r)$.

Similarly, $|x| < 3$ is satis
are less than 3—that is, the inte

$-3 <$

See **Figure 6.** Finally, $|x| > 3$ i
from 0 are greater than 3. These
the solution set is

$(-\infty$

Notice in **Figure 6** that the union o
$|x| > 3$ is the set of real numbers.
These observations support the
and inequalities summarized in the
fits the form of Case 1, 2, or 3, chan
solution set and its graph will look sin

For each equation or inequality
$k > 0$.

bsolute Value Equations (Case 1 and the Special Case
b|)

(b) $|4x - 3| = |x + 6|$

ression $5 - 3x$ to have absolute value 12, it must repre-
12. This equation fits the form of Case 1.

Don't forget this second possibility

$|5 - 3x| = 12$

= 12

= 7

= −

s −

e sol

ues c

valu

x −

5

$K > 0$

هنا بس موجب

$|\square| \geq K$

$|\square| \leq K$ او $<$

$|\square| < -K$ $|\square| > K$

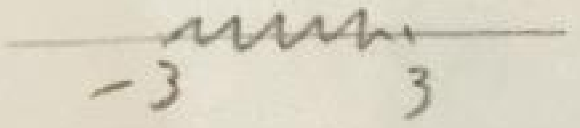
او

$-K < |\square| < K$

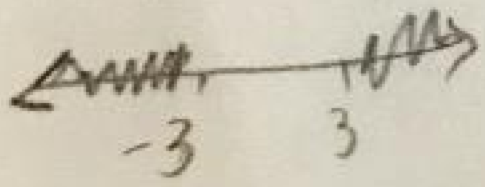
e.g

$|x| < 3$

ن نحل المعادلة



$|x| > 3$



هنا صفر وسالب

$K \leq 0$ او $<$

مالها D

$|\square| \geq$ او $\leq$ K

اعمل صح الي جو

اول 0 ...

اتوقع

T او F

اذا تحقق

always True $\mathbb{R}$

always false $\varnothing$

ualities

lving Absolute Value Inequalities (Cases 2 and 3)

y.

(b) $|2x + 1| > 7$

... require that the absolute value expression be isolated on

2.4: أنواع المتباينات وطريقة حل كل نوع.

الخصائص: ① $a<b \rightarrow a+c<b+c$

② $a<b \rightarrow c>0 \begin{cases} ca<cb \\ \frac{c}{a}<\frac{c}{b} \end{cases}$ العكس

③ $a<b \rightarrow c<0 \begin{cases} ca>cb \\ \frac{a}{c}>\frac{b}{c} \end{cases}$

تنعكس.

الأنواع:

Linear:

2 Parts:

$5-4(x-1) \geq -5(x-3)$

① نتخلص من الأقواس. ② نفك الأقواس.

③ نجمع الـ Like terms

④ intervals

$5-4x+4 \geq -5x+15$

$x \geq 6$

Solution Set:

$[6, \infty)$

3 Parts:

$3+x<\frac{1-3x}{2} \leq x+8$

① أخلي الـ x بالنص

② التخلص من الكسور

$6+2x<1-3x \leq 2x+16$
$-2x \quad -2x \quad -2x$

$6<1-5x \leq 16$
$-1 \quad -1 \quad -1$

$5<-5x \leq 15$
$\div -5$

$-1>x \geq -3$

$[-3,-1)$

Qudratic:

$(x-1)^2 \leq 4$

① ممنوع نأخذ الجذر في المتباينات.

② الحول أخذها من جهة واحدة

③ أفكها.

$x^2-2x+1-4 \leq 0$

$x^2-2x-3=0$

$x=-1, x=3$

x ✓ x

$[-1,3]$

rational:

① أخلي طرف يساوي صفر

② أصفار المقام والبسط

$\frac{3}{x-1} \leq \frac{5}{x+3}$

$\frac{3}{x-1}-\frac{5}{x+3} \leq 0$

$\frac{-2x+14}{(x-1)(x+3)} \leq 0$

$-2x+14=0$

$\boxed{x=7} \rightarrow$ closed

$x-1=0$ $\quad$ $x+3=0$

$\boxed{x=1}$ $\quad$ $\boxed{x=-3}$

open

✓ x ✓ x ✓

$(-3,1) \cup [7,\infty)$

Quadratic Inequalities

Rational Inequalities

$x>y, z<0$, then:

a value of the ... equality, and the set of all solutions ... inequalities with the same solution se...

Inequalities are solved with the p...

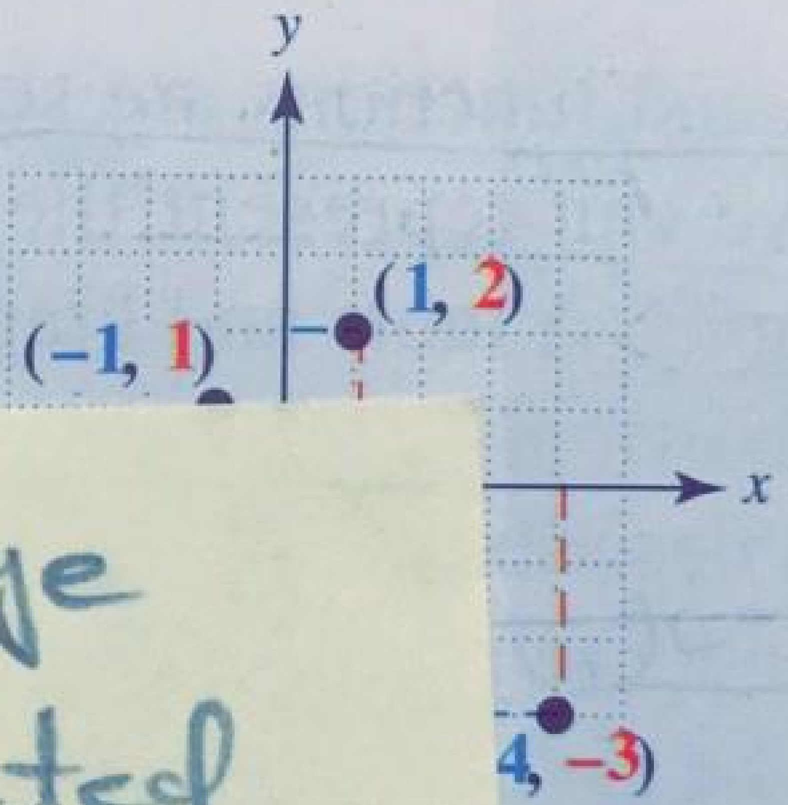
the properties of equality in Section

slope m of a linear function of the form $y = mx$.)

Finding Domains and Ranges from

Give the domain and range of each relation.

(a)



(b)

(d)

Domain → R

So $\frac{1}{x}$ $x \neq 0$

$\sqrt{[\ ]}$-ever

Range

Related

For (y)

P.90

provides a formula, in function form, for finding the slope of the tangent line to the graph of the function at a given point.

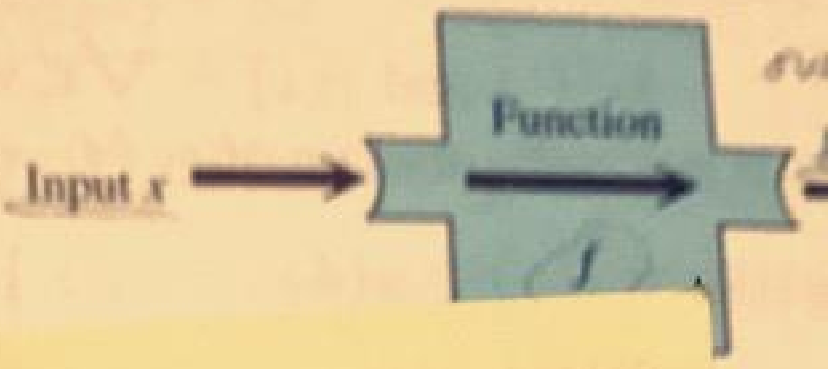
To illustrate, it is shown in calculus that the derivative of $f(x) = x^2 + 3$ is given by the function $f'(x) = 2x$. Now, $f'(0) = 2(0) = 0$, meaning that the slope of the tangent line to $f(x) = x^2 + 3$ at $x = 0$ is 0, which implies that the tangent line is horizontal. If you draw this tangent line, you will see that it is the line $y = 3$, which is indeed a horizontal line.

...expressions differ by $4xh$. ... *not* equivalent to $f(x) + f(h)$.

تركيب الدوال , مجال

Composition of Functions and Dom...

function f that assigns to each x in ... tion g assigns to each $f(x)$ in its do... takes an element x and produces a ...



Domine of Composition

$$D_{f \circ g} = D_{\underset{fn}{result}} - \{\ldots\}$$

* الدالة الداخلية

مثال

{ الدالة مثل (g) المركبة لحالها }

الي نخليها غير معرفة

صفر في المقام

سالب تحت الجذر الزوجي

P.108

...ues ... an...

...ons

then

(g

the ...
f g.

...ple ...

...ns is ...
A ... noon, the retailer *offers* ar... blue jeans?

8) x-intercept (Put y=0)

$ax^2+bx+c=0$

$b^2-4ac>0$ 2 Points | $b^2-4ac=0$ 1 points | $b^2-4ac<0$ No x-intercept

9) y-intercept (Put x=0)

$y=c \Rightarrow (0,c)$

10) Max. point (h,k) الرأس

Min. Point (h,k) الرأس

11) Max. Value (k)

Min. Value (k)

~~d Models~~

n is defined as follo

ion

ction f of degree n,

$= a_nx^n + a_{n-1}x^{n-}$

a_1, and a_0 are real

lyzing a polynomial
n important role. Th
des examples.

$x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040}$.

Polynomial Function	Function Nan
$f(x) = 2$	Constant
$f(x) = 5x - 1$	Linear
$f(x) = 4x^2 - x + 1$	Quadratic
$f(x) = 2x^3 - \frac{1}{2}x + 5$	Cubic
$f(x) = x^4 + \sqrt{2}x^3 - 3x^2$	Quartic

Note الباقي

1) Parabola

2) Domain $\mathbb{R}$, $(-\infty, \infty)$

3) Range (مدى) ∪ $[k,\infty)$, ∩ $(-\infty,k]$

4) vertex (0,0) (h,k) رأس القطع

5) open up الفتح a>0 up, a<0 down

6) y-axis ← محور التماثل → x=h

7) increase & decrease
فترات الزيادة والتناقص

$$\boxed{a(x-h)+k \qquad h=\frac{-b}{2a} \mid k=f(h)}$$

y | x تقطع مكان ... راح تدرس اللي

(h,k)

(h,k)

$[h,\infty)$ inc, $(-\infty,h]$ dec | $(-\infty,h]$ inc, $[h,\infty)$ dec

The function $f(x) = 0$ is the **zero polyn**

Quadratic Functions Earlier we dis
functions. Polynomial functions of degr

Quadratic Function

A function f is a **quadratic function**

$$f(x) = ax$$

where a, b, and c are real numbers, w

The simplest quadratic function is

$f(x) = x^2$, Squaring function

as shown in **Figure 1.** This graph is a **parabola.** Every quadratic function defined over the real numbers has a graph that is a parabola.

The domain of $f(x) = x^2$ is $(-\infty, \infty)$, and the range is $[0, \infty)$. The lowest point on the graph occurs at the origin $(0,0)$. Thus, the function decrea on the interval $[0, \infty)$. (Remember th

P. 116

Quot

Since the divisor $x - k$ has degr
one less than the degree of the po

$f(-2) = -10$

$$\frac{5x^3 - 6x^2 - 28x - 2}{x + 2}$$

The result of the division in

$5x^3 - 6x^2 - 28x - 2$

by multiplying each side by the
... this product

* If $\frac{f(x)}{x-k} \rightarrow r = 0$ than

① $f(k) = \text{zero} \rightarrow$ k is the x-intercept of $f(x)$

② k is a zero of $f(x)$

③ k is a root (solution) of $f(x) = 0$

④ $(x - k)$ is a factor of $f(x)$

4.2 نظرية

4.3

نظرية الباقي

f the Divisi

ial $f(x)$ and
and number

$f(x)$

statement

$x^2 - 28x -$

$f(x)$

nection. Thi
er theorem.

ion. This
deve

4.2 - 4.3

$\frac{f(x)}{x-k}$ $f(K) =$ reminder

Evaluating Polynomial Funct

is writt as $f(x) = (x$
values of x, so it true for x
$f(k) = ($

This proves the following r
evaluating polynomial functi

f is a one-to-one fn if

① $a \neq b \rightarrow f(a) \neq f(b)$ for all $a, b \in D$

② $f(a) = f(b) \rightarrow a = b$ for all $a, b \in D$ domine

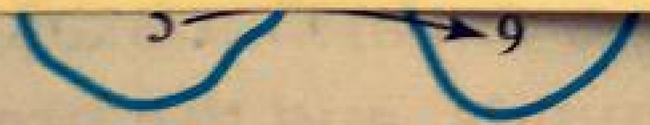
---

if = Not one to one fn

① $a \neq b \rightarrow f(a) = f(b)$

② $f(a) = f(b) \rightarrow a \neq b$

البداية نفهمها الاختلاف في مابعد انشالله

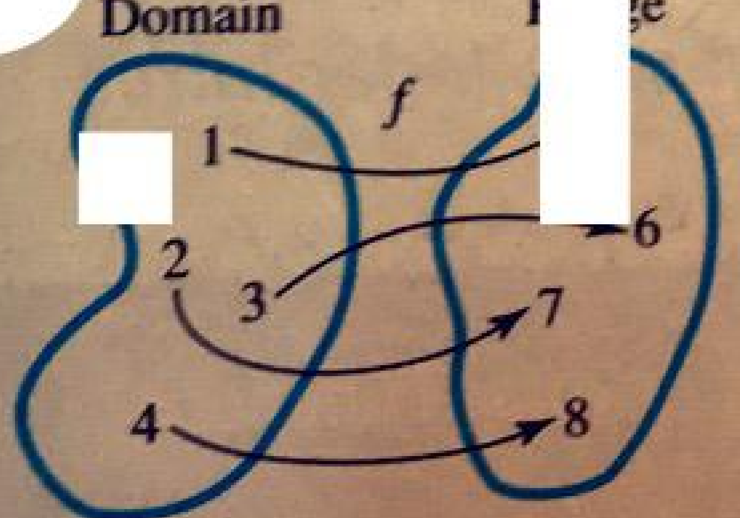


Not One-to-One

Figure 1

P.140

The function f shown
sponds to *two* x-values
ng t ction.



One-to-One

Figure 2

ne-t unct

A function f is a one
of f,

Using the concept
in the preceding box is

one to one

2.4 Exercises

2.4

linear

2 parts

$x < \square$

3 parts

$\square < x < \square$

في الحالتين

ابا x لحالها

- نشوف آخر حل وصلت له x لحالها واحدد الفترة

Quadratic

$(x-1)^2 \leq 4$

هنا غلط آخذ الطرفين بالجذر، عشانها متباينة مو معادلة !

طرف التربيع وأخليه معادلة درجة 2 وأطلع أقواس

$x^2 - 2x + 1 \leq 4$

$x^2 - 2x - 3 \leq 0$

$(x-3)(x+1) \leq 0$

$x = 3 \quad x = -1$

أطلعهم خط الأعداد وأشوف

$(-\infty, -1]$ -1 $[-1, 3]$ 3 $[3, \infty)$

-4	0	4
$25 \leq 4$	$1 \leq 4$	$9 \leq 4$
F	T	F

$S.S\ [-1, 3]$

هي الي حققت الحل

وإذا مثلاً جاتني الأول والأخيرة نربطهم باتحاد.

$(-\infty, -1] \cup [-3, \infty)$

rational

$$\frac{3}{x-1} \leq \frac{5}{x+3}$$

هنا غلط أضرب طرفين في بعض

لا متباينة !

- لازم يكون طرف = صفر

- ننقل الثاني ونوحد مقامات.

$$\frac{3}{x-1} - \frac{5}{x+3} \leq 0$$

نوحد

$$= \frac{3x+9-5x+5}{(x-1)(x+3)} \leq 0$$

$x = 1$

$x = -3$

- نأخذ أصفار المقام علشانها دايماً open

- ونطلع أصفار البسط وحسب المتباينة إذا مفتوح أو مغلق.

نحط الأصفار كلهم في خط الأعداد ونجرب زي هنا

- نأخذ رقم من داخل الفترة ونعوض في المعادلة ونشوف إذا

T F F

واشوف أي الفترة الي تحقق لي

أنتبه صفر المقام دايماً مفتوح

أما البسط في حسب المتباينة.

(1)

H.W 3

(a) $2x + 5y = 4$

$m = -\frac{A}{B} = -\frac{2}{5}$ لما انه موازيه فلهم نفس الميل

$y - y_1 = m(x - x_1)$

$y - 5 = -\frac{2}{5}(x - 3)$ $\times 5$

$5y - 25 = -2x + 6$

$2x + 5y = 6 + 25$

$2x + 5y = 31$

standard form

$5y = -2x + 6 + 25$

$5y = -2x + 31$

$y = \frac{-2x + 31}{5}$

$y = -\frac{2}{5}x + \frac{31}{5}$

slope intercept form

Table

(b) $2x + 5y = 4$

$\boxed{m = -\frac{2}{5}}$

بما انه متعامد لهم نفس الميل هو مقلوب المعكوس

$\boxed{m = \frac{5}{2}}$

$y - y_1 = m(x - x_1)$

$y - 5 = \frac{5}{2}(x - 3)$ $\times 2$

$2y - 10 = 5x - 15$

$2y - 5x = -15 + 10$

$2y - 5x = -5$

standard form

$2y = 5x - 5$

$y = \frac{5}{2}x - \frac{5}{2}$

slope intercept form

HOMEWORK 3 **Finding Eq**

Write the equation in both
passes through the point (3

(a) parallel to the line $2x$ -

(b) perpendicular to the lin

A summary of the vario

Equation	
$y = mx + b$	Slop Slop y-int
$y - y_1 = m(x - x_1)$	Poin Slop Line
$Ax + By = C$ $m = -\frac{A}{B}$	Stan (If th cons A, B as re with Slop x-int y-int
$y = b$	Hori Slope y-inte
$x = a$	Verti Slope x-inte

Modeling Data We can describe, or model, real data

5.1

1. f fn — $\underline{x}$ ماتتكرر → Vertical line test
2. One to one → Horizontal line test

$x \to \boxed{y}$
$x \to \boxed{y}$

* الحالات اللي دائماً one to one
1. $f(x) = x$ linear Equ $3x+2$
2. $f(x) = x^3, x^5, x^7$ الأس الفردي
3. $\boxed{f(x)} = \sqrt{x}$ x degree one

حالات ما فيها
One to one

$f(x) = x^2, x^4$ الزوجي

$f(x) = |x|$

$f(x) = 3$ constant

$f(x) = \sqrt{x^2+4}$

$D_f = R_{f^{-1}}$

$R_f = D_{f^{-1}}$ $f(x)$

$f(x)$, $g(x) =$

a	b
2	3

$f^{-1}(x)$

b	a
2	3

y=x

1. fn
2. one-to-one

find the invers

$f(x) \to y$

exchang y, x

Solve it $\boxed{y}$

$y \to f^{-1}(x)$

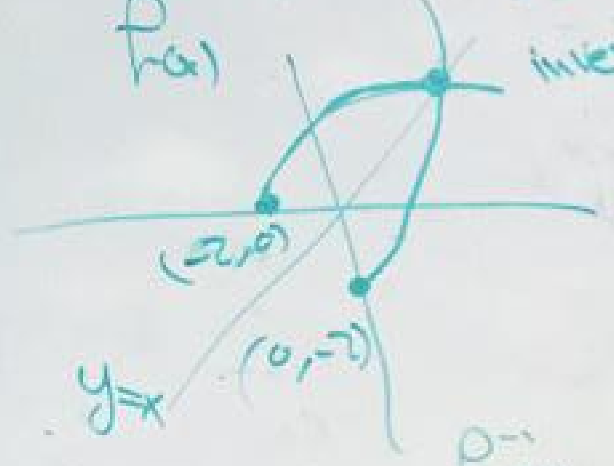
هل هذه الدالة معكوسة لبعض ؟

$f(x) \to g(x)$ ؟

$f\circ g(x) = x$

$g\circ f(x) = x$

$f(x)$ inverse



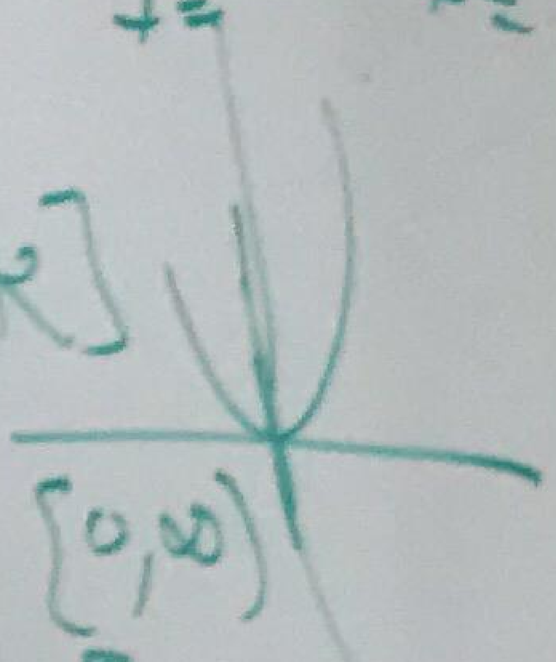
$f^{-1}(x)$

4.1

1. Domain $\mathbb{R}$
2. Range $[0,\infty) = [k,\infty)$
3. Des/inc ✓
4. (h,k) ✓
5. x-intercept
6. y-intercept $(0,c)$
7. open up / down
8. $a(x-h)^2 + k$
9. Max & min point
10. Max & min value k

$0 = -ax^2 + bx + c$

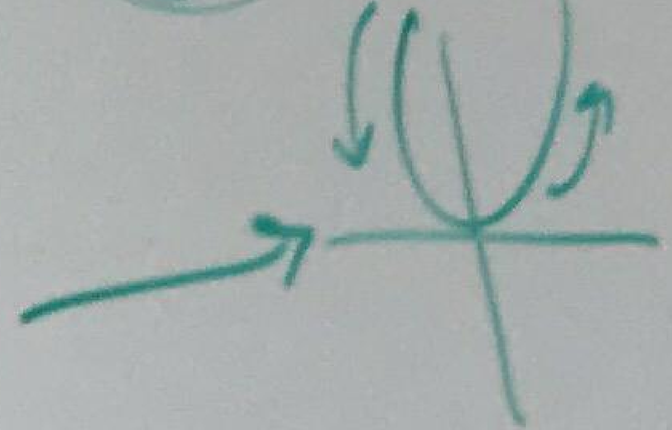
Range $(-\infty, k]$

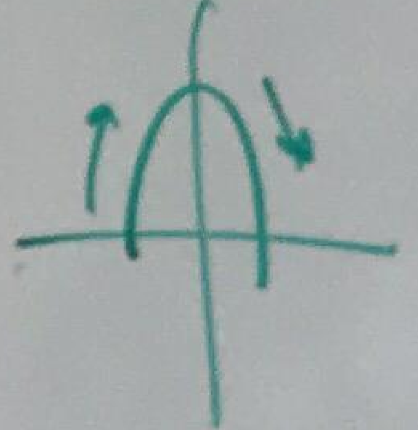


$[0,\infty)$

$[k,\infty)$

3





$(-\infty,0]$ dec $\quad$ $(-\infty,0]$ inc

$[0,\infty)$ inc $\quad$ $[0,\infty)$ dec

(h,k)

h

$h = \frac{-b}{2a}$

k

$f(h) = k$

$a(x-h)^2 + k$

$ax^2 + bx + c$

(1) $h = \frac{-b}{2a}$

$a(x-h)^2 + k$

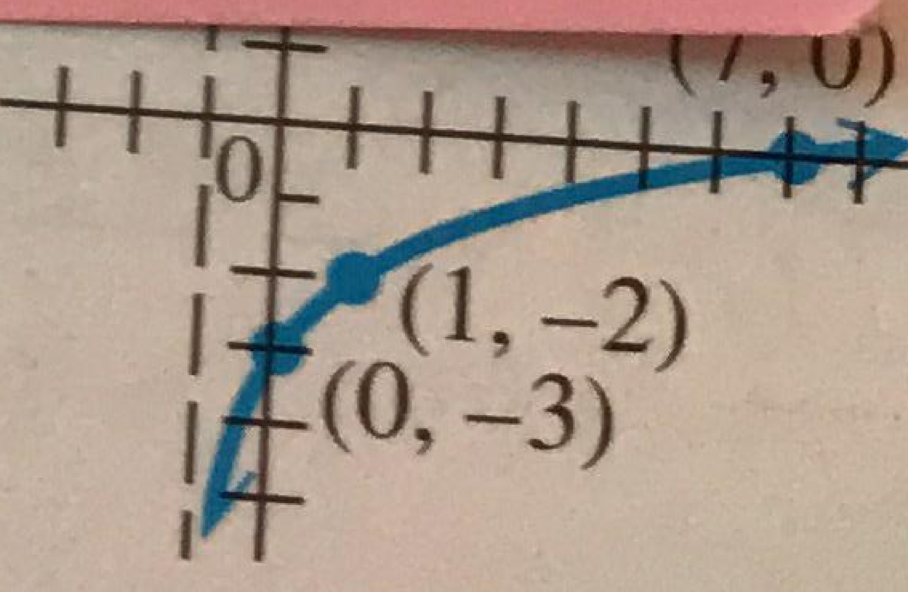
$\boxed{5.4}$

① $x = y \Leftrightarrow \log_a x = \log_a y$

② $\log_a b = c \Leftrightarrow \underline{a^c = b}$

$\ln 1 = 0$

$\log_{\square} 1 = 0$



Use the properties of lo
sible. Assume all variabl

32. $\log_2 \dfrac{6x}{y}$

35. $\log_m \sqrt{\dfrac{5r^3}{z^5}}$

Write each expression as
resent positive real numbe

38. $\log_a x + \log_a y - \log$

40. 1

$\log_{\square} \square \rightarrow$ لازم أساس و أقيف
مش أبا

$\log x \rightarrow \log_{10} x$

$\ln x \rightarrow \log_e x$

$\ln e = 1$

$\ln e^{\square} = \square$

$e^{\ln \square} = \square$

في العكسية $\square$ فوق كدوم أنه الا الحاسبة

Exercise 1, match the logarithm in Colu
er that $\log_a x$ is the exponent to which a mu

II

A. 0

B. $\frac{1}{2}$

C. 4

D. -3

E. -1

F. -2

الرجاء عدم تحريك
مكانها نهائيا
وعدم خروج الك

Quadratic Fn

$f(x) = ax^2 + bx + c$, $a, b, c \in R$

$a \neq 0$

is a graph of parabola

$\Downarrow$

$f(x) = a(x-h)^2 + K$

where $h = \frac{-b}{2a}$, $K = f(h)$

① vertex: (h, K)

② Eqn of axis: $x = h$

③ $a > 0$ opens up

$a < 0$ opens down

④ Domain: $(-\infty, \infty)$

⑤ Range: $\cup$ $[K, \infty)$

$\cap$ $(-\infty, K]$

⑥ interval of increasing: $\cup_{(h,k)}$ $[h, \infty)$

$\cap^{(h,k)}$ $(-\infty, h]$

⑦ interval of decreasing: $\cup$ $(-\infty, h]$

$\cap$ $[h, \infty)$

⑧ x-intercept: (Put $y = 0$)

$ax^2 + bx + c = 0$

$b^2 - 4ac > 0$ 2 points

$b^2 - 4ac = 0$ 1 point

$b^2 - 4ac < 0$ No x-intercept

⑨ y-intercept: (put $x = 0$) $\Rightarrow y = c$

$\Downarrow$

$(0, c)$

⑩ Maximum point: $\cap$

Minimum point: $\cup$

(h, k)

⑪ Maximum value $\cap$

Minimum value $\cup$

$= K$

⑫ narrower: $|a| > 1$

⑬ wider: $|a| < 1$

$f(x) = \log_a (x-h)+k$

- $D:(h,\infty)$ $\quad x-h>0$, $x>h$
- Rang $= \mathbb{R}$
- $a>1$ inc
- $0<a<1$ dec.
- Vertical asymptot $\rightarrow x=h$
- X intercept (Put $y=0$)
- y intercept: $h\geq 0$ No y intercept; $h<0$ put $y=0$