مقدمة

بسم الله والصلاة والسلام على رسول الله صلى الله عليه وسلم أما بعد نقدم لكم شرح وتبسيط كتاب الرياضيات :

Introduction to Real Numbers and Algebraic Expression

تناول هذا الملخص شرح وتبسيط كامل لمنهج الرياضيات مع وجود أمثلة توضيحية محلولة بالتفصيل لكل باب على حدة مع وجود اختبار تجريبي لكل باب.

مع تمنياتنا لكم بالتوفيق والنجاح

chapter (1)

Introduction to ALgebra (1.1) مقدمة فى علم الجبر

a- Evaluating ALgebra Expressions: (2 صفحة رقم)

Ex. No. 1 : Evaluate $x + y$ when $x = 37, y = 29$

بالنسبة للجمع نعوض عن كل رمز بقيمته ثم نجمع

$x + y = 37 + 29 = 66$. (جمع)

Ex. No. 2 : Evaluate $3y$ when $y = 14$

فى حالة الضرب $3y$ يعنى $3 \times y$ ، وموجود عندى قيمة y أعوض عنها.

$3y = 3 \times 14 = 42$. (ضرب)

Ex. No. 3 : Evaluate x when

$14.264 + x = 20.320$ (طرح)

لحل هذا النوع من المسائل عايزين x فى طرف وباقى الأرقام فى طرف والتخلص من الرقم الموجود بجوار x نضع المعكوس الجمعى يعنى نفس الرقم بإشارة مخالفة يعنى 14.264 معكوسة الجمعى 14.264- ، وذلك بالطرفين.

$14.264 + x = 20.320$

$14.264 - 14.264 + x = 20.320 - 14.264$

$x = 6.056$.

Ex. No. 4 : Evaluate $\frac{a}{b}$ when $a = 63, b = 9$

نعوض عن الرموز بقيمتها فى المعادلة ثم نقسم الرقمين

(قسمة)

$\frac{a}{b} = \frac{63}{9} = 7$.

b- Translating to Algebraic Expressions ترجمة العبارات الجبرية

(صفحة رقم 4)

EX. No. 5 : Twice (or Two times) some number.

تعني ضعف رقم ما ، أو مقداره مرتين ، نفرض هذا الرقم y

$= 2y$ أو $2 \times y$ أو $2(y)$.

EX. No. 6 : ANumber divided by 5

رقم ما مقسوم على خمسة نفرض الرقم m

$m \div 5$ or $\frac{m}{5}$ or $m/5$.

EX. No. 7 : Eieghteen more than anumber.

نفرض الرقم t يعني more than يعني نجمع

$18 + t$ or $t + 18$

EX. No. 8 : Half of anumber

يعني نصف الرقم ، نفرض الرقم t

$\frac{1}{2}\ t$ or $\frac{t}{2}$ or $t/2$

EX. No. 9 : (The difference of two numbers)

يعني الفرق بين رقمين ، يعني نطرح ، نفرض الرقمين x ، y

$= x - y$.

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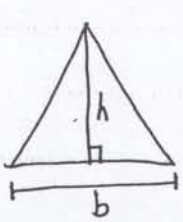
* $A = \frac{1}{2}\ b\ h$ = مساحة المثلث



The Real Numbers (1.2) الأعداد الحقيقية

1) NATURAL NUMBERS الأعداد الطبيعية

$= \{1, 2, 3, 4, \ldots\}$

2) whole numbers الأعداد الصحيحة

$= \{0, 1, 2, 3, 4, \ldots\}$

3) The Real numbers الأعداد الحقيقية.

$= \{\ldots -2, -1, 0, 1, 2, 3, \ldots\}$

4) The Rational Numbers الأعداد النسبية

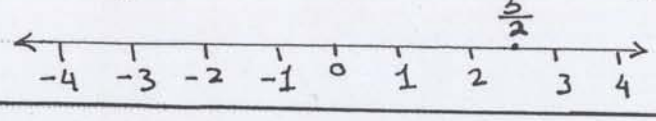
$\frac{2}{3}$, $-\frac{2}{3}$, $\frac{7}{1}$, -3 , 0 , $\frac{23}{-8}$

التي تأتي على صورة كسر وهي تكون موجبة وسالبة

$-\frac{a}{b}$, $\frac{a}{b}$, $\frac{a}{-b}$ $(b \neq 0)$

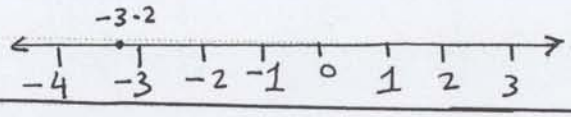
EX. No. 1 : Graph $\frac{5}{2}$ نرسم خط الأعداد ونحدد عليه مكان الرقم

$\frac{5}{2} = 2\frac{1}{2}$ or 2.5 نرسم بين 2 و 3



EX. No. 2 : Graph -3.2 نحدد فقط مكان الرقم.

$-3.2 =$ نرسم بين -3 و -4



EX. No. 3 : Convent to decimal notation $\frac{-5}{8}$

$$\begin{array}{r|l} & 0.625 \\ 8 & 5.000 \\ & 48 \\ \hline & 20 \\ & 16 \\ \hline & 40 \\ & 40 \\ \hline & 00 \end{array}$$

للتحويل من صيغة الكسر العشري إلى صيغة ديسيمل نستخدم طريقة القسمة المطولة.

Thus $\frac{5}{8} = 0.625$

so $\frac{-5}{8} = -0.625$

أو باستخدام الحاسبة.

EX. No. 4 . $\frac{0}{8} = 0$

باستخدام الحاسبة .

* $\frac{27}{100} = 0.27$

* $-8\frac{3}{4} = -8.75$.

ترجمة بعض المصطلحات لأرقام

EX. No. 5 : -8 less than 6

يعني -8 أقل من 6 نترجمها لأكبر أو أقل من $> <$

$= -8 < 6$

لأن السالب دائما أقل من الموجب مهما كبرت قيمته .

EX. No. 6 -3 is greater than -7

يعني -3 أكبر من -7

$-3 > -7$

كلما كبر الرقم السالب قلت قيمته

Absalate value القيمة المطلقة السالبة = الموجبة

1) $|-7| = 7$ لأنه لا توجد قيمة مطلقة سالبة .

2) $|12| = 12$

أي رقم بين $|\ |$ يساوي قيمته الموجبة وتحذف الإشارة .

EX. No. 7 use $> \text{or} <$

* $2 \boxed{<} 9$ بالنسبة للأعداد الموجبة كلما كبرت كبرت القيمة

* $-7 \boxed{<} 3$ والسالبة كلما كبرت قلت القيمة .

* $-18 \boxed{<} -5$

Addition of Real Numbers (1.3). جمع الأعداد الحقيقية.

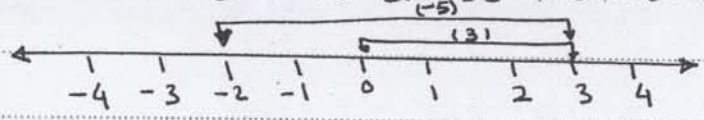
$a + b$ لو b موجب تحرك على خط الأعداد ناحية اليمين

لو b سالب تحرك ناحية اليسار على خط الأعداد

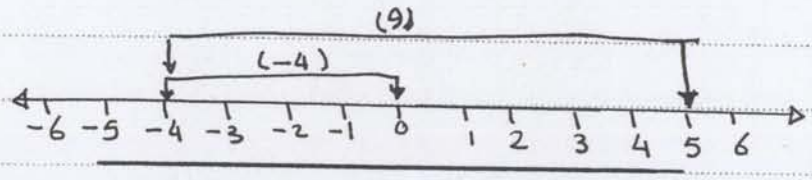
لو b = صفر ابقى عند نقطة a

Ex. No. 1 : add $3 + (-5)$

نتحرك على خط الأعداد من صفر إلى 3 ثم نتحرك ناحية اليسار وندعد (-5).



Ex. No. 2 : add $-4 + 9$



Addition without the Number Line الجمع بدون خط الأعداد

Ex. No. 3 : $-12 + (-7) = -19$

عند تشابه الإشارات نجمع

Ex. No. 4 : $-1.4 + 8.5 = 7.1$ عند اختلاف الإشارات

نطرح ونحط إشارة الرقم الأكبر.

Ex. No. 5 : Add $15 + (-2) + 7 + 14 + (-5) + (-12)$.

$a = 15 + 7 + 14 = 36$ ← نجمع الموجب

$b = -2 - 5 - 12 = -19$ ← نجمع السالب

$36 + (-19) = 17$ ← نطرح ونحط إشارة الرقم الأكبر

قاعدة الإشارات

موجب + موجب = موجب .

سالب + سالب = سالب .

سالب + موجب = إشارة الرقم الأكبر ونطرح .

موجب + سالب = إشارة الرقم الأكبر ونطرح .

Ex. No-6 : $-9-2+3.1$

نضع إشارة الرقم الأكبر اللي هو سالب ثم نطرح =

$= -6.1$.

Ex. No. 7 : $\frac{-4}{3} + \frac{2}{3}$

نضع إشارة الأكبر اللي هو سالب ثم نطرح

$\frac{-4}{3} + \frac{2}{3} = \frac{-4+2}{3} = \frac{-2}{3}$.

Ex. No. 8 : $T = -2+1+(-5)+3$

نجمع السالب معاً والموجب معاً أولاً

$= -2 + (-5) + 1 + 3$

$= -7 + 4$

ثم نجمع

$= -3$.

ثم نضع إشارة الأكبر ونطرح

Ex. No. 9 : $\frac{-2}{3} + \frac{5}{8}$

عند اختلاف المقام نوحد المقامات عن طريق الحاسبة أو بيدوى بضرب المقامين

$\frac{-2}{3} + \frac{5}{8} = \frac{-16}{24} + \frac{15}{24} = \frac{-16+15}{24}$

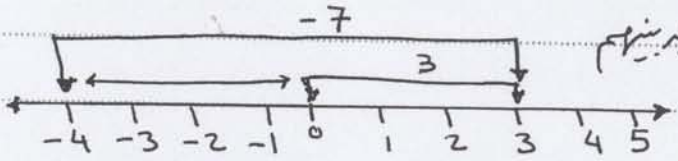
ثم نستخدم قواعد الطرح .

$= \frac{-1}{24}$

Subtraction of Real Numbers (1.4) طرح الأعداد الحقيقية

EX. No. 1 subtract $3 - 7 = -4$

الناتج يكون الفرق بينهم



EX. No. 2 : $5 - 8 = -3$

$5 + (-8) = -3$ الجمع والطرح في هذه الحالة تتساوى

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a) EX. No. 3 : subtract $2 - 6 = -4$

b) $4 - (-9) = 4 + 9 = 13$

c) $-4.2 - (-3.6) = -4.2 + 3.6 = -0.6$

d) $\frac{1}{8} - \frac{7}{8} = \frac{1-7}{8} = \frac{-6}{8}$ or $\frac{-3}{4}$

EX. No. 4 : $8 - (-4) - 2 - (-4) + 2 =$

$8 + 4 + (-2) + 4 + 2 = 16$

EX. No. 5 : $\frac{3}{4} - (-\frac{1}{12}) - \frac{5}{6} - \frac{2}{3}$

بتوحيد المقام على 12

$$\frac{9 + 1 + (-10) + (-8)}{12} =$$

$$\frac{10 + (-18)}{12} = \frac{-8}{12} = \frac{-2}{3}$$

EX. No. 6. $-6 - (-2) - (-4) - 12 + 3 =$

$-6 + 2 + 4 + (-12) + 3 = -9$

Multiplication of Real Numbers (1.5) ضرب الأعداد الحقيقية

+ X + = +	موجب X موجب = موجب
− X − = +	سالب X سالب = موجب
− X + = −	سالب X موجب = سالب
+ X − = −	موجب X سالب = سالب

. or x or ( ) تمثل علامة الضرب في المسائل يكون

EX. No. 1 : $4 \cdot 5 = 20$

الرقمين موجبين يكون الناتج موجب .

EX. No. 2 : $-3 \cdot 5 = -15$

سالب X موجب = سالب ونضرب الرقمين .

EX. No. 3 : $\frac{-1}{3} \cdot \frac{5}{7}$

نضرب البسط في البسط والمقام في المقام

$$\frac{-1}{3} \cdot \frac{5}{7} = \frac{-1 \cdot 5}{3 \cdot 7} = \frac{-5}{21}$$

EX. No. 4 : $-3 \cdot (-5) =$

سالب X سالب = موجب ونضرب الرقمين

$$-3 \cdot (-5) = 15$$

EX. No. 5 : Evaluate $2x^2$ when $x = 3$, when $x = -3$

التربيع يجعل الإشارة لرمزه موجبة وتخرج القيمة موجبة طالما داخل القوس فقط .

when $x = 3$, $2x^2 = 2(3)^2 = 2(9) = 18$.

when $x = -3$, $2x^2 = 2(-3)^2 = 2(9) = 18$

لأن $2(9) = 2(-3 \cdot -3)$ فتكون الإجابة موجبة لأن الأس مربع

Division of Real Numbers (1.6) قسمة الأعداد الحقيقية.

$\frac{+}{+} = +$ موجب ÷ موجب = موجب

سالب ÷ سالب = موجب

$\frac{-}{-} = +$ سالب ÷ موجب = سالب

$\frac{-}{+} = -$ موجب ÷ سالب = سالب

$\frac{+}{-} = -$

* $a \div b$ or $\frac{a}{b}$ or a/b when $b \neq 0$.

Ex. No. 1 : Divide, if possible. Check your integers.

$14 \div (-7) = \frac{14}{-7} = -2$

لأنه موجب ÷ سالب = سالب ونقسم الأرقام.

Ex. No. 2 : $\frac{-32}{-4} = 8$

لأنه $\frac{-}{-} = +$ ونقسم الأرقام

check $8(-4) = -32$.

Ex. No. 3: Divide

$0 \div -6 = 0$

Ex. No. 4 : $-3 \div 0$ is not defined

أي حاجة مقسومة على صفر = قيمة غير معلومة لأنه المقام لا يمكن يكون صفر.

Ex. No. 5 : $\frac{7}{8}$ المعكوس الضربي للعدد هو عكسه وبس

$\frac{7}{8} \cdot \frac{8}{7} = \frac{7 \cdot 8}{8 \cdot 7} = \frac{56}{56} = 1$ الإشارة

ضرب العدد في معكوسه = 1.

Ex No. 3.1) $\frac{6}{-7} = 6\left(-\frac{1}{7}\right)$

2) $\frac{x+2}{5} = (x+2)\left(\frac{1}{5}\right)$

3) $\frac{-17}{1/b} = -17 \cdot \frac{b}{1} = -17 \cdot b$

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* $\frac{2}{3} = \frac{2}{3} \cdot 1 = \frac{2}{3} \cdot \frac{-1}{-1} = \frac{-2}{-3}$ Thus $\frac{2}{3} = \frac{-2}{-3}$

* $-\frac{2}{3} = -1 \cdot \frac{2}{3} = \frac{-1}{1} \cdot \frac{2}{3} = \frac{-1 \cdot 2}{1 \cdot 3}$ Thus $-\frac{2}{3} = \frac{-2}{3}$

* $\frac{-2}{3} = \frac{-2}{3} \cdot \frac{-1}{-1} = \frac{-2}{3} \cdot 1 = \frac{-2(-1)}{3(-1)} = \frac{2}{-3}$ Thus $\frac{-2}{3} = \frac{2}{-3}$

---

* $\left(\frac{3y}{8x}\right)\left(\frac{8x}{3y}\right) = 1$

* $-\frac{3}{4} \div \frac{3}{10} = -\frac{3}{4} \cdot \frac{10}{3} = \frac{-10}{4} = \frac{-5}{2}$

* $\frac{4}{7} \div \left(\frac{-3}{5}\right) = \frac{4}{7} \cdot \left(\frac{-5}{3}\right) = \frac{-20}{21}$

حاصل ضرب العدد في معكوسه الضربي ينتج 1 .

عند القسمة يمكن أن نحولها لضرب ونقلب الكسر مثل

* $\frac{-3}{4} \div \frac{3}{10} = \frac{-3}{4} \cdot \frac{10}{3} = \frac{-30}{12} = \frac{-5}{2}$

نقلب الكسر الثاني ونضرب ونبسط المعادلة .

properties of Real Numbers (1.7) خصائص الأعداد الحقيقية.

١- الإبدال $a + 0 = 0 + a$

يعني إبدال أي عنصر بآخر في مكانه لا يغيره في الناتج النهائي.

* $\frac{2}{3} = \frac{2}{3} \cdot 1 = \frac{2}{3} \cdot \frac{x}{x} = \frac{2x}{3x}$.

٢- التبسيط.

* Ex. No. 1 :→ simplify $-\frac{20x}{12x}$

Solve :→ $-\frac{20x}{12x} =$

يعني نحذف المتشابه في الطرفين.

$-\frac{5 \cdot 4\cancel{x}}{3 \cdot 4\cancel{x}} = -\frac{5}{3}$.

---

* simplify $\frac{14ab}{56a}$

نحذف المتشابه بسطًا ومقامًا.

solve :→ $\frac{14\cancel{a}}{14\cancel{a}} \cdot \frac{b}{4} = \frac{b}{4}$

Ex. No. 3 Evaluate xy and yx when $x = 3, y = -12$

$xy = 3 \cdot (-12) = -36$ عند إبدال الرموز في الضرب

$yx = (-12) \cdot 3 = -36$ لا يتغير الناتج

Ex. No. 3 :→ $(x + 5) + y = x + (5 + y) = (y + x) + 5$

Ex. No. 4 : Compute in two way $5 \cdot (4 + 8)$

a) Solve :→ $5 \cdot (4 + 8) =$

$5 \cdot 12 = 60$

b) $5(4 + 8) = (5 \cdot 4) + (5 \cdot 8)$

$= 20 + 40 = 60$

٢ - خاصية التوزيع

* $a(b-c) = (a.b) - (a.c)$

Ex. No. 5 → $9(x-5) = (9.x) - (9.5)$

$= 9x - 45$. توزع الضرب على القوس

Ex. No. 6 → $\frac{2}{3}(w+t) = \frac{2}{3}.w + \frac{2}{3}.t$

Ex. No. 7 → Multiply $-4(x-2y+3z)$.

$-4(x-2y+3z) = -4.x - (-4)(2y) + (-4).(3z)$

$= -4x + 8y - 12z$.

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* Collecting Like Terms :- نجمع المتشابه في الرموز

* $4x + 2x = (4+2)x = 6x$.

* $2x + 3y - 5x - 2y = (2x-5x) + (3y-2y)$

$= -3x + y$

---

* $4x - 7y + 9x - 5 + 3y - 8 =$ نجمع الرموز المتشابهة.

$(4x + 9x) + (-7y + 3y) + (-5-8) =$

$13x - 4y - 13$.

---

* $3.3x + 3.9y - 3.3 = 3(3x+9y-3)$

$= 3.3(x+3y-1)$

$= 9(x+3y-1)$

في هذه المسألة أخذنا عامل مشترك وهو 3 لأن الرموز مختلفة.

Simplifying Expressions; order of operations. (1-8)

Examples:- عملية تبسيط المقادير الجبرية.

1) $-(3+X) = (-1 \cdot 3) + (-1 \cdot X)$ ندخل الإشارة على جانب الخل

$= -3 - X$ القوس مثل الضرب في 1-

---

2) $-(5-y) = -5+y$

---

3) $3X - (4X+2) = 3X - 4X - 2$

$= -X - 2.$

---

4) $3X - 2 - (5X-8) = 3X - 2 - 5X + 8$

$= -2X + 6.$

---

5) $(2a+3b-7) - 4(-5a-6b+12) =$ بالتوزيع

نجمع المتشابه. $2a + 3b - 7 + 20a + 24b - 48 =$

$22a + 27b - 55.$

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6) $\left[(-4) \div \left(-\frac{1}{4}\right)\right] \div \frac{1}{4} =$

بقلب القسمة-ضرب نقلب الكسر $\left[(-4) \cdot (-4)\right] \div \frac{1}{4} =$

$16 \div \frac{1}{4} = 16 \cdot 4 = 64.$

---

7) $-2^4 + 51 \cdot 4 - (37 + 23 \cdot 2) =$

$-2^4 + 204 - (37 + 46) =$

$-2^4 + 204 - 83 =$

$= -16 + 204 - 83 = 105.$

Exam. No. 1. اختبار تجريبي على الفصل الأول

1 - Evaluate $\frac{x}{y}$ when $x = 10$ and $y = 5$.

2 - Translate to an algebraic expression.
Nine less than some number.

3 - use either < or > :

* $-3 \square -8$
* $-10 \square 2$

4 - write true or False
$-13 \leq -3$

5 - Find the opposite
$\frac{2}{3}$, -1.4

6 - simplify :
$|-7|$, $\left|\frac{9}{4}\right|$, $|-2.7|$

7 - Compute and simplify :.

* $3.1 - (-4.7)$
* $-8 + 4 + (-7) + 3$
* $-\frac{1}{5} + \frac{3}{8}$

8 - Evaluate $-x$ when $x = -8$.

chapter (2)

Solving Equations The Addition principle (2.1).

حل المعادلات بقاعدة الجمع .

لكل معادلة طرفين متساويين سواء كانت أعداد رموز .

$X = 2$

في هذه المسائل المطلوب إيجاد قيمة المجهول سواء كانت العملية جمع أو طرح أو قسمة أو ضرب

Ex. No. 1 : $X + 6 = 13$, Find X .

$7 + 6 = 13$ (جمع) نعوض عن المجهول بقيمته .

$\therefore X = 7$

Ex. No. 2 : $7X = 133$

$7(19) = 133$

$\therefore X = 19$

الطريقة الثانية عن طريق إضافة المعكوس الجمعي للعدد المجاور للمجهول بالطرح أو الجمع أو القسمة أو الضرب ، وبذلك يكون المجهول في طرف لوحده ويسهل إيجاد قيمته

Ex. No. 3 : $X + 6 = 13$

$X + 6 + (-6) = 13 + (-6)$ الطرح بالتساوي في الطرفين .

$\therefore X = 7$

EX.No-4 :- $x + 5 = -7$

$x + 5 + (-5) = -7 + (-5)$ الطرح بالتساوى

$\therefore x = -12$

EX.No.5 :- $a - 4 = 10$

$a - 4 + 4 = 10 + 4$ الجمع بالتساوى

$\therefore a = 16$

check $a - 4 = 10$

$14 - 4 = 10$ True #

EX.No.6 :- $-6.5 = y - 8.4$

$8.4 - 6.5 = y - 8.4 + 8.4$ الجمع بالتساوى

$\therefore 1.9 = y$

$\therefore y = 1.9$

check $-6.5 = y - 8.4$

$-6.5 = 1.9 - 8.4$ True #.

EX.No.7 :- $-\frac{2}{3} + x = \frac{5}{2}$

$-\frac{2}{3} + \frac{2}{3} + x = \frac{5}{2} + \frac{2}{3}$ الجمع بالتساوى

بتوحيد المقام على 6 $x = \frac{15+4}{6} = \frac{19}{6}$

$\therefore x = \frac{19}{6}$

check $-\frac{2}{3} + \frac{19}{6} = \frac{5}{2}$

$\frac{-4+19}{6} = \frac{5}{2}$

$3 \div \frac{15}{6} = \frac{5}{2}$ True #.

Solving Equations : The Multiplication principle (2.2)

حل المعادلات : ضرب المعادلات بطريقة الضرب .

* $a \cdot c = b \cdot c \quad \therefore a = b \quad c \neq 0$.

Ex.No.1 → solve $5X = 70$

$\frac{5}{5}X \cdot \frac{1}{5} = 70 \cdot \frac{1}{5}$ الضرب بالمساوى فى المعكوس الضربى للرقم المجاور للمجهول وذلك فى طرفى المعادلة .

$X = \frac{70}{5}$

$X = 14$.

check $5X = 70$

$5 \cdot 14 = 70$ True #.

Ex.No.2 → $-4X = 92$

$\frac{-4X}{-4} = \frac{92^{23}}{-4}$ القسمة بالمساوى على الرقم المجاور للمجهول وذلك بالمساوى فى الطرفين

$+X = -23$

$\therefore X = -23$

check $-4X = 92$

$-4(-23) = 92$ True #.

Ex.No.3 → ---

$-X = 9$

$-X \cdot -1 = 9 \cdot -1$

$\therefore X = -9$

$-X- = +$
$+X+ = +$
$-X+ = -$
$+X- = -$

المعكوس الضربى هو قلب الكسر بنفس الإشارة يعنى 5 معكوسه $\frac{1}{5}$

$\frac{2}{3}$ معكوسه الضربى $\frac{3}{2}$ ، والمعكوس رقم ضربه فى الرقم ينتج 1 .

EX No. 4 → Solve $\frac{3}{8} = -\frac{5}{4}x$

المعكوس الضربي $-\frac{4}{5} \cdot \frac{3}{8} = +\frac{5}{4}x \cdot -\frac{4}{5}$

للعدد مقلوبه لكنه يشبهه في الإشارة $-\frac{3}{10} = x$

$x = -\frac{3}{10}$

check → $\frac{3}{8} = -\frac{5}{4} \cdot -\frac{3}{10}$

$\frac{3}{8} = \frac{15}{40}$

$\frac{15}{40_8}$ بالقسمة على 5

$\therefore x = -\frac{3}{10}$ True #.

EX. No. 5 → Solve. $\frac{-y}{9} = 14$

$\frac{-y}{9} \cdot 9 = 14 \cdot 9$ بالضرب في المعكوس الضربي لـ $\frac{1}{9}$ هو 9

$-y = 126$

$-y \cdot -1 = 126 \cdot -1$

$\therefore y = -126$

check → $\frac{-y}{9} = 14$

$\frac{-(-126)}{9} = \frac{126}{9}$

$= 14$

$\therefore y = -126$ True #.

المعكوس الضربي لأي رقم هو مقلوب الرقم و يشبهه في الإشارة

مثل $-\frac{3}{20}$ مقلوب $-\frac{20}{3}$ ، $-\frac{4}{5}$ مقلوب $-\frac{5}{4}$ ، و مقلوب $\frac{1}{9}$

و يُضرب بكل ن في طرفي المعادلة.

Using the Principles Together (2-3)

a- Applying Both Principles: استخدام قواعد حل المعادلات معًا من ضرب وطرح وجمع وقسمة.

Ex-No-1 → solve $3x + 4 = 13$

$3x + 4 - 4 = 13 - 4$ الطرح بالتساوى

$\frac{3x}{3} = \frac{9}{3}$ بالقسمة على 3

$\therefore x = 3$

check $3x + 4 = 13$

$3 \cdot 3 + 4 = 13$

$9 + 4 = 13$

$\therefore x = 3$ True #.

Ex-No-2 → solve $45 - t = 13$

$-45 + 45 - t = 13 - 45$ الطرح بالتساوى

$-t = -32$

$-1 \cdot -t = -32 \cdot -1$ الضرب فى 1- فى الطرفين

$\therefore t = 32$

check $45 - t = 13$

$45 - 32 = 13$

$\therefore t = 32$ True #.

Ex-No-3 → $8x - 4 = 28$

$8x - 4 + 4 = 28 + 4$ الجمع بالتساوى

$\frac{8x}{8} = \frac{32}{8}$ بالقسمة على 8 فى الطرفين.

$\therefore x = 4$

b - Collecting like Terms :- جمع المتشابه

(صفحة رقم 99)

Ex. No. 4 :→ Solve $3X + 4X = 14$

$3X + 4X = 14$ الرموز متشابهه إذن نجمع المتشابه

$7X = 14$

$\frac{7X}{7} = \frac{14}{7}$ بالقسمة ÷ 7 فى الطرفين

$\therefore X = 2$

Ex. No. 5 :→ Solve : $2X - 2 = -3X + 3$

$2X - 2 + 2 = -3X + 3 + 2$ الجمع بالتساوى

$2X = -3X + 5$

$2X + 3X = -3X + 3X + 5$ الجمع بالتساوى

$5X = 5$

$\frac{5X}{5} = \frac{5}{5}$ بالقسمة على 5

$\therefore X = 1$

check → $2X - 2 = -3X + 3$

$2(1) - 2 = -3(1) + 3$

$2 - 2 = -3 + 3$

$0 = 0$ True #

$\therefore X = 1$

فى حالة تشابه الرموز نجمع الرقم اللى أمام الرمز ثم نضع الرمز

C - Equations containting Parentheses : ← (صفحة رقم 102)

توزيع الضرب على ما بداخل الأقواس.

EX. No. 6 → $8X = 2(12 - 2X)$

$8X = 24 - 4X$ الضرب بالتوزيع

$8X + 4X = 24 - 4X + 4X$ الجمع بالتساوى

$12X = 24$

$\frac{12X}{12} = \frac{24}{12}$ بالقسمة على 12

$X = \frac{24}{12} = 2$

$\therefore X = 2$ #.

EX. No. 7 Solve : $2 - 5(X + 5) = 3(X - 2) - 1$

$2 - 5X - 25 = 3X - 6 - 1$ الضرب بالتوزيع

$-5X - 23 = 3X - 7$ بجمع المتشابه

$-5X - 23 + 7 = 3X - 7 + 7$ بالجمع بالتساوى

$-5X - 16 = 3X$

$-5X + 5X - 16 = 3X + 5X$ بالجمع بالتساوى

$-16 = 8X$

$\frac{-16^{2}}{8} = \frac{8X}{8}$ بالقسمة على 8

$\therefore X = -2$

check → $2 - 5(-2 + 5) = 3(-2 - 2) - 1$ بالتعويض المباشر بالمعادلة.

$2 - 5(3) = 3(-4) - 1$

$2 - 15 = -12 - 1$

$-13 = -13$ True #.

$\therefore X = -2$

Equations with infinitely Many solutions :→

المعادلات والعدد اللا نهائي من الحلول.

EX. No. 8 $3 + x = x + 3$

$-x + 3 + x = -x + x + 3$ بإضافة $x -$ للطرفين

$\therefore 3 = 3$ True #.

EX. No. 9 solve $7x - 17 = 4 + 7(x - 3)$

$7x - 17 = 4 + 7x - 21$

$7x - 17 = 7x - 17$

$-7x + 7x - 17 = -7x + 7x - 17$ بإضافة $7x -$ للطرفين

$-17 = -17$ True #.

Equations with No soluation :→ معادلات لا حل لها.

EX. No. 10 → $3 + x = x + 8$

$-x + 3 + x = -x + x + 8$ بإضافة $x -$ للطرفين

$\therefore 3 = 8$ False $\therefore 3 \neq 8$

EX. No. 11 → $3x + 4(x + 2) = 11 + 7x$ الضرب بالتوزيع

$3x + 4x + 8 = 11 + 7x$

$7x + 8 = 11 + 7x$

$-7x + 7x + 8 = 11 + 7x - 7x$ إضافة $7x -$ للطرفين

$8 = 11$ False

$\therefore 8 \neq 11$

Formulas (2.4)

a- Evaluating Formulas: → تقدير (الصيغ) أو طرق الحل (التخمين)

EX-No-1 : socks from cotton Consider the formula $s = 4321x$ where s is the number of socks of average size that can be producted from x bales of cotton. You see a shipment of 300 bales of cotton taken off a ship. How many socks can be made from the cotton?

Solve : $s = 4321x$, $x = 300$

$\therefore s = 4321(300)$

$s = 1,296,300$ socks

Thus, 1296300 socks can be made from 300 bales of cotton.

EX. No. 2 → Distance, Rate and Time. The distance d that a car will travel at a rate, or speed, r in time t is given by → $d = rt$, A car travels at 75 miles per hour (mph) for 4.5hr. How Far will it travel?

$\therefore t = 4.5\ hr \qquad r = 75$

$d = r.t$

$d = (4.5)(75) = 337.5\ mi.$

The car will travel 337.5 mi.

في هذا النوع من المسائل نترجم الجمل إلى أرقام ثم نحل المعادلة.

b- Solving Formulas :→ (صفحة رقم 110)

EX-No : 3 solve for x : $y = x - a$

$y = x - a$

$y + a = x - a + a$ بإضافة $+a$ في الطرفين .

$y + a = x$

$\therefore x = y + a$

EX-No. 4. solve for r : $c = 2\pi r$. This is a formula for the circumference c of a circle of radius r.

$\frac{c}{2\pi} = \frac{2\pi r}{2\pi}$ بالقسمة على 2π في الطرفين

$\frac{c}{2\pi} = r$

$\therefore r = \frac{c}{2\pi}$

EX-No. 5 : Solve for a : $A = \frac{a+b+c}{3}$. This is a formula for the average A of three numbers a, b, and c.

$A = \frac{a+b+c}{3}$

$3 \cdot A = \frac{a+b+c}{3} \cdot 3$ بالضرب بالتساوي في الطرفين $\times 3$

$3A = a + b + c$

$3A - b - c = a + b - b + c - c$ بإضافة $-b-c$ في الطرفين .

$3A - b - c = a$

$\therefore a = 3A - b - c$.

Applications of percent (2.5) استعمال النسب المئوية.

a- Translating and solving

(of) translates to "." or "x"

(IS) translate to "="

(what number) or (what percent) translates to any letter.

(%) translates to " $\times \frac{1}{100}$ " or " $\times 0.01$ ".

EX-No.1 . Translate :.

28 % of 5 is what number?

28 % . 5 = a

Ex-No.2 Translate

45 % of what number is 28?

45 % . b = 28

---

* Finding the Amount :.

Ex.No. 3 :→ what number is 11 % of 49?

what number is 11 % of 49

a = 11 % X 49

$a = 11 \times \frac{1}{100} \times 49$

a = 5.39

The answer is 5.39.

طريقة الحل تكون بترجمة المسألة من كلام إلى رموز وأرقام.
ثم حلها بعد ذلك.

* Finding the Base :-

Ex-No. 4 → 3 is 16% of what number ?

Translate : 3 = 16% x b

3 = 0.16 x b

لإيجاد قيمة b

$\frac{3}{0.16} = \frac{0.16 \times b}{0.16}$ بالقسمة على 0.16 للطرفين

b = 18.75

The answer is 18.75.

---

* Finding the percent Number :→

Ex. No. 5 $32 is what percent of $50 ?

$32 = p x $50

$\therefore \frac{32}{50} = \frac{p \times 50}{50}$ بالقسمة للطرفين على 50

p = 0.64

p = 64%

∴ $32 is 64% of $50.

EX. No. 6 :- Finding the percent number

Translate 15 is what percent of 60 ?

15 = b x 60

$\frac{15}{60} = \frac{b \times 60}{60}$ بالقسمة على 60 للطرفين

0.25 = b

∴ b = 25% ∴ 15 is 25% of 60

Applications and problem solving :→ (2.6)

التطبيقات وحل المشاكل

a- Five steps for solving problems.

1- Familiarize yourself with the problem situation.

2- Translate the problem to an equation.

3- solve the equation.

4- Check the answer in the original problem.

5- state the answer to the problem clearly.

Ex. No. 1 :→ knitted scarf. Lily knitted a scarf in three shades of blue, starting with a light-blue section, then a medium-blue section, and finally a dark-blue section. The medium-blue section is one-Half the length of the light-blue section. The dark-blue section is one-fourth the length of the light-blue section. The scarf is 7 ft Long. Find the length of each section of scarf.

solve :→ 1- Familiarize :→

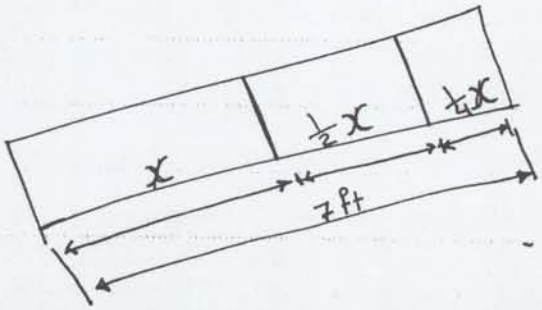
x = the length of light-blue section.

$\frac{1}{2}x$ = the length of the medium-blue section.

$\frac{1}{4}x$ = " " " " dark- " "

7 = " " " scarf.

to draw this..



لحل هذا النوع من المسائل
يتم ترجمة الجمل إلى
أرقام ونسب .
ثم حل المعادلة بعد ذلك .

2- Translate :

$$x + \frac{1}{2}x + \frac{1}{4}x = 7$$

3- solve : $x + \frac{1}{2}x + \frac{1}{4}x = 7$

$x\left(1 + \frac{1}{2} + \frac{1}{4}\right) = 7$ بأخذ x عامل مشترك

$x\left(\frac{4+2+1}{4}\right) = 7$ بتوحيد المقام

$x\left(\frac{7}{4}\right) = 7$ بالضرب x $\frac{4}{7}$ للطرفين

$x\left(\frac{7}{4}\right)\left(\frac{4}{7}\right) = 7 \cdot \frac{4}{7}$

$\therefore \boxed{x = 4}$

4- Check → $x = 4$

$\frac{1}{2}x = \frac{1}{2} \cdot 4 = 2$

$\frac{1}{4}x = \frac{1}{4} \cdot 4 = 1$

$4 + 2 + 1 = 7$ True

∴ The light_blue = 4 ft

∴ The medium_blue = 2 ft

∴ The dark_blue = 1 ft

لحل هذه المسائل نترجم الجمل إلى معادلات ثم نحلها .
أهم شيء هو التركيز في معنى الجمل حتى يسهل ترجمتها لمعادلة .

دعاء المذاكرة : اللهم لا سهل إلا ما جعلته سهلاً وأنت تجعل الحزن إن شئت سهلاً .

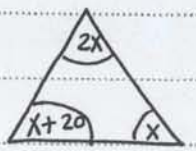
EX.No.2 :→ Roof Gable. In a triangular gable end of a roof, the angle of the peak is twice as large as the angle of the back side of the house. The measure of the angle on the front side is 20° greater than the angle on the back side. How large are the angles?

1 - Farmiliaize :→

x = measure of back side angle.

$2x$ = " " peak "

$x+20$ = " " front "



2- Translate to Equation :→

$$(x)+(2x)+(x+20) = 180$$

3- Solve :→

$$x + 2x + x + 20 = 180$$

$$4x + 20 = 180$$ نجمع المتشابه

$$4x + 20 - 20 = 180 - 20$$ بإضافة -20 للطرفين

$$\frac{4x}{4} = \frac{160}{4} \quad \therefore \boxed{x = 40}$$

$\therefore$ Back angle $= x = 40°$.

$\therefore$ peak angle $= 2x = 2 \cdot 40 = 80°$.

$\therefore$ front angle $= x + 20 = 20 + 40 = 60°$.

check → $80 + 40 + 60 = 180°$ True #.

$40 = \frac{1}{2}\ 80$

$60 = 20 + 40$ True #.

Solving Inequalities (2.7) حل المتباينات أو المتراجحات

a- Soluations of inequalities :→

> (is greater than) أكبر من

≥ (is greater than or equal to) أكبر من أو يساوى

< (is less than) أصغر من

≤ (is less than or equal to) أصغر من أو يساوى

Ex. No. 1 → Determine wherher each number is a solution of $x < 2$

1 - $-2.7 < 2$ True, -2.7 is a solution

2 - $2 < 2$ false, 2 is not a solution

Ex. No. 2 → Determine wherher each number is a solution of $y \geq 6$

1 - $6 \geq 6$ is true, 6 is a solution.

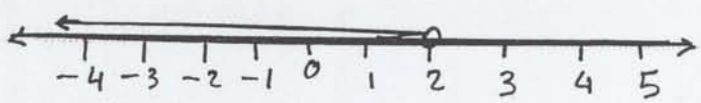
2 - $-3 \geq 6$ is false, 6 in not a solution.

---

b- Graphs of Inequalities →

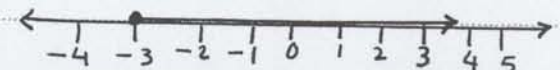
Ex. No. 3 → Graph $x < 2$

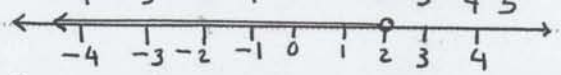
x أقل من 2 رسمتها على خط الأعداد متجهة ناحية السالب علشان

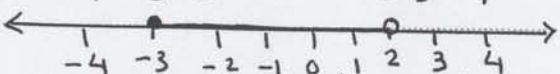


أقل من والدائرة عند الرقم 2 فارغة بهذا الشكل ○ لأنه لا توجد علامة يساوى ولو وجدت يكون الشكل ● دائرة مغلقة وغامقة.

EX. No. 4. Graph $-3 \leq x < 2$

$x \geq -3$  الدائرة غامقة على الـ 3 =

$x < 2$  الدائرة فارغة لأنه مفتوح =

$-3 \leq x < 2$  المفترة من 2 : 3 - ممثلة على خط الأعداد

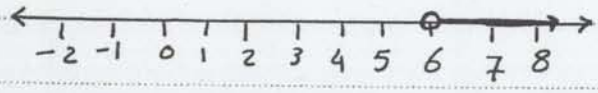
C - Solving inequalities using the Addition principle.

EX. No. 5 : Solve $x + 2 > 8$. Then graph

$x + 2 > 8$

$x + 2 - 2 > 8 - 2$ بإضافة 2- للطرفين

$\therefore x > 6$ الدائرة فارغة لأنها لا توجد علامة يساوى

graph → 

check : →

$x + 2 > 8$

$7 + 2 > 8$

$9 > 8$ True #

EX. No. 6 : → Solve $3x + 1 \leq 2x - 3$ Then graph

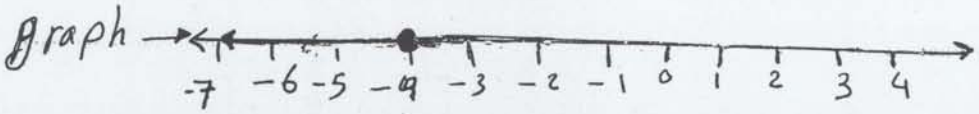
$3x + 1 \leq 2x - 3$

$3x + 1 - 1 \leq 2x - 1 - 3$ بإضافة 1- للطرفين

$3x \leq 2x - 4$

$3x - 2x \leq 2x - 4 - 2x$ بإضافة 2x- للطرفين

$x \leq -4$

graph → 

لحل المتباينة ورسمها إذا كانت ≤ أو ≥ تكون الدائرة عند الرقم على خط الأعداد ● . وإذا كانت < أو > بدون يساوى تكون الدائرة على خط الأعداد ○ .

---

d_ Solving Inequatities using the Multiplication principle.

$a < b \quad \therefore ac < bc$

$a > b \quad \therefore ac > bc$

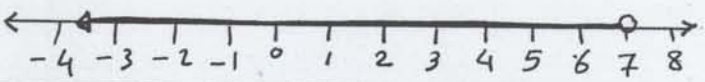
EX.No.7 : Solve $4x < 28$ Then graph

$4x < 28$

$\frac{4x}{4} < \frac{28}{4}$ بالقسمة على 4

$x < 7 \qquad \{x \mid x < 7\}$

graph →



---

e_ using the principles together:

EX.No.8. Solve $3(x-2) - 1 < 2 - 5(x+6)$

$3x - 6 - 1 < 2 - 5x - 30$ الضرب بالتوزيع

$3x - 7 < -5x - 28$

$3x - 7 + 5x < -5x + 5x - 28$ بإضافة $+5x$ للطرفين

$8x - 7 < -28$

$8x - 7 + 7 < -28 + 7$ بإضافة $+7$ للطرفين

$\frac{8x}{8} < \frac{-21}{8}$ بالقسمة على 8

$\therefore x < -\frac{21}{8}$

$\therefore \{x \mid x < -\frac{21}{8}\}$

Applications and problem solving with inequalities (2.8)

تطبيقات ومشاكل حل المتباينات أو المتراجحات.

a- Translating to inequalities :→ ترجمة مصطلحات

* Is at least ⟶ $b \geq 21$
* Is at most ⟶ $n \leq 5$
* cannot exceed ⟶ $r \leq 12.000$
* must exceed ⟶ $s > 15$
* Is less than ⟶ $w < 50$
* Is more than ⟶ $d > 200$
* Is between ⟶ $90 < t < 100$
* no more than ⟶ $w \leq 90$
* no less than ⟶ $s \geq 8.3$

Five steps for solving can be used for problems involving inequalities :→ مراحل الحل.

1- Familiaize.
2- Translate
3- solve
4- check
5- state

أهم شيء في المسائل اللفظية هو ترجمة الجمل لأرقام ورموز بشكل صحيح حتى يسهل حلها.

b- Solving Problems:

Ex. No-1 :→ Catering costs : To cater a party, Curtis, Barbeque charges a $150 setup fee plus $15.50 per person. The cost of Betre Manufacturing's anual picnic cannot exceed $2100. How many can attend the picnic?

1- Familiarize :→

The cost would then be $150 + $15.50(110)

n = numbers of people.

2- Translate :→

The setup	plus	The cost of The meals	Cannot exceed 2100
150	+	15.50n	≤ 2100

3- solve →

$$150 + 15.50n \leq 2100$$

$$-150 + 150 + 15.50n \leq 2100 - 150$$ بإضافة -150 للطرفين

$$15.50n \leq 1950$$

$$\frac{15.50n}{15.50} \leq \frac{1950}{15.50}$$ بالقسمة على 15.50

$$n \leq 125.8 \quad \therefore \text{people number} = 125$$

4- check

$$150 + 15.50(125) \leq 2100$$

$$2087.5 \leq 2100 \quad \text{True}$$

5- state At most 125 people can attend the picnic.

Exam. No. 2. اختبار تجريبي على الفصل الثاني

1- solve :

* $x + 7 = 15$
* $t - 9 = 17$
* $-3x - 6(x - 4) = 9$

2- solve, write set nation for the answer..

* $x + 6 \leq 2$.
* $5 - 9x \geq 19 + 5x$.

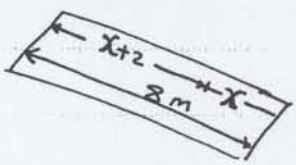
3- Graph on the number Line..

* $y \leq 9$.
* $6x - 3 < x + 2$.

4- solve

* what number is 24 % of 75 ?
* 800 is 2% of what number ?

5- Board cutting. An 8 m Board is cut into two pieces, one piece is 2m Longer than the other. How Long are the pieces ?



6- solve :.

* $\frac{-2}{5} + x = \frac{-3}{4}$.
* $3(x + 2) = 27$.

Chapter (3)

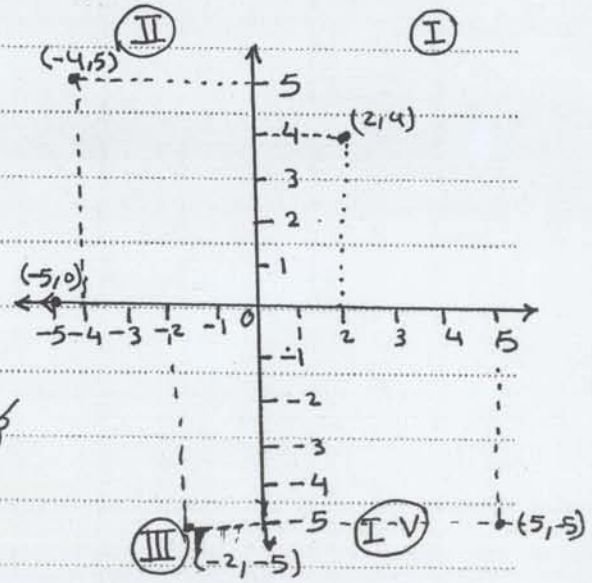
Graphs of Linear Equations:- رسم المعادلة الخطية

Introduction to Graphing (3-1)

a- plotting ordered pairs:→ (صفحة رقم 174)

Ex. No. 1 :- In which quadrant, if any, are the points (-4,5), (5,-5), (2,4), (-2,-5), and (-5,0) Located?

الربع الثاني	الربع الأول
الربع الثالث	الربع الرابع



يتم تمثيل النقط على خط الأعداد
كل نقطة الأولى على المحور الأفقي X
والثانية على المحور الرأسي Y

(-4,5) في الربع الثاني

(5,-5) في الربع الرابع ، (2,4) في الربع الأول

(-5,0) على محور X ، (-2,-5) في الربع الثالث

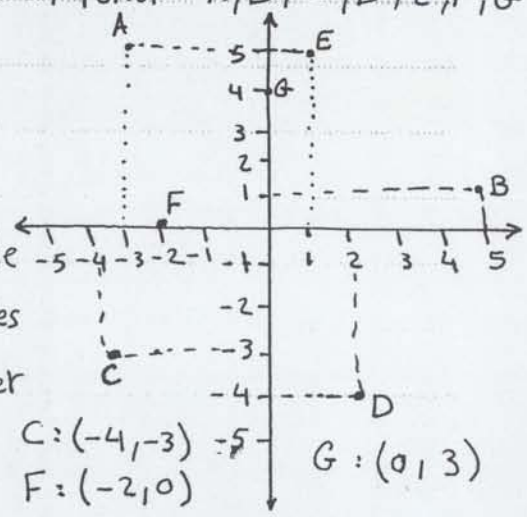
---

b- Finding Coordinates:→ (صفحة رقم 176)

EX. No. 2 : Find the coordinates of point A, B, C, D, E, F, G

Point A is 3 units to the left (horizontal direction) and 5 units up (vertical direction). Its Coordinates are (-3,5). Point D is 2 units to the right and 4 units down. Its Coordinates are (2,-4). The coordinates of the other Points are as follows.

A: (-3,5) , B: (5,1) ,



C: (-4,-3)
F: (-2,0)
G: (0,3)

C- Solutions of Equations. (صفحة رقم 176)

EX. No: 3 → Determine whether each of the following pairs is a solution of $4q - 3p = 22$

$(2, 7)$ and $(-1, 6)$

* بالتعويض بـ $(2, 7)$ عن $p \to 2$ ، $q \to 7$.

$4q - 3p = 22$

$4 \cdot 7 - 3 \cdot 2 = 22$

$28 - 6 = 22$

$= 22$ true #.

$(2, 7)$ is a solution of the equation.

* بالتعويض بـ $(-1, 6)$ عن $p \to -1$ ، $q \to 6$.

$4q - 3p = 22$

$4 \cdot 6 - 3 \cdot -1 = 22$

$24 + 3 = 22$

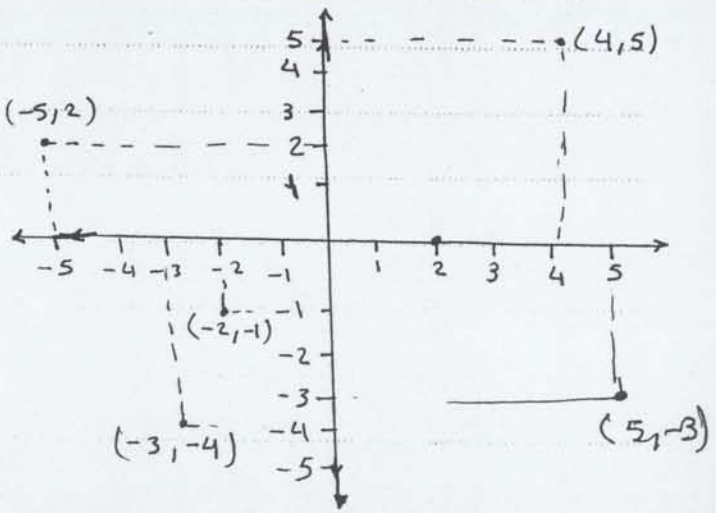
$= 27$ False

$(-1, 6)$ is not solution of the equation.

EX. No- 4 → plot the point $(-5, 2)$, $(4, 5)$

$(-2, -1)$, $(2, 0)$

$(-3, -4)$, $(5, -3)$



Graphing Linear Equations (3.2) (صفحة رقم 181)

a - Graphs of Linear Equations :→ المعادلة الخطية .

Ex. No. 1 Graph $y = 2x$

نعوض بأى رقم فى المعادلة والناتج يطلع معادلة تُرسم خطية .

$y = 2x$

$y = 2 \cdot 3 = 6$ نعوض بـ 3

(x, y)

$(3, 6)$

$y = 2x$

$y = 2 \cdot 0 = 0$ نعوض بـ 0

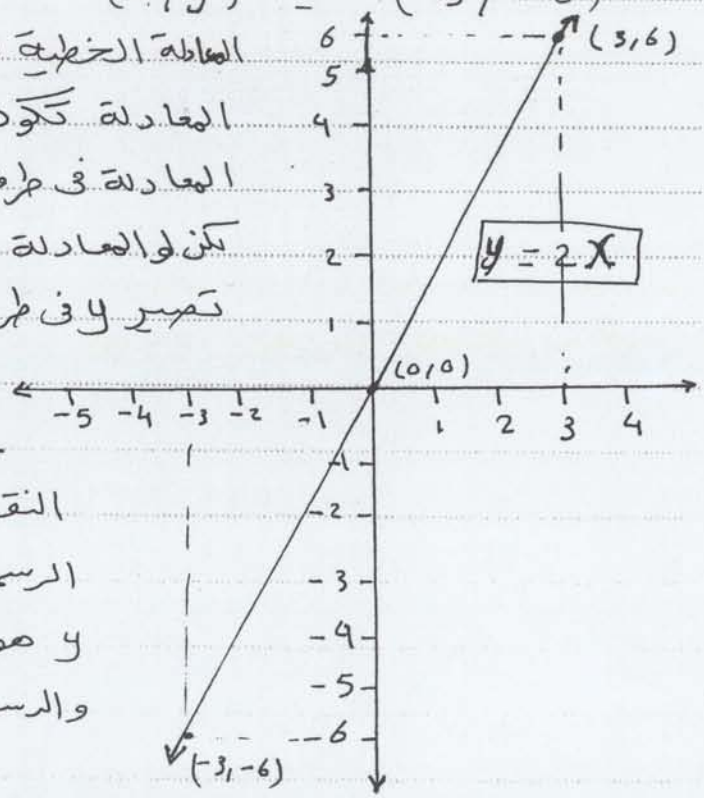
$(x, y) = (0, 0)$

$y = 2x$

$y = 2 \cdot -3 = -6$ نعوض بـ -3

$(x, y) = (-3, -6)$

المعادلة الخطية لابد قبل التعويض فى المعادلة تكون y فى طرف وباقى المعادلة فى طرف آخر $y = mx + b$ لكن لو المعادلة معقدة أحلها أولاً بحيث تصير y فى طرف وباقى الأجزاء فى الطرف الآخر .



النقطة التى أعوض بها وتظهر على الرسم x هى الفرض الذى أعوض به y هو الناتج الذى ينتج من المعادلة والرسم يظهر على شكل خط

EX. No. 2 Graph $3y + 5x = 0$

لحل المعادلة لابد تكون y فى طرف وباقى المعادلة فى طرف ثم بعد ذلك نعوض

$$3y + 5x = 0$$

بإضافة $-5x$ للطرفين $\quad 3y + 5x - 5x = -5x$

$$3y = -5x$$

بالقسمة على 3 $\quad \frac{3y}{3} = \frac{-5x}{3}$

$$\therefore y = -\frac{5x}{3}$$

بعد ذلك نعوض فى المعادلة $\quad y = -\frac{5x}{3}$

* $x = 0 \quad \therefore y = \frac{-5x}{3}$

$$y = \frac{-5 \cdot 0}{3}$$

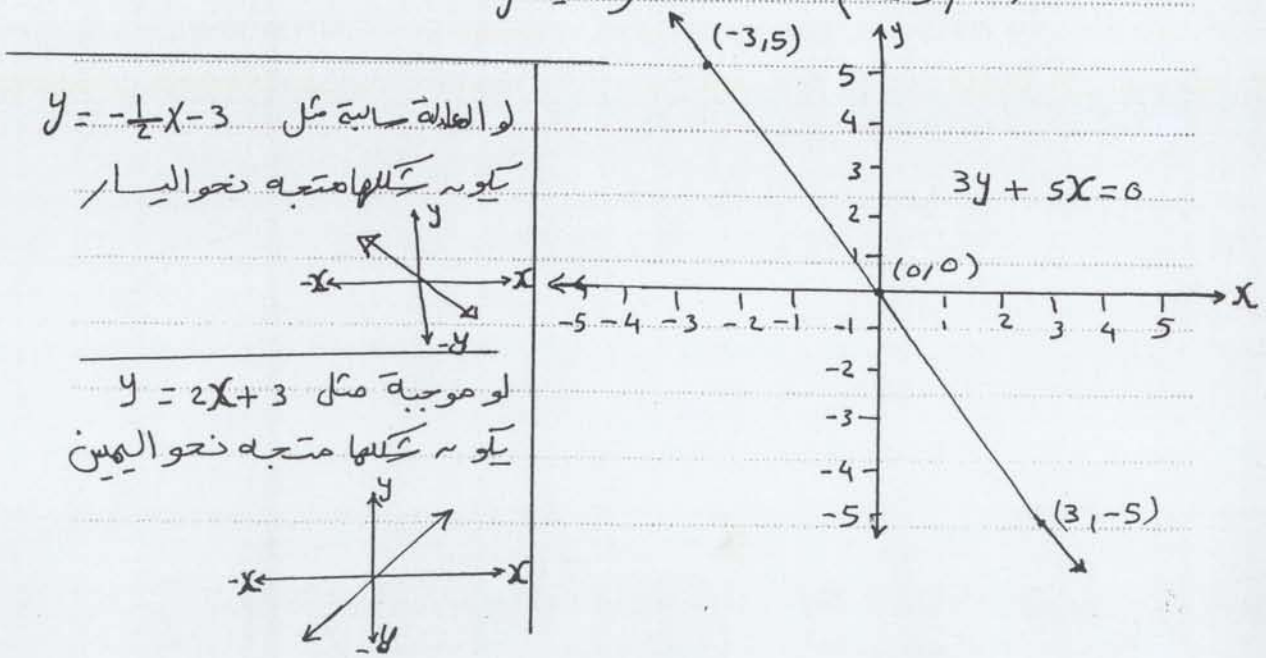
$\therefore y = 0 \quad (0,0)$

* $x = 3 \quad \therefore y = \frac{-5 \cdot 3}{3}$

$\therefore y = -5 \quad (3, -5)$

* $x = -3 \quad y = \frac{-5 \cdot -3}{3} = \frac{15}{3} = 5$

$\therefore y = 5 \quad (-3, 5)$



لو المعادلة سالبة مثل $y = -\frac{1}{2}x - 3$

يكون شكلها متجه نحو اليسار

لو موجبة مثل $y = 2x + 3$

يكون شكلها متجه نحو اليمين

More with Graphing and intercepts - (3-3)

a- Graphing using intercepts :→ نقاط التقاطع مع المحاور.

EX.No- 1 : Consider $4x + 3y = 12$ Find the intercepts Then graph the equation using the intercepts.

نعوض عن x بصفر لإيجاد قيمة y.

$$4x + 3y = 12$$

$$4 \cdot 0 + 3y = 12$$

$$\frac{3y}{3} = \frac{12}{3} \quad \therefore \boxed{y = 4} \quad (0, 4)$$

نعوض عن y بصفر لإيجاد قيمة x

$$4x + 3y = 12$$

$$4x + 3 \cdot 0 = 12$$

$$\frac{4x}{4} = \frac{12}{4} \quad \therefore \boxed{x = 3} \quad (3, 0)$$

ثم نرسم على axis.

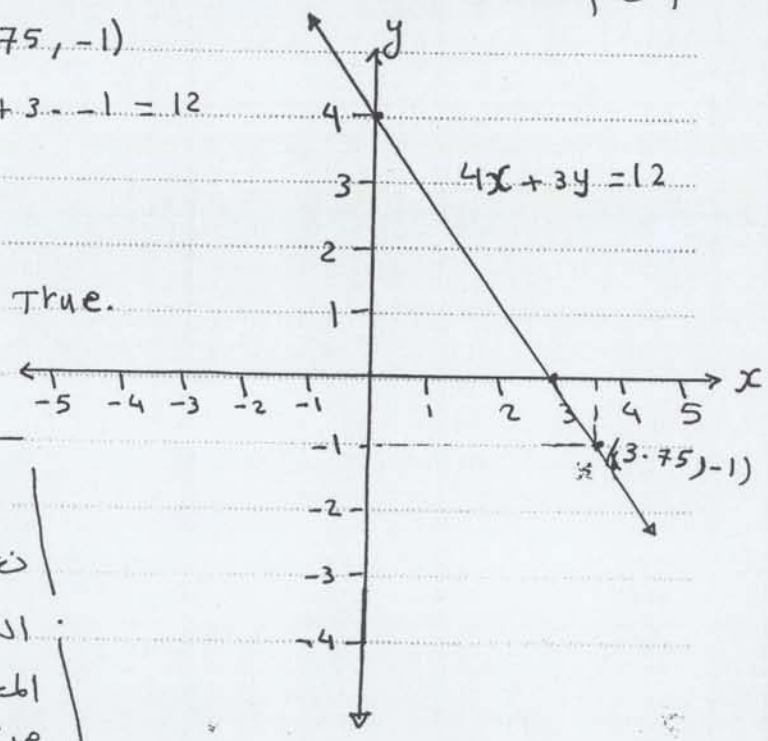
checkpoint $(3.75, -1)$

$$4x + 3 \cdot -1 = 12$$

$$4x - 3 = 12$$

$$4x = 15$$

$$\boxed{x = 3.75}$$ True.



في هذا النوع من المسائل نعوض عن x، y بقيم لإيجاد أماكن النقط على المحاور ثم الرسم على المحاور ثم اختيار نقطة للتأكد من صحة الرسم.

b- Equations whose Graphs Are Horizontal or vertical Lines :→ (صفحة رقم 194)

لما المعادلة تكون تساوى رقم تكون المعادلة توازى محور X أو محور y حسب المعادلة.

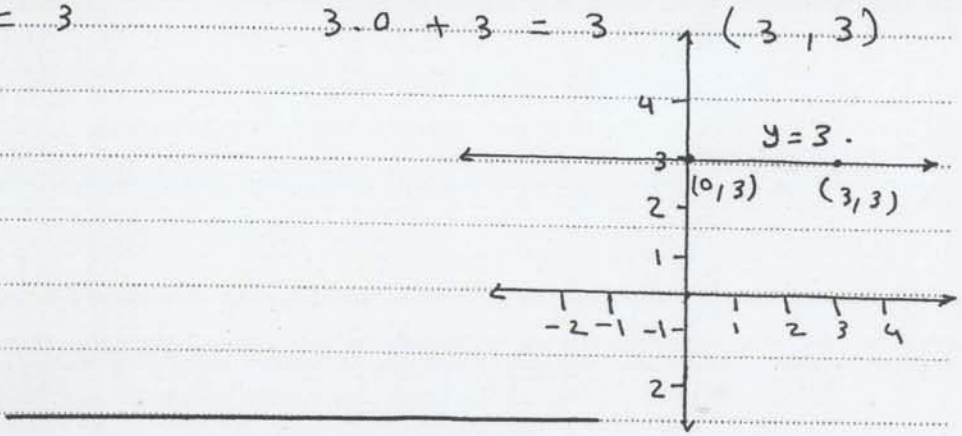
EX. No. 2 → Graph $y = 3$.

فى هذه المعادلة بغض النظر عن X بأى رقم مضروب فى 0 ، $y = 3$

$x = -2$ $\quad -2 \cdot 0 + 3 = 3 \quad (-2, 3)$

$x = -5$ $\quad -5 \cdot 0 + 3 = 3 \quad (-5, 3)$

$x = 3$ $\quad 3 \cdot 0 + 3 = 3 \quad (3, 3)$



EX. No. 3 → graph $x = -4$

$\therefore x + y \cdot 0 = -4$

$y = 3 \quad x + 0 \cdot 3 = -4 \quad \therefore x = -4 \quad (-4, 3)$

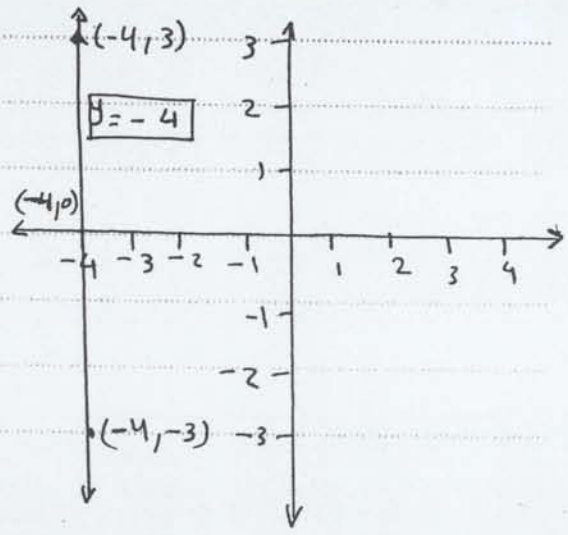
$y = -3$

$\therefore x + 0 \cdot -3 = -4$

$\therefore x = -4$

$(-4, -3)$.

∴ المعادلة إما توازى المحور X أو المحور y



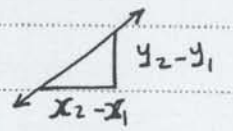
Slope and Applications (3-4) الميل وتطبيقاته

a- slope. الميل (صفحة رقم 205)

$y = mx + b$ معادلة المستقيم

$m = \frac{y_2 - y_1}{x_2 - x_1}$ ميل المستقيم وهو فرق y على فرق x

$\frac{\text{فرق } y}{\text{فرق } x} = \text{slope} = (\text{هو الميل } m)$



EX. No 1

Graph the line containing the point $(-4, 3)$ and $(2, -6)$ and Find the slope.

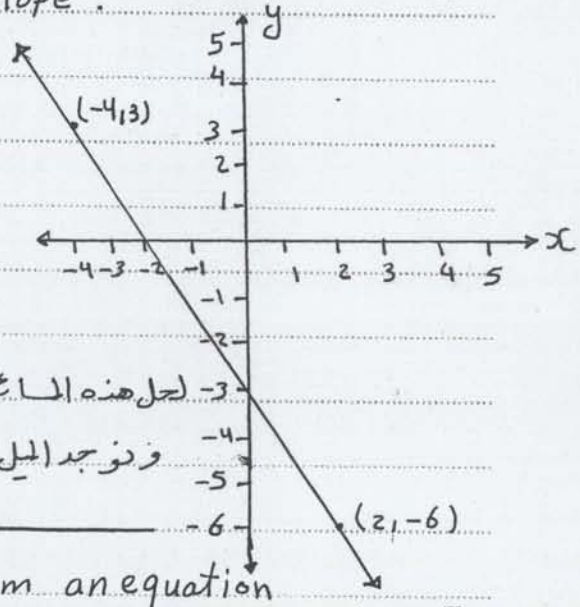
$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{-6 - 3}{2 + 4} = \frac{-9}{6}$

$m = \frac{-3}{2}$

لحل هذه المسائل نمثل النقاط على المحاور، ثم نعوض في المعادلة slope. ونوجد الميل.

$\therefore \text{slope} = \frac{-3}{2}$



b- Finding the slope from an equation (ص رقم 207)

EX. No. 2 :→ Find the slope of each line

* $y = -3x + \frac{2}{9} \rightarrow y = mx + b$

$\therefore m = -3 = \text{slope}.$

* $y = x + 6 \rightarrow y = mx + b$

$\therefore m = 1 = \text{slope}.$

EX. No. 3 Find the slope of the line $y = 5$

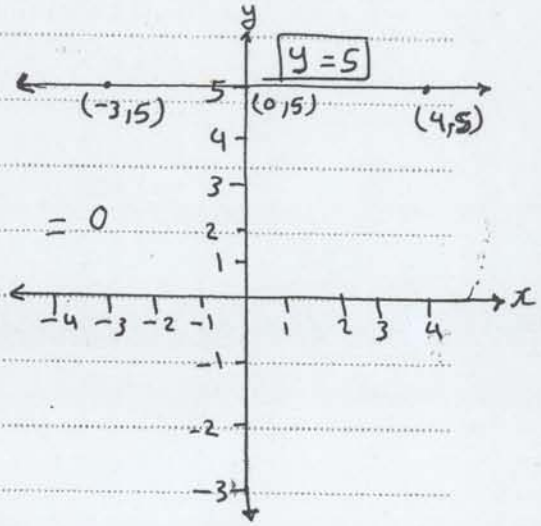
في هذه الحالة دخنا، أي رقم ونمثله لـ X مثل لأن قيمة y ثابتة

$(-3, 5)$, $(4, 5)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{5 - 5}{4 + 3} = \frac{0}{7} = 0$$

slope = 0



C- Applications of slope ; Rates of change. (209 رقم، up)

EX. No. 4. Skiing. Among the steepest skiable terrain in North America, the Headwall on Mt. washington, in New Hampshire, drops 720 ft over a horizontal distance of 900ft. Find the grade of the Headwall.

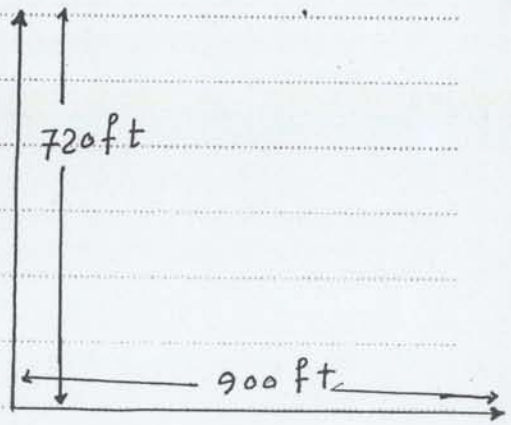
The grade of the Head wall is its slope expressed as a persent.

$$m = \frac{y}{x}$$

$$m = \frac{720}{900}$$

$$m = \frac{8}{10} = 80\ \%$$

$\therefore$ slope = 80 %.



Exam. No- 3. اختبار تجريبي على الفصل الثالث

1- In which quadrant is each point Located:.

* $(-\frac{1}{2}, 7)$
* $(-5, -6)$

2- Graph each equation . Identify the y-intercept

* $y = 2x - 1$
* $y = -\frac{3}{2}x$

3- Find the intercepts of each equation . Then graph the equation .

* $2x - 4y = -8$.
* $2x - y = -3$.

4- Graph each equation.

* $2x + 8 = 0$
* $y = 5$

5- Find the slope , if it exists, of each Line.

* $2x - 5y = 10$
* $x = -2$
* $3y = \frac{1}{9}$

6- Solve :.

* $2(x+2) \geq 5(2x+3)$.

Chapter (4)

Polynomials: operations. (4.1) (صفحة رقم 228)

a- Exponential Notation. (الأسس).

* الأساس هو الرقم اللي تحت a^n و a عدد.
* الأس هو اللي فوق الرقم ويرمز له بالرمز n مثل 5^n , 3^n و n عدد. ، يعني أن العدد يكون مضروب في نفسه عدد مرات الأس

$n^3 = n.n.n$ يعني عدد مضروب في نفسه ٣ مرات

$a^4 = a.a.a.a$ وهكذا.

---

b- one and zero as Exponents.→ (ص رقم 228)

EX. No. 1 → Evaluate 5^1 , $(-8)^1$, 3^0 , $(-7.3)^0$

$5^1 = 5$, $(-8)^1 = -8$

$3^0 = 1$ $(-7.3)^0 = 1$

أي حاجه أس صفر = ١ ، افكر المثل الشعبي أي حاجه بجنيه ، ريال.

$3^3 = 3.3.3 = 27$

---

c- Evaluating Algebraic Expressions.. (ص رقم 229)

EX. No. 2 → Evaluate $1000 - x^4$ when $x = 5$

$1000 - x^4$

$1000 - 5^4$ يعني مضروب في نفسه أربع مرات.

$1000 - (5.5.5.5)$

$1000 - 625 = 375$.

EX. No. 3 → Evaluate $(5x)^3$ when $x = -2$

$(5 . -2)^3 = (-10)^3 = -1000$

d- Multiplying powers with Like Bases. (ص رقم 230)

* المضروب : في الضرب نجمع الأس في المتشابه من الرموز ، لازم يكون الأساس متشابه علشان أجمع الأسس .
* في الضرب نجمع ، في القسمة نطرح .

$a^3 \cdot a^2 = a^5$ لأن الأساس واحد نجمع

$b^4 \cdot b^3 = b^7$

في الضرب نجمع الأسس

EX. No. 4 . Multiplying and simplity.

* $5^6 \cdot 5^2 = 5^{6+2} = 5^8$
* $x \cdot x^8 = x^{1+8} = x^9$
* $(4y)^6 \cdot (4y)^3 = (4y)^{6+3} = (4y)^9$

---

e- Diving powers with Like Bases. (ص رقم 231)

$a^5 / a^2 = a^{5-2} = a^3$ في القسمة نطرح الأسس

* EX. No. 5 :→ $\frac{6^5}{6^3} = 6^{5-3} = 6^2$
* $\frac{(3t)^{12}}{(3t)^2} = (3t)^{12-2} = (3t)^{10}$
* $\frac{p^5 q^7}{p^2 q^5} = \frac{p^5}{p^2} \cdot \frac{q^7}{q^5} = p^{5-2} \cdot q^{7-5} = p^3 \cdot q^2$

---

f- Negative integers as Exponents. (ص رقم 232)

في الأمثلة السابقة كان أس البسط أكبر من أس المقام لكن عندما يكون أس المقام أكبر يكون كالتالي :

$a^5 / a^{10} = a^{5-10} = a^{-5} = \frac{1}{a^5}$

EX. No. 6 → $\frac{5^4}{5^{-2}} = 5^{4+2} = 5^6$

* $\frac{x}{x^7} = x^{1-7} = x^{-6} = \frac{1}{x^6}$
* $y^{-4} \cdot y^{-8} = y^{-4-8} = y^{-12} = \frac{1}{y^{12}}$ هذه ضرب لكن الأسس سالبة
* $\frac{5^3}{57} = 5^{3-7} = 5^{-4} = \frac{1}{5^4}$.

Exponents and scientific Notation. (4.2).

a- Raising Powers to Powers. → (صفحة رقم 238)

$(a^m)^n = a^{mn}$ لما يكون الأس داخل القوس والأس خارج القوس نضرب بعضهم في بعض.

Ex. No. 1 → simplify Express the answers using positive exponents. →

1- $(3^5)^4 = 3^{5\cdot4} = 3^{20}$

2- $(y^{-5})^7 = y^{-5\cdot7} = y^{-35} = \frac{1}{y^{35}}$

3- $(a^{-4})^{-6} = a^{(-4)(-6)} = a^{24}$

4- $(2^2)^5 = 2^{2\cdot5} = 2^{10}$

---

b- Raising a aproduct or a Quotient apower.

* $2(a)^3$ معناها أن 2 مضروب مرة واحدة ويختلف عن ما بداخل القوس

والنتيجة تكون $2(a)^3 = 2 \cdot a \cdot a \cdot a = 2a^3$

لاحظ الفرق ← $(2a)^3 = 2^3 \cdot a^3 = 2 \cdot 2 \cdot 2 \times a \cdot a \cdot a$

$= 8a^3$.

لو الأس على كل القوس يبقى كل حاجة مضروبة في نفسها عدد مرات الأس.

Ex. No. 2

1- $(4x^2)^3 = 4^3 \cdot x^{2\cdot3} = 4^3 x^6 = 64x^6$

2- $(-5x^4y^3)^3 = (-5)^3 \cdot (x^{4\cdot3}) \cdot (y^{3\cdot3})$

توزع الأس على كل ما بداخل القوس $= -5^3 \cdot x^{12} \cdot y^9$

$= -125x^{12}y^9$

3- $((-x)^{25})^2 = -x^{25\cdot2} = -x^{50} = x^{50}$

هنا حذفنا الإشارة لأن الأس زوجي. لو عدده فردي تبقى الإشارة ولو عدده زوجي يحذف الإشارة لأنه يضرب المرفق نفسه عدد مرات تكون النتيجة + = -x-

c- Scientific Notation ← (صارقم 240) استخدامات علمية

تستخدم الأسس عوضاً عن أرقام كبيرة جداً يصعب علينا حفظها مثل حجم الأرض والمسافة بين الأرض والشمس وهذه الأشياء تقدر بالملايين والأسس تسهل ذلك.

Ex. No. 3 - The mass of the earth

$= 6.615 \times 10^{21}$ tons $= 6,615,000,000,000,000,000,000$ طن ضخم جداً.

∴ للحل بسهولة المكتوب هو $(M \times 10^n)$.

Ex. No - 4 → Convert to scientific notation.

$78,000 = 7,8 \times 10^4$

$= 7, \underbrace{8,000}$ → 4 places.

Ex. No. 5 → Large number, so the exponent is positive

$0.0000057 = 5.7 \times 10^{-6}$

$= 0.000005,7$ → 6 places.

---

d- Multiplying and Diving using scientific Notation.

Ex. No. 6 → Multiply $(1.8 \times 10^6) \cdot (2.3 \times 10^{-4})$

$= (1.8)(2.3)(10^6)(10^{-4})$

$= 4.14 \times 10^2$.

Ex. No. 7 → Multiply $(3.1 \times 10^5) \cdot (4.5 \times 10^{-3})$

$= (3.1)(4.5)(10^5)(10^{-3})$

$= 13.95 \times 10^2$

$= 1.395 \times 10^3$

Diving .→

EX. No. 8 → divide $(3.41 \times 10^{5}) \div (1.1 \times 10^{-3})$

$= \frac{3.41 \times 10^{5}}{1.1 \times 10^{-3}} = \frac{3.41}{1.1} \cdot \frac{10^{5}}{10^{-3}}$

$= 3.1 \times 10^{5-(-3)} = 3.1 \times 10^{8}$

EX. No. 9 → Divide $(6.4 \times 10^{-7}) \div (8.0 \times 10^{6})$

$= \frac{6.4 \times 10^{-7}}{8.0 \times 10^{6}} = \frac{6.4}{8} \cdot \frac{10^{-7}}{10^{6}}$

$= 0.8 \times 10^{-7-6} = 0.8 \times 10^{-13}$

$= 8 \times 10^{-14}$ #.

---

e- Applications with scientific Notation :→ (243 رقم ص)

EX. No. 10 → DNA . A strand of DNA (deoxyibonucleicacid is about 150 cm long and 1.3×10^{-10} cm wide. The length of a strand of DNA is how many times the width?

* To determine how many times longer DNA is than it is wide we divide the length by the width. →

$\frac{150}{1.3 \times 10^{-10}} = \frac{150}{1.3} \times \frac{1}{10^{-10}}$

$= 115.385 \times 10^{10}$ كل ما تحرك بالعلامة العشرية رقم تزود أس

$= 11.5385 \times 10^{11}$

$= 1.15385 \times 10^{12}$

∴ The length of DNA is about 1.15385×10^{12} times its width.

Introduction to polynomials :→ (4.3) (من رقم 250)

a- Evaluating polynomials and Applications :→

هذا النوع من المسائل تعويض مباشر عن قيمة x أو المجهول بالرقم الموجود بالسؤال

EX. No. 1 → Evaluate the polynomial when $x = 2$

* $2x^2 - 7x + 3 = 2 \cdot 2^2 - 7 \cdot 2 + 3$

$= 2 \cdot 4 - 7 \cdot 2 + 3 =$

$= 8 - 14 + 3 = -3$

*

EX. No. 2 → Evaluate the polynomial when $x = -4$

$2 - x^3 = 2 - (-4)^3$

$= 2 - (-64)$

$= 2 + 64 = 66$

---

b- Identifying Terms :→ (من رقم 253)

EX. No. 3 → Find an equivalent polynomial using only additions :→

$-5x^2 - x = -5x^2 + (-x)$.

EX. No. 4 → Identify the terms of the polynomial.

$4t^4 - 5t^6 - 4t + 2$ تبين عدد أجزاء المعادلة فقط

Terms : $4t^4$, $-5t^6$, $-4t$ and 2.

$4x^7 + 3x + 12 + 8x^3 + 5x$

Terms : $4x^7$, $3x$, 12, $8x^3$ and $5x$.

C- Like Terms :→ (ص رقم 253)

* EX.No. 5 → Identify the like terms in the polynomials.

$4x^3 + 5x - 4x^2 + 2x^3 + x^2$ نجمع الأجزاء المتشابهة في رقم الأس

Like terms :→ $4x^3$ and $2x^3$

" " :→ $-4x^2$ and x^2

---

* $6 - 3a^2 - 8 - a - 5a$

Like terms → 6 and -8

" " → $-a$ and $-5a$

---

d- Coefficients :→ (ص رقم 253)

يعني نوجد أسماء كل جزء من المسألة وليس الأس .. يعني الرقم اللي في الأول.

EX. No. 6 :. Identify the coefficients of each term in the polynomial. $3x^4 - 4x^3 + \frac{1}{2}x^2 + x - 8$

3 , -4 , $\frac{1}{2}$, 1 and -8

---

e- Collecting Like terms :→ (ص رقم 254)

EX. No. 7 → Collect like terms يعني نجمع المتشابهة (الأس)

* $2x^3 - 6x^3 = (2-6)x^3 = -4x^3$

لو الأس متشابه نجمع الأسس.

* $3x^5 + 2x^2 - 3x^5 + 8$

$= x^5(3-3) + 2x^2 + 8$

$= 0 \cdot x^5 + 2x^2 + 8 = 2x^2 + 8$

f- Descending and Ascending order → (ص رقم 255)

Ex. No. 8 → Arrange to polynomial in descening order.

* $6x^5 + 4x^7 + x^2 + 3x^3 =$ رتب المعادلة حسب رقم الأس ودرجته

$= 4x^7 + 6x^5 + 3x^3 + x^2$.

* $2x^2 - 4x^3 + 3 - x^2 - 3x^3$ من الأكبر للأصغر.

$= (2x^2 - x^2)(-4x^3 - 3x^3) + 3$

$= x^2 - 7x^3 + 3$

الترتيب $= -7x^3 + x^2 + 3$ يعني ذلك أننا رتبنا المعادلة حسب الأس من الأكبر للأصغر.

---

g- Degrees (ص رقم 255)

Ex. No. 9 → $8x^4 - 3x + 7$ يعني درجة الأس

$8x^4 \longrightarrow 4$ درجة كل جزء والأس.

$-3x \longrightarrow 1$

$7 \longrightarrow 0$

---

h- Missing Terms.. (ص رقم 256)

Ex. No. 10 → Identify the Missing terms in the polynomial → $8x^5 - 2x^3 + 5x^2 + 7x + 8$

الأس x^4 ليس موجود و بذلك نعوض عنه بقيمة $0x^4 = 0$

$= 8x^5 + 0.x^4 - 2x^3 + 5x^2 + 7x + 8$

Ex. No. 11 → $x^4 - 6x^3 + 2x - 1$

* $x^4 - 6x^3 + 0.x^2 + 2x - 1$ نعوض عنها بـ $0.x^2$.

$x^4 - 6x^3 + 2x - 1$ أو نترك المكان فارغ

Addition and subtraction of polynomials :- (4-4)

جمع وطرح المعادلات التى جها أسس عن طريق ترتيب المعادلات تباعاً حسب الدرجة

a- Addition of polynomials (ص رقم 263)

EX. No: 1 :→ Add $(-3x^3 + 2x - 4) + (4x^3 + 3x^2 + 2)$

$(-3 + 4)x^3 + 3x^2 + 2x + (-4 + 2)$

$= x^3 + 3x^2 + 2x - 2$

EX. No. 2 → Add $(9x^5 - 2x^3 + 6x^2 + 3)$ and $(5x^4 - 7x^2 + 6)$ and $(3x^6 - 5x^5 + x^2 + 5)$

أولاً نرتب المعادلة ثم نجمع

$9x^5 \quad - 2x^3 + 6x^2 + 3$

$5x^4 \quad - 7x^2 + 6$

$3x^6 - 5x^5 \quad + x^2 + 5$

$3x^6 + 4x^5 + 5x^4 - 2x^3 + 14$

$= (3x^6 + 4x^5 + 5x^4 - 2x^3 + 14)$

---

b- opposites of Polynomials :- (ص رقم 264)

EX. No. 3 → simplify $-(x^2 - 3x + 4)$ .. تبسيط المعادلات

$= -x^2 + 3x - 4$

EX. No. 4 → simplify $-(-7x^4 - \frac{5}{9}x^3 + 8x^2 - x + 67)$

$= 7x^4 + \frac{5}{9}x^3 - 8x^2 + x - 67.$

يعني ذلك ندخل الإشارة على المعادلة فتتغير مثلما ذلك للتبسيط

* $-(x - 2y + 5)$

$= -x + 2y - 5.$

للتذكير
$+ = -x -$
$+ = +x +$
$- = +x -$
$- = -x +$

C- subtraction of polynomicals. → (ص رقم 264)

$a - b = a + (-b)$ يعني الإشارة خارج القوس تدخل على ما بداخله وتغير الإشارة. وهذا التبسيط.

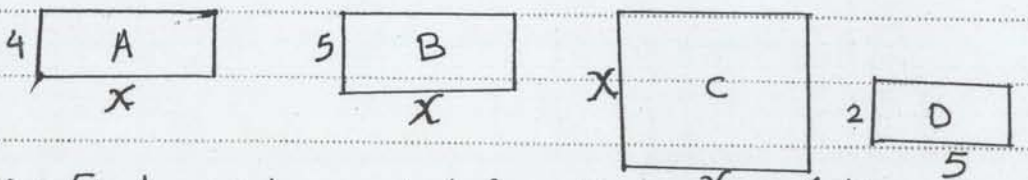
EX. No. 5 → subtract $(9x^5 + x^3 - 3x^2 + 4)$
$-(2x^5 + x^4 - 4x^3 - 2x^2)$

$= 9x^5 + x^3 - 2x^2 + 4 - 2x^5 - x^4 + 4x^3 + 3x^2$

$= x^5(9-2) - x^4 + x^3(1+4) + x^2(-2+3) + 4$

$= 7x^5 - x^4 + 5x^3 + x^2 + 4.$

D- polynomials and Geometry. → (ص رقم 265)



EX. No6 → Find a polynomial for the sum of the areas of these rectangle: → نحسب كل المساحات ثم نجمعها

$m = L \cdot w$

$= 4 \cdot x \;,\; 5 \cdot x \;,\; x \cdot x \;,\; 5 \cdot 2$

$m = 4x + 5x + x^2 + 10$ جمع

$= 9x + x^2 + 10$

$= x^2 + 9x + 10$ # ترتيب

EX-No.7 → Add $(3x + 2) + (-4x + 3)$

$= 3x + 2 - 4x + 3$

$= -x + 5.$

Multiplication of polynomials: (4.5)

a- Multiplying Monomials: → الضرب (حد وحد) (273)

Ex. No. 1 : → Multiply قاعدة: فى الضرب نجمع الأسس.

* $5X \cdot 6X = (5 \cdot 6)(X \cdot X) = 30X^2$.

---

* $(3X)(-X) = (3 \cdot -1)(X \cdot X) = -3X^2$.

---

* $(-7X^5)(4X^3) = (-7 \cdot 4)(X^5 \cdot X^3) = -28X^8$

---

b- Multiplying a Monomial and Any polynomial. →

Ex. No. 2 : → Multiply $2X(5X+3)$

$= 2X(5X+3) = (2X \cdot 5X) + (2X \cdot 3)$

$= 10X^2 + 6X$.

---

* $5X(2X^2 - 3X + 4)$ توزع الضرب على القوس

$= (5X \cdot 2X^2) - (5X \cdot 3X) + (5X \cdot 4)$

$= 10X^3 - 15X^2 + 20X$.

---

* $-2X^2(X^3 - 7X^2 + 10X - 4)$

$= (-2X^2 \cdot X^3) + (-2X^2 \cdot -7X^2) + (-2X^2 \cdot 10X) + (-2X^2 \cdot -4)$

$= -2X^5 + 14X^4 - 20X^3 + 8X^2$

للتذكير فى الضرب نجمع الأسس ، فى القسمة نطرح الأسس .

C - Multiplying Two Binomials.. (ص رقم 274)

Ex. No. 3 → Multiply $(x+5)(x+4)$

$= x(x+4) + 5(x+4)$

$= x \cdot x + x \cdot 4 + 5x + 20$

$= x^2 + 9x + 20.$

---

* $(4x+3)(x-2)$

$= 4x(x-2) + 3(x-2)$

$= 4x \cdot x - 4x \cdot 2 + 3x - 6$

$= 4x^2 - 8x + 3x - 6$

$= 4x^2 - 5x - 6.$

---

d - Multiplying Any Two Polynomials.. (ص رقم 275)

Ex. No. 4 → Multiply $(x^2 + 2x - 3)(x^2+4)$

$= x^2(x^2+4) + 2x(x^2+4) - 3(x^2+4)$

$= x^2 \cdot x^2 + 4x^2 + 2x \cdot x^2 + 8x - 3x^2 - 12$

$= x^4 + 4x^2 + 2x^3 + 8x - 3x^2 - 12$

$= x^4 + 2x^3 + x^2 + 8x - 12$

---

* $(2x^2 + 3x - 4)(2x^2 - x + 3)$

$= 2x^2(2x^2 - x + 3) + 3x(2x^2 - x + 3) - 4(2x^2 - x + 3)$

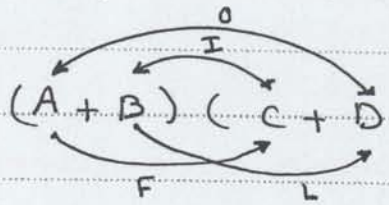
$= 4x^4 - 2x^3 + 6x^2 + 6x^3 - 3x^2 + 9x - 8x^2 + 4x - 12$

$= 4x^4 + 4x^3 - 5x^2 + 13x - 12$ #.

Special proudets. (4.6) (ص رقم 280)

a- products of Two Binomials using FoIL

$(A+B)(C+D) = AC + AD + BC + BD$



لحل هذه العملية

الأول × الأول + الثاني × الثاني + الطرفين + الوسطين .

في هذه المسألة المقوسين غير متشابهين.

Ex. No. 1 → Multiply $(X+8)(X^2-5)$

$= X \cdot X^2 + 8X^2 - 8 \cdot 5 + 5X$

$= X^3 + 8X^2 - 5X - 40$

---

b- Multiplying sums and Differences of Two Terms..

لو المقوسين متشابهين مع اختلاف الإشارة يكون الحل كالتالي

Ex. No. 2 → $(X+2)(X-2)$

$= X^2 - 4 + 2X - 2X$ نلاحظ الوسطين والطرفين مختلفين في الإشارة يعني لا نلزم لضربهما أصلاً.

$= X^2 - 4$.

---

* $(3X-5)(3X+5) = 9X^2 - 25$. (الأول تربيع + الثاني تربيع بإشارة سالبة)

---

* $\left(X+\frac{3}{8}\right)\left(X-\frac{3}{8}\right) = X^2 - \frac{9}{64}$.

---

* $(X+6)(X-6) = X^2 - 36$

---

* $(X+7)(X+4) = X^2 + 28 + 7X + 4X$

$= X^2 + 11X + 28$

c- squaring Binomials.. (ص رقم 82)

Ex. No. 3 → $(x+3)^2 = (x+3)(x+3)$

$= x^2 + 9 + 6x$

$(A+B)^2$ طريقة الحل لفك القوس التربيعي

$A^2 + B^2 + 2AB$

(الأول تربيع + الثاني تربيع + (الأول في الثاني × 2)

Ex. No. 4 → $(t-5)^2 = t^2 + 25 - 10t$

$= t^2 - 10t + 25.$

---

* $(5x - 3x^2)^2 = (5x)^2 + (-3x^2)^2 + (5x \cdot -3x^2 \cdot 2)$

$= 25x^2 + 9x^4 - 30x^3$

$= 9x^4 - 30x^3 + 25x^2$

---

d- Multiplication of various Types:→

Ex. No. 5 → $(x+7)(x+7) = (x+7)^2$

$= x^2 + 14x + 49$

---

* $(x+7)(x-4) = x^2 - 28 + 7x - 4x$

$= x^2 + 3x - 28$

---

* $(x^2 - 3x + 2)(x+7) = x^2(x+7) - 3x(x+7)$

$+ 2(x+7) = x^3 + 7x^2 - 3x^2 - 21x + 2x + 14$

$= x^3 + 4x^2 - 19x + 14.$

operations with polynomials in several variables (4.7)

(ص رقم 291)

a- Evaluating polynomials :→

EX. No. 1 → Evaluate the polynomial $4 + 3x + xy^2 + 8x^3y^3$ when $x = -2$, $y = 5$

نعوض في المعادلة

$$4 + 3x + xy^2 + 8x^3y^3 = 4 + (3 \cdot -2) + (-2 \cdot 5^2) + (8 \cdot -2^3 \cdot 5^3)$$

$$= 4 - 6 + (-2 \times 25) + (8 \cdot -8 \cdot 125)$$

$$= 4 - 6 - 50 - 8000$$

$$= -8052.$$

---

b- Coefficients and Degrees : (ص رقم 292)

EX. No. 2 → Identify the coefficient and degree of each term and the degree of the polynomial.

$$9x^2y^3 - 14xy^2z^3 + xy + 4y + 5x^2 + 7$$

المطلوب الأسس ، الأسس وأجزاء المعادلة ، عدد أجزاء المعادلة

Term	Coefficients	Degree	Degree of the Polynomial
$9x^2y^3$	9	5	6
$-14xy^2z^3$	-14	6	
xy	1	2	
$4y$	4	1	
$5x^2$	5	2	
7	7	0	

C- Collecting like Terms. جمع المتشابهة من الصور والأسس

EX. No. 3 → Gllect Like terms. (ص رقم 292)

* $5x^2y + 3xy^2 - 5x^2y - xy^2 = (5-5)x^2y + (3-1)xy^2$

$= 2xy^2$.

* $7xy - 5xy^2 + 3xy^2 - 7 + 6x^3 + 9xy - 11x^3 + y - 1$

$= xy(7+9) + xy^2(-5+3) + x^3(6-11) + (-7-1) + y$

$= 16xy - 2xy^2 - 5x^3 - 8 + y$ (بالطرح أو الجمع)

$= -5x^3 - 2xy^2 + 16xy + y - 8$. أو المرتب حسب الطرحية المطلوبة بالسؤال

D- Addition → (ص رقم 293)

Ex. No. 4 → Add $(-5x^3 + 3y - 5y^2) + (8x^3 + 4x^2 + 7y^2)$

$= -5x^3 + 3y - 5y^2 + 8x^3 + 4x^2 + 7y^2$

$= 3x^3 + 4x^2 + 2y^2 + 3y$.

E- Subtraction → (ص رقم 293)

EX. No. 5 → subtract $(4x^2y + x^3y^2 + 3x^2y^3 + 6y + 10)$

$-(4x^2y - 6x^3y^2 + x^2y^2 - 5y - 8)$

$= 4x^2y + x^3y^2 + 3x^2y^3 + 6y + 10 - 4x^2y + 6x^3y^2 - x^2y^2 + 5y + 8$

$= 18 + 11y - x^2y^2 + 7x^3y^2 + 3x^2y^3$ #.

f- Multiplication → (ص رقم 294)

EX. No. → 6 → Multiply $(3x^2y - 2xy + 3y)(xy + 2y)$

$= 3x^2y(xy + 2y) - 2xy(xy + 2y) + 3y(xy + 2y)$

$= 3x^3y^2 + 6x^2y^2 - 2x^2y^2 - 4xy^2 + 3xy^2 + 6y^2$

$= 3x^3y^2 + 4x^2y^2 - xy^2 + 6y^2$. #.

Division of polynomials : (4.8)

a- Diving by a Monomial : → (صـ رقم 300)

Ex.No.1 → divide نقسم الرقم على الرقم ، والرمز على الرمز

* $\frac{10x^2}{2x} = \frac{10}{2} \cdot \frac{x^2}{x} = 5x^{2-1} = 5x$ ثم نطرح الأسس

---

* $(9x^8 + 12x^6) \div (3x^2) = \frac{9x^8}{3x^2} + \frac{12x^6}{3x^2}$

$= 3x^5 + 4x^4$.

---

b- Diving by a Binomial : →

Ex.No.2 → divide $3711 \div 8$

قسمة مطولة

خطواتها :

١- قسمة

٢- ضرب

٣- طرح ، بتغيير الإشارات

٤- تنزيل رقم جديد

```
      0463   → divide
   8 | 3711
     -32     → Multiply
      51     → subtract
     -48
      31  ← Bring down the one
     -24
       7
```

$\frac{x+3}{x+2}$ divide (x^2+5x+6) by $(x+2)$

```
x+2 | x² + 5x + 6
    -x² - 2x
    ---------
      0 + 3x + 6
        -3x - 6
      ---------
        0   0   0
```

الحل هو $(x+3)$

$\therefore (x+2)(x+3)$

$= (x^2+5x+6)$

Exam - No. 4 - اختبار تجريبي على الفصل الرابع

1- Multiply and simplify :-

* $6^{-2} \cdot 6^{-3}$.

* $(4a)^3 \cdot (4a)^8$.

---

2 - Divide and simplify :-

* $\frac{3^5}{3^2}$, * $\frac{x^3}{x^8}$.

---

3 - simplify :- * $(3x^2)^3 \cdot (-2x^5)^3$.

* $2x^2(-3x^2)^4$.

---

4 - Collect like terms :-

* $4a^2 - 6 + a^2$.

* $y^2 - 3y - y + \frac{3}{4}y^2$.

---

5 - Add :- $(3x^5 + 5x^3 - 5x^2 - 3)$

$+ (x^5 + x^4 - 3x^3 - 3x^2 + 2x - 4)$.

---

6 - subtract :- $(x^3 - 0.4x^2 - 12)$

$- (x^5 - 0.3x^3 + 0.4x^2 + 9)$.

---

7 - Divide :- $(12x^4 + 9x^3 - 15x^2) \div (3x^2)$

---

8 - Multiply :- $(3x^5 - 4y^5)(3x^5 + 4y^5)$.

chapter (5)

Polynomials : Factoring

Introduction to Factoring (5.1)

a - Finding the Greatest Common Factor :→ (ص رقم 318)

إيجاد العامل المشترك الأكبر ← يعني أكثر شيء تشترك فيه هذه الأرقام والرموز ويكون ذلك بالتحليل :

EX. No. 1 → Finding GCF of 20 and 30

$20 = 2 \cdot 2 \cdot 5$

$30 = 2 \cdot 3 \cdot 5$

المشترك بينهم هو 2 ، 5 ∴ يكون العامل المشترك الأكبر هو $10 = 2 \cdot 5$

EX. No. 2 → Finding GCF of 180 and 420

∴ GCF = $2 \cdot 2 \cdot 3 \cdot 5 = 60$

المشترك بينهم هو العامل المشترك الأكبر .

$240 = 2^2 \cdot 3 \cdot 5 \cdot 7$

$180 = 2^2 \cdot 3 \cdot 3 \cdot 5$

بالتحليل :

← 2	180
← 2	90
← 3	45
3	15
← 5	5
	1

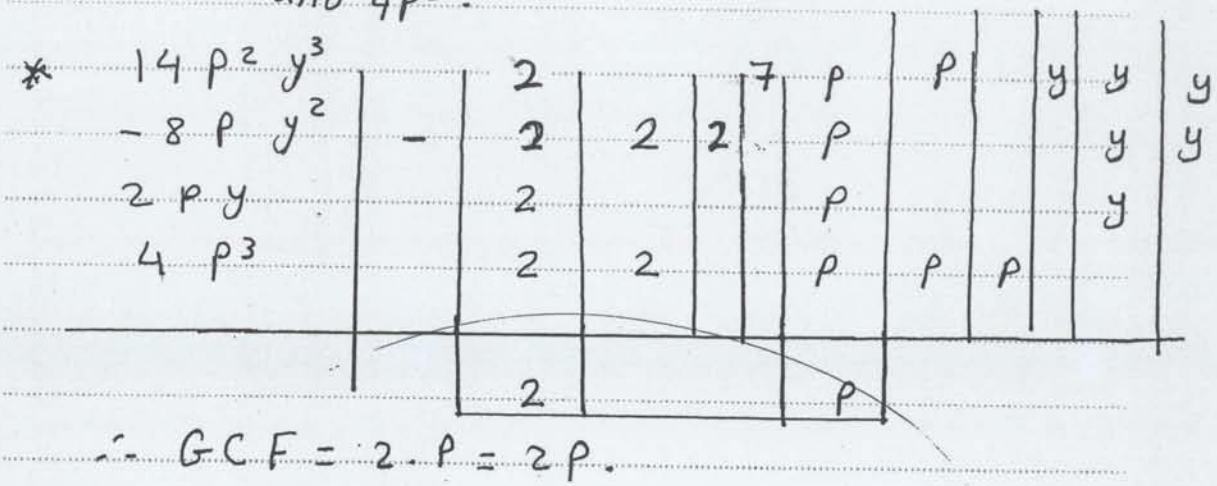
← 2 ÷	420
← 2	210
← 3	105
← 5	35
7	7
	1

EX. No. 3 → Finding GCF of $15x^5$, $-12x^4$, $27x^3$ and $-3x^2$.

* $15x^5 = 3 \cdot 5 \cdot x \cdot x \cdot x \cdot x \cdot x$
* $-12x^4 = -1 \cdot 3 \cdot 4 \cdot x \cdot x \cdot x \cdot x$
* $27x^3 = 3 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x$
* $-3x^2 = 3 \cdot -1 \cdot x \cdot x$

∴ GCF = $3 \cdot x \cdot x = 3x^2$

Ex. No. 4 → Find the GCF of $14p^2y^3$, $-8py^2$, $2py$ and $4p^3$.



$\therefore$ GCF = 2 . p = 2p.

يعني المشترك بين الأربعة.

---

b- Factoring when Terms Have a common Factor: (صورة 321)

$a(b+c) = ab + ac$

Ex. No. 5 → Factor $7x^2 + 14$

$7x^2 = 7 . x^2$ نأخذ عامل مشترك

$14 = 7 . 2$

$\therefore 7x^2 + 14 = 7(x^2 + 2)$

check → Multiply → $7(x^2 + 2) = 7x^2 + 14$.

Ex. No. 6 → $16x^3 + 20x^2$

$16x^3 + 20x^2 = 4x^2(4x + 5)$. نأخذ عامل مشترك .

check → Multiply $4x^2(4x+5)$

$= 16x^3 + 20x^2$.

C- Factoring by Grouping : Four Terms → (صارقم 323)

EX-No. 7 → Factor : $x^2(x+1) + 2(x+1)$

$= (x+1) \cdot (x^2+2)$ فأخذ المشابه عامل مشترك.

The factorization is $(x+1)(x^2+2)$

EX-No. 8 → Factor by Grouping

$6x^3 - 9x^2 + 4x - 6$ حل هذه المسائل نفرق المشابه

$= (6x^3 - 9x^2) + (4x - 6)$ من الأجناد ونجمعها

$= 3x^2(2x-3) + 2(2x-3)$

$= (2x-3)(3x^2+2)$

EX-No. 9 → $12x^5 + 20x^2 - 21x^3 - 35$

$= (12x^5 + 20x^2) + (-21x^3 - 35)$

$= 4x^2(3x^3+5) + (-7)(3x^3+5)$

$= (3x^3+5)(4x^2-7)$. #

EX. No. 10 → $2x^3 - 6x^2 - x + 3$

$= (2x^3 - 6x^2) + (-x+3)$

$= 2x^2(x-3) - 1(x-3)$

$= (x-3)(2x^2-1)$. #

Factoring Trinomials of the Type $x^2 + bx + c$, (5-2)

(ص 327 رقم)

a - Factoring $x^2 + bx + c$

لحل هذه المسائل
الأول × الأول + الثاني الثاني
+ الوسطين + الطرفين .

EX. No. 1 → Factor $x^2 + 5x + 6$

PAIRS of factor	sums of factor
1, 6	7
-1, -6	-7
2, 3	5
-2, -3	-5

Pairs of factor	sums of fact
1, 6	7
2, 3	5

$2 \cdot 3 = 6$, $2 + 3 = 5$ True

$\therefore x^2 + 5x + 6 = (x+2)(x+3)$.

check → $(x+2)(x+3) = x^2 + 6 + 3x + 2x$

$= x^2 + 5x + 6$.

EX. No. 2 → Factor $y^2 - 8y + 12$

محاولات الحل

Pairs	Sums
-1, -12	-13
-3, -4	-7
-2, -6	-12 ↑

$\therefore -2, -6$ True

$\therefore y^2 - 8y + 12$

$= (y - 2)(y - 6)$

To check → $(y-2)(y-6)$

$= y^2 + 12 - 2y - 6y$

$= y^2 - 8y + 12$ #.

Ex. No. 3 → Factor $10 - 3x - x^2$

$\therefore -(x^2 + 3x - 10)$

$= -1(x - 2)(x + 5)$

check → $(-x + 2)(x + 5)$

$= -x^2 + 10 - 5x + 2x$

$= -x^2 + 10 - 3x$ #.

Pairs	Sums
الضرب	المجموع
1, 10	11
2, 5	7
→ 2, -5	-3
1, -10	-9
→ -2, 5	3

Ex. No. 4 → Factor $x^4 - x^2 + 110$

طالما الفرق 1 لا نحتاج للتفكير كثيراً

$= (x^2 - 11)(x^2 + 10)$

رقمين حاصل ضربهم 110 والفرق بينهم 1 ←

check → $(x^2 - 11)(x^2 + 10)$

$= x^4 - 11x^2 + 10x^2 - 110$

$= x^4 - x^2 - 110$.

Ex. No. 5 → Factor $x^2 - x + 5$

رقمين حاصل ضربهم 5 والفرق بينهم 1 هذا مستحيل لأن $\boxed{\begin{matrix} 5 \times 1 = 5 \\ -5 \times -1 = 5 \end{matrix}}$

ومجموعهم = 7

$\therefore x^2 - x + 5$ لا يمكن أن تكون ---

لا يمكن أن تحل في حوسبة

لا بد من التفكير جيداً في حاصل ضرب أي رقمين في أي معادلة، والفرق بينهما أو مجموعهما.

Factoring $ax^2 + bx + c$, $a \neq 1$: The FOIL Method

(5-3). (ص رقم 337)

a- The FOIL Method: → F O I

$(2x + 5)(3x + 4) = 6x^2 + 8x + 15x + 20$

في هذا النوع من المعادلات، حيث $a \neq 1$، يعني أن أمام x^2 رقم، تكون طريقة الحل ← (الأول × الأول) + (الثاني × الثاني) + (ضرب الوسطين) + (ضرب الطرفين) عند حلهم بالعكس، يعني معادلة وعايزين نفكها في قوسين لابد أن نفكر في الرقم الذي أمام x^2، ثم الرقم الأوسط، ثم الطرف.

EX. No. 1 :→ Factor $3x^2 - 10x - 8$ //

نفكر في $3x^2$، $3 \cdot 1 = 3$، $x^2 = x \cdot x$

$\therefore (3x + \square)(x + \square)$

نفكر في الطرفين، عددين حاصل ضربهم 8-، والفرق بينهم 10.

جعلنا إشارة الأكبر سالب لأن الأوسط سالب،

$-4, +2$

أو

$-8, 1$

ثم نفكر بالوسطين. ونجرب مثلاً

$(3x + 1)(x - 8) =$

$3x^2 - 8 - 23x$ False #

نجرب $-4, 2$

$(3x + 2)(x - 4)$

$= 3x^2 - 10x - 8$ True #.

∴ لابد من التفكير جيداً في هذا النوع من المسائل ويا هيا لو حفظت جدول الضرب جيداً فهو يفيد في هذا النوع من المسائل ويجعلك أسرع في الحل ←.

EX. No. 2 → Factor $10x^2 + 37x + 7$

رقم 7 عدد أولى لا يقبل القسمة إلا على نفسه والواحد الصحيح ∴ الطرفين

$(\square x + 1)(\square x + 7)$ ، 7, 1

نستقل بعد ذلك لـ $10x^2$ 10.1 أو 5.2 = 10

1.10 أو 2.5 = 10

* نجرب $(10x + 1)(x + 7) = 10x^2 + 71x + 7$

False # 1.10 False

* نجرب $(5x + 1)(2x + 7) = 10x^2 + 37x + 7$

True

∴ الحل الصحيح هو 5,2 كما مع ترتيب الطرفين 1,7

∴ $(5x+)(2x+7) = 10x^2 + 37x + 7$

EX. No. 3 → Factor $6p^2 - 13pq - 28q^2$

أول شيء نعمل هكذا $= (\square p + \square q)(\square p + \square q)$

عددين حاصل ضربهم 28 2, 14 أو 4, 7 أو 28, 1

عددين حاصل ضربهم 6 2, 3 أو 1, 6

ثم نجرب 1,6 ، 28,1

$(p - 28q)(6p + q)$

$= 6p^2 - 167pq - 28q^2$ False #

نجرب 2,3 ، 4,7

$(2p - 7q)(3p + 4q)$

$= 6p^2 - 21pq - 28q^2 + 8pq$

$= 6p^2 - 13pq - 28q^2$ True #

$\therefore (2p - 7q)(3p + 4q)$

$= 6p^2 - 13pq - 28q^2$ #.

Factoring ax^2+bx+c, $a \neq 1$: The ac-Method. (5.4)

a- The ac-Method. → (ص رقم 345)

EX. No: 1 → Factor $8x^2+8x-6$

أولاً: ننظر إذا كان يوجد عامل مشترك لأن هذا النوع من المعادلات لا يحل في خطوة واحدة

$2(4x^2+4x-3)$

ثم نأخذ ما بداخل القوسين وننساها لفترة، نضرب الأول في الأخير ونحن بالتجربة، عندنا حاصل ضربهما 12 والفرق بينهم 4

∴ الاحتمالات هي

11	-1, 12
-1	-4, 3
1	-3, 4
4	→ -2, 6
-4	-6, 2

ثم ننظر ∴ الحل هو 6, -2

حاصل ضربهم 12 والفارق بينهم 4

∴ $4x^2+4x-3$

$= 4x^2+6x-2x-3$

ثم نقسمهم لقوسين حسب قابلية القسمة 3, -6, 1 يقبلون القسمة على 3

2, -4, 1 يقبلون على 2

∴ $(4x^2-2x)+(6x-3)$

نأخذ عامل مشترك $= 2x(2x-1)+3(2x-1)$

أوجدنا عامل مشترك جديد

$= (2x-1)(2x+3)$

ثم نضيف العامل المشترك اللي في أول المسألة

∴ $= 2(2x-1)(2x+3)$

Check → $= 2(2x-1)(2x+3)$

$= (4x-2)(2x+3) = 8x^2-6-4x+12x$

$= 8x^2+8x-6$ # True.

Factoring Thinomial squares and Differences of squares (5.5)

a- Recognizing Thinomial squares: → (ص رقم 353)

$(A+B)^2 = A^2 + 2AB + B^2$

$(A-B)^2 = A^2 - 2AB + B^2$

لغك هذا القوس التربيعي ← الأول تربيع + الثاني تربيع + الأول في الثاني X 2

بإشارة المتصرف .

EX. No. 1 : → Determine whether $X^2 + 6X + 9$ is a thinomial square :

$X^2 + 6X + 9$ ← عددا بحاصل ضربهم 9 ومجموعهم 6

$= (X + 3)(X + 3)$

$= (X+3)^2$ #.

EX. No. 2 → $16X^2 + 49 - 56X$

$= 16X^2 - 56X + 49$

عددا بحاصل ضربهم 49 ، 7 , 7 ، عددا بحاصل ضربهم 16 ← 4 , 4

$\therefore = (4X - 7)(4X - 7)$

الإشارة سالبة لأنه إشارة الحد الأوسط سالبة .

$= (4X - 7)^2$ #.

EX. No. 3 → Factor $X^2 + 6X + 9$

$= (X^2) + (2 \cdot 3X) + (3^2) = A^2 + 2AB + B^2$

$= (X + 3)^2 = (A + B)^2$ #.

EX. No. 4 → Factor $t^4 + 20t^2 + 100$

$= t^4 + 20t^2 + 100 = (t^2)^2 + (2 \cdot 10)t^2 + 10^2$

$= (t^2 + 10)^2$ #.

EX. No. 5 → $x^2 - 9$ لو كانت الإشارة سالبة.

$= A^2 - B^2 = (A - B)(A + B)$ وكل بهم حد أوسط.

$= (x + 3)(x - 3)$ #.

EX. No. 6 → $25 - t^3$ ليس تربيعاً.

t^3 is not a squre

The expression is not adiffrance of square

EX. No. 7. Factor $x^2 - 4$

$= x^2 - 4 = x^2 - 2^2 = (A^2 - B^2)$

$= (x + 2)(x - 2) = (A - B)(A + B)$

EX. No. 8 → Factor $\frac{1}{16}x^8 - 81$

$= (\frac{1}{4}x^4 + 9)(\frac{1}{4}x^4 - 9)$

$= (\frac{1}{4}x^4 + 9)(\frac{1}{2}x^2 - 3)(\frac{1}{2}x^2 + 3)$

في هذه المسألة تم فك القوس مرتين، نلاحظ فك القوس الى إشارتك سالبة

EX. No. 9. → Factor $y^4 - 16x^{12}$

$= (y^2 - 4x^6)(y^2 + 4x^6)$

$= (y^2 + 4x^6)(y - 2x^3)(y + 2x^3)$ #.

Factoring sums or Differences of cubes (5-6).

a- sums or Differences of cubs (من رقم 363)

القوس التكعيبي يتم فكه للقوسين أحدهما مثل حكل المعادلة والثاني

الأول تربيع + الثاني تربيع + الأول في الثاني عكس إشارة الوسط.

$$A^3 + B^3 = (A+B)(A^2 - AB + B^2)$$

$$A^3 - B^2 = (A-B)(A^2 + AB + B^2)$$

لاحظ أن الحد الأوسط عكس إشارة وسط المعادلة.

EX. No. 1 → Factor $125x^3 + y^3$

$$= (5x + y)(25x^2 - 5xy + y^2)$$

$$= (A + B)(A^2 - AB + B^2)$$

القوس الأول هو الجذر التكعيبي للمعادلة الأصلية.

القوس الثاني الأول² + الثاني² + الأول × الثاني عكس إشارة الحد الأوسط

EX. No. 2 → Factor → $64a^6 - 72b^6$

$$= (8a^3)^2 - (27b^3)^2$$ قوس تربيعي

$$= (8a^3 - 27b^3)(8a^3 + 27b^3)$$ قوس تكعيبي

$$= (2a - 3b)(4a^2 + 6ab + 9b^2)(2a + 3b)(4a^2 - 6ab + 9b^2)$$

EX. No. 3 Factor $x^3 - 27$

$$= (x - 3)(x^2 + 3x + 9)$$

ملاحظة القوس الثاني لا يمكن فكه أكثر من ذلك ، إلا عند ضرب القوسين فتعود المعادلة كما كانت ، مع أن هذا القوس تربيعي #.

Factoring : A General strategy (5.7) (ص رقم 368)

يوجد بهذا الباب أنواع مسائل كثيرة وعلينا التفكير في نوع المسألة حتى نتمكن من حلها.

EX. No. 1 → Factor $5x^4 - 80$

هذه المعادلة يمكنه أنه تحل فروعيه مربعيه لكنه بعد أخذ عامل مشترك

$$\therefore 5x^4 - 80 = 5(x^4 - 16)$$
$$= 5(x^2 - 4)(x^2 + 4)$$
$$= 5(x^2 + 4)(x - 2)(x + 2) \quad \#$$

check → $5(x^2+4)(x-2)(x+2)$

$$= (5x^2 + 20)(x^2 - 4)$$
$$= 5x^4 - 80 + 20x^2 - 20x^2$$
$$= 5x^4 - 80 \quad \text{True} \#.$$

EX. No. 2 → Factor $2x^3 + 10x^2 + x + 5$

هذه المعادلة مكونة من أربعة أجزاء إذن يمكنه تقسيمها في قوسين

$$2x^3 + 10x^2 + x + 5$$
$$= (2x^3 + 10x^2) + (x + 5)$$ ثم نأخذ عامل مشترك
$$= 2x^2(x + 5) + (x + 5)$$ نوجد عامل مشترك جديد
$$= (x + 5)(2x^2 + 1)$$

check → $(x+5)(2x^2+1)$

$$= 2x^3 + 5 + 10x^2 + x \quad \text{True} \#.$$

EX. No. 3 → Factor → $x^5 - 2x^4 - 35x^3$

$$= x^3(x^2 - 2x - 35) = x^3(x + 5)(x - 7)$$

check → $x^3(x+5)(x-7) = (x^4 + 5x^3)(x-7)$

$$= x^5 - 7x^4 + 5x^4 - 35x^3$$
$$= x^5 - 2x^4 - 35x^3 \quad \text{True} \#.$$

Ex. No. 3 → Factor $40t^3 - 5b^3$

$= 5(8t^3 - b^3)$ بأخذ عامل مشترك

$= 5(2t - b)(4t^2 + 2tb + b^2)$

Ex. No. 4 → Factor $p^2q^2 + 7pq + 12$

$= (pq + 4)(pq + 3)$ بالتحليل

Check $= p^2q^2 + 4pq + 3pq + 12$ بالجمع للتشابه

$= p^2q^2 + 7pq + 12.$

Ex. No. 5 → $x^4 - 10x^2 + 25$

$= (x^2 - 5)(x^2 - 5)$

$= (x^2 - 5)^2$

Check $= x^4 + 25 - 2 \cdot 5x^2$

$= x^4 - 10x^2 + 25$ #.

Ex. No. 6 → Factor $px + py + qx + qy$ بأخذ عامل مشترك

$= (px + py) + (qx + qy) = p(x + y) + q(x + y)$

$= (x + y)(p + q)$

Check $(x + y)(p + q) = (xp + xq + py + qy)$ True

Ex. No. 7 → Factor $25x^2 + 20xy + 4y^2$

$= (5x)^2 + 2 \cdot 5x \cdot 2y + (2y)^2$

$= (5x + 2y)^2$

Check $= 25x^2 + 4y^2 + 2 \cdot 5 \cdot 2xy$

$= 25x^2 + 20xy + 4y^2$ True.

Solving Quadratic Equations by Factoring (5.8)

a - The principle of zero prouducts.. (ص رقم 377)

$ax^2 + bx + c = 0 \quad , a \neq 0$

EX. No. 1 → solve $(x+3)(x-2) = 0$

$\therefore x + 3 = 0 \quad$ or $\quad x - 2 = 0$

$\therefore x = -3 \quad$ or $\quad x = 2$

$x = 2 \quad$ or $\quad x = -3$ للتأكد من صحة هذه الحلول

check $(x+3)(x-2) = 0$ For $x = -3$

$(-3+3)(-3-2)$

$= 0(-5) = 0$ True #

$(2+3)(2-2) = 0$ For $x = 2$

$5(0) = 0$ True #.

$\therefore x = -3 \quad$ or $\quad x = 2$

---

b - Using Factoring to solve Equations:..

EX-No-2 → solve $x^2 - 8x + 16 = 0$

$(x - 4)(x - 4) = 0$

$x = 4 \quad$ or $\quad x = 4$

solving each equation $x = 4$.

EX-No-3 → solve $x^2 = -5x$

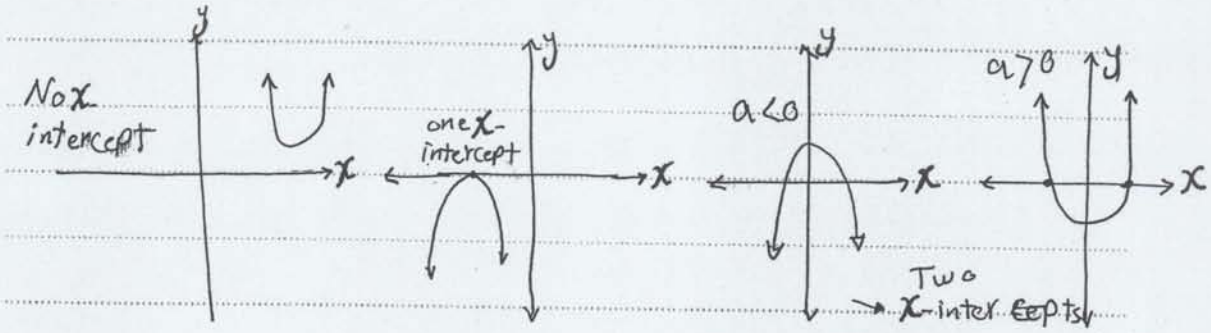
$\therefore x^2 + 5x = 0$

$x(x+5) = 0$

$x = 0 \quad$ or $\quad x + 5 = 0$

$\therefore x = 0$

$x = -5$



الدالة التربيعية هذا هو شكلها عند الرسم.

EX. No. 4 → Find the x-intercepts of the graph of

$y = x^2 - 4x - 5$ ، $y = 0$

$0 = x^2 - 4x - 5$

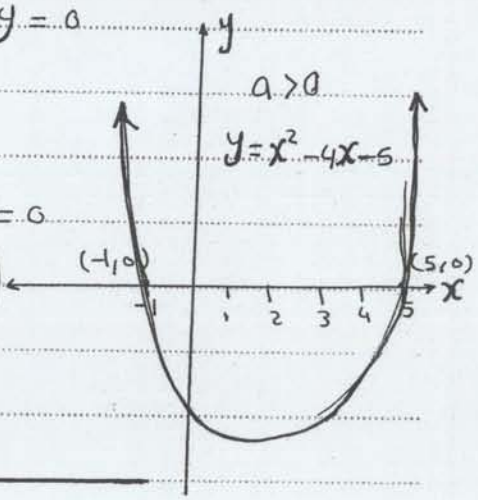
$0 = (x - 5)(x + 1)$

$x - 5 = 0$ or $x + 1 = 0$

$(x = 5)$ or $(x = -1)$

نعوض في بالقيم ونرسم الدالة.

$(-1, 0)$ ، $(5, 0)$



EX. No. 5 → Solve $-5x^2 + 2x + 3 = 0$

$-1(-5x^2 + 2x + 3) = 0 \cdot -1$ بالضرب في -1

$5x^2 - 2x - 3 = 0$

$(x - 1)(5x + 3) = 0$

$x = 1$ or $5x = -3$

$x = 1$ or $x = \frac{-3}{5}$

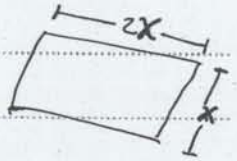
Applications of Quadratic Equations. (5.9)

a- Applied problems, Quadratic Equations and Factoring → (ص رقم 386)

EX. No.1 → Lisa buys a kitchen island with a butcher-block top as part of a remodeling project. The top of the island is a rectangle that is twice as long as it is wide and that has an area of 800 in^2. what are the dimensions of the top of the island?

1- Familiarize : we Let x = wide of the top

Length = $2x$, $A = 800$



2- Translate → Area of rectangle

$A = L \cdot w$

$A = 2x \cdot x$

$800 = 2x^2$

$2x^2 - 800 = 0$

$2(x^2 - 400) = 0$

$2(x - 20)(x + 20) = 0$

$x = 20$ or $x = -20$

$x = -20$ False لأن الأطوال لا تكون سالبة

$\therefore x = 20$ True #

x = wide = 20 in.

$2x$ = Length = $2 \cdot 20 = 40$ in.

Ex. No. 2 → The hight of atriangular sail on aracing asailboat is 9 ft more than the base. The area of the triange is 110 ft^2. Find the height and the base of the sail.

1- Familiarize: $A = 110\ ft^2$, hight = $b+9$

base = b

2- Translate → A of atriangule $= \frac{1}{2} b \cdot h$

$A = \frac{1}{2} b \cdot (b+9)$

$110 = \frac{1}{2}(b^2+9b)$ بالضرب 2X

$2 \cdot 110 = 2 \cdot \frac{1}{2}(b^2+9b)$

$220 = (b^2+9b)$

$b^2 + 9b - 220 = 0$

$(b - 11)(b + 20) = 0$

$b - 11 = 0$ or $b + 20 = 0$

$b = 11$ or $b = -20$

check → $\frac{1}{2} \cdot 11(11+9)$ ↓

$= \frac{1}{2} \cdot 11 \cdot 20 = 11 \cdot 10 = 110$ True

→ $\frac{1}{2} \cdot -20(-20+9)$

$-10(-11) = 110$ True

state: ∴ $b = 11$ ft, $h = b+9 = 20$ ft.

Five steps For problem solving

1- Familiarize, 2- Translate

3- Solve, 4- Check, 5- state.

Exam. No. 5. اختبار تجريبي على الفصل الخامس

1- Find the G C F :

* $28x^3$, * $48x^7$

2- Factor Completely :

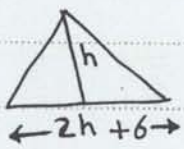
* $x^2 - 7x + 10$.
* $3w^2 - 75$.
* $6t^3 + 9t^2 - 15t$.

3- Solve :

* $x^2 - 3x = 0$
* $x(x - 3) = 28$
* $x^2 - x - 20 = 0$

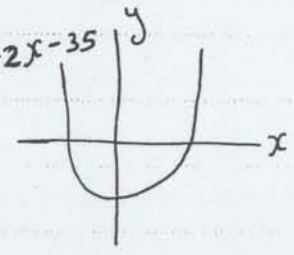
4- solve :

* The base of a triangle is 6cm greater than twice the hieght. The area is $28\ cm^2$. Find the hight and the base.

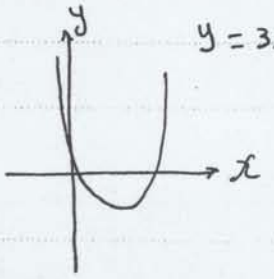


5- Find the x - intercepts of the graph of each equation.

$y = x^2 - 2x - 35$



$y = 3x^2 - 5x + 2$



chapter (6)

Rational Expressions and Equations. حل المقادير الجبرية والمعادلات

Multiplying and simplifying Rational Expressions (6-1).

a- Rational Expressions and Replacements :→ (ص رقم 410)

Ex. No. 1 → $\frac{x+4}{x^2-3x-10}$

نحلل المقام لقوسين

$= \frac{x+4}{(x+2)(x-5)}$

ثم نوجد قيم x

$(x+2)(x-5)=0$

$x+2=0$ or $x-5=0$

$x=-2$ or $x=5$

---

b- Multiplying by 1 :→ (ص رقم 411)

$\frac{A}{B} \cdot \frac{C}{D} = \frac{Ac}{BD}$

Ex. No. 2 → Multiply $\frac{3x+2}{x+1} \cdot 1$ using $\frac{2x}{2x}$

* $\frac{3x+2}{x+1} \cdot 1 = \frac{3x+2}{x+1} \cdot \frac{2x}{2x}$

$= \frac{(3x+2)2x}{(x+1)2x}$

---

* $\frac{x+2}{x-7} \cdot \frac{x+3}{x+3} = \frac{(x+2)(x+3)}{(x-7)(x+3)}$. $(x+3)/(x+3)=1$

$= \frac{(x+2)}{(x-7)}$.

أي حاجة مقسومة على نفسها خارج القسمة = 1 .

$\frac{x}{x}=1$, $\frac{\text{أي مقدار}}{\text{نفس}}=1$

C- simplifying Rational Expressions :→ تبسيط المقادير الجبرية

Ex. No. 3 → simplify $\frac{8x^2}{24x}$ (412 صارم)

$\frac{8x^2}{24x} = \frac{8 \cdot x \cdot x}{8 \cdot 3 \cdot x} = \frac{8x}{8x} \cdot \frac{x}{3}$ نأخذ عامل مشترك طالما وقعاً بحيث يكون خارج قسمتهم = 1

$= \frac{1}{1} \cdot \frac{x}{3} = \frac{x}{3}$.

Ex. No. 4 → simplify $\frac{5a + 15}{10}$

* $\frac{5a + 15}{10} = \frac{5(a + 3)}{5 \cdot 2} = \frac{1 \cdot (a+3)}{1 \cdot 2} = \frac{(a+3)}{2}$

* $\frac{x^2 + 3x + 2}{(x^2 - 1)} = \frac{(x+1)(x + 2)}{(x+1)(x - 1)} = \frac{(x+2)}{(x-1)}$

D- Multiplying and simplifing... →

Ex. No- 5 → $\frac{x^2 + 6x + 9}{x^2 - 4} \cdot \frac{x-2}{x+3}$

$= \frac{(x + 3)(x + 3)}{(x - 2)(x + 2)} \cdot \frac{(x-2)}{(x+3)}$ نحذف المتشابه للتبسيط.

$= \frac{x+3}{x+2}$ # لأن $\frac{(x+3)(x-2)}{(x-2)(x+3)} = 1$

---

* $\frac{5a^3}{4} \cdot \frac{2}{5a} = \frac{5a \cdot a^2}{2} \cdot \frac{2}{5a} = \frac{a^2}{2}$.

* $\frac{8x^3}{32x} = \frac{8 \cdot x \cdot x \cdot x}{8 \cdot 4 \cdot x} = \frac{8x \cdot x^2}{8x \cdot 4} = \frac{x^2}{4}$.

* $\frac{5a - 40}{5} = \frac{5(a - 8)}{5} = a - 8$.

Division and Reciprocals (6.2) القسمة التبادلية

المقلوب في المعكوس

a- Finding Reciprocals → (ص 420 رقم)

Ex. No. 1 → reciprocal of $\frac{2}{5}$ is $\frac{5}{2}$

This because $\frac{2}{5} \cdot \frac{5}{2} = \frac{10}{10} = 1$

∴ العدد مضروب في معكوسه الضرب = 1

---

* $\frac{2x^2-3}{x+4}$ this reciprocal is $\frac{x+4}{2x^2-3}$

---

b- Division →

$\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \cdot \frac{D}{C} = \frac{AD}{BC}$

علشان نقلب القسمة ضرب بنقلب الكسر الثاني والعكس صحيح.

Ex. No. 2 → $\frac{2}{x} \div \frac{3}{x}$

$= \frac{2}{x} \cdot \frac{x}{3} = \frac{2}{\cancel{x}} \cdot \frac{\cancel{x}}{3} = \frac{2}{3}$ نقلب الكسر والقسمة ضرب

because $\frac{x}{x} = 1$

---

* $\frac{x+1}{x+2} \div \frac{x-1}{x+3} = \frac{x+1}{x+2} \cdot \frac{x+3}{x-1}$

$= \frac{(x+1)(x+3)}{(x+2)(x-1)}$. لم تختصر أكثر من هذا

---

* $\frac{4}{x^2-7x} \div \frac{28x}{x^2-49} = \frac{4}{x(x-7)} \cdot \frac{x^2-49}{28x}$

$= \frac{4(x-7)(x+7)}{x(x-7) \cdot 4 \cdot 7 \cdot x} = \frac{(x+7)}{7x^2}$ #. ∴

Least Common Multiples and Denominators - (6-3)

a- Least Common Multiples. (ص رقم 425)

* توحيد المقامات يكون بشرط أن تقبل كل المقامات الموجودة القسمة على هذا الرقم وتعطي نتائج ليس فيها كسور.

EX. No. 1 → Find the LCM of 24, 36

$24 = 2 \cdot 2 \cdot 2 \cdot 3$

$36 = 2 \cdot 2 \cdot 3 \cdot 3$ ∴ L.C.M = $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$

نأخذ أكبر الأرقام التي تقبل القسمة على الرقمين LCM = 72

Check $72 \div 24 = 3$

$72 \div 36 = 2$

---

b- Adding using the LCD

EX. No. 2 → Add $\frac{5}{12} + \frac{11}{18}$

$12 = 2 \cdot 2 \cdot 3$

$18 = 2 \cdot 3 \cdot 3$

نختار رقم يقبل القسمة على الرقمين ويعطي أرقام صحيحة.

∴ LCD = $2 \cdot 2 \cdot 3 \cdot 3 = 36$

$$\frac{5}{12} + \frac{11}{18} = \frac{15+22}{4,36} = \frac{37}{36} \#$$

EX. No. 3 → 12, 30

$12 = 2 \cdot 2 \cdot 3$

$30 = 2 \cdot 3 \cdot 5$ } L.C.M = $2 \cdot 2 \cdot 3 \cdot 5 = 60$

c-LCMs of Algebraic Expressions :→ (4 26 ص، م)

Ex. No. 4 → Find the LCM of $12x$, $16y$, $8xyz$

$12x = 2 \cdot 2 \cdot 3 \cdot x$

$16y = 2 \cdot 2 \cdot 2 \cdot 2 \cdot y$

$8xyz = 2 \cdot 2 \cdot 2 \cdot x \cdot y \cdot z$

$\therefore LCM = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot y \cdot z$

$= 48xyz$

Ex. No. 5 → $x^2 + 5x - 6$ and $x^2 - 1$

* $x^2 + 5x - 6 = (x + 6)(x - 1)$
* $x^2 - 1 = (x + 1)(x - 1)$

$\therefore LCM = (x-1)(x+6)(x+1)$

الملاحظة: ان الشئ المكرر نأخذ منه واحد فقط.

Ex. No. 6 → $x^2 - 25$ and $2x - 10$

$x^2 - 25 = (x - 5)(x + 5)$

$2x - 10 = 2(x - 5)$

$\therefore LCM = 2(x-5)(x+5)$

Ex. No. 7 → $x^2 - 4y^2$, $x^2 - 4xy + 4y^2$ and $x - 2y$

$x^2 - 4y^2 = (x + 2y)(x - 2y)$

$x^2 - 4xy + 4y^2 = (x - 2y)(x - 2y)$

$x - 2y = (x - 2y)$

$\therefore LCM = (x - 2y)^2 (x + 2y)$.

Adding Rational Expressions (6.4)

جمع المقادير الجبرية

a- Adding Raional Expressions. (ص رقم 429)

Ex. No. 1 Add

* $\frac{x}{x+1} + \frac{2}{x+1} = \frac{x+2}{x+1}$ نجمع جمع عادى للمتشابه فقط

* $\frac{2x^2+3x-7}{2x+1} + \frac{x^2+x-8}{2x+1} = \frac{3x^2+4x-15}{(2x+1)}$

$= \frac{(x+3)(3x-5)}{(2x+1)}$

* $\frac{x-5}{x^2-9} + \frac{2}{x^2-9} = \frac{x-3}{x^2-9} = \frac{(x-3)}{(x-3)(x+3)}$

$= \frac{1}{x+3}$

* $\frac{5x^2}{8} + \frac{7x}{12}$

$8 = 2 \cdot 2 \cdot 2$
$12 = 2 \cdot 2 \cdot 3$ } $LCD = 2 \cdot 2 \cdot 2 \cdot 3 = 24$

$= \frac{5x^2}{2 \cdot 2 \cdot 2} \cdot \frac{3}{3} + \frac{7x}{2 \cdot 2 \cdot 3} \cdot \frac{2}{2}$

$= \frac{15x^2}{24} + \frac{14x}{24} = \frac{15x^2+14x}{24}$

$= \frac{x(15x+14)}{24}$

جمع المقادير الجبرية يكون بجمع x^2 مع مثيلاتها ، x مع مثيلاتها ، والأرقام معاً

Ex. No. 2 → Add $\frac{x}{x^2+11x+30} + \frac{-5}{x^2+9x+20}$

$= \frac{x}{(x+5)(x+6)} + \frac{-5}{(x+5)(x+4)}$ ∴ LCD = $(x+5)(x+4)(x+6)$

$= \frac{x(x+4) - 5(x+6)}{(x+5)(x+6)(x+4)}$

$= \frac{x^2+4x-5x-30}{(x+5)(x+6)(x+4)} = \frac{x^2-x-30}{(x+5)(x+4)(x+6)}$

$= \frac{(x-6)(x+5)}{(x+5)(x+4)(x+6)} = \frac{(x-6)}{(x+4)(x+6)}$.

---

* $\frac{x+4}{x-2} + \frac{x-7}{x+5}$ LCD = $(x-2)(x+5)$

$= \frac{(x+4)(x+5) + (x-7)(x-2)}{(x-2)(x+5)}$

$= \frac{x^2+20+9x+x^2+14-9x}{(x-2)(x+5)}$

$= \frac{2x^2+34}{(x-2)(x+5)} = \frac{2(x^2+17)}{(x-2)(x+5)}$.

---

* $\frac{2a}{a^2-1} + \frac{1}{a^2+a}$.

$\frac{2a}{(a-1)(a+1)} + \frac{1}{a(a+1)}$ LCD = $a(a+1)(a-1)$

$= \frac{2a^2+a(a-1)}{a(a-1)(a+1)} = \frac{(2a^2+a-1)}{a(a-1)(a+1)} =$

$= \frac{(2a-1)(a+1)}{a(a-1)(a+1)} = \frac{2a-1}{a(a-1)}$ #.

Subtracting Rational Expressions (6-5).

الطرح.

a- subtracting Rational Expressions. (صورة 437)

Ex. No-1 → subtract $\frac{8}{x} - \frac{3}{x}$

$= \frac{8-3}{x} = \frac{5}{x}$.

* $\frac{3x}{x+2} - \frac{x-2}{x+2} = \frac{3x+x-2}{x+2} = \frac{2x-2}{x+2}$

$= \frac{2(x-1)}{x+2}$.

* $\frac{x}{x^2+5x+6} - \frac{2}{x^2+3x+2}$

$= \frac{x}{(x+2)(x+3)} - \frac{2}{(x+1)(x+2)}$

$LCD = (x+2)(x+3)(x+1)$

$= \frac{x(x+1)-2(x+3)}{(x+2)(x+3)(x+1)} = \frac{x^2+x-2x-6}{(x+2)(x+3)(x+1)}$

$= \frac{x^2-x-6}{(x+2)(x+3)(x+1)} = \frac{(x+2)(x-3)}{(x+2)(x+3)(x+1)}$

$= \frac{(x-3)}{(x+3)(x+1)}$.

نطرح المقادير الجبرية المتشابهة ، ثم نبسط الحل بحذف المتشابه والتحليل

b- Combined Additions and subtractions. (مهارة 440)

Ex. No. 2 → perform the indicated operations and simplify:

$$\frac{x+9}{x^2-4} + \frac{5-x}{4-x^2} - \frac{2+x}{x^2-4} =$$

$$\frac{x+9}{(x-2)(x+2)} + \frac{5-x}{(2-x)(2+x)} - \frac{2+x}{(x-2)(x+2)}$$

$$LCD = (x-2)(x+2)$$

$$= \frac{x+9-5+x-2-x}{(x-2)(x+2)} = \frac{(x+2)}{(x-2)(x+2)}$$

$$= \frac{1}{(x-2)} .$$

Ex. No. 3 → $\frac{1}{x} - \frac{1}{x^2} + \frac{2}{x+1}$

$$LCD = x^2(x+1)$$

$$\frac{1}{x} - \frac{1}{x^2} + \frac{2}{x+1} = \frac{x(x+1) - (x+1) + 2x^2}{x^2(x+1)}$$

$$= \frac{x^2+x-x-1+2x^2}{x^2(x+1)} = \frac{3x^2-1}{x^2(x+1)} .$$

أول شيء نوحد المقامات ثم نجمع ونطرح المقادير الجبرية المتشابهة ثم نحلل البسط ونحذف المتشابه بسطاً ومقاماً ثم نبسط المعادلة.

Complex Rational Expressions: (6-6)

a- simplifying Complex Rational Expressions :→ (447 من رقم)

Ex-No-1 → simplify $\dfrac{\frac{1}{2}+\frac{3}{4}}{\frac{5}{6}-\frac{3}{8}}$

في هذه الآلة يتم تحديد LCM لكل المقامات (مقام البسط ومقام المقام).

$2 = 2$, $4 = 2 \cdot 2$, $6 = 2 \cdot 3$, $8 = 2 \cdot 2 \cdot 2$

$\therefore LCM = 2 \cdot 2 \cdot 2 \cdot 3 = 24$

بالضرب × 24 بسطاً ومقاماً

$$\therefore \frac{\frac{1}{2}+\frac{3}{4}}{\frac{5}{6}-\frac{3}{8}} = \frac{\frac{1}{2}\cdot 24 + \frac{3}{4}\cdot 24}{\frac{5}{6}\cdot 24 - \frac{3}{8}\cdot 24} = \frac{12+18}{15-9}$$

$$= \frac{30}{11}$$

---

* $\dfrac{1-\frac{1}{x}}{1-\frac{1}{x^2}}$, $LCM = x^2$

بالضرب × $\frac{x^2}{x^2}$

$$\frac{1\cdot x^2 - \frac{1}{x}\cdot x^2}{1\cdot x^2 - \frac{1}{x^2}\cdot x^2} = \frac{x^2 - x}{x^2 - 1} = \frac{x(x-1)}{(x-1)(x+1)}$$

$$= \frac{x}{(x+1)}$$

---

* $\dfrac{\frac{3}{x}+\frac{1}{2x}}{\frac{1}{3x}-\frac{3}{4x}}$, $LCM = 12x$

بالضرب × $\frac{(12x)}{(12x)}$

$$= \frac{\frac{3}{x}\cdot 12x + \frac{1}{2x}\cdot 12x}{\frac{1}{3x}\cdot 12x - \frac{3}{4x}\cdot 12x}$$

$$= \frac{36+6}{4-9} = -\frac{42}{5} \#$$

Solving Rational Equations: (6.7)

a- Rational Equations :→ (ص 453 رقم)

EX. No. 1 → Solve $\frac{2}{3} + \frac{5}{6} = \frac{X}{9}$

$3 = 3$, $6 = 2 \cdot 3$, $9 = 3 \cdot 3$ نوجد LCM

$LCM = 2 \cdot 3 \cdot 3 = 18$

$\frac{2}{3} \cdot 18 + \frac{5}{6} \cdot 18 = \frac{X}{9} \cdot 18$

$\frac{36}{3} + \frac{90}{6} = \frac{18X}{9}$ نبسط المعادلة

$12 + 15 = 2X$ نجمع

$\frac{27}{2} = \frac{2X}{2}$ نقسم بالقسمة ÷ 2

$\therefore X = \frac{27}{2}$

The solution is $\frac{27}{2}$ #.

EX. No. 2 → Solve $\frac{2}{3X} + \frac{1}{X} = \frac{10}{1}$

$LCM = 3X$ نوجد المقام

$\therefore \frac{2}{3X} \cdot 3X + \frac{1}{X} \cdot 3X = 10 \cdot 3X$

$2 + 3 = 30X$ نبسط المعادلة

$\frac{5}{30} = \frac{30X}{30}$

$\therefore X = \frac{5}{30} = \frac{1}{6}$ نوجد قيمة X

The solution is $\frac{1}{6}$

Ex. No. 3 → Solve $x + \frac{6}{x} = -5$

LCM = x بالضرب في x

$x \cdot x + \frac{6}{x} \cdot x = -5 \cdot x$

$x^2 + 6 = -5x$

$x^2 + 6 + 5x = -5x + 5x$ بإضافة $5x$ للطرفين

$x^2 + 5x + 6 = 0$

$(x + 2)(x + 3) = 0$

$x + 2 = 0$ or $x + 3 = 0$

$x = -2$ or $x = -3$

Check $-2 + \frac{6}{-2} = -2 - 3 = -5$ true #.

$-3 + \frac{6}{-3} = -3 - 2 = -5$ true

The solution is -2 and -3.

---

* Solve $\frac{3}{x-5} + \frac{1}{x+5} = \frac{2}{x^2 - 25}$

LCM = $(x-5)(x+5)$

$\frac{3(x+5) + (x-5)}{(x-5)(x+5)} = \frac{2}{(x-5)(x+5)}$

المقام متساوي الطرفين كأنها قسم.

$\therefore 3(x+5) + (x-5) = 2$

$3x + 15 + x - 5 = 2$

$4x + 10 = 2$

$\frac{4x}{4} = \frac{-8}{4}$ $\therefore x = -2$

The solution is -2.

Applications using Rational Equations and proportions (6-8)
التطبيقات .

a- Solving Applied problems :→ (مسائل كلامية 462)

Ex. No. 1 → Mileage - A 2009 chevrolet cobalt SS can travel 176 mi in city driving on 8 gal of gas. Find the amount of gas required for 242 mi of city driving.

1- Familiarize : 176 mi on 8 gal of gas.

$176 \longrightarrow 8$

$242 \longrightarrow x$

2- Translate : $\frac{\text{Miles}}{\text{gas}} = \frac{176}{8} = \frac{242}{x} \leftarrow \text{gas}$

3- Solve :

$\frac{176}{8} = \frac{242}{x}$ بالضرب $8x$.

$\frac{176}{8} \cdot 8x = \frac{242}{x} \cdot 8x$

$176x = 1936 \quad \therefore x = \frac{1936}{176}$

$\therefore x = 11$

The solution is 11.

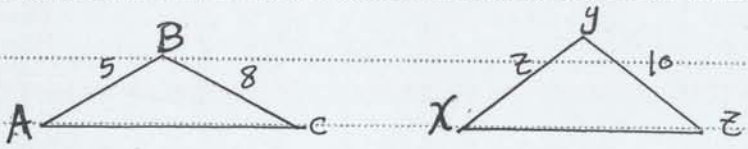
4- Check → $242 \div 11 = 22$

$176 \div 8 = 22$ Thue

5- state → chevrolet Cobalt will require 11 gal of Gas for 242 mi of city driving.

Ex. No. 2 → similar Triangles. Triangles ABC and XYZ below are similar triangles solve for if $a = 8$, $c = 5$ and $x = 10$

1- draw



we have

$$\frac{z}{5} = \frac{10}{8}$$

LCM = 40

$$\frac{z}{5} \cdot 40 = \frac{10}{8} \cdot 40$$

$$8z = \frac{400}{8}$$

$$8z = 50$$

$$z = \frac{50}{8} = \frac{25}{4} = 6.25$$

Variation and Applications ... (6.9)

a- Equations of Direct Variation: → (478 ص, up)

EX. No. 1 :→ Find the variation constant and an equation of variation in which y varies directly as x and $y = 32$ when $x = 2$.

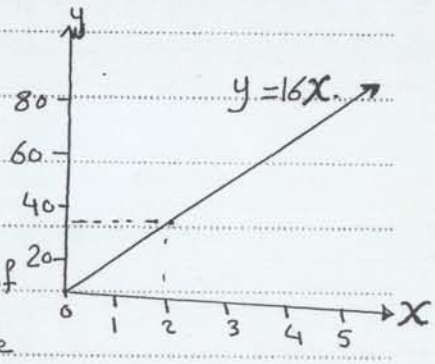
we know that (2, 32) is a solution of $y = kx$. Thus,

$y = kx$

$32 = k \cdot 2$

$\frac{32}{2} = k$

$\therefore k = 16$



The variation constant, 16, is the rate of change of y with respect to x. The equation of variation is $y = 16x$.

---

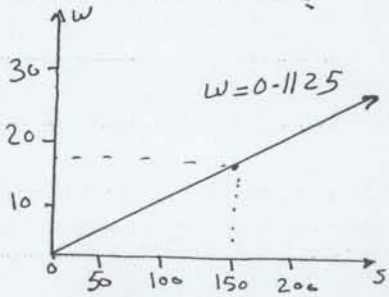
b- Applications of Direct variation:

EX. No. 2 → water from melting snow. The number of centimeters W of water produced from melting snow varies directly as S, the number of centimeters of snow. Meteorologists have found that, under certain conditions, 150cm of snow will melt to 16.8 cm of water. To how many centimeters of water will 200 cm of snow melt?

$W = kS$

$16.8 = k \cdot 150$

$k = \frac{16.8}{150}$ $\therefore k = 0.112$.



* when $S = 200$

$w = 200(0.112) = 22.4$, Thus 200 cm of snow will melt to 22.4cm of water

c- Equations of inverse variation. (ص 480 وتمرين)

Ex. No. 3 → Find the variation constant and an equation of variation in which y varies inversely as x, and $y = 32$ when $x = 0.2$.

we know that $(0.2, 32)$ is a solution of $y = k/x$.

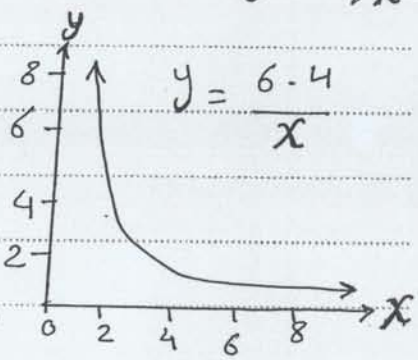
$$y = \frac{k}{x}$$

$$32 = \frac{k}{0.2}$$

$$(0.2)(32) = k$$

$$\therefore k = 6.4$$

$$y = \frac{6.4}{x}$$



---

d- Applications of inverse variation (ص 481 وتمرين)

Ex. No. 4 → Musical pitch. The pitch p of a musical tone varies inversely as its wavelength w. one tone has a pitch of 550 vibrations per second and wavelength of 1.92 ft. Find the pitch of another tone that has a wave-length of 3.2 ft.

we first find the variation constant using the data given and then find an equation of variation.

$$p = \frac{k}{w}$$

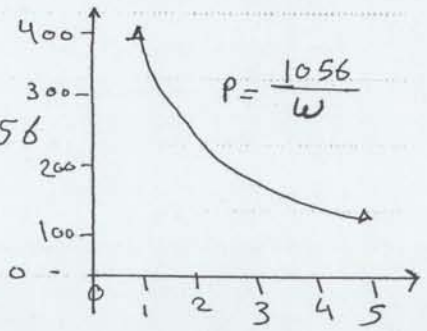
$$550 = \frac{k}{1.92}$$

$$\therefore k = 1056$$

when $w = 3.2$

$$p = \frac{1056}{3.2}$$

$$\therefore p = 330$$



e- other kinds of variation :→ (482 ص, ب.)

Ex. No. 5 → Find an equation of variation in which y varies jointly as x and z and $y = 42$ when $x = 2$ and $z = 3$.

$y = k x z$

$42 = k \cdot 3 \cdot 2$

$42 = 6k$ $\quad \therefore k = \frac{42}{6} = 7$

Thus $y = 7xz$.

Different types of variation can be combined. For example, the equation $y = k \frac{xz^2}{w}$.

---

f- other Applications of variation :

Ex. No. 6 → volume of a Tree, The volume of wood V in a tree varies jointly as the height h and the square of the girth g (girth is distance around). If the volume of a redwood tree is 216 m^3 when the hight is 30 m and the gnith is 1.5 m. what is the height of a tree whose volume is 960 m^3 and girth is 2 m ?

$V = k h g^2$

$216 = k \cdot 30 \cdot 1.5^2$

$k = 3.2$

when $V = 960$

$V = k h g^2$

$960 = 3.2 \cdot h \cdot 2^2$

$h = 75$. The height of the tree is 75 m.

Exam. No. 6. اختبار تجريبي على الفصل السادس

1- Find all numbers for which the raional expression is not defined.

* $\frac{5}{x+8}$, * $\frac{x-7}{x^2-49}$

2- simplify :- $\frac{6x^2+17x+7}{2x^2+7x+3}$

3- Divide and simplify $\frac{25x^2-1}{9x^2-6x} \div \frac{5x^2+9x-2}{3x^2+x-2}$

4- Find the LCM :-

y^2-9 , $y^2+10y+21$, $y^2+4y-21$.

5- Add or subtract . simplify if possible :-

* $\frac{16+x}{x^3} + \frac{7-4x}{x^3}$

* $\frac{x-4}{x-3} - \frac{x-1}{3-x}$

6- This pair of triangles is similar . Find the missing length x .

7- Train travel. The distance d traveled by a train varies directly as the time t that it travels. The train travels 60 km in $\frac{1}{2}$ hr . How far will it travel in 2 hr ?

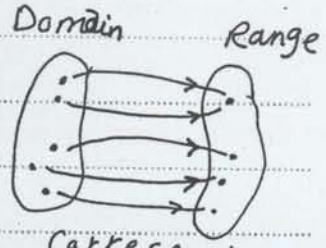
Chapter (7)

Graphs, Functions, and Applications:.

رسم الدوال وتطبيقاتها.

Functions and Graphs. (7.1).

(500 تقريباً)



a - Identifying Functions.

نعوض عن x بقيم مختلفة ونوجد قيم الدالة
ثم نرسم الدالة.

---

b - Finding Function values.

Ex. No. 1 → Find the indicated function value.

1) $f(5)$, for $f(x) = 3x + 2$

$f(5) = 3(5) + 2 = 17$.

---

2) $g(-2)$. for $g(x) = 7$

$g(x) = 7$ Constant function.

---

3) $F(a+1)$, for $F(x) = 5x - 8$

$F(a+1) = 5(a+1) - 8$

$= 5a + 5 - 8 = 5a - 3$.

---

4) $f(a+h)$, for $f(x) = -2x + 1$

$f(a+h) = -2(a+h) + 1$

$= -2a - 2h + 1$.

C- Graphs of Functions. (ص رقم 504)

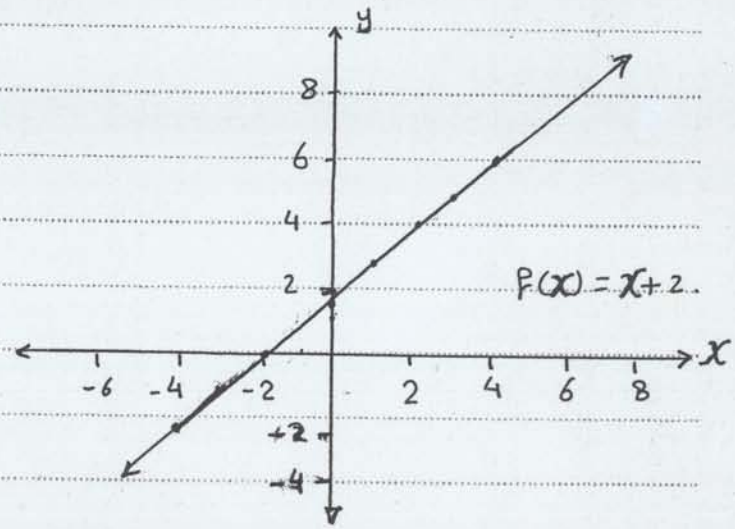
EX. No. 2 → Graph $f(x) = x + 2$

$f(x) = -4 + 2 = -2$

$f(x) = -3 + 2 = -1$

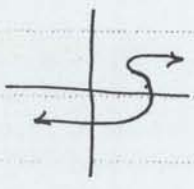
$f(x) = -2 + 2 = 0$ ... بالتعويض باستمرار في المعادلة

x	$f(x)$
-4	-2
-3	-1
-2	0
-1	1
0	2
1	3
2	4
3	5
4	6

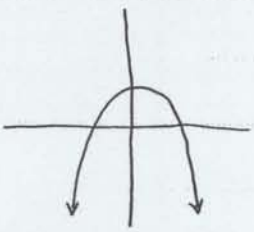


d- The vertical - Line Test. (ص رقم 505)

EX. No. 3 → Determine whether each of the following is the graph of a function.

1) 

The graph is not that a function because a vertical line can cross the graph at more than one point

2) 

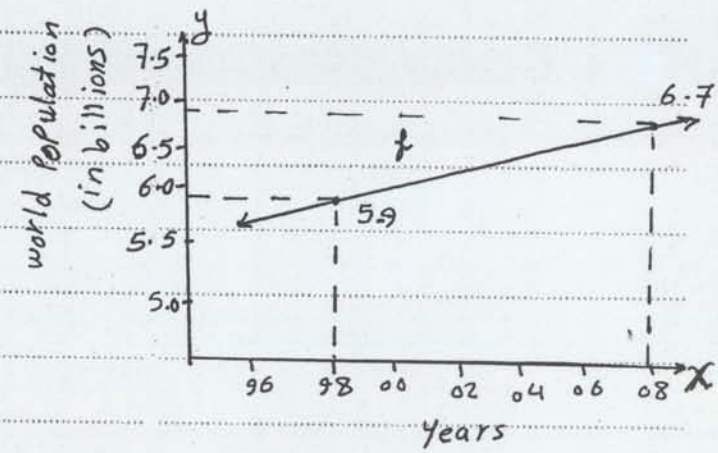
The graph is that of a function because a vertical line can cross the graph mor than once.

c- Applications of Functions and Their Graphs:

Ex. No. 4 → world population, The following graph represents the world population, in billions, The population is a function of the year. Note that no equation is given for the function.

a) what was the world population in 1998 ? That is find $f(1998)$.

b) $f(2008)$

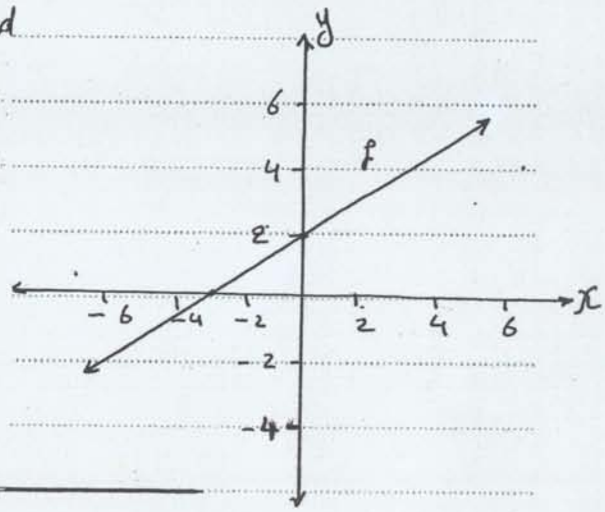


Finding Domain and Range (7-2).

a- Finding Domain and Range. (ص رقم 514)

EX-No-1 → Find the domain and the range of the function, f whose graph is shown below.

since no endpoints are indicated the graph extends indefinitely both horizontally and vertically. Thus the domain is the set of all real numbers. Like wise, the range is the set of all numbers.



EX No. 2 → Find the domain : $f(x) = |x|$

The answer cannot calculate $|x|$. Thus the domain of f is the set of all real numbers.

EX. No. 3 → Find the domain $f(x) = \frac{3}{2x-5}$

we cannot calculate $3/2x-5$, So we solve

$2x - 5 = 0$

$2x - 5 = 0$

$2x = 5$

$x = \frac{5}{2}$

Thus , $\frac{5}{2}$ in not the domain, whereas all other real numbers are. The domain of f is $\{x | x \text{ is real number and } x \neq \frac{5}{2}\}$

لأن المقام وقتها سيكون = 0 .

Linear Function Graphs and slope . (7.3).

الدالة الخطية وميل المستقيم .

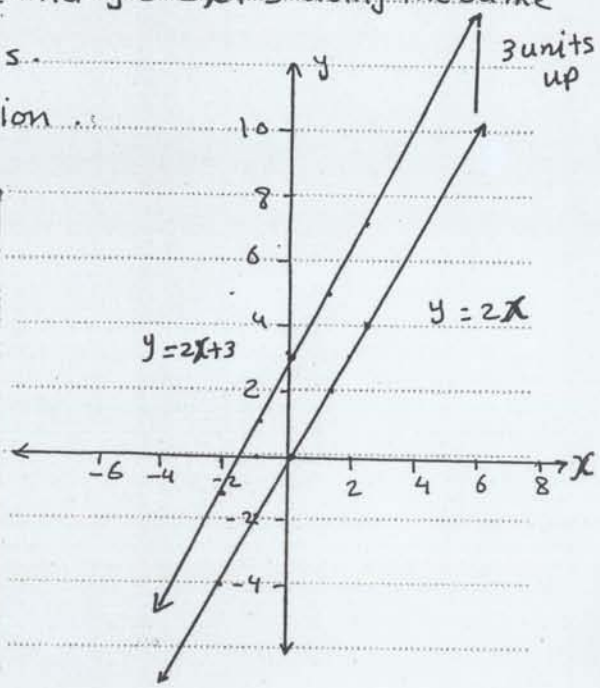
A Linear Function $f(x) = mx + b$

a- The constant b: The y-Intercept.

EX. No. 1 → Graph $y = 2x$ and $y = 2x + 3$ using the same set of axes compare the graphs.

we list make a table of solution :

x	$y = 2x$	$y = 2x+3$
0	0	3
1	2	5
-1	-2	1
2	4	7
-2	-4	-1

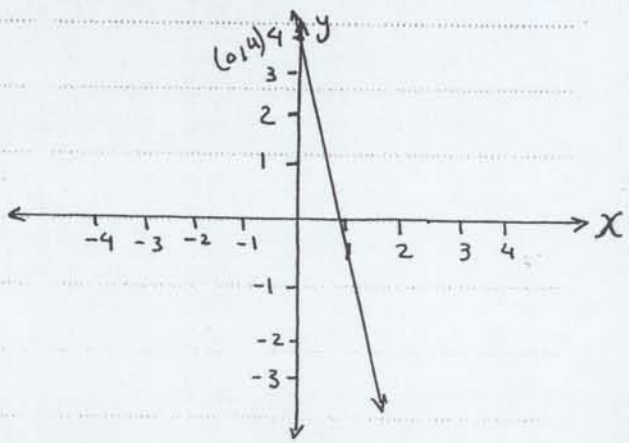


EX. No. 2 → Find the y-Intercept : $y = -5x + 4$.

$y = -5x + 4$ (0,4) is the y intercept.

$f(x) = mx + b$.

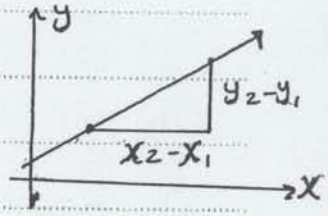
the point (0, b).



b- The Constant m : slope. (524 ص، ب)

slope :.

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{Change in } y}{\text{Change in } x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

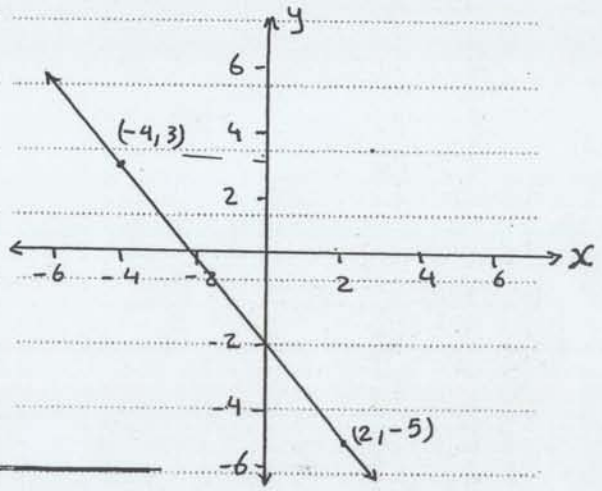


EX. No. 3 → Graph the Line Containing the points $(-4, 3)$ and $(2, -5)$ and find the slope.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-5 - 3}{2 + 4} = \frac{-8}{6}$$

$$\text{slope} = -\frac{4}{3}$$



EX. No. 4 → Find the slope and the y-intercept of

$$2x + 3y = 8$$

$$3y = 8 - 2x$$

$$\frac{3y}{3} = \frac{-2x + 8}{3}$$

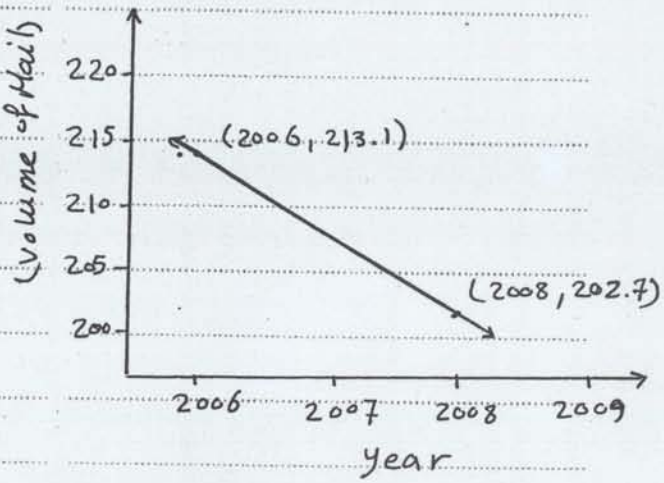
$$\therefore y = \frac{-2x}{3} + \frac{8}{3} \quad \text{when } y = mx + b$$

The slope is $\frac{-2}{3}$

The y-intercept is $(0, \frac{8}{3})$.

C - Applications:. (527 ص, ق)

EX. No. 5 → volume of Mail, The volume of mail though the u.s postal service has been dropping since 2006, as shown in the graph below.



Rate of change =

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{202.7 - 213.1}{2008 - 2006}$$

$$= \frac{-10.4}{2}$$

$$= -5.2 \text{ billion per year.}$$

∴ The volume of mail through the u.s postal service is decreasing at a rate of about 5.2 billion pieces per year.

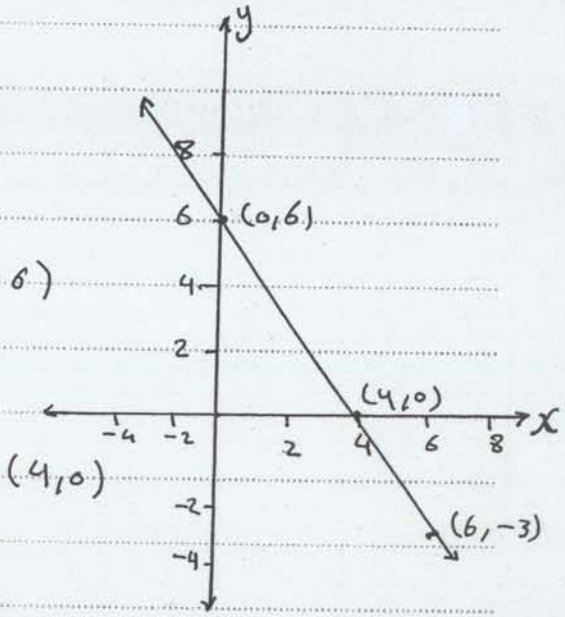
Note on Graphing Linear Equations. (7-4).

توجد التقاطع مع محور السينات ومحور الصادات ثم نرسم الدالة .

a- Graphing using Intercepts:

X and y- intercepts. (0, b) and (a, 0)

Ex. No. 1 → Find the intercepts of $3x + 2y = 12$ and then graph the line.



to find y- intercept

welet $x = 0$

$3x + 2y = 12$

$2y = 12$, $\therefore \boxed{y = 6}$ (0, 6)

welet $y = 0$

$3x + 2y = 12$

$3x = 12$ $\therefore \boxed{x = 4}$ (4, 0)

choose $x = 6$

$\therefore 3x + 2y = 12$

$18 + 2y = 12$

$2y = -6$ $\therefore \boxed{y = -3}$ (6, -3)

---

b- Graphing using the slope and the y- intercept.

Ex. No. 2 → Graph $y = -\frac{2}{3}x + 1$

$m = \frac{-2}{3}$ 2 units down

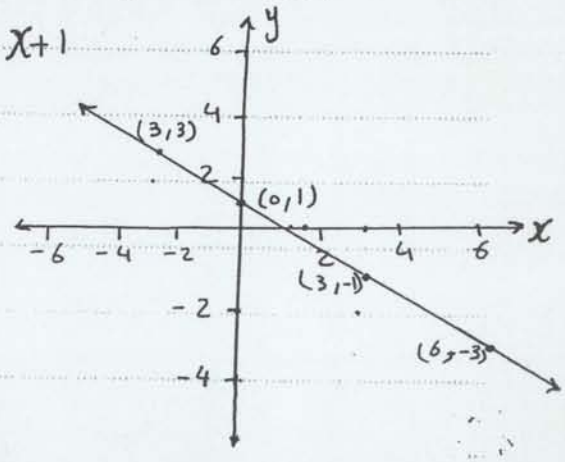
3 units right

$m = \frac{2}{-3}$ 2 units up

3 units left

when $x = 3$, $y = -1$

when $x = 6$, $y = -3$



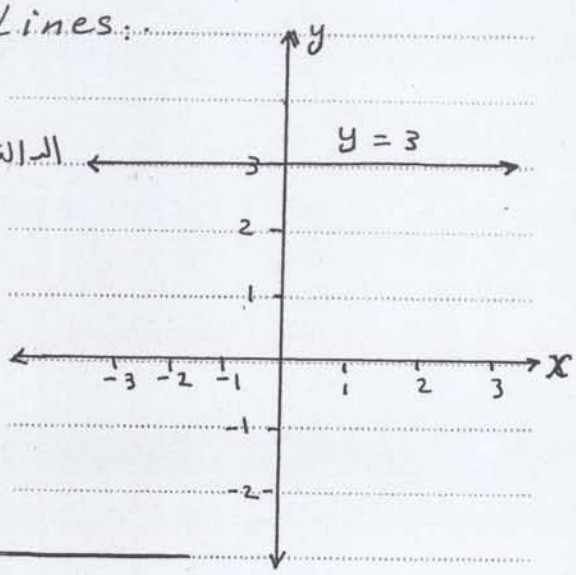
C- Horizontal and vertical Lines:

EX. No. 3 → Graph : $y = 3$.

الدالة التي تساوي رقم تركم خط موازي للمحور.

x	y
-1	3
0	3
2	3

$m = \frac{3-3}{-1-2} = 0$



d- Parallel and perpendicular Lines:

EX. No. 4 → Deter mine whether the graph of

$y - 3x = 1$ and $3x + 2y = -2$

$y - 3x = 1$

$y = 1 + 3x$

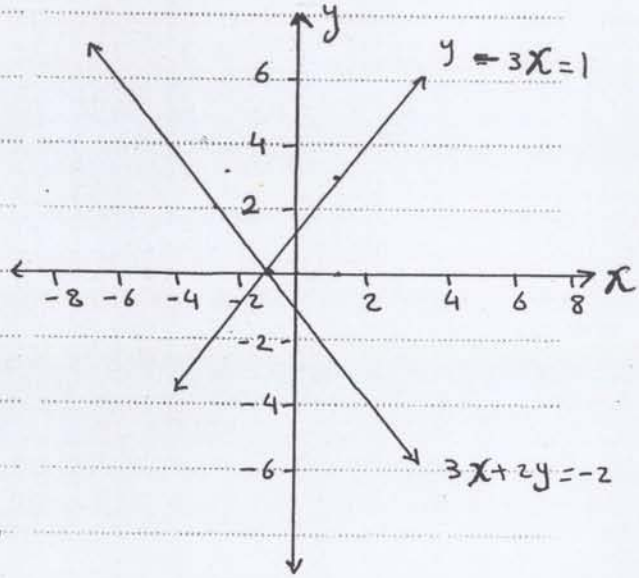
$y = 3x + 1 \rightarrow ①$

$2y = -3x - 2$

$y = \frac{1}{2}(-3x - 2)$

$y = \frac{-3}{2}x - 1$.

The slopes 3, and $\frac{-3}{2}$ are different. Thus the lines are not parallel, as the graphs at right confirm.



نخرج من قيمة x أو y المقيمين للآخر.

Finding Equations of Lines Applications. (7-5).

معادلة المستقيم وتطبيقاتها .

a- Finding an Equation of a Line when the slope and the y-intercept Are Given. (ص 544 رقم)

Ex. No. 1 → Aline has slope -0.7 and y-intercept (0, 13). Find an equation of the line.

$y = mx + b$ معادلة المستقيم والمنحنى .

$y = 0.7x + 13$

---

b- Finding an Equation of a Line when the slope and a point Are Given. $\frac{y - y_1}{x - x_1} = m$, slope $= \frac{rise}{run}$.

$y - y_1 = m(x - x_1)$

Ex. No. 2 → Find an Equation of the line with slope 5 and containing the point $(\frac{1}{2}, -1)$

$y - y_1 = m(x - x_1)$

$y - (-1) = 5(x - \frac{1}{2})$

$y + 1 = 5x - \frac{5}{2}$

$y = 5x - \frac{5}{2} - \frac{2}{2}$

$y = 5x - \frac{7}{2}$. $y = 5x - \frac{7}{2}$

$y = mx + b$

$-1 = 5(\frac{1}{2}) + b$

$b = \frac{-2}{2} - \frac{5}{2}$

$b = -\frac{7}{2}$ ∴ $y = 5x - \frac{7}{2}$.

c- Finding an Equation of a line when Two points Are given. (ص رقم 546)

Ex. No. 3 → Find an Equation of the line containing the Points (2, 3) and (−6, 1).

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{3-1}{2+6} = \frac{2}{8} = \frac{1}{4}.$$

→ $y - y_1 = m(x - x_1)$

$y - 3 = \frac{1}{4}(x - 2)$

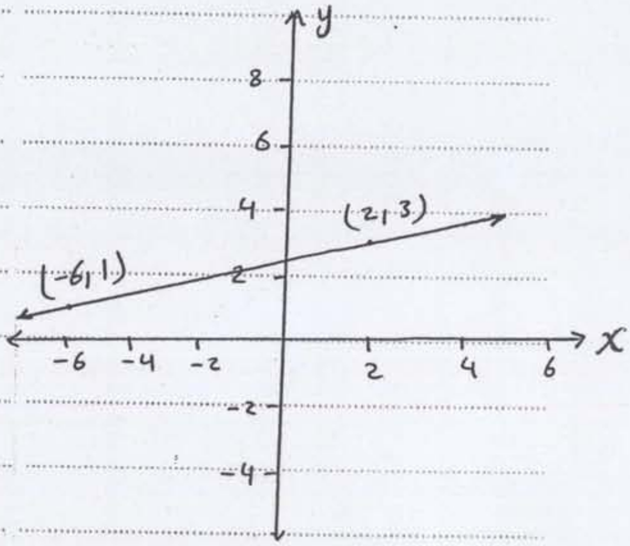
$\therefore y = \frac{1}{4}x + \frac{5}{2} \rightarrow ①$

→ $y = mx + b$

$3 = \frac{1}{4} \cdot 2 + b$

$b = \frac{5}{2}$

$\therefore y = \frac{1}{2}x + \frac{5}{2} \rightarrow ②$



d- Finding an Equation of a line parallel or perpendicular to a given line through a point off the line.

Ex. No. 4 → Find an Equation of the line containing the point (−1, 3) and parallel to the line $2x + y = 10$

$y = -2x + 10$

$3 = -2(-1) + b$

$b = 1$

$\therefore y = -2x + 1 \rightarrow ①$

$y - y_1 = m(x - x_1)$

$y - 3 = -2(x - (-1))$

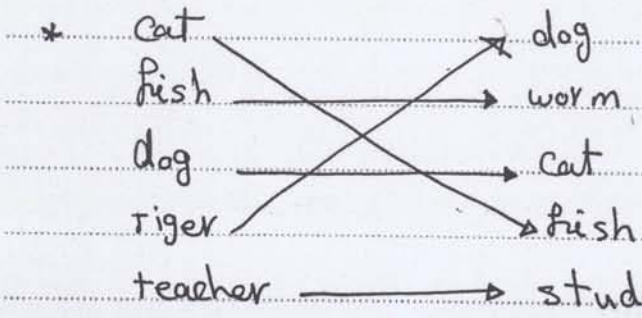
$y - 3 = -2x - 2$

$y = -2x + 1 \rightarrow ②$

استخدام معادلة الميل والمقطع

Exam. No. 7. اختبار تجريبي على الفصل السابع

1- Determine whether each correspondence is function.



2- Find the function values.

* $f(x) = -3x - 4$; $f(0)$ and $f(-2)$
* $f(x) = |x + 7|$; $f(-10)$ and $f(-7)$.

3- Graph:

* $h(x) = -2x - 5$
* $f(x) = \frac{-3}{5}x$
* $f(x) = x^2 + 2x - 3$

4- Graph using the slope and the y-intercept.

$f(x) = -\frac{2}{3}x - 1$

5- Find the intercepts. Then graph the equation.

$2x + 3y = 6$.

6- Find an equation of the line containing the given pair of points: $(4, -6)$ and $(-10, 15)$.

chapter(8)

systems of Equations.

systems of Equations in Two variables:. (8.1).

حل المعادلات التى بها متغيرين (x, y)

a- Solving systems of Equations Graphically:. (بيانيا) (5 ص رقم 71)

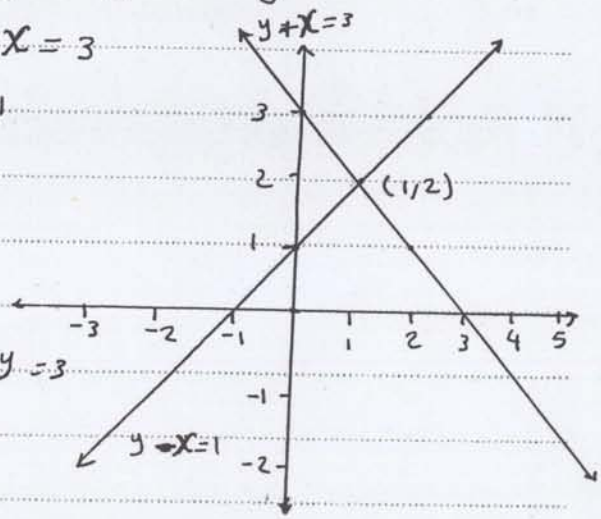
Ex. No. 1 → Solve this system graphically.

$y - x = 1$, $y + x = 3$

for $x = 0$, $y - 0 = 1$ $y = 1$

for $x = 1$, $y = 2$

for $x = 2$, $y = 3$.



For $x = 0$, $y + 0 = 3$ $y = 3$

for $x = 1$, $y = 2$

for $x = 2$, $y = 1$.

Check → $2 - 1 = 1$ True

$2 + 1 = 3$ True The solution is $(1, 2)$.

Ex. No. 2 → Solve graphically $3y - 2x = 6$

$-12y + 8x = -24$.

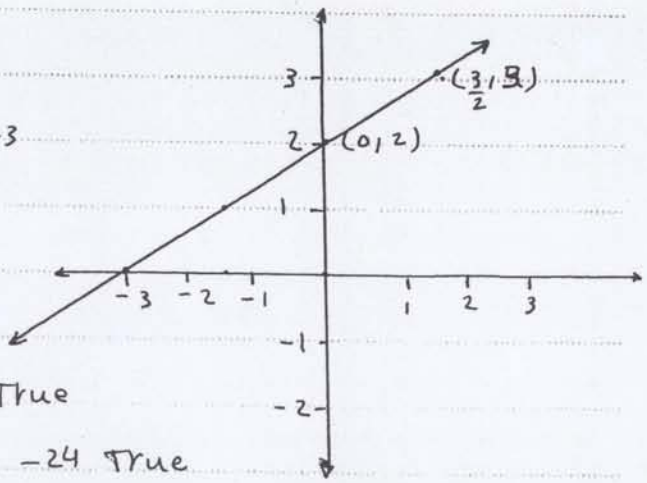
$x = \frac{-6 + 3y}{2}$

when $y = 0$, $x = \frac{-6}{2} = -3$

when $y = 1$, $x = \frac{-3}{2}$.

when $y = 2$, $x = 0$

when $y = 3$, $x = \frac{3}{2}$.



Check $(3)(2) - 2(0) = 6$ True

$-12(2) + 8(0) = -24$ True

The solution is $(0, 2)$.

Solving by substitution (8-2). (صفحة رقم 579)

a- The substitution Method. الحل بالتعويض من معادلة لأخرى

EX-No.1 → solve this system.

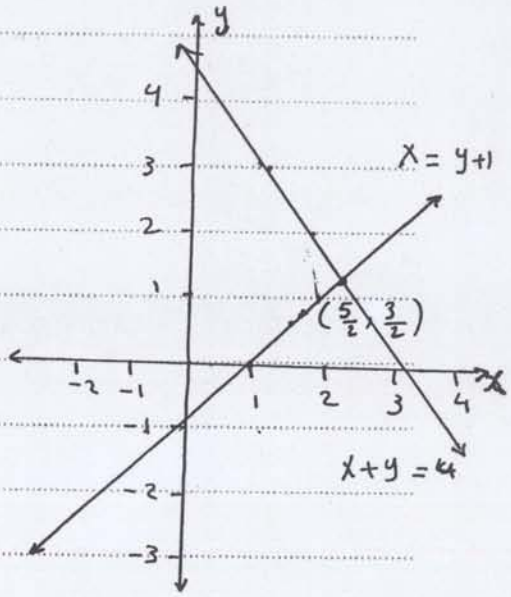
$x + y = 4$ $\quad x = y + 1$.

$\therefore x = y + 1$

$(y+1) + y = 4$

$2y = 3$, $y = \frac{3}{2}$ $\quad \therefore y = \frac{3}{2}$

$\therefore x = \frac{3}{2} + 1 = \frac{5}{2}$ $\quad x = \frac{5}{2}$.



Check $\frac{5}{2} = \frac{3}{2} + 1$

$\frac{5}{2} = \frac{3}{2} + \frac{2}{2} = \frac{5}{2}$ True

$\frac{5}{2} + \frac{3}{2} = \frac{8}{2} = 4$ True.

The solution is $(\frac{5}{2}, \frac{3}{2})$.

$x = 1$ $\quad y = 0$

$x = 2$ $\quad y = 1$

$x = 1$ $\quad y = 3$

$x = 2$ $\quad y = 2$

b- solving Applied problems Involving Two Equations.

EX. No. 2 → Architecture. The architects who designed the John Han cock Building in chicago created a visually appealing building that slants on the sides, The ground floor is a rectangle that is larger that the rectangle formed by the top floor. The ground floor has a perimeter of 860 ft. the length is 100 ft more than the width. Find the length and the width.

$P = 2L + 2w$

$860 = 2L + 2w$, $L = w + 100$

$860 = 2(w+100) + 2w$

$860 = 2w + 200 + 2w$

$4w = 660$ $\quad \therefore w = 165$.

$L = 165 + 100$

$\therefore L = 265$.

Solving by Elimination. (8-3).

الحل بالتخلص من أحد المتغيرين

a - The Elimination Method. (طريقة الحذف)

EX. No. 1 → solve this system

$2x - 3y = 0$ ①

$-4x + 3y = -1$ ②

بجمع المعادلتين

$-2x = -1 \quad \therefore x = \frac{1}{2}$

بالتعويض في أحد المعادلتين

$2x - 3y = 0$

$2 \cdot \frac{1}{2} - 3y = 0$

$-3y = -1$

$\therefore y = \frac{1}{3}$

check → $2(\frac{1}{2}) - 3(\frac{1}{3}) = 0$; $-4(\frac{1}{2}) + 3(\frac{1}{3}) =$

$1 - 1 = 0$ True ; $-2 + 1 = -1$ true

The solution is $(\frac{1}{2}, \frac{1}{3})$.

EX. No. 2 → solve this system

$2x + 3y = 17$ ①

$5x + 7y = 29$ ②

بالضرب X 5 في الأولى و X 2 في الثانية

$10x + 15y = 85$

$-10x - 14y = -58$

$0 + y = 27$

$\therefore y = 27$

بالتعويض في المعادلة الأولى

$2x + 3(27) = 17$

$2x = -64$

$\therefore x = -32$

بالجمع check → $2(-32) + 3(27) = 17$ True

$5(-32) + 7(27) = 29$ True.

$(-32, 27)$. The solution.

b - Solving Applied problems using Elimination. (5.90 ص ٢٤٠)

EX. No. 3 → Stintulating the Home town Economy. To stimulate the economy in his town of Brewton, Alabama, in 2009, Danny Cottrell, Co-owner of The Medical center pharmacy, gave each of his full-time employees $700 and each part-time employee $300. He asked that each person donate 15% to a charity of his or her choice and spend the rest locally. The money was paid in $2 bills, a rarely used currency. so that business Comunity could easily see how the money circulated. Courell gave away a total of $16.000 to his 24 employees. How many full-time employees and how many part-time employees were there?

f = number of full-time. , P = the number of part-time

total = 700f + 300p.

$700f + 300P = 16000$ 1

$f + P = 24$

$700f + 300p = 16000$ ①

$-300f - 300P = 7200$ ②

$400f = 8800$ $\therefore f = 22$ →

بالتعويض المعادلة الأولى

$f + P = 24$

$22 + P = 24$ $\therefore P = 2$

* check → $22 + 2 = 24$ True

state → There were 22 full-time employees and 2 part-time employees.

Solving Applied problems: Two Equations. (8-4).

a- Total - value problems and mixure problems. (5.94) (رقم، مس)

EX. No. 1 → Blending Flower seeds. Tara's web site. Garden Edibles specializes in the sale of herbs and flowers for colorful meals and garnishes. Tara sells packets of nasturtium seeds for $0.95 each and packets of Johnny-Jump-up seeds for $1.43 each. she decides to offer a 16-packet spring-garden mixture, combining packets of both types of seeds at $1.10 per packet. How many packets of each type of seed should be put in her garden mix?

lets try 12 and 4

$0.95(12) + $1.43(4) or 17.12.

$a + b = 16$ ①

$0.95a + 1.43(b) = 17.6$ ② , $95a + 143b = 1760$

$b = 16 - a$ بالتعويض في المعادلة الثانية

$95a + 143(16 - a) = 1760$

$95a + 2288 - 143a = 1760$

$-48a = -528$

$\therefore a = 11$

$\therefore b = 16 - a$

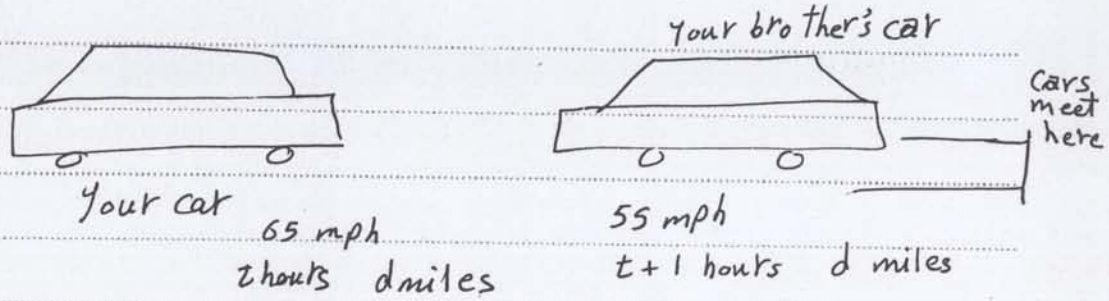
$b = 16 - 11 = 5$

Check: $0.95(11) + 1.43(5) = 17.6$ True.

State: The spring-garden mixture can be made by combining 11 packets of nasturtium seeds with 5 packets of Johnny-jump-up seeds.

b - Motion problems :.. (مسائل الحركة)

EX. No. 2 → Auto travel, your brother leaves your home on a trip, for-getting his suitcase. you know that he normally drives a speed of 55 mph. you don't discover the suitcase until I hr after he has left. If you follow him at a speed of 65 mph. how long will it take you to catch up with him ?



$d = r \cdot t$

$d = 55(t+1)$ ①

$d = 65t$ ②

$\therefore 65t = 55(t+1)$

$65t = 55t + 55$

$10t = 55$

$t = 5.5$ hr. $\quad t+1 = 6.5$ hr.

state : you will catch up your brother in 5.5 hr.

system of Equations in three variables. (8-5).

حل المعادلات التي بها ثلاث متغيرات.

a- Solving systems in Three variables:

Ex. No. 1 → solve the following system of equations.

$x + y + z = 4 \rightarrow ①$

$x - 2y - z = 1 \rightarrow ②$

$2x - y - 2z = -1 \rightarrow ③$ بجمع المعادلة الأولى والثانية

$x + y + z = 4$

$x - 2y - z = 1$

$2x - y = 5 \rightarrow ④$

بجمع المعادلة الأولى والثالثة

$2x + 2y + 2z = 8$

$2x - y - 2z = -1$ بالضرب × 2

$4x + y = 7 \rightarrow ⑤$

بجمع المعادلة ④، ⑤

$2x - y = 5$

$4x + y = 7$

$6x = 12 \quad \therefore \boxed{x = 2}$

بالتعويض في المعادلة ④

$2x - y = 5$

$4 - y = 5 \quad \therefore \boxed{y = -1}$

$x + y + z = 4$

$2 - 1 + z = 4 \quad \therefore \boxed{z = 3}$

Check → $2 - 1 + 3 = 4 \rightarrow ①$ True

$2 + 2 - 3 = 1 \rightarrow ②$ True

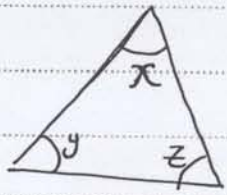
$4 + 1 - 6 = -1 \rightarrow ③$ True

The solution is $(2, -1, 3)$.

Solving Applied probelms: Three Equations. (8-6).

a - using systems of Three Equations. (617 ص، ج)

Ex. No. 1 → Jetoelry Design - kim is designing a triangular - shaped pendant for aclient of her custom jewelry business. The largest angle of the triangle is 70° greater than the smallest angle, The largest angle is twice as large as the remaining angle. Find the measure of each angle.



let smallestangle = x

TheLargest angle = z

remaining angle = y

$x + y + z = 180 \rightarrow ①$

$z = 70 + x \rightarrow ②$

$z = 2y \rightarrow ③$

$x + y + 70 + x = 180$ بالتعويض في المعادلة ①

$2x + y = 110 \rightarrow ④$

$z - 2y = 0$

$70 + x - 2y = 0$

$x - 2y = -70$ $\quad x-2$

$-2x + 4y = 140 \rightarrow ⑤$ ④ ، ⑤ بالجمع

$2x + y = 110$

$5y = 250$ $\quad \therefore \boxed{y = 50^\circ}$

$2y = z$

$\boxed{100 = z}$

$\therefore x = 180 - 150 = 30^\circ$ The solution is (30, 50, 100).

Exam. No. 8 اختبار تجريبي على الفصل الثامن

1- Solve graphically. Then classify the system as consistent or inconsisent and the equations as dependent or independent.

* $y = 3x + 7$, $3x + 2y = -4$.
* $y - 3x = 6$, $6x - 2y = -12$.

2- solve by the substitution method.

* $4x + 3y = -1$, $y = 2x - 7$.
* $x + 2y = 6$, $2x + 3y = 7$.

3- solve by the elimination method.

* $2x + 5y = 3$, $-2x + 3y = 5$.
* $\frac{2}{3}x - \frac{4}{5}y = 1$, $\frac{1}{3}x - \frac{2}{5}y = 2$.

4- Air Travel. An airplake flew for 5 hr with a 20 km/h tailwind and returned in 7 hr against the same wind. Find the speed of the plane in still air.

5- solve:.

* $6x + 2y - 4z = 15$
* $-3x - 4y + 2z = -6$
* $4x - 6y + 3z = 8$

chapter (9)

Sets, Inequalities, and Interval Notation (9-1)

a- Inequalities :- (أو المتراجحات) حل المتباينة على فترة

An inequality is a sentence containing $<, >, \leq, \geq$ or $\neq$

Ex. No. 1. Determine whether the given number, حل المتباينات ، is a solution of the inequality.

1) $x + 3 < 6 \; ; \; 5$ يعني نعوض عن قيمة x بـ 5

$5 + 3 < 6$

$8 < 6$ False, Therefore 5 in not a solution.

---

2) $4x - 1 \leq 3x + 2 \; ; \; -3$ نعوض عن x بـ -3

$4(-3) - 1 \leq 3(-3) + 2$

$-12 - 1 \leq -9 + 2$

$-13 \leq -7$ True, There fore -3 is a solution.

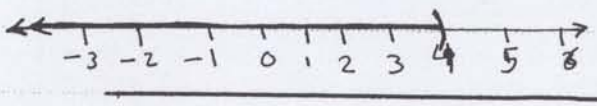
---

b- Inequalities and Interval Notation :- (ص رقم 636)

Ex. No. 2 → Graph $x < 4$

إذا كانت لا توجد علامة $\leq$ ، $\geq$ ، علامة التساوي نرسم القوس (،)

إذا كانت موجودة نرسم القوس [،] .



أشكال الرسم على خط الأعداد وعلاقتها بكتابة المجموعة

* (a,b) , $\{x \mid a < x < b\}$

* $[a,b]$, $\{x \mid a \leq x \leq b\}$

* $[a,b)$, $\{x \mid a \leq x < b\}$

* $(a,b]$, $\{x \mid a < x \leq b\}$

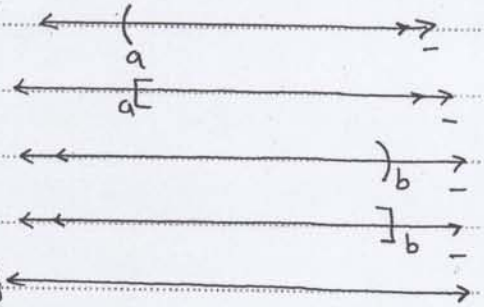
(a, ∞) , $\{x | x > a\}$

$[a, \infty)$, $\{x | x \geq a\}$

$(-\infty, b)$, $\{x | x < b\}$

$(-\infty, b]$, $\{x | x \leq b\}$

$(-\infty, \infty)$, $\{x | x$ is a real number$\}$



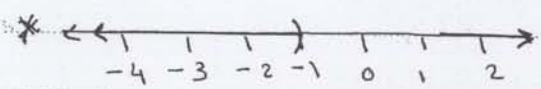
EX. No. 3 → $\{x | -4 < x < 5\}$

$= (-4, 5)$

---

* $\{x | x \geq -2\} = [-2, \infty)$

---

*  $= (-\infty, -1)$.

C- Solving inequalities. (ص رقم 638)

EX. No. 4 → Solve and graph.

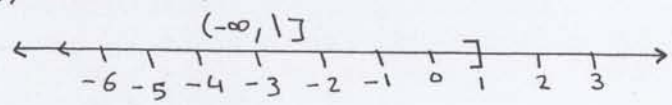
* $4x - 1 \geq 5x - 2$

$4x - 1 + 2 \geq 5x - 2 + 2$

$4x + 1 \geq 5x$

$-4x + 4x + 1 \geq 5x - 4x$

$1 \geq x$



---

* Solve $16 - 7y \geq 10y - 4$

$16 - 16 - 7y \geq 10y - 4 - 16$

$-7y \geq 10y - 20$

$-10y - 7y \geq 10y - 10y - 20$

$-17y \geq -20$

عند تغيير الإشارة تقلب ≥ إلى ≤

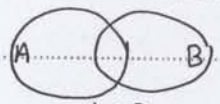
$\therefore y \leq \frac{20}{17}$

$(-\infty, \frac{20}{17}]$

Intersections, unions, and Compound Inequalities (9.2).

a- Intersections of sets and conjunction of inequalities:

(صـ رقم 652)

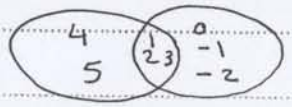
$A \cap B$ 

علامة التقاطع يعني أنه هناك جزء مشترك بين المجموعتين.

Ex. No. 1 → Find the intersection:

$\{1,2,3,4,5\} \cap \{-2,-1,0,1,2,3\}$

The intersection $\{1,2,3\}$



---

* Ex. No. 2 → Solve and graph $-1 \leq 2x+5 < 13$

نقسمها لمتباينتين

$$\begin{cases} -1 \leq 2x+5 \\ 2x+5 < 13 \end{cases}$$

* $-1 \leq 2x+5$

$-6 \leq 2x$ $\quad \therefore \frac{-6}{2} \leq x$

$\therefore x \geq -3$

* $2x+5 < 13$

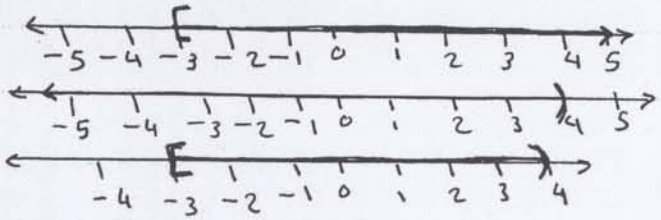
$\frac{2x}{2} < \frac{8}{2}$

$\therefore x < 4$

$x \geq -3$

$x < 4$

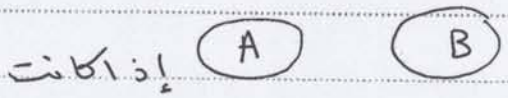
$-3 \leq x < 4$



$[-3, \infty) \cap (4, \infty) = [-3, 4)$.

$A \cap B = \emptyset$

إذا كانت المجموعتان لا تتقاطعان



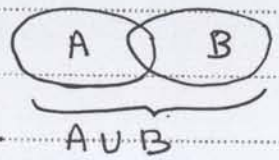
∴ تتقاطعان في مجموعة فاي $\emptyset$ وهي مجموعة فارغة أو خالية.

---

b - unions of sets and Disjunctions of inequatities.

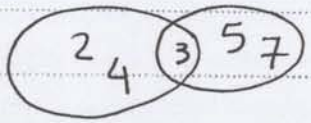
$A \cup B$ علامة الاتحاد.

يعني كل المجموعتين



EX. No. 3 → Find the union

$\{2,3,4\} \cup \{3,5,7\} = \{2,3,4,5,7\}$



Ex. No. 4 → Solve and graph $7+2x < -1$

or $13 - 5x \leq 3$

$7 + 2x < -1$

$-7 + 7 + 2x < -1 - 7$

$\frac{2x}{2} < \frac{-8}{2}$

$x < -4$

or $13 - 5x \leq 3$

$13 - 13 - 5x \leq 3 - 13$

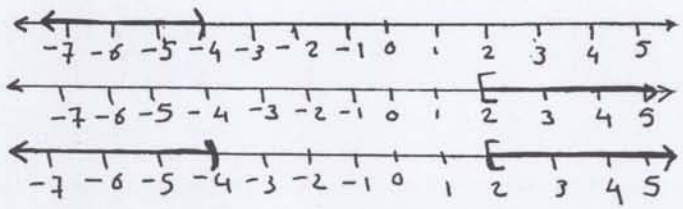
$-5x \leq -10$

$x \geq 2$

$x < -4$

$x \geq 2$

$(-\infty, -4) \cup [2, \infty)$



Absolute_value Equations and Inequalities (9.3).

a - properties of Absolute value: → (ص رقم 666)

القيمة المطلقة لا يوجد فيها سالب ، أى رقم أمامه إشارة سالبة تلغى

$|ab| = |a| \cdot |b|$.

$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ $b \neq 0$

$|-a| = |a|$

EX. No. 1 → simplify

1) $|5x| = |5| \cdot |x| = 5|x|$

2) $|-3y| = |-3| \cdot |y| = 3|y|$

3) $|7x^2| = |7| \cdot |x^2| = 7|x^2| = 7x^2$ لأن x^2 دائماً موجبة

$\left|\frac{6x}{-3x^2}\right| = \left|\frac{2}{-x}\right| = \frac{|2|}{|-x|} = \frac{2}{|x|}$.

---

b - Distance on Number Line: →

$|a-b|$. this is The distance between them.

EX. No. 2 → Find the distance between

1) -8 and -92

$|-8-(-92)| = |-8+92| = |84| = 84$.

or $|(-92)-(-8)| = |-84| = 84$.

---

2) $|x-0| = |x|$

3) -3 and 2 $= |-3-2| = |-5| = 5$.

C- Equations with Absolute value:→ (667 ص، ق)

EX-No. 3 → Solve

1) $|x| = 0$, The solution set is $\{0\}$

2) $|x| = -7$, " " " " " ϕ لأن لا توجد قيمة مطلقة سالبة

3) $2|x| + 5 = 9$

$2|x| = 4$

$|x| = 2$

$\therefore x = 2$ or $x = -2$

The solution set is $\{2, -2\}$.

---

d- Equations with Two Absolute value Expressions:

EX-No. 4 → Solve $|2x - 3| = |x + 5|$

$2x - 3 = x + 5$ or $2x - 3 = -(x + 5)$

$2x = x + 8$ | $2x - 3 = -x - 5$

$x = 8$ | $3x = -2$

$x = \frac{-2}{3}$

The solutions are 8 and $\frac{-2}{3}$, The solution set is $\{8, \frac{-2}{3}\}$

---

e- Inequalities with Absolute value:

EX-No. 5 → Solve $|3x - 2| < 4$ Then graph.

$|x| < p$

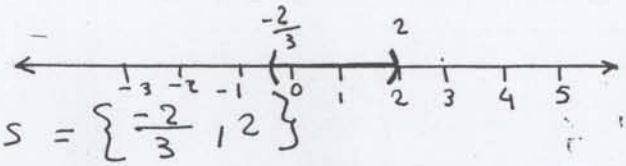
$|3x - 2| < 4$

$-4 < 3x - 2 < 4$

$\frac{-2}{3} < x < 2$

The solution set is $= \{\frac{-2}{3}, 2\}$

$|3x - 2| < 4$



System of Inequalities in Two variables (9-4).

حل متباينة بها متغيرين.

a- Solution of Inequalities in Two variables. → (أ، ب) حل المتباينة

Ex. No. 1 → Determine whether the ordered pair is a solution of the inequality $5x - 4y > 13$, $(-3, 2)$, $(4, -3)$

1) $(-3, 2)$, 2) $(4, -3)$

$5x - 4y > 13$

$5(-3) - 4(2) \;\big|\; 13$

$-15 - 8$

-23 False

$-23 > 13$ False $(-3, 2)$ is not a solution.

$5x - 4y > 13$

$5(4) - 4(-3) \;\big|\; 13$

$20 + 12$

32 True

$32 > 13$

$(4, -3)$ is a solution.

---

b- Graphing Inequalities in Two variables →

Ex. No. 2 → Graph $6x - 2y < 12$

$6x - 2y < 12$

$6(0) - 2(0) \;\big|\; 12$

$0 - 0$

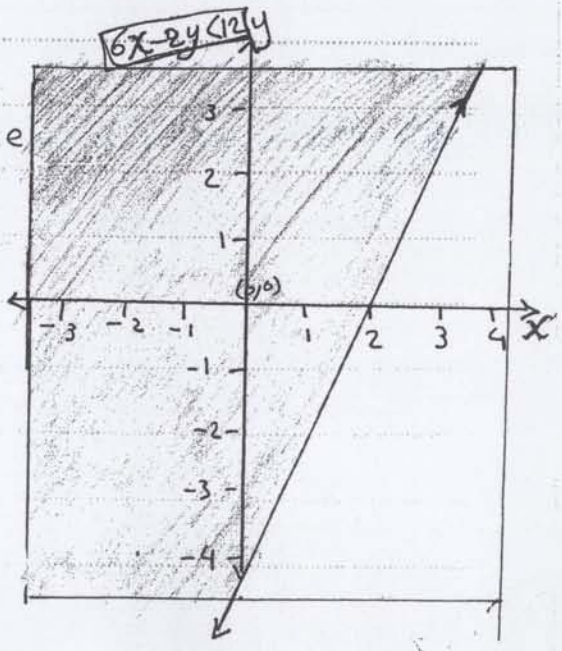
0 $\quad 0 < 12$ True

$(2, 0)$ $\quad < 12$ True

$(3, 0)$ $\quad < 18$ false

$(0, -\infty)$ $\quad < 12$ True

we try $(0, 0)$ and substitue.



C- systems of Linear Inequalities :→ (ص 881 رقم 1)

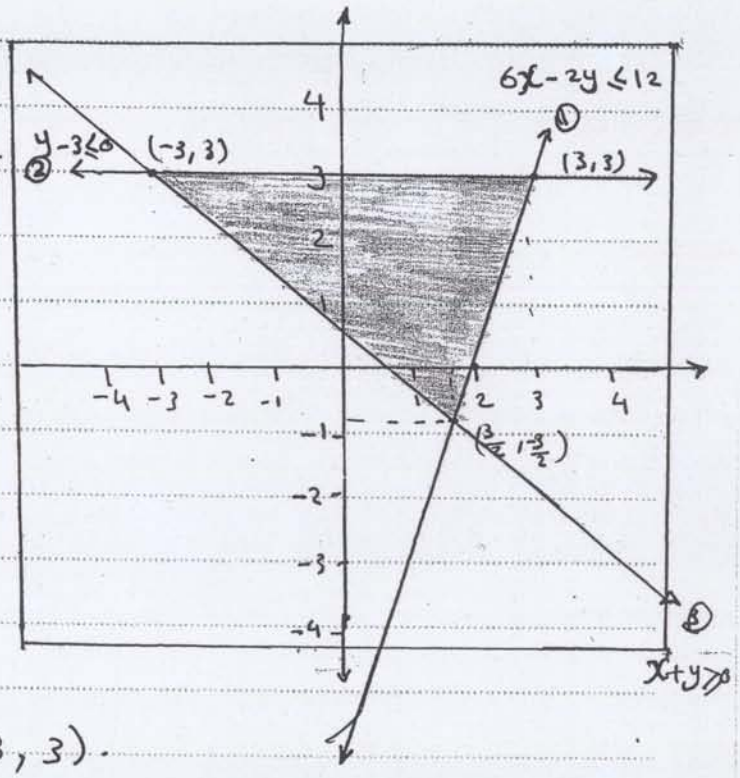
EX. No. 3 → Graph the following system of inequalities. Find nates of any vertices formed.

$6x - 2y \leq 12$ (1)

$y - 3 \leq 0$ (2)

$x + y \geq 0$ (3)

Then graph this lines.



→ To find the vertices

$6x - 2y = 12$ ①

$y - 3 = 0$ ②

$y = 3$ →

بالتعويض في المعادلة الأولى

$6x - 2y = 12$

$6x - 2 \cdot 3 = 12$

$6x - 6 = 12$

$6x = 18$

$\therefore x = 3$ $(3, 3)$.

---

* $y - 3 = 0$ ② $\quad y = 3$

$x + y = 0$ ③ $\quad x = -y = -3$ $(-3, 3)$.

---

* $6x - 2y = 12$ ①

$x + y = 0$ ③

$\boxed{x = -y}$

$6x + 2(x) = 12$

$8x = 12$

$x = \frac{3}{2}$

$\therefore y = -x = \frac{-3}{2}$

$(\frac{3}{2}, \frac{-3}{2})$.

Exam. No. 9. اختبار تجريبي على الفصل التاسع

1- write interval notation for the given set or graph.

* $\{x \mid -3 < x \leq 2\}$

2- solve and graph, write interval notation for the solution set.

* $x - 2 \leq 4$
* $-4y - 3 \geq 5$

3- solve:.

* $x - 4 \geq 6$.
* $3a - 5 \leq -2a + 6$.

4- solve:.

* $5 - 2x \leq 1$ and $3x + 2 \geq 14$
* $x - 7 \leq -5$ or $x - 7 \geq -10$

5- graph and find the coordinates of any vertices formed.

* $x - 6y < -6$.
* $x + y \geq 3$, $x - y \geq 5$.
*

6- solve:.

* $|4x - 1| < 4.5$
* $|x - 3| = 9$.

chapter(10)

Radical Expressions, Equations, and Functions.

Radical Expressions and Functions (10.1).

(ص، قم 701)

a- square Roots and square-Root Functions:→

Ex. No. 1. simplify.

الجذر التربيعي

1) $\sqrt{25} = 5$

يعني أن التربيع عكس الجذر التربيعي

$5 \cdot 5 = 5^2 = 25$

$\sqrt{25} = 5$.

2) $-\sqrt{25} = -5$

3) $\sqrt{\frac{81}{64}} = \frac{9}{8}$ because $\left(\frac{9}{8}\right)^2 = \frac{81}{64}$

4) $\sqrt{11} = 3.317$

5) $\sqrt{487} = 22.068$

Ex. No. 2 → For the given function find the indicated function values.

$f(x) = \sqrt{3x-2}$; $f(1)$, $f(5)$, and $f(0)$

* $f(1) = \sqrt{3 \cdot 1 - 2} = \sqrt{1} = 1$

* $f(5) = \sqrt{3 \cdot 5 - 2} = \sqrt{15-2} = \sqrt{13} \approx 3.606$.

* $F(0) = \sqrt{3 \cdot 0 - 2} = \sqrt{-2}$ #.

b- Finding $\sqrt{a^2}$

$a \geq 0 \quad \sqrt{a^2} = a$

$a < 0 \quad \sqrt{a^2} = -a$

EX. No. 3 → Find each of the following Assume that letters can represent any real number:

1) $\sqrt{(-16)^2} = |-16| = 16$

2) $\sqrt{(3b)^2} = |3b| = 3|b|$.

---

3) $\sqrt{(x-1)^2} = |x-1|$.

4) $\sqrt{x^2+8x+16} = \sqrt{(x+4)^2} = |x+4| = 4+|x|$.

---

c- Cube Roots: الجذر التكعيبي

$\sqrt[3]{125} = 5$ يعني العدد مضروب 3 مرات $5.5.5 = 125$

EX. No. 4 → Find each of the following:

1) $\sqrt[3]{8} = 2$ because $2.2.2 = 8$.

2) $\sqrt[3]{x^3} = x$.

3) $\sqrt[3]{0} = 0$.

4) $\sqrt[3]{-27} = -3$

5) $\sqrt[3]{-8y^3} = \sqrt[3]{(-2y)^3} = -2y$.

6) $\sqrt[3]{-8} = -2$

7) $\sqrt[3]{-\frac{216}{125}} = -\frac{6}{5}$.

---

d- odd and Even xth Roots:

EX. No. 5 → Find each of the following

1) $\sqrt[5]{32} = 2$.

2) $\sqrt[7]{x^7} = x$.

3) $\sqrt[7]{-128} = -2$.

4) $\sqrt[5]{a^5} = a$

5) $\sqrt[4]{16} = 2$

6) $\sqrt[6]{(x+7)^6} = |x+7|$.

Rational numbers as Exponents: (10.2)

الأعداد الحقيقية والأسس

a- Rational Exponents: → (صـ رقم 713)

$a^{\frac{1}{n}} = \sqrt[n]{a}$

Ex. No. 1 →

1) $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

2) $(abc)^{\frac{1}{5}} = \sqrt[5]{abc}$

3) $\sqrt{x} = x^{\frac{1}{2}}$

4) $(27)^{\frac{2}{3}} = \sqrt[3]{(27)^2} = 3^2 = 9$.

5) $(4)^{\frac{3}{2}} = \sqrt{4^3} = 2^3 = 8$.

---

b- Negative Rational Exponents:

$a^{-m/n} = \frac{1}{a^{m/n}}$

Ex. No. 2 → Rewrite with positive exponents, and simplify

1) $9^{-\frac{1}{2}} = \frac{1}{9^{\frac{1}{2}}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$.

2) $(5xy)^{-\frac{4}{5}} = \frac{1}{(5xy)^{\frac{4}{5}}}$.

3) $64^{-\frac{2}{3}} = \frac{1}{64^{\frac{2}{3}}} = \frac{1}{(\sqrt[3]{64})^2} = \frac{1}{4^2} = \frac{1}{16}$.

4) $\left(\frac{3r}{7\sigma}\right)^{-5/2} = \frac{1}{\left(\frac{3r}{7\sigma}\right)^{5/2}} = \left(\frac{7\sigma}{3r}\right)^{\frac{5}{2}}$.

5) $4x^{-2/3} \cdot y^{\frac{1}{5}} = \frac{1}{4x^{\frac{2}{3}}} \cdot y^{\frac{1}{5}} = \frac{y^{\frac{1}{5}}}{4x^{\frac{2}{3}}}$.

C - Laws of Exponents.. (قوانين الأسس (صـ 715

1) $a^m \cdot a^n = a^{m+n}$. في الضرب نجمع الأسس .

2) $\frac{a^m}{a^n} = a^{m-n}$. في القسمة نطرح الأسس .

3) $(a^m)^n = a^{m \cdot n}$.

4) $(ab)^m = a^m \cdot b^m$.

5) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$.

EX. No. 3 → use the laws of exponents to simplify.

1) $3^{\frac{1}{5}} \cdot 3^{\frac{3}{5}} = 3^{\frac{1}{5}+\frac{3}{5}} = 3^{\frac{4}{5}}$.

2) $7^{\frac{1}{4}} \div 7^{\frac{1}{2}} = 7^{\frac{1}{4}-\frac{1}{2}} = 7^{-\frac{1}{4}} = \frac{1}{7^{\frac{1}{4}}}$.

3) $(7 \cdot 2^{\frac{2}{3}})^{\frac{3}{4}} = 7 \cdot 2^{\frac{2}{3} \cdot \frac{3}{4}} = 7 \cdot 2^{\frac{6}{12}} = 7 \cdot 2^{\frac{1}{2}}$.

---

d - simplifying Radical Expressions..

EX. No. 4 → use radical exponents to simplify.

1) $\sqrt[6]{x^3} = x^{\frac{3}{6}} = x^{\frac{1}{2}} = \sqrt{x}$.

---

2) $\sqrt[6]{4} = 4^{\frac{1}{6}} = (2^2)^{\frac{1}{6}} = 2^{\frac{2}{6}}$

$= 2^{\frac{1}{3}} = \sqrt[3]{2}$.

---

3) $\sqrt[8]{a^2 b^4} = (a^2 b^4)^{\frac{1}{8}} = a^{\frac{2}{8}} \cdot b^{\frac{4}{8}}$

$= a^{\frac{1}{4}} \cdot b^{\frac{2}{4}} = (ab^2)^{\frac{1}{4}}$

$= \sqrt[4]{ab^2}$.

---

4) $\sqrt[3]{5} \cdot \sqrt{2} = 5^{\frac{1}{3}} \cdot 2^{\frac{1}{2}} = 5^{\frac{2}{6}} \cdot 2^{\frac{3}{6}}$

$= (5^2 \cdot 2^3)^{\frac{1}{6}} = \sqrt[6]{5^2 \cdot 2^3}$

$= \sqrt[6]{200}$.

Simplifying Radical Expressions. (10-3) -

(ص رقم 720)

a - Multiplying and simplifying Radical Expressions:

$\sqrt[k]{a} \cdot \sqrt[k]{b} = \sqrt[k]{ab}$ or $a^{\frac{1}{k}} \cdot b^{\frac{1}{k}}$

EX. No. 1 → Multiply.

1) $\sqrt{3} \cdot \sqrt{5} = \sqrt{3 \cdot 5} = \sqrt{15}$.

2) $\sqrt{5a}\ \sqrt{2b} = \sqrt{5a \cdot 2b} = \sqrt{10ab}$.

3) $\sqrt[3]{4}\ \sqrt[3]{5} = \sqrt[3]{4 \cdot 5} = \sqrt[3]{20}$.

EX-No. 2 → simplify by factoring

1) $\sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{25} \cdot \sqrt{2} = = 5\sqrt{2}$

هذا النوع من المسائل مثل $\sqrt{2}$ لا يخرج من تحت الجذر ويبقى كما هو.

2) $\sqrt[3]{32} = \sqrt[3]{8 \cdot 4} = 2\sqrt[3]{4}$.

---

3) $\sqrt[4]{48} = \sqrt[4]{16 \cdot 3} = \sqrt[4]{16} \cdot \sqrt[4]{3} = 2\sqrt[4]{3}$.

---

4) $\sqrt{18x^2y} = \sqrt{9 \cdot 2 \cdot x^2 y} = 3x\sqrt{2y}$.

---

5) $\sqrt{20}\ \sqrt{8} = \sqrt{20 \cdot 8} = \sqrt{4 \cdot 5 \cdot 4 \cdot 2} = 4\sqrt{10}$.

---

6) $\sqrt[3]{18y^3} \cdot \sqrt[3]{4x^2} = \sqrt[3]{18y^3 \cdot 4x^2}$

$= \sqrt[3]{9 \cdot 2y^3 \cdot 4x^2} = 2y\sqrt[3]{9x^2}$.

---

7) $\sqrt[3]{16a^7b^{11}} = \sqrt[3]{8 \cdot 2 \cdot a^6 \cdot a \cdot b^9 \cdot b^2} = 2a^2b^3\sqrt[3]{2ab^2}$.

b- Diving and simplifying Radical Expressions:- (723 ص رقم)

Ex. No. 3 → Divide and simplify. Assume that no radicands were formed by raising negative numbers to even powers:.

1) $\frac{\sqrt{80}}{\sqrt{5}} = \sqrt{\frac{80}{5}} = \sqrt{16} = 4$.

2) $\frac{5\sqrt[3]{32}}{\sqrt[3]{2}} = 5\sqrt[3]{16} = 5\sqrt[3]{8 \cdot 2} = 2 \cdot 5\sqrt[3]{2}$

$= 10\sqrt[3]{2}$.

3) $\frac{\sqrt{72xy}}{2\sqrt{2}} = \frac{1}{2}\sqrt{\frac{72xy}{2}} = \frac{1}{2}\sqrt{36xy} = \frac{1}{2} \cdot 6\sqrt{xy}$

$= 3\sqrt{xy}$

4) $\sqrt[3]{\frac{27}{125}} = \frac{\sqrt[3]{27}}{\sqrt[3]{125}} = \frac{3}{5}$.

5) $\sqrt{\frac{25}{y^2}} = \frac{\sqrt{25}}{\sqrt{y^2}} = \frac{5}{y}$.

6) $\sqrt{\frac{16x^3}{y^4}} = \frac{\sqrt{16x^3}}{\sqrt{y^4}} = \frac{\sqrt{16x^2} \cdot \sqrt{x}}{y^2} = \frac{4x\sqrt{x}}{y^2}$.

7) $\sqrt[3]{a^2b^4} \div \sqrt{ab} = \frac{(a^2b^4)^{\frac{1}{3}}}{(ab)^{\frac{1}{2}}} = \frac{a^{\frac{2}{3}} \cdot b^{\frac{4}{3}}}{a^{\frac{1}{2}} \cdot b^{\frac{1}{2}}}$

$= a^{\frac{2}{3}-\frac{1}{2}} \cdot b^{\frac{4}{3}-\frac{1}{2}} = a^{\frac{1}{6}} \cdot b^{5/6}$

$= (ab^5)^{\frac{1}{6}} = \sqrt[6]{ab^5}$.

Addition, Subtraction, and More Multiplication (10-4)

جمع وطرح وضرب الجذور والأسس

a- Addition and subtraction:- (مسألة 729)

Ex. No. 1 → Add or subtract.

1) $6\sqrt{7} + 4\sqrt{7} = 10\sqrt{7}$.

---

2) $3\sqrt{8} - 5\sqrt{2} = 3\sqrt{4 \cdot 2} - 5\sqrt{2}$

$= 3 \cdot 2\sqrt{2} - 5\sqrt{2}$

$= 6\sqrt{2} - 5\sqrt{2} = \sqrt{2}$.

---

3) $5\sqrt[3]{16y^4} + 7\sqrt[3]{2y} = 5\sqrt[3]{8y^3 \cdot 2y} + 7\sqrt[3]{2y}$

$= 5 \cdot 2y\sqrt[3]{2y} + 7\sqrt[3]{2y}$

$= 10y\sqrt[3]{2y} + 7\sqrt[3]{2y} = \sqrt[3]{2y}(10y + 7)$

---

b- More Multiplication:- (مسألة 730)

Ex. No. 2 → Multiply.

1) $\sqrt{3}(x - \sqrt{5}) = \sqrt{3} \cdot x - \sqrt{3} \cdot \sqrt{5}$

$= x\sqrt{3} - \sqrt{15}$.

---

2) $(4\sqrt{3} + \sqrt{2})(\sqrt{3} - 5\sqrt{2})$

$= 4\sqrt{3} \cdot \sqrt{3} + \sqrt{2} \cdot \sqrt{3} - 20\sqrt{3} \cdot \sqrt{2} - 5\sqrt{2} \cdot \sqrt{2}$

$= 4\sqrt{9} + \sqrt{6} - 20\sqrt{6} - 5\sqrt{4}$

$= 4 \cdot 3 + \sqrt{6} - 20\sqrt{6} - 5 \cdot 2$

$= 12 - 19\sqrt{6} - 10$

$= 2 - 19\sqrt{6}$

More on Division of Radical Expressions (10-5).

a- Rationalizing Denominators: (ص رقم 737)

EX. No. 1 → Rationalize the denominator.

1) $\sqrt{\frac{7}{3}} = \frac{\sqrt{7}}{\sqrt{3}}$ هذه الطريقة نضرب المقام في المرافق حتى لا يكون هنا جذر في المقام نضرب في مرافق المقام.

$= \frac{\sqrt{7}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{21}}{3}$

2) $\sqrt[3]{\frac{7}{25}} = \frac{\sqrt[3]{7}}{\sqrt[3]{25}} = \frac{\sqrt[3]{7}}{\sqrt[3]{5 \cdot 5}} \cdot \frac{\sqrt[3]{\quad}}{\sqrt[3]{5}} =$

$\frac{\sqrt[3]{35}}{\sqrt[3]{5^3}} = \frac{\sqrt[3]{35}}{5}$

3) $\frac{\sqrt[3]{9}}{\sqrt[3]{9x}} = \frac{\sqrt[3]{9}}{\sqrt[3]{3 \cdot 3 \cdot x}} \cdot \frac{\sqrt[3]{3x^2}}{\sqrt[3]{3x^2}} = \frac{\sqrt[3]{9 \cdot 3x^2}}{\sqrt[3]{3^3 x^3}}$

$= \frac{\sqrt[3]{9 \cdot 3x^2}}{3x}$

b- Rationalizing when There Are Two Terms:

EX. No. 2 →

1) $\frac{4}{\sqrt{3}+x} = \frac{4}{\sqrt{3}+x} \cdot \frac{\sqrt{3}-x}{\sqrt{3}-x}$ عندما يكون المقام من جزءين يكون المرافق مثل المقام مع جعل إشارة الوسط سالبة.

$= \frac{4\sqrt{3}-x}{3-x^2}$

2) $\frac{3}{x+\sqrt{7}} = \frac{3}{x+\sqrt{7}} \cdot \frac{x-\sqrt{7}}{x-\sqrt{7}}$

$= \frac{3(x-\sqrt{7})}{x^2-7}$

solving Radical Equations: (10-6).

a-The principle of powers: (ص 742 وقم)

EX. No. 1 → solve $\sqrt{x} - 3 = 4$

$\sqrt{x} = 7$ use (squaring)

$(\sqrt{x})^2 = (7)^2$

$x = 49$

check $\sqrt{49} - 3 = 4$

$7 - 3 = 4$ True

The solution is 49.

---

2) $x - 7 = 2\sqrt{x+1}$ use (squaring)

$(x-7)^2 = (2\sqrt{x+1})^2$

$x^2 - 14x + 49 = 4(x+1)$ ثم نحل المعادلة.

$x^2 - 14x + 49 = 4x + 4$

$x^2 - 18x + 45 = 0$ فوجد قيم x

$(x - 3)(x - 15) = 0$

$x = 3$ or $x = 15$

check ..

* $3 - 7 = 2\sqrt{3+1}$

$-4 = 2\sqrt{4}$

$-4 = 4$ False

* $15 - 7 = 2\sqrt{15+1}$

$8 = 2\sqrt{16}$

$8 = 8$ True

The solution is 15

b- Equations with Two Radical Terms:. (746 ص وقـ)

EX. No. 2 → solve

1) $\sqrt{x-3} + \sqrt{x+5} = 4$

$(\sqrt{x-3}) = (4 - \sqrt{x+5})$

$(\sqrt{x-3})^2 = (4 - \sqrt{x+5})^2$

$x - 3 = 16 + (x+5) - 8\sqrt{x+5}$

$x - 3 = 21 + x - 8\sqrt{x+5}$

$\frac{-24}{-8} = \frac{-8\sqrt{x+5}}{-8}$

$3 = \sqrt{x+5}$

$3^2 = (\sqrt{x+5})^2$

$x + 5 = 9$

$x = 4$ 4 is the solution.

C - Applications:. (748 ص وقـ)

EX. No. 3 → out door concent. The geologically formed open-air Red Rocks Amphitheatre near Denver, Colorado, hosts a series of concerts. A scientific instrument at one of these concerts determined that the sound of the music was traveling at a rate of 1170 ft/sec. what was the air temperature at the concent ? we substitute 1170 for v in the formula $v = 21.9\sqrt{5t+2457}$

$1170 = 21.9\sqrt{5t+2457}$

$\left(\frac{1170}{21.9}\right)^2 = (\sqrt{5t+2457})^2$

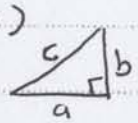
$397.2 \simeq 5t$

$\therefore t \simeq 79$

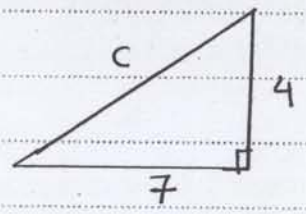
Applications Involving Powers and Roots (10.7).

(753 ص، ق)

a - Applications :. Use $a^2 + b^2 = c^2$



EX. No. 1 → Find the length of the hypotenuse of this right triangle. Give an exact answer and an approximation to three decimal places.



$A^2 + b^2 = c^2$

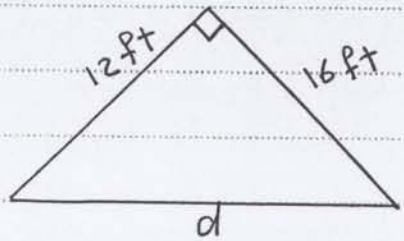
$7^2 + 4^2 = c^2$

$49 + 16 = c^2$

$65 = c^2$

$c = \sqrt{65} \simeq 8.062.$

EX. No. 2 → Construction: Darla is laying out the footer of a house. To see if the corner is square, she measures 16 ft from the corner along one wall and 12 ft from the corner along the other wall. How long should the diagonal be between those two points if the corner is a right angle?



$d^2 = 12^2 + 16^2$

$d^2 = 144 + 256$

$d^2 = 400$

$d = \sqrt{400}$

$d = 20$ ft.

The length of the diagonal should be 20 ft.

The Complex Numbers (10-8)

a- Imaginary and Complex Numbers:. (760 ص وقم)

EX. No. 1 → Express in terms of i $\quad i = \sqrt{-1}$, $i^2 = -1$

1) $\sqrt{-7} = \sqrt{-1 \cdot 7} = \sqrt{-1} \cdot \sqrt{7} = \sqrt{7}\, i$.

2) $\sqrt{-16} = \sqrt{-1 \cdot 16} = 4 \cdot \sqrt{-1} = 4i$.

3) $\sqrt{-48} = \sqrt{-1 \cdot 48} = i\sqrt{3 \cdot 16} = 4i\sqrt{3}$.

---

b- Addition and subtraction:. (761 ص وقم)

EX. No. 2 → Addor subtract.

1) $(8 + 6i) + (3 + 2i) = 11 + 8i$

2) $(3 + 2i) - (5 - 2i) = 3 + 2i - 5 + 2i$

$= -2 + 4i$

---

c- Multiplication:. (761 ص وقم)

EX. No. 3 → Multiply

1) $\sqrt{-49} \cdot \sqrt{-16} = \sqrt{-1} \cdot \sqrt{49} \cdot \sqrt{-1} \cdot \sqrt{16}$

$= 7i \cdot 4i = 28 i^2$

---

2) $-2i \cdot 5i = -10 i^2 = 10$.

---

3) $(3 - 2i)^2 = 3^2 + (2i)^2 - 2 \cdot 3 \cdot 2i$

$= 9 + 4i^2 - 12i = 9 + (4 \cdot -1) - 12i$

$= 5 - 12i$.

d - powers of i :- (ص ٧٦٣ وقم)

$\sqrt{-1} = i$, $i^2 = -1$, $i^3 = -i$

$i^4 = 1$, $i^5 = i$, $i^6 = -1$

EX. No. 4 → simplify.

1) $i^{37} = i^{36} \cdot i = (i^2)^{18} \cdot i = (-1)^{18} \cdot i = 1 \cdot i = i$

2) $i^{58} = (i^2)^{29} = (-1)^{29} = -1$

3) $i^{23} + i^{98} = (i^{22}) \cdot i + (i^2)^{24} = -1^{24} + (i^2)^{11} \cdot i$

$= 1 + (-1)^{11} \cdot i$

$= 1 - 1 \cdot i = 1 - i$.

---

e - Conjugates and Division:- (ص ٧٦٣ وقم)

EX. No. 5 → Multiply

1) $(5 + 7i)(5 - 7i) = 5^2 - (7i)^2$

$= 25 - 49i^2 = 25 - 49(-1)$

$= 74$

---

f - Solutions of Equations:- (ص ٧٦٦ وقم)

EX. No. 6 → Determine whether $1+i$ is a solution of the equation $x^2 - 2x + 2 = 0$

we substitute $1+i$ for x in the equation.

$x^2 - 2x + 2 = 0$

$(1+i)^2 - 2(1+i) + 2 = 0$

$1 + 2i + i^2 - 2 - 2i + 2 = 0$

$1 + i^2 = 0$ $\quad 1 + (-1) = 0$ True.

Exam . No. 1o اختبار تجريبي على الفصل العاشر

1- use acalculator to approximate $\sqrt{148}$ to three decimal places.

2 - For the given function, find theindicated function.
$f(x) = \sqrt{8-4x}$; $f(1)$ and $f(3)$.

3 - simplify :.
* $\sqrt{(-39)^2}$
* $\sqrt[5]{x^5}$
* $\sqrt[16]{(-4)^{10}}$

4 - Add :. $3\sqrt{128} + 2\sqrt{18} + 2\sqrt{32}$

5 - Multiply :. * $(3+\sqrt{x})^2$
* $(\sqrt{20}+2\sqrt{5})(\sqrt{20}-3\sqrt{5})$

6 - solve :.
* $\sqrt{x-6} = \sqrt{x+9} - 3$
* $\sqrt{x-1} + 3 = x$

7 - Multiply :. * $(3-4i)(3+7i)$
* i^{95}

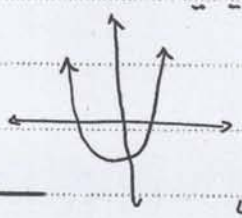
8 - Divide :. $\frac{-7+14i}{6-8i}$

Chapter (11)

Quadratic Equations and Functions:.

The Basics of solving Quadratic Equations (11 - 1)

$f(x) = ax^2 + bx + c \quad a \neq 0$ الدالة التربيعية



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a - The principle of square Roots:.

EX. No. 1 → 1) solve: $x^2 = 25$

$x^2 - 25 = 0$

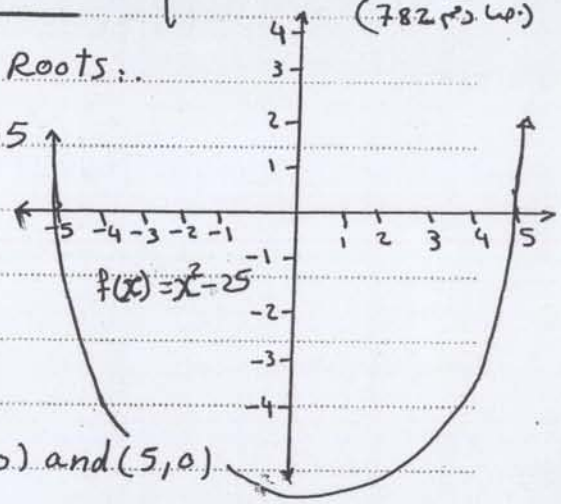
$(x + 5)(x - 5) = 0$

$x = -5 \quad or \quad x = 5$

The solution are 5, -5

2) $f(x) = x^2 - 25$ are $(-5, 0)$ and $(5, 0)$

(782 ص)



EX. No. 2 → solve $-5x^2 + 2 = 0$

$-5x^2 = -2$

$x^2 = \frac{2}{5} \qquad x = \sqrt{\frac{2}{5}} \quad or \quad x = -\sqrt{\frac{2}{5}}$

$x = \frac{\sqrt{10}}{5} \quad or \quad -\frac{\sqrt{10}}{5}$

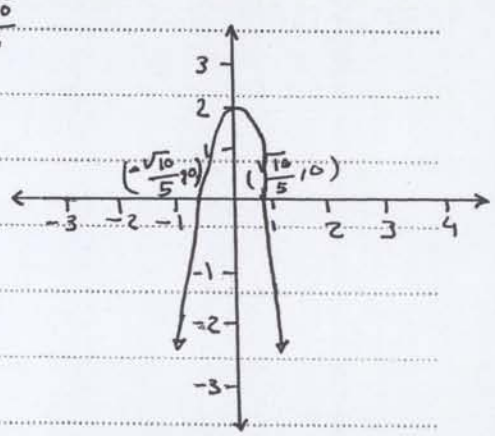
$-5\left(\pm\frac{\sqrt{10}}{5}\right)^2 = -2$

$-5\left(\frac{10}{25}\right) = -2$

$-\frac{10}{5} = -2$

$-2 = -2$ True

The solution are $\frac{\sqrt{10}}{5}$ and $-\frac{\sqrt{10}}{5}$



b - Completing the square:.. (ص رقم 787)

EX. No. 3 → Solve $x^2 - 6x + 8 = 0$

$x^2 - 6x = -8$

$x^2 - 6x + 9 = -8 + 9$ بإضافة 9+ للطرفين

$(x - 3)^2 = 1$

$x - 3 = 1$ or $x - 3 = -1$

$x = 4$ or $x = 2$

The solution are 4 and 2.

C - Applications and problem solving:. (ص رقم 791)

EX. No. 4 → Hang Time. one of the most exciting plays in basket ball is the dunk shot. The amount of time T That passes from the moment a player leaves the ground, goes up, makes the shot, and arrives back on the ground. is called hang time. A function relating an athlete's vertical leap V, in inches, to hang time T, in seconds, in given by $(V)\ T = 48T^2$

$T = 0.889$ sec.

$V(0.889) = 48(0.889)^2 \simeq 37.9$ in

whe $v = 44$

$44 = 48T^2$

$T^2 = \frac{44}{48}$

$T \simeq 0.957$

The Quadratic Formula (11.2).

a - Solving using the Quadratic Formula. (796 ص، وص)

The solution of $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

EX. No.1 → solve $5x^2 + 8x = -3$ using the quadratic formula.

$5x^2 + 8x + 3 = 0$

$a = 5 \quad , b = 8 \quad , c = 3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-8 \pm \sqrt{64 - 4 \cdot 5 \cdot 3}}{2 \cdot 5} =$$

$$x = \frac{-8 \pm \sqrt{64 - 60}}{10}$$

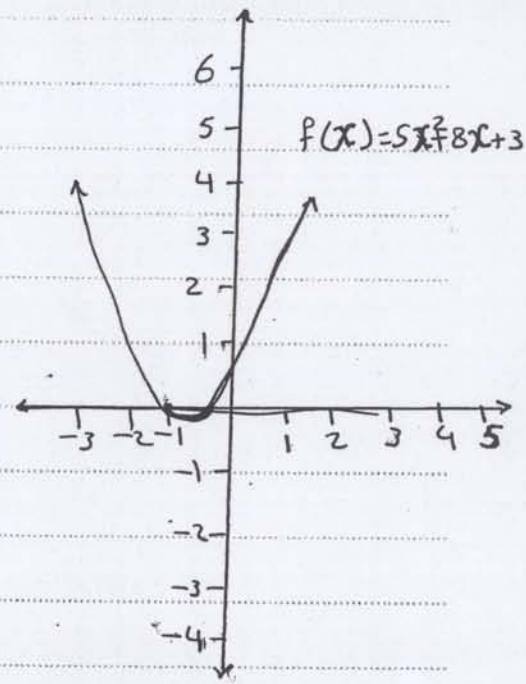
$$x = \frac{-8 \pm 2}{10} =$$

$$x = \frac{-8 + 2}{10} \quad \text{or} \quad x = \frac{-8 - 2}{10}$$

$$x = \frac{-6}{10} \quad \text{or} \quad x = \frac{-10}{10}$$

$$x = \frac{-3}{5} \quad \text{or} \quad x = -1$$

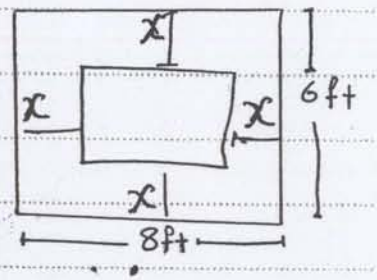
The solution are $\frac{-3}{5}$ and -1.



Applications Involving Quadratic Equations. (11-3).

a- Applications and problem solving:→ (صارق 803)

EX. No. 1 → Quilt Dimensions. Michelle is making a quilt for a wall hanging at the entrance of a state museum. The finished quilt will measure 8 ft by 6 ft. The quit has a border of uniform width around it. The area of the interior rectangular section is one-half the area of the entire quilt. How wide is the border



Area entire quilt = 8.6

Area of interior section

$= (8-2x)(6-2x)$

interior section is one-Half

The Area of the entire quilt.

$(8-2x)(6-2x) = \frac{1}{2} \cdot 8 \cdot 6$

$48 - 12x + 4x^2 - 16x = 24$

$4x^2 - 28x + 24 = 0$ بالعشرة على 4

$x^2 - 7x + 6 = 0$

$(x - 1)(x - 6) = 0$

$x - 1 = 0$ or $x - 6 = 0$

$x = 1$ or $x = 6$

check → $(8-2x) = (8-2) = 6$ ft

length = 6 ft , wide = 4 ft

$(8-2x) = (6-12) = -6$ ft

$(6-2) = 4$ ft

State → The border of the quilt is 1 ft wide.

b- Solving Formulas.. (ص رقم 806)

EX. No. 2 → Falling Distance. An object that is tossed downward with an initial speed (velocity) of v_0 will travel a distance of s meters, where $s = 4.9t^2 + v_0t$ and t is measured in seconds, solve for t.

$$4.9t^2 + v_0t = s$$

$$4.9t^2 + v_0t - s = 0$$

$$a = 4.9 \quad , b = v_0 \quad , c = -s$$

use $$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-v_0 \pm \sqrt{(v_0)^2 - 4(4.9)(-s)}}{2(4.9)}$$

$$t = \frac{-v_0 \pm \sqrt{(v_0)^2 + 19.6s}}{9.8}$$

More on Quadratic Equations (11.4).

a - The Discriminat :. (815 وقم ص)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ → use $b^2 - 4ac$.

Ex. No. 1 → Determine the nature of solutions of

$9x^2 - 12x + 4 = 0$.

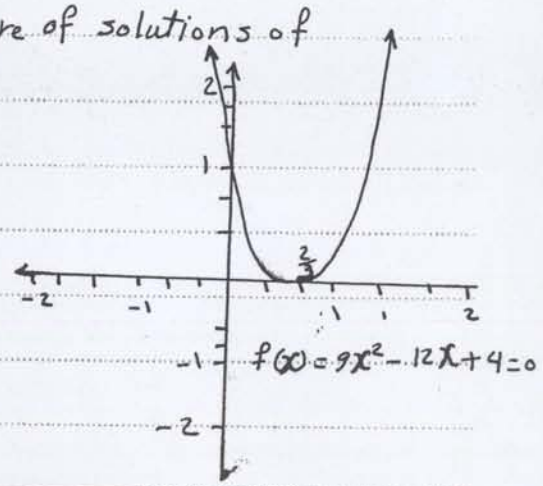
$a = 9 \quad b = -12 \quad c = 4$

we compute the discriminant

$b^2 - 4ac = (-12)^2 - 4 \cdot 9 \cdot 4$

$= 144 - 144 = 0$

when $y = 0 \quad x = \frac{2}{3}$.



b - writing Equations from Solutions:.

Ex. No. 2 → Find aquadratic equation whose solutions are 3 and $\frac{-2}{5}$.

$x = 3 \quad or \quad x = -\frac{2}{5}$

$x - 3 = 0 \quad or \quad x + \frac{2}{5} = 0$

$x - 3 = 0 \quad or \quad 5x + 2 = 0$

$(x - 3)(5x + 2) = 0$

$5x^2 - 15x + 2x - 6 = 0$

$5x^2 - 13x - 6 = 0$.

EX. No. 3 → write aquadratic equation whose solution are $\sqrt{3}$ and $-2\sqrt{3}$

$x = \sqrt{3}$ or $x = -2\sqrt{3}$

$x - \sqrt{3} = 0$ or $x + 2\sqrt{3} = 0$

$(x - \sqrt{3})(x + 2\sqrt{3}) = 0$

$x^2 - \sqrt{3}x + 2\sqrt{3}x - 2.3 = 0$

$x^2 + \sqrt{3}x - 6 = 0$.

---

c- Equations Quadratic in form.. (u)

EX. No 4 → Solve $x^4 - 9x^2 + 8 = 0$

let $u = x^2$

$u^2 - 9u + 8 = 0$

$(u - 8)(u - 1) = 0$

$u = 8$ or $u = 1$

---

* solve :→ $y^{-2} - y^{-1} - 2 = 0$

let $u = y^{-1}$, $u^2 = y^{-2}$

$u^2 - u - 2 = 0$

$(u - 2)(u + 1) = 0$

$u = 2$ or $u = -1$

$u = y^{-1}$

$2 = \frac{1}{y}$ $\therefore y = \frac{1}{2}$

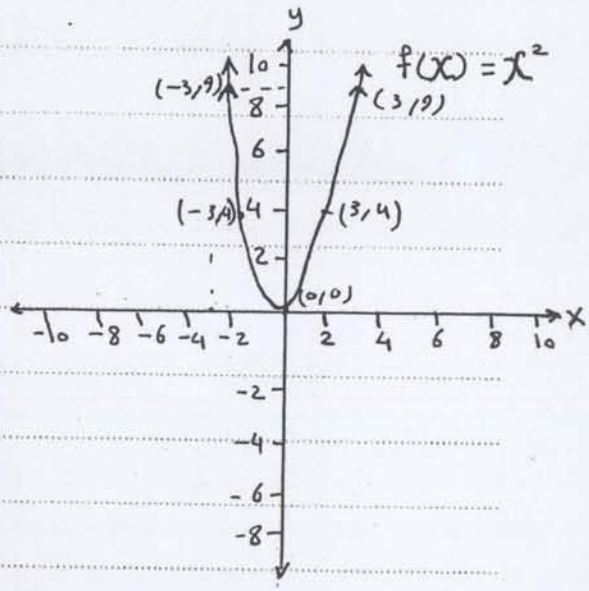
$\frac{1}{y} = -1$ $\therefore y = -1$

Graphing $f(x) = a(x-h)^2 + k$. (11-5).

a- Graphs of $f(x) = ax^2$.

EX. No. 1 → Graph. $f(x) = x^2$

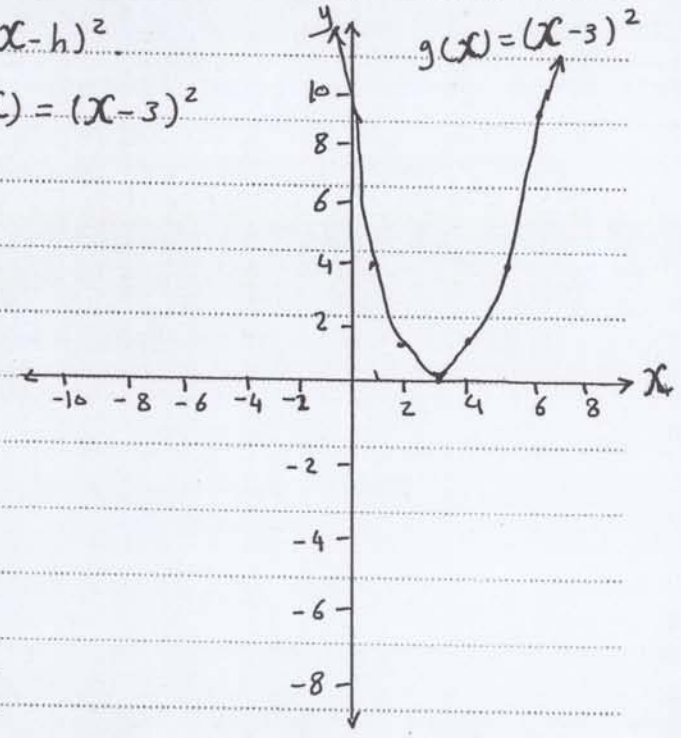
x	$f(x) = x^2$	$(x, f(x))$
-3	9	(-3, 9)
-2	4	(-2, 4)
-1	1	(-1, 1)
0	0	(0, 0)
1	1	(1, 4)
2	4	(2, 4)
3	9	(3, 9)



b- Graphs of $f(x) = a(x-h)^2$.

EX. No. 2 → Graph $g(x) = (x-3)^2$

x	$g(x) = (x-3)^2$
3	0
4	1
5	4
6	9
2	1
1	4
0	9



C- Graphs of $f(x) = a(x-h)^2 + k$. (ص رقم 829)

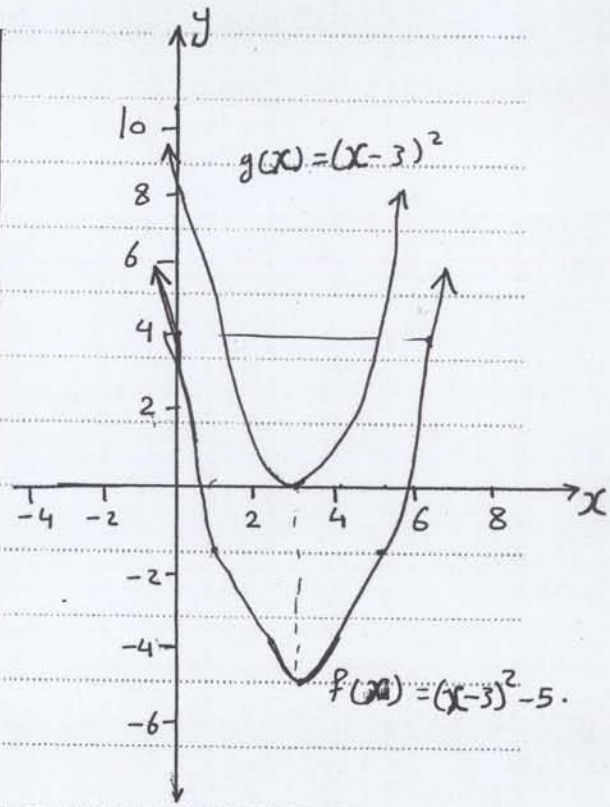
EX. No. 3 → Graph $f(x) = (x-3)^2 - 5$, and find the minimum function value.

$f(4) = (4-3)^2 - 5 = -4$

$g(4) = (4-3)^2 = 1$

$\therefore h = 3$, $k = -5$ $(3, -5)$

x	$f(x) = (x-3)^2 - 5$
3	-5
4	-4
5	-1
6	4
2	-4
1	-1
0	4



Graphing $f(x) = ax^2 + bx + c$. (11.6)

a- Analyzing and Graphing $f(x) = ax^2 + bx + c$. (835 صفحة)

EX. No. 1 → For $f(x) = 3x^2 + 12x + 13$, find the vertex, the line of symmetry, and the maximum or the minimum value, Then graph. since the coefficient of x^2 is not 1, we factor out 3 from only the first two terms of the expression, Remember that we want to get to the form

$f(x) = a(x-h)^2 + k$

$f(x) = 3x^2 + 12x + 13$

$= 3(x^2 + 4x) + 13$

$f(x) = 3(x^2 + 4x + 4 - 4) + 13$

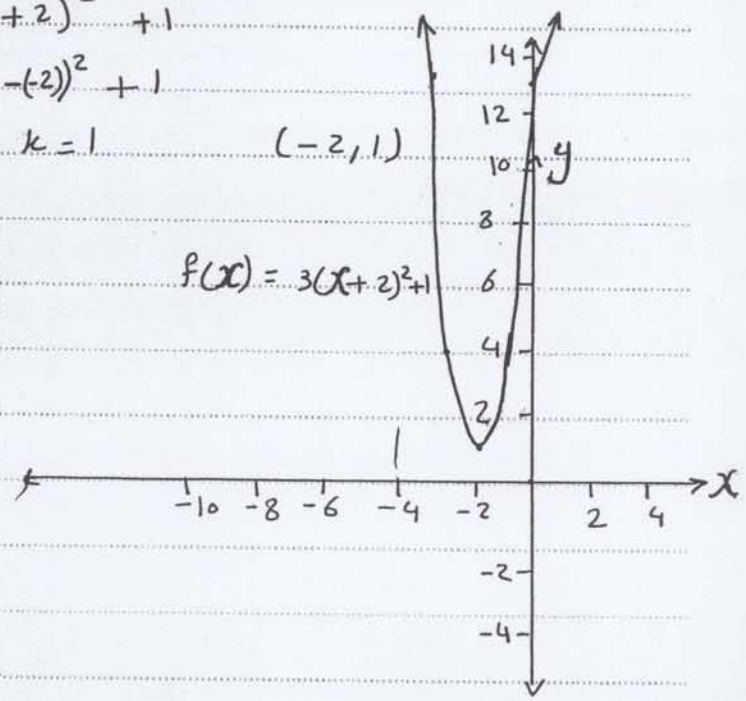
$= 3(x^2 + 4x + 4) + 13 + (3 \cdot -4)$

$= 3(x+2)^2 + 1$

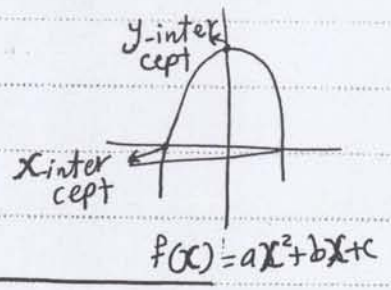
$= 3(x-(-2))^2 + 1$

$h = -2$, $k = 1$ $(-2, 1)$

x	$f(x)$
-2	1
-1	4
-3	4
0	13
-4	13



6- Finding the inter cepts of a Quadratic Function:



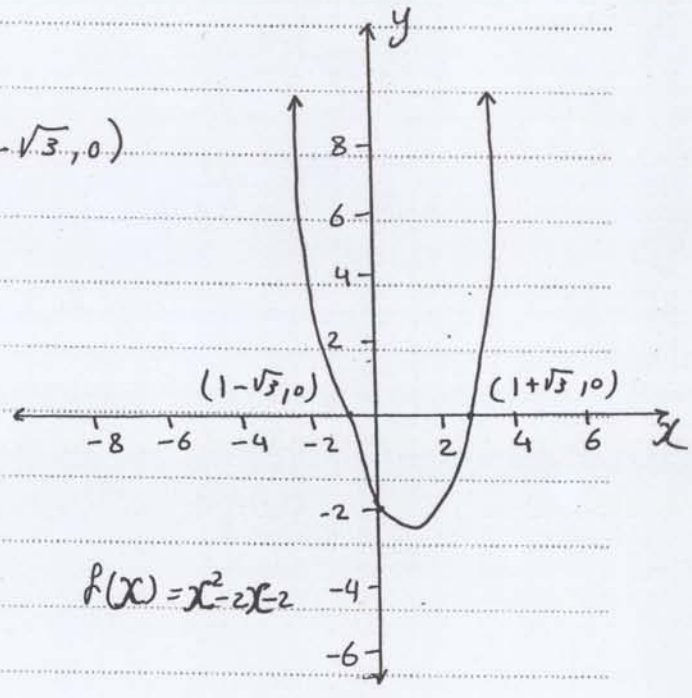
EX. No. 2 → Find the inter cepts of $f(x) = x^2 - 2x - 2$

The y-intercept is $(0, f(0))$. since $f(0) = 0^2 - 2.0 - 2 = -2$ the y-intercept is $(0, -2)$. To find the x-intercepts, we solve

$a = x^2 - 2x - 2$.

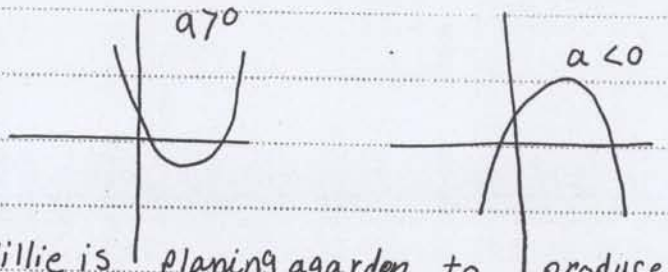
$x = 1 \pm \sqrt{3}$.

x-intercepts are $(1-\sqrt{3}, 0)$ and $(1+\sqrt{3}, 0)$



Mathematical Modeling with Quadratic Functions. (11.7).

a - Maximum - Minimum problems. (ص وقت 8 44)



Ex. No: 1 →

Bordered Garden, Millie is planing a garden to produce vegetables and fruit for the local food bank. she has enough raspberry plants to edge a 64-yd perimeter and wants to maximize the area within to plant the most vegtables possible. what are the dimensions of the largest rectangular garden that Millie can enclose with the raspberry plants?

perimeter: $2L + 2w$

Area: $L \cdot w$

$2L + 2w = 64$

$2L = 64 - 2w$

$L = \frac{64 - 2w}{2}$

$L = 32 - w$.

بالتعويض في معادلة المساحة

* $A = L \cdot w$

$A = (32 - w)w$.

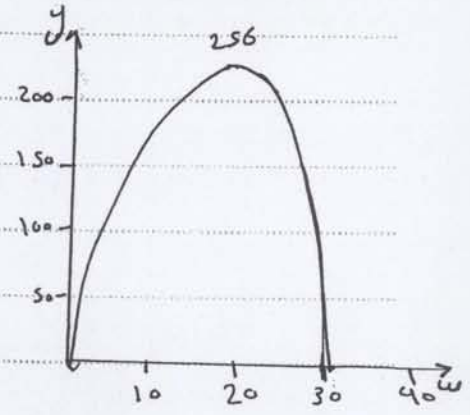
$= 32w - w^2$

$A = -(w^2 - 32w)$

$= -(w^2 - 32w + 256 - 256)$

$= -(w - 16)^2 + 256$.

$w = 16$, $L = 32$



Polynomial Inequalities and Rational Inequalities (11.8).

a - Quadratic and other Polynomial Inequalities :- (ص 855 وقد)

Ex. No. 1 → Solve $x^2 + 3x - 10 \geq 0$

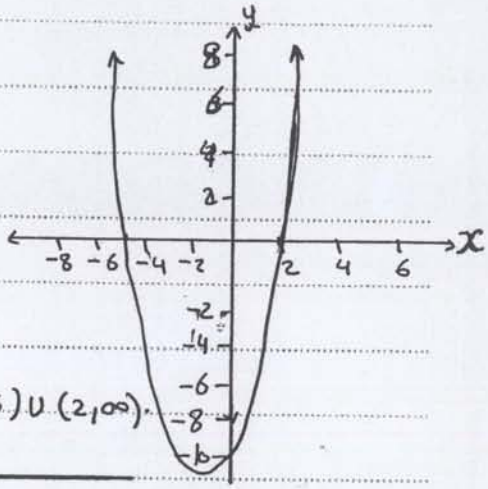
$x^2 + 3x - 10 = 0$

$(x+5)(x-2) = 0$

$x + 5 = 0$ or $x - 2 = 0$

$x = -5$ or $x = 2$

$\{x \mid x \leq -5 \text{ or } x \geq 2\}$ or $(-\infty, -5] \cup [2, \infty)$.



b - Rational Inequalities :-

Ex. No. 2 → Solve $\frac{x-3}{x+4} \geq 2$

we change $\geq$ to $=$

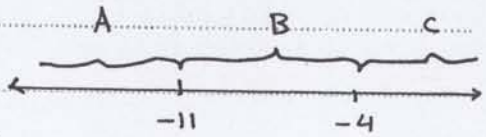
$\frac{x-3}{x+4} = 2$

$(x+4) \cdot \frac{x-3}{x+4} = 2 \cdot (x+4)$

$x - 3 = 2x + 8$

$-11 = x$

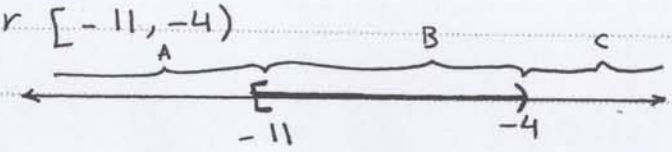
$x + 4 = 0$ $\quad x = -4$.



* Test -8 $\quad \frac{-8-3}{-8+4} = \frac{11}{4} \geq 2$ True

Test 1 $\quad \frac{1-3}{1+4} = \frac{-2}{5} \not\geq 2$ False

$\therefore \{x \mid -11 \leq x < -4\}$ or $[-11, -4)$



Exam . No. 11. اختبار تجريبي على الفصل الحادى عشر

1 - solve : $2x^2 - 4 = 0$

* Find the x-intercepts of $F(x) = 3x^2 - 4$

---

2 - solve : * $x^2 + x + 1 = 0$.

* $4x(x-2) - 3x(x+1) = -18$.

---

3 - solve : $V = 48T^2$ for T.

---

4 - Determine the natural of the solutions of the equation : $x^2 + 5x + 17 = 0$.

---

5 - solve : $x^2 - 4x + 1 = 0$ by completing the square show your work.

---

6 - $x^2 + 4x = 2$. Give exact solutions and approximate solutions to three decimal places.

---

7 - Free Falling Objects. The peachtree plaza in Atlanta Georgia, is 732 ft tall. Use the function $s(t) = 16t^2$ to approximate how long it would take an object to fall from the top.

chapter (12)

Exponential and Logarithmic Functions.

الأسس واللوغاريتمات .

Exponential Functions: → (12-1).

a- Graphing Exponential Functions. (874 ص)

Ex. No. 1 → Graph the exponential function $f(x) = 2^x$

$f(0) = 2^0 = 1$

$f(1) = 2^1 = 2$

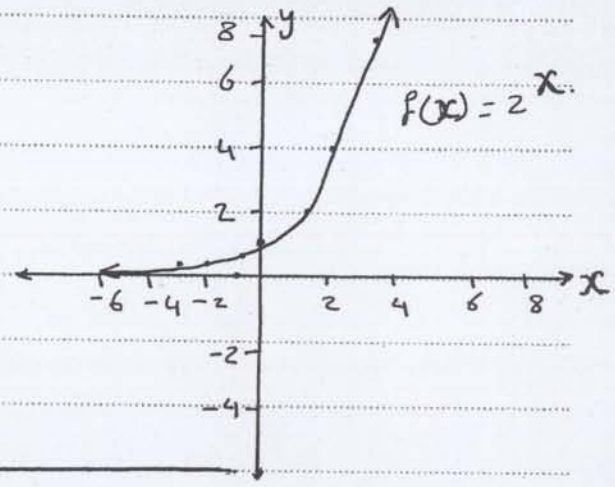
$f(2) = 2^2 = 4$

$f(3) = 2^3 = 8$

$f(-1) = 2^{-1} = \frac{1}{2}$

$f(-2) = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

$f(-3) = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$



Ex. No. 2 → Graph $f(x) = 2^{x-2}$

$f(0) = 2^{0-2} = 2^{-2} = \frac{1}{4}$

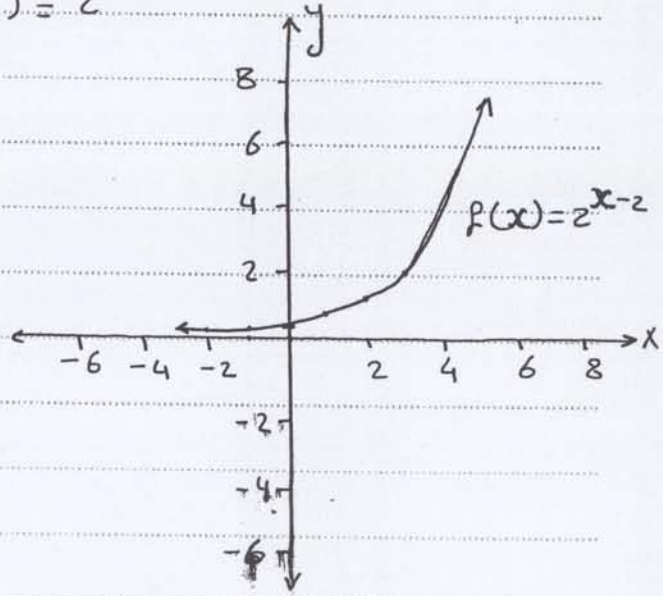
$f(1) = 2^{1-2} = 2^{-1} = \frac{1}{2}$

$f(2) = 2^0 = 1$

$f(3) = 2^1 = 2$

$f(-1) = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

$f(-2) = 2^{-4} = \frac{1}{2^4} = \frac{1}{16}$



b - Equations with x and y Interchanged... (صفحة 879)

Ex. No. 3 → Graph $x = 2^y$

For $y = 0$, $x = 2^0 = 1$.

For $y = 1$, $x = 2^1 = 2$.

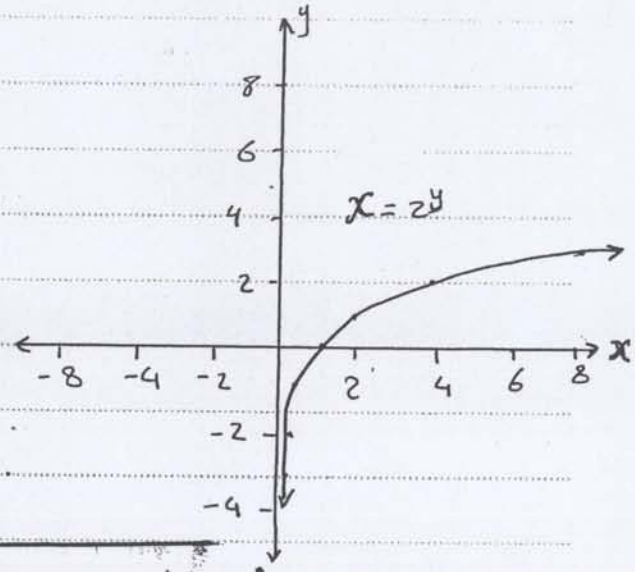
For $y = 2$, $x = 2^2 = 4$.

For $y = 3$, $x = 2^3 = 8$.

For $y = -1$, $x = 2^{-1} = \frac{1}{2}$.

For $y = -2$, $x = 2^{-2} = \frac{1}{4}$.

For $y = -3$, $x = 2^{-3} = \frac{1}{8}$.



C - Applications of Exponential Functions... (صفحة 880)

Ex. No. 4 → The Ibsens invest $4000 in an account paying $2\frac{5}{8}\%$ compounded quarterly. Find the amount in the account after $2\frac{1}{2}$ years.

$A = P\left(1 + \frac{r}{n}\right)$

$P = 4000$

$r = 2\frac{5}{8}\%$

$n = 4$

$t = \frac{5}{2}$

$A = 4000\left(1 + \frac{2\frac{5}{8}\%}{4}\right)^{4 \cdot \frac{5}{2}}$

$A = 4000\left(1 + \frac{0.02625}{4}\right)^{10}$

$A = 4000\,(1.00656\,25)^{10}$

$A = \$\,4270.39$.

The amount in the account after $2\frac{1}{2}$ years is $4270.39.

Inverse Functions and composite Functions (12-2).

a- Inverse:.. (888 صـ)

Ex. No. 1 → Find an equation of the inverse of the relation

$y = 3x - 4$

inverse $x = 3y - 4$.

for $y = 0$, $x = 3.0 - 4 = -4$

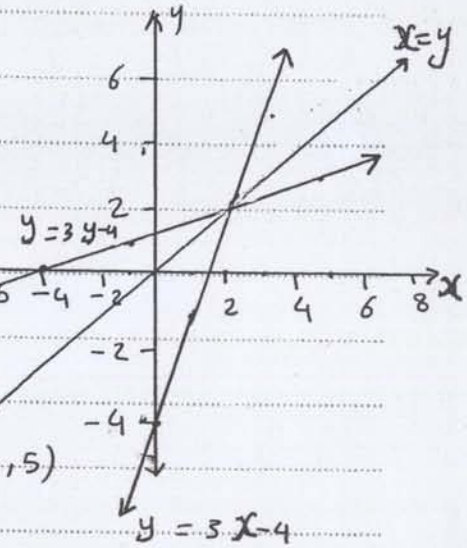
for $y = 1$, $x = 3.1 - 4 = -1$

for $y = 2$, $x = 3.2 - 4 = 2$

for $y = 3$, $x = 3.3 - 4 = 5$.

Relation y, x نعكس

$(0,-4)$, $(1,-1)$, $(2,2)$, $(3,5)$



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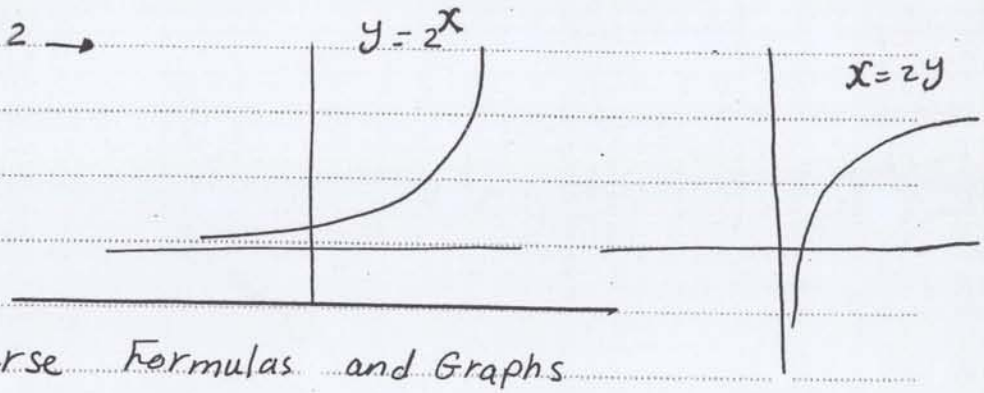
b- Inverses and One-To-One Functions:..

Ex. No. 2 →



c- Inverse Formulas and Graphs

Ex. No. 3 → Given $f(x) = 2x - 3$.

a) Determine whether the function is one-to-one.

b) If it is one-to-one. find a formula for $f^{-1}(x)$.

c) Graph the inverse function, If it exists.

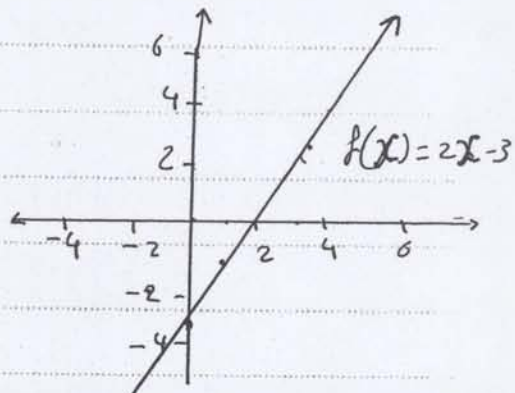
a) Graph $f(x) = 2x - 3$.

$f(0) = 2 \cdot 0 - 3 = -3$

$f(1) = 2 - 3 = -1$

$f(2) = 4 - 3 = 1$

$f(3) = 6 - 3 = 3$.



b) Replace $f(x)$ with y.

$y = 2x - 3$

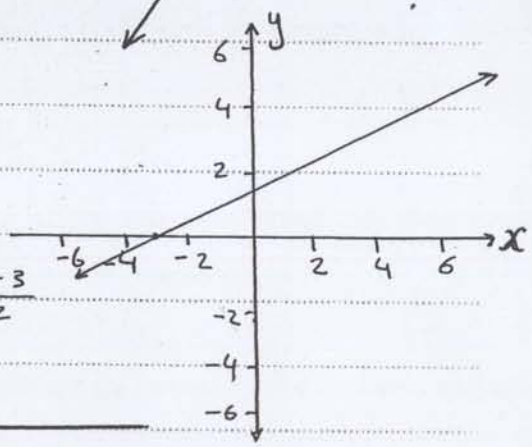
$x = 2y - 3$

$x + 3 = 2y$

$y = \frac{x+3}{2}$

we Replace y with $f^{-1}(x) = \frac{x+3}{2}$

$y = \frac{1}{2}x + \frac{3}{2}$



---

d_ Composite Functions.. (890 ص د ق)

Ex. No. 4 → Given $f(x) = \sqrt{3}$ and $g(x) = x - 1$

find $(f.g)(x)$ and $(g.f)(x)$.

$(f.g)(x) = f(g(x)) = f(x-1) = \sqrt{x-1}$.

$(g.f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt{x} - 1$.

---

e_ Inverse Function and composition.. (892 ص د ق)

Ex. No. 5 → let $f(x) = 2x - 3$ use composition to show that $f^{-1}(x) = \frac{x+3}{2}$

$(f^{-1} \cdot f)(x) = f^{-1}(f(x) = 2x - 3$

$= \frac{(2x-3)+3}{2}$

$= \frac{2x}{2} = x$

$(f^{-1} \cdot f)(x) = x$.

Logarithmic Functions (12.3)

الدوال اللوغاريتمية

a- Graphing Logarithmic Functions: (ص 905 وجه)

EX. No. 1 → Graph $y = f(x) = \log_5 x$.

For $y = 0$, $x = 5^0 = 1$

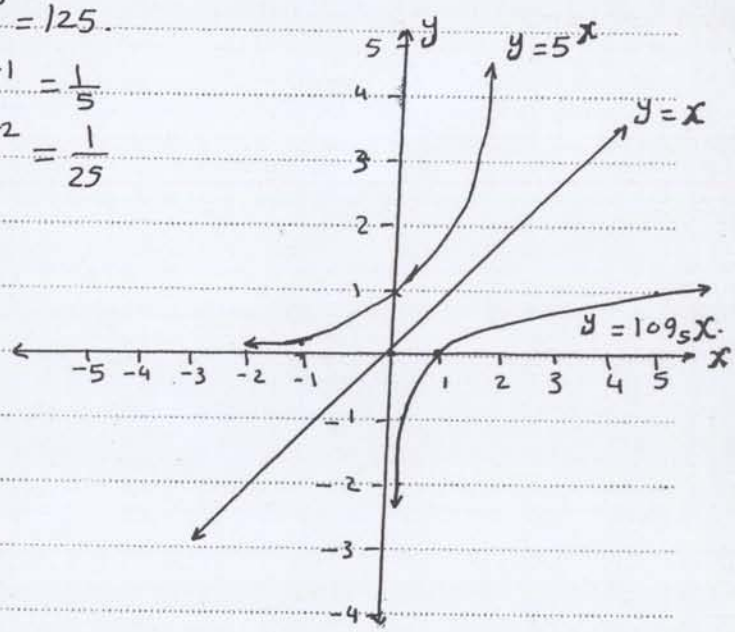
For $y = 1$, $x = 5^1 = 5$

For $y = 2$, $x = 5^2 = 25$

For $y = 3$, $x = 5^3 = 125$.

for $y = -1$, $x = 5^{-1} = \frac{1}{5}$

for $y = -2$, $x = 5^{-2} = \frac{1}{25}$



$\log_5 1 = 0$

$\log_5 5 = 1$

$\log_5 25 = 2$

$\log_5 125 = 3$

$\log_5 \frac{1}{5} = -1$

$\log_5 \frac{1}{25} = -2$

---

b- Converting Between Exponential Equations and logarithmic Equations: $y = \log_a x \longrightarrow a^y = x$

EX. No → 2 → Convert to logarithmic equation

1) $8 = 2^x \longrightarrow x = \log_2 8$.

2) $y^{-1} = 4 \longrightarrow -1 = \log_y 4$.

3) $a^b = c \longrightarrow b = \log_a c$.

4) $y = \log_3 5 \longrightarrow 3^y = 5$.

C- Solving Certain logarithmic Equations:... (ص ٥٥٨ وقم ٥٥٨)

EX. No. 3 → solve $\log_2 x = -3$

$\log_2 x = -3$

$x = 2^{-3}$

$x = \frac{1}{2^3}$

$x = \frac{1}{8}$.

---

* $\log_x 16 = 2$

$16 = x^2$

$x = 4$ or $x = -4$.

all Logarithm bases must be positive

$x = 4$ is a solution, $x = -4$ is not solution.

---

* EX. No. 4 → Find $\log_{10} 1000$

let $\log_{10} 1000 = x$

$10^x = 1000$

$10^x = 10^3$

$x = 3$.

$\therefore \log_{10} 1000 = 3$

---

* $\log_5 1$

let $\log_5 1 = x$

$5^x = 1$

$5^x = 5^0$

$x = 0$ $\therefore \log_5 1 = 0$

properties Logarithmic Functions (12.4)

a- Logarithms of products:. (ص 916 رقم)

$\log (M \cdot N) = \log M + \log N$.

EX. No. 1 → Express as asum of logarithms.

1) $\log_2 (4 \cdot 16)$

$\log_2 (4 \cdot 16) = \log_2 4 + \log_2 16$

---

2) $\log_{10} 0.01 + \log_{10} 1000 = \log_{10} (0.01 \cdot 1000)$

$= \log_{10} 10$.

---

b- Logarithms of powers:.

$\log_a M^k = k \log_a M$.

EX. No. 2 → EXpress as aproduct.

1) $\log_a 9^{-5} = -5 \log_a 9$.

---

2) $\log_a \sqrt[4]{5} = \log_a 5^{\frac{1}{4}} = \frac{1}{4} \log_a 5$.

---

c- Logarithms of Quotients:.

$\log_a \frac{M}{N} = \log_a M - \log_a N$.

---

* $\log_t \frac{6}{u} = \log_t 6 - \log_t u$.

---

* $\log_b 17 - \log_b 27 = \log_b \frac{17}{27}$.

d- Using the properties Together... (ص 918)

Ex. No. 3 → Express in terms of logarithms of w., x, y and z

1) $\log_a \frac{x^2 y^3}{z^4} = \log_a (x^2 y^3) - \log_a z^4$

$= \log_a (x^2) + \log_a (y^3) - \log_a z^4$

$= 2\log_a x + 3\log_a y - 4\log_a z$.

---

2) $\log_a \sqrt[4]{\frac{xy}{z^3}} = \log_a \left(\frac{xy}{z^3}\right)^{\frac{1}{4}}$

$= \frac{1}{4}\log_a \frac{xy}{z^3} = \frac{1}{4}(\log_a xy - \log_a z^3)$

$= \frac{1}{4}(\log_a x + \log_a y - 3\log_a z)$

$= \frac{1}{4}\log_a x + \frac{1}{4}\log_a y - \frac{3}{4}\log_a z$.

---

3) $\frac{1}{2}\log_a x - 7\log_a y + \log_a z$

$= \log_a x^{\frac{1}{2}} - \log_a y^7 + \log_a z$

$= \log_a \frac{\sqrt{x}}{y^7} + \log_a z$

$= \log_a \frac{z\sqrt{x}}{y^7}$.

---

e- The Logarithm of the Base to a power.

$\log_a a^k = k$. , for any base a.

---

1) $\log_3 3^7 = 7$.

2) $\log_{10} 10^{5.6} = 5.6$.

3) $\log_e e^{-t} = -t$.

Natural Logarithmic Functions (12.5).

a - The Base e and Natural Logarithms:. (924 صفحة)

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$A(n) = \left(1-\frac{1}{n}\right)^{n}$

$e \simeq 2.71828$ by calculator.

EX. No. 1 → Find $e^{12.5691}$ using a calculator

we might enter 12.5691 and press (ex) on a graphing calculator, we press (2ND) (ex), followed by 12.5691 (enter)

$e^{12.5691} \simeq 287,535.0371$.

---

b- Changing Logarithm Bases:.

$\text{Log}_b M = \frac{\text{Log}_a M}{\text{Log}_a b}$.

EX. No. 2 → Find $\log_4 7$ using common logarithms.

we let $a = 10$, $b = 4$, $M = 7$

$\text{Log}_b M = \frac{\text{Log}_a M}{\text{Log}_a b}$

$\text{Log}_4 7 = \frac{\text{Log}_{10} 7}{\text{Log}_{10} 4} = \frac{\log 7}{\log 4} \simeq 1.4037$.

---

* $\log_5 29$ using Natural logarithms.

$\log_5 29 = \frac{\log_e 29}{\log_e 5} = \frac{\ln 29}{\ln 5} \simeq 2.0922$.

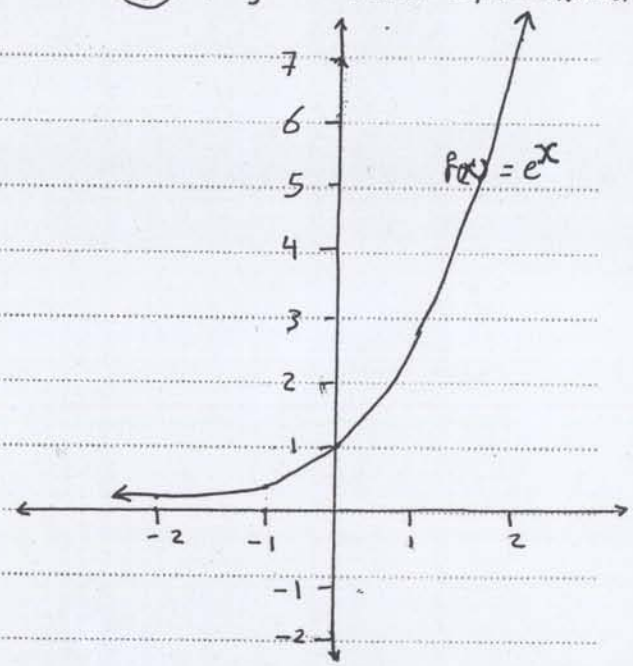
C – Graphs of exponential and Logarithmic Functions, Base e.

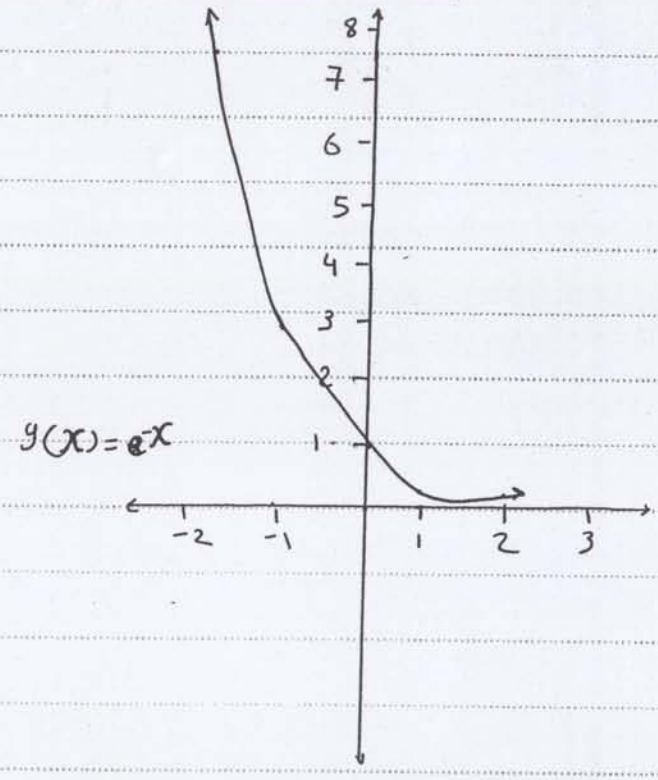
(الدوال الأسية واللوغاريتمية، الأساس e)

EX. No. 3 → Graph $f(x) = e^x$ and $g(x) = e^{-x}$.

We use a calculator with an e^x key to find approximate values of e^x and e^{-x}

x	e^x	e^{-x}
0	1	1
1	2.7	0.4
2	7.4	0.1
−1	0.4	2.7
−2	0.1	7.4





Solving Exponential and Logarithmic Equations (12-6)

a - Solving Exponential Equations: (صفحة 933)

$a^x = a^y \quad \therefore x = y$ قاعدة لو الأساس = الأساس ∴ الأس = الأس

Ex. No. 1 → Solve $2^{3x-5} = 16$.

$2^{3x-5} = 2^4$, is the same base

$3x - 5 = 4$

$3x = 9 \qquad x = 3$

check $2^{9-5} = 2^4 = 16$ True.

The solution is 3.

Ex. No. 2 → Solve $5^x = 12$

$5^x = 12$

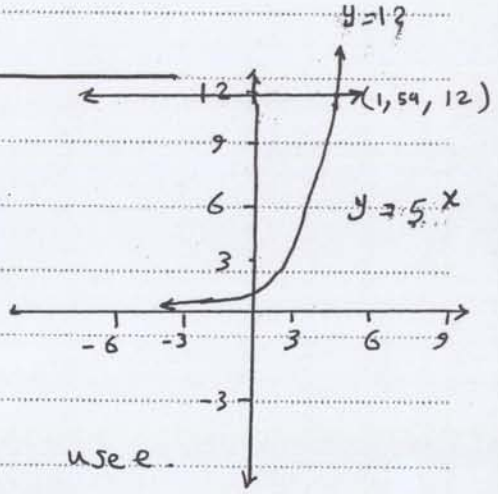
$\log 5^x = \log 12$

$x \log 5 = \log 12$

$x = \frac{\log 12}{\log 5}$

$x \simeq 1.5440$.

check: $5^{1.5440} \simeq 12.000785$.



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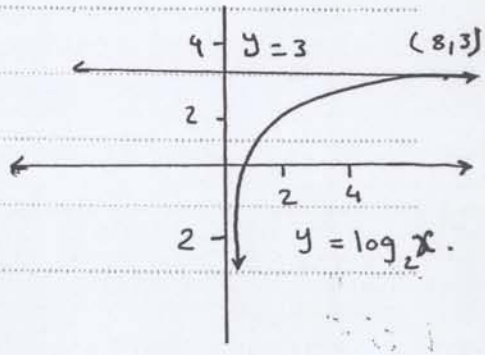
b - solving logarithmic Equations.

Ex. No. 3 → solve $\log_2 x = 3$

$x = 2^3$

$\therefore x = 8$

The soution is 8.



Mathematical Modeling with Exponential and Logarithmic Functions (12.7).

a- Applications of Logarithmic Functions... (صفحة 940)

Ex. No. 1 → sound levels. To measure the "loudness" of any particular sound, the decibel scale is used. The Loudness (I) in decibels (dB) of a sound is given by

$L = 10 \cdot \log \frac{I}{I_0}$ $\qquad I_0 = 10^{-12}\ w/m^2$

a volume can damage the hearing $= 10^{-5}\ w/m^2$

$L = 10 \log \frac{I}{I_0}$ $\qquad L = 90\ dB$

$$L = 10 \log \frac{10^{-0.5}}{10^{-12}}$$

$$= 10 \cdot \log 10^{11.5}$$

$$= 10 \cdot 11.5 = 115.$$

The solution is 115 decibels.

* we substitute and solve for: I

$L = 10 \log \frac{I}{I_0}$

$90 = 10 \cdot \log \frac{I}{10^{-12}}$

$9 = \log \frac{I}{10^{-12}}$

$9 = \log I - \log 10^{-12}$

$9 = \log (I - (-12))$ $\qquad$ Add -12

$-3 = \log I$

$I = 10^{-3}$.

b- Applications of exponential Functions. (942 ص)

EX. No. 2 → Interest Compalended Amnually.

Suppose that $ 30.000 is invested at 4% interest, Compounded annually. In t years. it will grow to the amount A given by the function.

$$A(t) = 30000(1.04)^t$$

we set $A(t) = 150,000$

$$150000 = 30000(1.04)^t$$

$$(1.04)^t = \frac{150000}{30000}$$

$$(1.04)^t = 5$$

$$\log 5 = \log(1.04)^t$$

$$\log 5 = t \log(1.04)$$

$$t = \frac{\log 5}{\log 1.04} \qquad \therefore t \simeq 41.04$$

about 41 years for the $3000 to grow to $150000

* we replace $A(t) = 60000$

$$60000 = 30000(1.04)^T$$

$$(1.04)^T = \frac{60000}{30000}$$

$$(1.04)^T = 2$$

$$\log(1.04)^T = \log 2$$

$$T \log(1.04) = \log 2$$

$$T = \frac{\log 2}{\log(1.04)}$$

$$T \simeq 17.7$$

The doubling time is about 17.7 years.

Exam. No. 12 اختبار تجريبي على الفصل الثاني عشر

1 - Graph

* $f(x) = 2^{x+1}$
* $f(x) = e^{x-2}$

2- Find the inverse of the relation $\{(-4, 3), (5, -8), (-1, -3), (10, 12)\}$

3- Find $(f \cdot g)$ and $(g \cdot f)(x)$ if $f(x) = x + x^2$ and $g(x) = 5x - 2$

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4 - Convert to algarithmic equation.

$256^{\frac{1}{2}} = 16$

---

5 - Find each of the following

* $\log_5 125$
* $\log_t t^{23}$

6- Tomatoes . what is th pH of tomatoes if the hydrogen ion concentration is 6.3×10^{-5} moles per Liter ? (use $pH = -\log(H^+)$

---

7 - solve :- * $\log_x 25 = 2$

* $\log(x^2 - 1) - \log(x - 1) = 1$
* $\log_5 x \div \log_5 (x + 4) = 1$