ENG. Mahdi Alsiddig
0563938879

3.1 Maxima and Minima

- لإيجاد اقصى وادنى نقطة max and min :

1- نوجد النقاط الحرجة **critical points** :

نوجد المشتقة الاولى للدالة ونساويها بالصفر , ونوجد قيم x.

2- نعوض في الدالة الاصلية بقيم النقاط الحرجة , ونقارن بين القيم , اكبر قيمة تكون هي max واقل قيمة هي min.

Example 1 : Find the critical points of $f(x) = -2x^3 + 3x^2$ on $\left[-\frac{1}{2}, 2\right]$.

Solution

$$f'(x) = -6x^2 + 6x = 0$$

$$a = -6, \quad b = 6, \quad c = 0$$

$$x_{1,2} = \frac{-6 \pm \sqrt{36}}{-12} = \frac{-6 \pm 6}{-12}$$

$$x_1 = \frac{-6 + 6}{-12} = \frac{0}{-12} = 0$$

$$x_2 = \frac{-6 - 6}{-12} = \frac{-12}{-12} = 1$$

$$\therefore \textit{The critical points are} : -\frac{1}{2}, 0, 1, 2$$

Example 3 : Find the maximum and minimum values of $f(x) = -2x^3 + 3x^2$ on $\left[-\frac{1}{2}, 2\right]$.

Solution

$$f\left(-\frac{1}{2}\right) = -2\left(-\frac{1}{2}\right)^3 + 3\left(-\frac{1}{2}\right)^2 = -2\left(-\frac{1}{8}\right) + 3\left(\frac{1}{4}\right) = \frac{2}{8} + \frac{3}{4} = \frac{2}{8} + \frac{6}{8} = \frac{8}{8} = \mathbf{1}$$

$$f(0) = -2(0)^3 + 3(0)^2 = \mathbf{0}$$

$$f(1) = -2(1)^3 + 3(1)^2 = -2 + 3 = \mathbf{1}$$

$$f(2) = -2(2)^3 + 3(2)^2 = -2(8) + 3(4) = -16 + 12 = \mathbf{-4}$$

$$\therefore \textit{the max value} = \mathbf{1}$$

$$\therefore \textit{the min value} = \mathbf{-4}$$

Example 2 : Find the maximum and minimum values of $f(x) = x^3$ on $[-2,2]$.

Solution

- اولا نوجد النقاط الحرجة :

$$f'(x) = 3x^2 = 0 \quad \to \frac{3x^2}{3} = \frac{0}{3} \quad \to x^2 = 0 \quad \to x = \mathbf{0}$$

$$\textit{The critical points are}: \ \mathbf{-2, 0, 2}$$

- الان نوجد *max and min* :

$$f(-2) = (-2)^3 = \mathbf{-8}$$

$$f(0) = (0)^3 = \mathbf{0}$$

$$f(2) = (2)^3 = \mathbf{8}$$

$$\therefore \textit{the max value} = \mathbf{8}$$

$$\therefore \textit{the min value} = \mathbf{-8}$$

ENG. Mahdi Alsiddig
0563938879

Example 4 : The function $F(x) = x^{2/3}$ is continuous everywhere . Find its maximum and minimum values on $[-1,2]$.

Solution

- **اولا نوجد النقاط الحرجة :**

$$F'(x) = \frac{2}{3}\, x^{-1/3}$$

$$\textit{The critical points are}: \ -1, 0, 2$$

- **الان نوجد *max and min* :**

$F(-1) = (-1)^{\frac{2}{3}} = \mathbf{1}$

$F(0) = (0)^{\frac{2}{3}} = \mathbf{0}$

$F(2) = (2)^{\frac{2}{3}} = \sqrt[3]{2^2} = \sqrt[3]{\mathbf{4}} \approx \mathbf{1.59}$

$$\therefore \textit{the max value} = \sqrt[3]{4} \approx 1.59$$

$$\therefore \textit{the min value} = 0$$

Example 5 : Find the maximum and minimum values of $f(x) = x + 2\cos x$ on $[-\pi, 2\pi]$.

Solution

- اولا نوجد النقاط الحرجة :

$$f'(x) = 1 - 2\sin x = 0$$
$$2\sin x = 1$$
$$\sin x = \frac{1}{2} \quad \rightarrow \quad x = \frac{\pi}{6} \; and \; \frac{5\pi}{6}$$

$$\textit{The critical points are}: -\pi, \frac{\pi}{6}, \frac{5\pi}{6}, 2\pi$$

- الان نوجد max and min :

$$f(-\pi) = -\pi + 2\cos(-\pi) = -\pi - 2 \approx -5.14$$
$$f\left(\frac{\pi}{6}\right) = \frac{\pi}{6} + 2\cos\left(\frac{\pi}{6}\right) = \frac{\pi}{6} + 2\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6} + \sqrt{3} \approx 2.26$$
$$f\left(\frac{5\pi}{6}\right) = \frac{5\pi}{6} + 2\cos\left(\frac{5\pi}{6}\right) = \frac{5\pi}{6} + 2\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6} - \sqrt{3} \approx 0.89$$
$$f(2\pi) = 2\pi + 2\cos(2\pi) = 2\pi + 2 \cong 8.28$$

$$\therefore \textit{the max value} = 2\pi + 2 \cong 8.28$$

$$\therefore \textit{the min value} = -\pi - 2 \approx -5.14$$