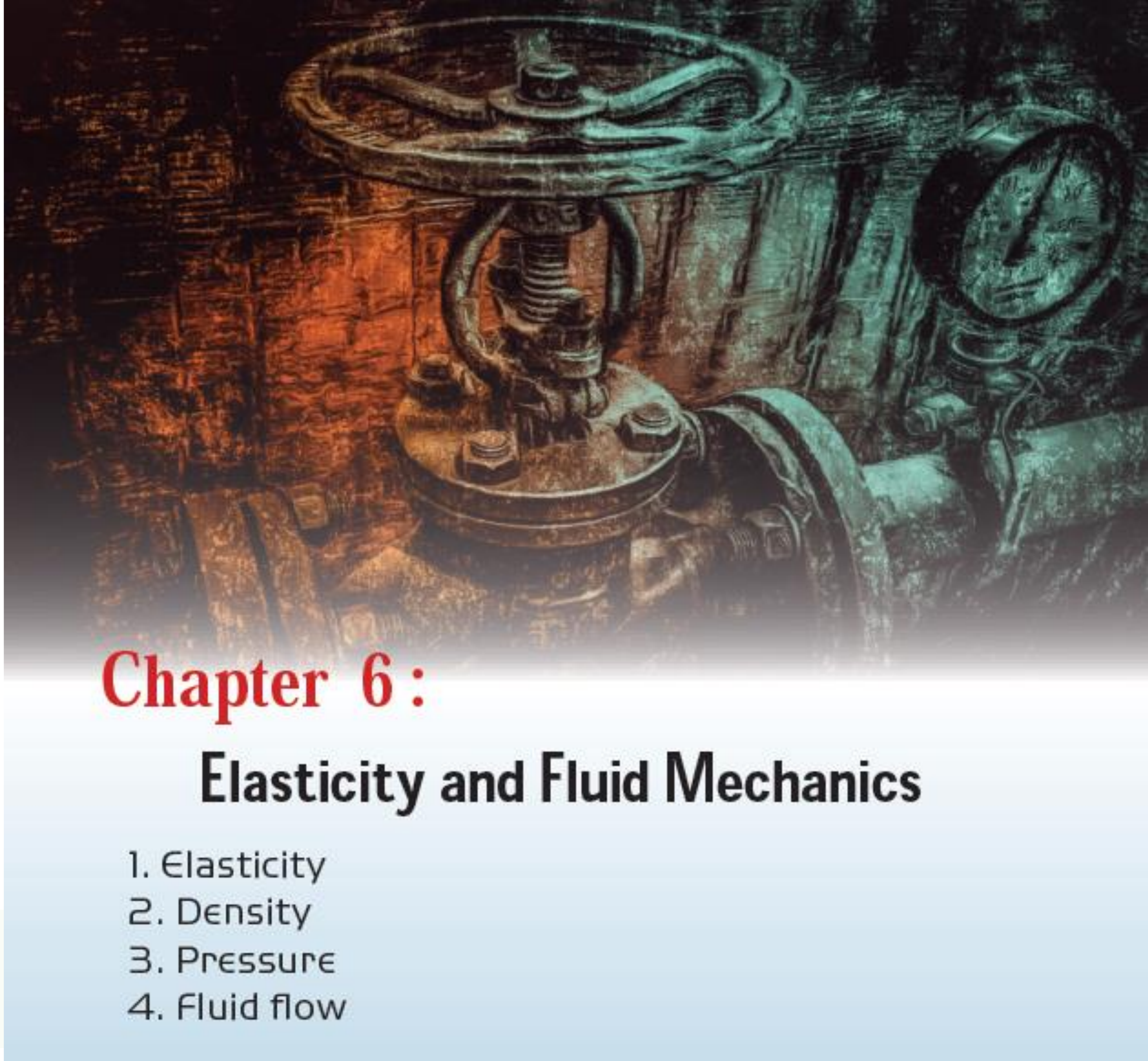
Elasticity and Fluid Mechanics



Prepared by

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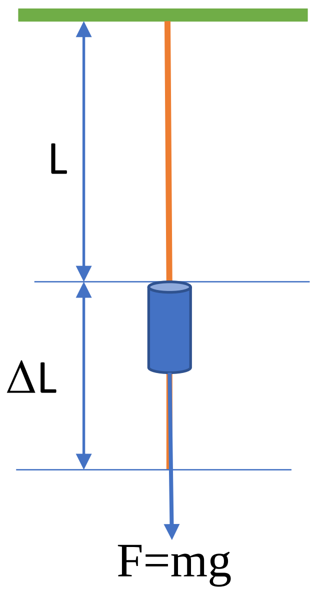
1. Elasticity

- Young's modulus **Y** : Elasticity of length

$$Y = \frac{\text{stress}}{\text{strain}}$$

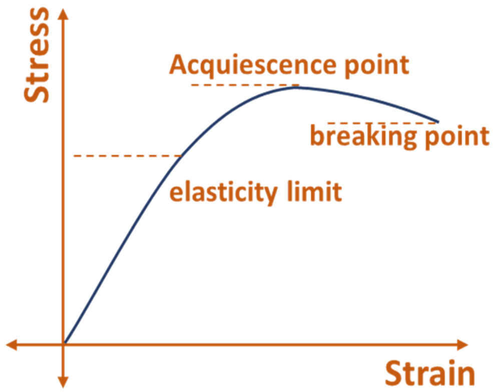


Stress is the force affecting on the unit of areas from the wire.

$$\text{stress} = \frac{F}{A}$$ measured by $\frac{N}{m^2}$

Strain is the amount of expansion of the wire relative to its original length.

$$strain = \frac{\Delta L}{L}$$ which is unitless



$$\frac{F}{A} = Y\frac{\Delta L}{L}$$

1. Elasticity

Example: 6.1

A force of 2500 N affected a metal wire 10 m long and 3.5 mm in diameter, extending by 0.5 cm find:

a. Stress.
b. Strain.
c. Young modulus

Solution

$$Stress = \frac{F}{A}$$

$$= \frac{2500}{\pi \times (1.75 \times 10^{-3})^2} = 2.6 \times 10^8 \, \frac{N}{m^2}$$

$$Strain = \frac{\Delta L}{L}$$

$$= \frac{0.5 \times 10^{-2}}{10} = 5 \times 10^{-4}$$

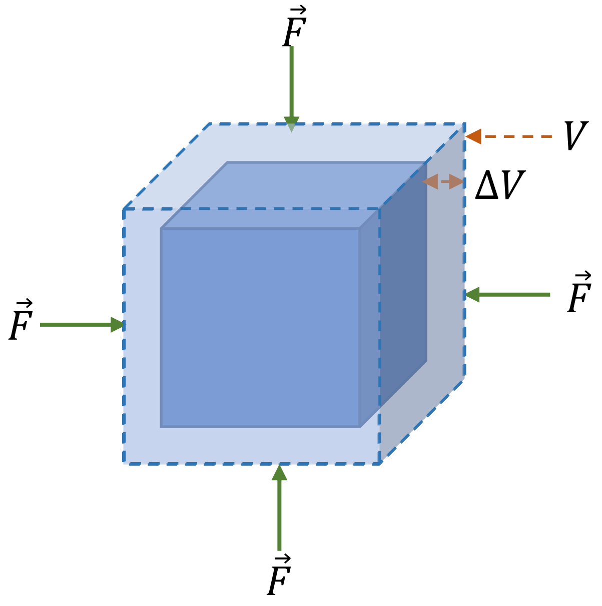
$$Y = \frac{Stress}{Strain}$$

$$= \frac{2.6 \times 10^8}{5 \times 10^{-4}} = 5.2 \times 10^{11} \, \frac{N}{m^2}$$

1. Elasticity

- Bulk modulus: Elasticity of volume

$$\frac{F}{A} = -B\frac{\Delta V}{V}$$



Example: 6.2

A copper cube with a side length of $5 \times 10^{-2}\ cm$, if we affect on it with a vertical force of $1.5 \times 10^{3}\ N$, its volume reduced by $5 \times 10^{-6}\ cm^{3}$. Calculate the Bulk modulus for this cube?

Solution

$$\frac{F}{A} = B\frac{\Delta V}{V}$$

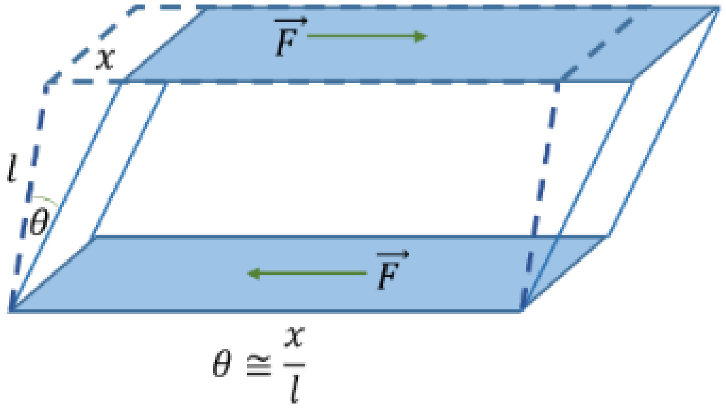
$$B = \frac{F\ V}{A\ \Delta V}$$

$$= \frac{1.5 \times 10^{3} \times 125 \times 10^{-6}}{25 \times 10^{-4} \times 5 \times 10^{-6}} = 1.5 \times 10^{7}\,\frac{N}{m^{2}}$$

1. Elasticity

- Shear modulus: Elasticity of shape

$$\frac{F}{A} = S\frac{x}{L}$$



Example: 6.3

Cube 10 cm long, affected by a tangential force of its upper surface of 10^6 N, causing a displacement of 0.03 cm for the upper side related to the lower side. Calculate the value of the shear modulus.

Solution

$$\frac{F}{A} = S\frac{x}{L}$$

$$S = \frac{F\,L}{A\,x}$$

$$= \frac{10^6 \times (10 \times 10^{-2})}{(10 \times 10^{-2})^2 \times (0.03 \times 10^{-2})} = 3.3 \times 10^{10}\,\frac{N}{m^2}$$

2. Density

Density ρ is defined as mass per unit volume.

$$\rho = \frac{m}{V}$$

The SI unit of the density is $\frac{kg}{m^3}$

Example: 6.4

Calculate the density of Glycerol if the size of 100 gm of it is equal to 79.3 cm^3.

Solution

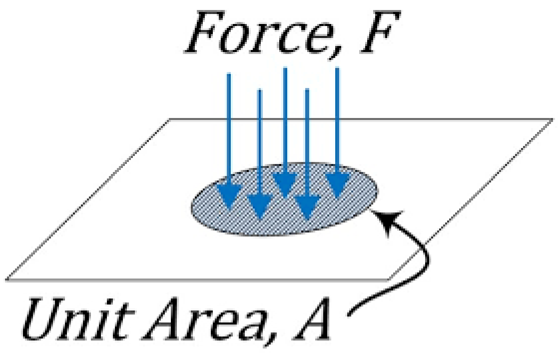
$$\rho = \frac{m}{V}$$

$$= \frac{100 \times 10^{-3}}{79.3 \times 10^{-6}} = 1261 \frac{kg}{m^3}$$

3. Pressure

The pressure is the force per unit area.

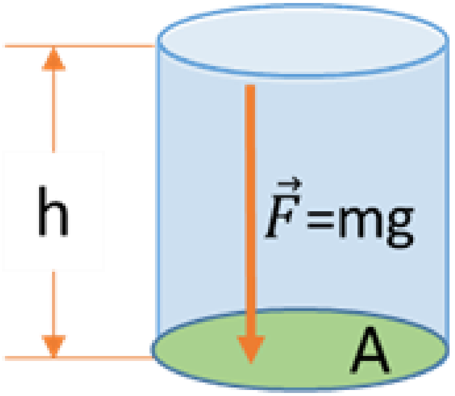
$$P = \frac{F}{A}$$



The SI unit of the pressure is Pascal $P_a = \frac{N}{m^2}$

- The pressure of the column of liquid

$$P = \frac{F}{A} = \rho\ g\ h$$



3. Pressure

Example: 6.5

Submarine at a depth of 30 m below sea level. Find the amount of force that seawater pressure affects on a cover at the top of the submarine with area 2 m^2. The density of seawater is 1025 kg/m^3.

Solution

$F = A\,P, \qquad P = h\,\rho\,g + P_0$

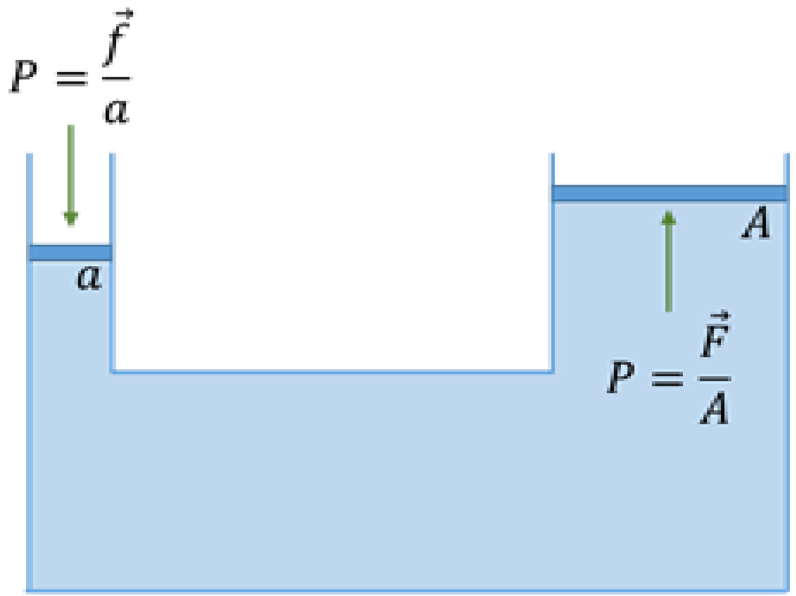
$F = A\,(h\,\rho\,g + P_0)$

$= 2 \times [(30 \times 1025 \times 9.8) + 10^5] = 802700\ N$

3. Pressure

Pascal's Law: A change in pressure at any point in an enclosed fluid at rest is transmitted undiminished to all points in the fluid.

$$\frac{f}{a} = \frac{F}{A}$$



Example: 6.6

In the hydraulic piston, the radius of the small and large piston is 2 cm, 20 cm respectively. A force of 2000 N is generated on the large piston. Calculate the force acting on the small piston.

Solution

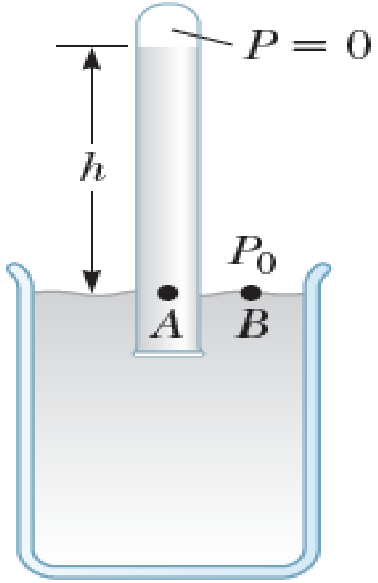
$$\frac{f}{a} = \frac{F}{A}$$

$$f = F\ \frac{a}{A}$$

$$= \frac{2000 \times \pi\ (2 \times 10^{-2})^2}{\pi\ (20 \times 10^{-2})^2} = 20\ N$$

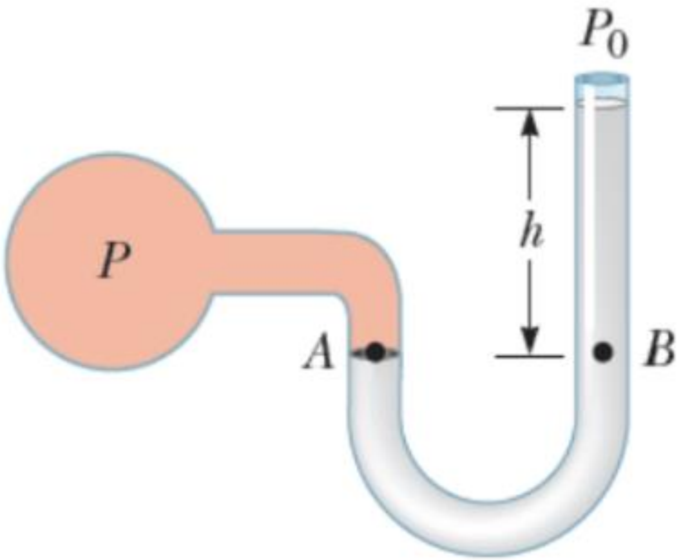
3. Pressure gauges

- The Barometer



$$P_0 = \rho g h$$

- The Manometer



$$P = P_0 + \rho g h$$

3. Pressure gauges

Example: 6.7

How long is the tube that we need to make a water barometer?

Note: (Atmospheric pressure is $1 \times 10^5\ Pa$ - gravitational acceleration $g = 9.8\ \frac{m}{s^2}$ - water density 1000 $\frac{kg}{m3}$)

Solution

$$P_a = \rho\ g\ h$$

$$h = \frac{P_a}{\rho\ g}$$

$$= \frac{1 \times 10^5}{1000 \times 9.8} = 10.2\ \ m$$

3. Pressure gauges

Example: 6.8

The height of mercury in open branch of the manometer relative to the surface of mercury in container branch is 40 cm, the mercury density is 13600 $\frac{kg}{m^3}$, atmospheric pressure $1 \times 10^5 Pa$ and the gravitational acceleration g= 9.8 $\frac{\mathrm{m}}{\mathrm{s}^2}$.

Calculate the pressure of the trapped gas in the container.

Solution

$$P = P_a + \rho \, g \, h$$

$$= 1 \times 10^5 + (13600 \times 9.8 \times 0.4) = 153312 \; Pa$$

4. Fluid flow

- Equation of Continuity

Fluid flow rate $R = Av$: Defined as the size of the fluid that crosses the fluid stream section area in the unit of time, measured in a unit $\frac{m^3}{s}$.

Example: 6.9

The water flows by pressure $3 \times 10^5\ P_a$ into a horizontal tube with velocity $1\ \frac{m}{s}$. The radius of tube is narrows from 0.2 m to 0.1 m. Calculate the flow speed in the narrow part of the tube.

Solution

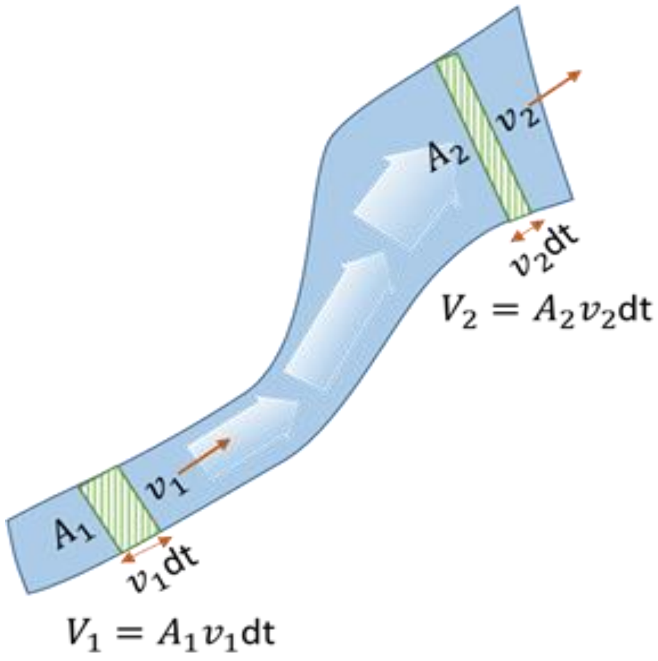
$$A_1\, v_1 = A_2\, v_2$$

$$v_2 = \frac{A_1}{A_2} v_1$$

$$= \frac{\pi\, r_1^2}{\pi\, r_2^2}\ v_1$$

$$= \frac{0.2^2}{0.1^2} \times 1 = 4\ m/s$$

$$A_1 v_1 = A_2 v_2$$

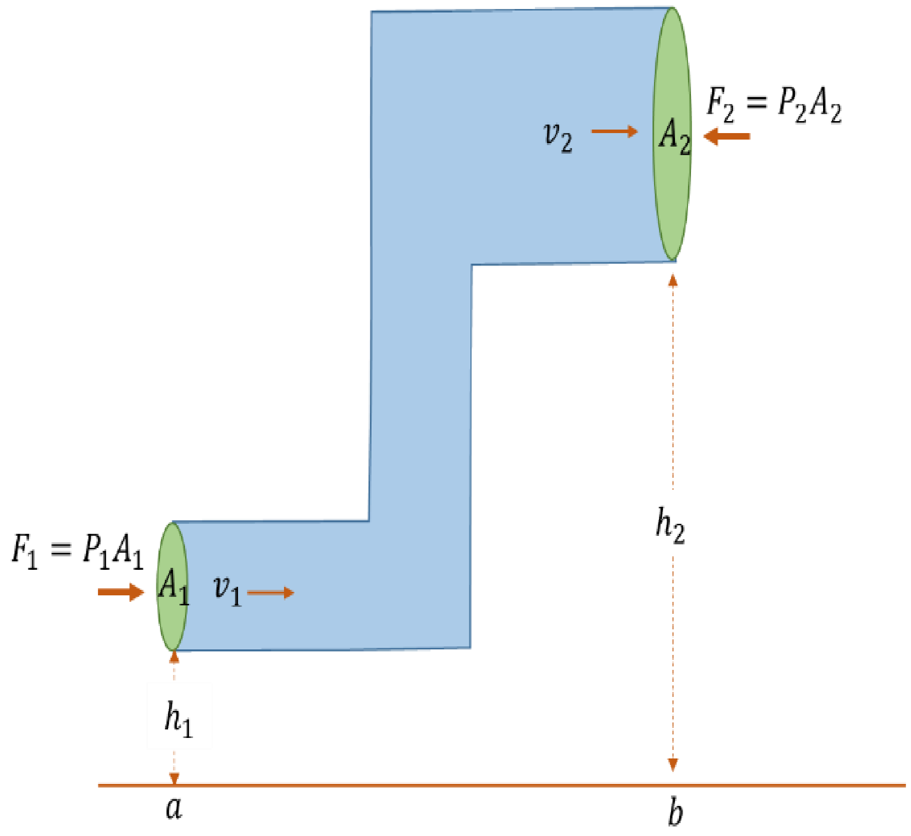


4. Fluid flow

Bernoulli's Equation

This relationship is known as the Bernoulli equation, which can be formulated that at all points on the flow line, the amount $P + h\,\rho\,g + \frac{1}{2}\,\rho\,v^2$ remains constant.

$$\boldsymbol{P_1 + h_1\,\rho\,g + \frac{1}{2}\,\rho\,v_1^2 \;=\; P_2 + h_2\,\rho\,g + \frac{1}{2}\,\rho\,v_2^2}$$



4. Fluid flow

Example: 6.10

An irregular horizontal tube in which the water flows, so if the pressure 1332.8 P_a is in the part where the speed of the water is 0.5 $\frac{m}{s}$. Calculate the pressure in the part where the speed is 0.8 $\frac{m}{s}$.

Solution

$$P_1 - P_2 = \frac{1}{2}\rho\,(v_2^2 - v_1^2)$$

$$P_2 = P_1 - \frac{1}{2}\rho\,(v_2^2 - v_1^2)$$

$$= 1332.8 - \frac{1}{2} \times 10^3 \times [(0.8)^2 - (0.5)^2] = 1137.8\ N/m^2$$