Chapter 16

Statistical Quality Control

To accompany
Quantitative Analysis for Management, Eleventh Edition,
by Render, Stair, and Hanna
Power Point slides created by Brian Peterson

Learning Objectives

After completing this chapter, students will be able to:

- Define the quality of a product or service.
- Develop four types of control charts: $\bar{x}$, R, p, and c.
- Understand the basic theoretical underpinnings of statistical quality control, including the central limit theorem.
- Know whether a process is in control.

Chapter Outline

Introduction

- ***Quality*** **is often the major issue in a purchase decision, as poor quality can be expensive for both the producing firm and the customer.**
- **Quality management, or *quality control* (*QC*), is critical throughout the organization,**
- **Quality is important for manufacturing and services.**
- **We will be dealing with the most important statistical methodology, *statistical process control* (*SPC*).**

Defining Quality and TQM

- ***Quality of a product or service*** is the degree to which the product or service meets specifications.
- Increasingly, definitions of ***quality*** include an added emphasis on meeting the customer's needs.
- ***Total quality management*** (***TQM***) refers to a quality emphasis that encompasses the entire organization from supplier to customer.
- Meeting the customer's expectations requires an emphasis on TQM if the firm is to complete as a leader in world markets.

Defining Quality and TQM

Several definitions of quality:

- "Quality is the degree to which a specific product conforms to a design or specification." (Gilmore, 1974)
- "Quality is the totality of features and characteristics of a product or service that bears on its ability to satisfy stated or implied needs." (Johnson and Winchell, 1989)
- "Quality is fitness for use." (Juran, 1974)
- "Quality is defined by the customer; customers want products and services that, throughout their lives, meet customers' needs and expectations at a cost that represents value." (Ford, 1991)
- "Even though quality cannot be defined, you know what it is." (Pirsig, 1974)

Table 16.1

Statistical Process Control

- ***Statistical process control*** **involves establishing and monitoring standards, making measurements, and taking corrective action as a product or service is being produced.**
- **Samples of process output are examined.**
- **If sample results fall outside certain specific ranges, the process is stopped and the assignable cause is located and removed.**
- **A *control chart* is a graphical presentation of data over time and shows upper and lower limits of the process we want to control.**

Patterns to Look for in Control Charts

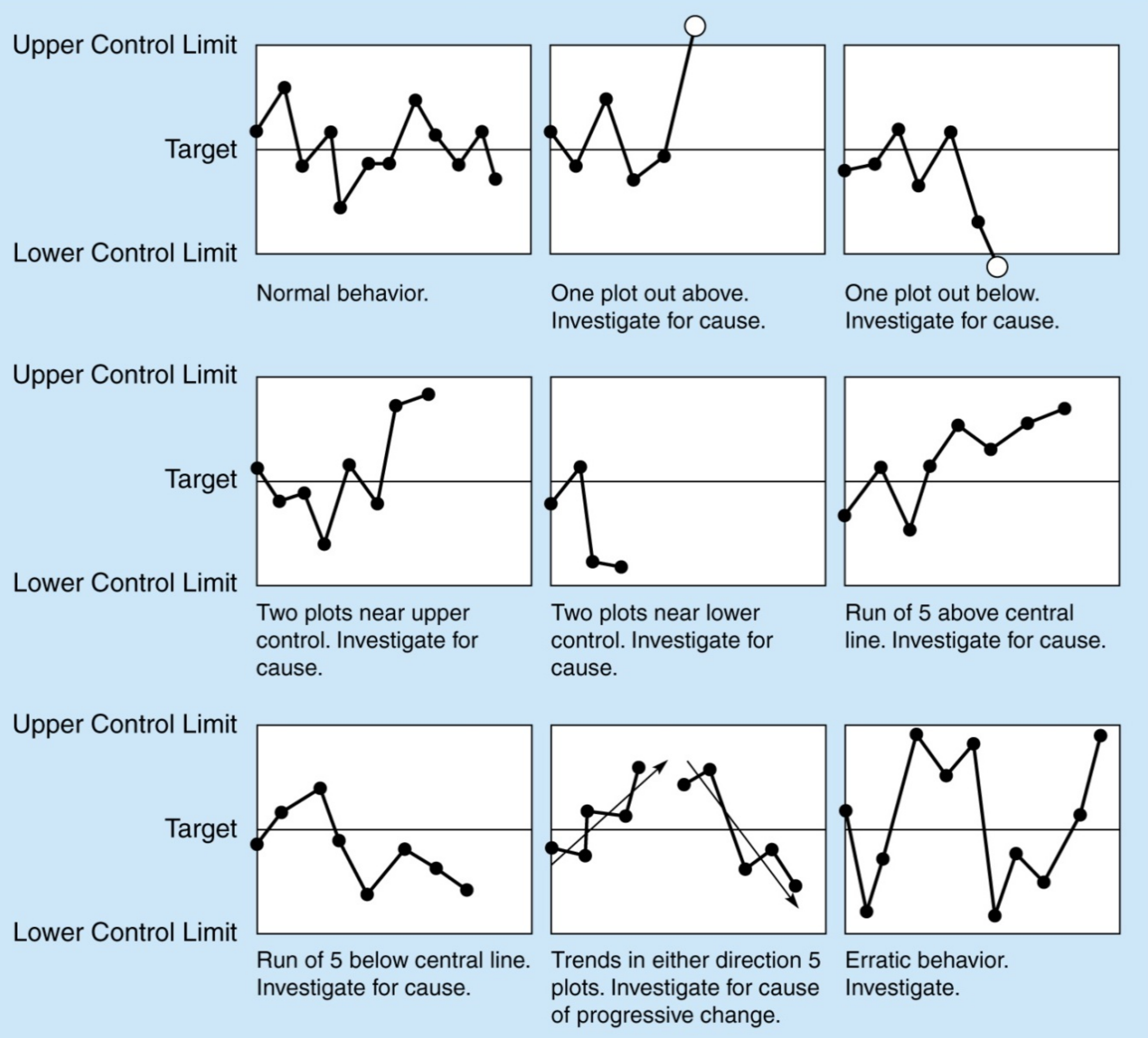


Figure 16.1

Building Control Charts

- Control charts are built using averages of small samples.
- The purpose of control charts is to distinguish between ***natural variations*** and ***variations due to assignable causes.***

Building Control Charts

- Natural variations
  - ***Natural variations*** affect almost every production process and are to be expected, even when the process is in statistical control.
  - They are random and uncontrollable.
  - When the distribution of this variation is ***normal*** it will have two parameters.
    - Mean, μ (the measure of central tendency of the average).
    - Standard deviation, σ (the amount by which smaller values differ from the larger ones).
  - As long as the distribution remains within specified limits it is said to be “in control.”

Building Control Charts

- Assignable variations
  - When a process is not in control, we must detect and eliminate special (*assignable*) causes of *variation.*
  - The variations are not random and can be controlled.
  - Control charts help pinpoint where a problem may lie.
  - The objective of a process control system is *to provide a statistical signal when assignable causes of variation are present.*

Control Charts for Variables

- The $\bar{x}$-chart (mean) and R-chart (range) are the control charts used for processes that are measured in continuous units.
- The ***$\bar{x}$-chart*** tells us when changes have occurred in the central tendency of the process.
- The ***R-chart*** tells us when there has been a change in the uniformity of the process.
- Both charts must be used when monitoring variables.

The Central Limit Theorem

- The central limit theorem is the foundation for $\bar{x}$-charts.
- The central limit theorem says that the distribution of sample means will follow a normal distribution as the sample size grows large.
- Even with small sample sizes the distribution is nearly normal.

The Central Limit Theorem

- The central limit theorem says:
  1. The mean of the sampling distribution will equal the population mean.
  2. The standard deviation of the sampling distribution will equal the population standard deviation divided by the square root of the sample size.

$$\mu_{\bar{x}} = \mu \qquad \text{and} \qquad \sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}}$$

- We often estimate $\mu_{\bar{x}}$ and μ with the average of all sample means ($\bar{\bar{x}}$).

The Central Limit Theorem

- Figure 16.2 shows three possible population distributions, each with their own mean (μ) and standard deviation (σ_x).
- If a series of random samples ($\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4$, and so on) each of size n is drawn from any of these, the resulting distribution of the $\bar{x}_i$'s will appear as in the bottom graph in the figure.
- Because this is a normal distribution:
    1. 99.7% of the time the sample averages will fall between $\pm 3\sigma_{\bar{x}}$ if the process has only random variations.
    2. 95.5% of the time the sample averages will fall between $\pm 2\ \sigma_{\bar{x}}$ if the process has only random variations.
- If a point falls outside the $\pm 3\ \sigma_{\bar{x}}$ control limit, we are 99.7% sure the process has changed.

The Central Limit Theorem

Population and Sampling Distributions

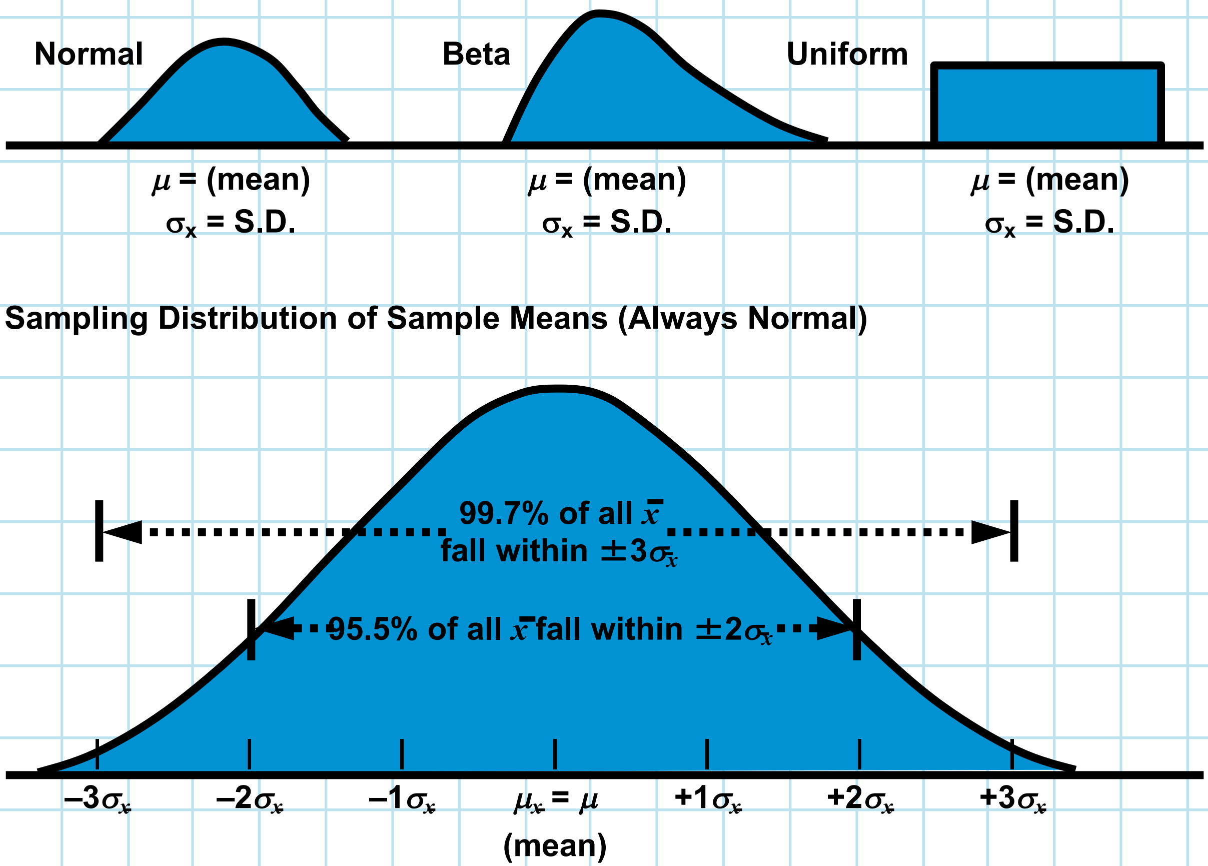


Figure 16.2

Standard error $= \sigma_{\bar{x}} = \dfrac{\sigma_x}{\sqrt{n}}$

Setting the $\bar{x}$-Chart Limits

- **If we know the standard deviation of the process, we can set the control limits using:**

$$\text{Upper control limit } (UCL) = \bar{\bar{x}} + z\sigma_{\bar{x}}$$

$$\text{Lower control limit } (LCL) = \bar{\bar{x}} - z\sigma_{\bar{x}}$$

where

$\bar{\bar{x}}$ **= mean of the sample means**

z **= number of normal standard deviations**

$\sigma_{\bar{x}}$ **= standard deviation of the sampling distribution of the sample means =** $\dfrac{\sigma_x}{\sqrt{n}}$

Box Filling Example

- A large production lot of boxes of cornflakes is sampled every hour.
- To set control limits that include 99.7% of the sample, 36 boxes are randomly selected and weighed.
- The standard deviation is estimated to be 2 ounces and the average mean of all the samples taken is 16 ounces.
- So $\bar{\bar{x}} = 16, \sigma_{\bar{x}} = 2, n = 36, z = 3$ and the control limits are:

$$UCL_{\bar{x}} = \bar{\bar{x}} + z\sigma_{\bar{x}} = 16 + 3\left(\frac{2}{\sqrt{36}}\right) = 16 + 1 = 17 \text{ ounces}$$

$$LCL_{\bar{x}} = \bar{\bar{x}} - z\sigma_{\bar{x}} = 16 - 3\left(\frac{2}{\sqrt{36}}\right) = 16 - 1 = 15 \text{ ounces}$$

Box Filling Example

- If the process standard deviation is not available or difficult to compute (a common situation) the previous equations are impractical.
- In practice the calculation of the control limits is based on the *average range* rather than the standard deviation.

$$UCL_{\bar{x}} = \bar{\bar{x}} + A_2 \bar{R}$$

$$LCL_{\bar{x}} = \bar{\bar{x}} - A_2 \bar{R}$$

where

$\bar{R}$ = average of the samples

A_2 = value found in Table 16.2

$\bar{\bar{x}}$ = mean of the sample means

Factors for Computing Control Chart Limits

SAMPLE SIZE, n	MEAN FACTOR, A_2	UPPER RANGE, D_4	LOWER RANGE, D_3
2	1.880	3.268	0
3	1.023	2.574	0
4	0.729	2.282	0
5	0.577	2.115	0
6	0.483	2.004	0
7	0.419	1.924	0.076
8	0.373	1.864	0.136
9	0.337	1.816	0.184
10	0.308	1.777	0.223
12	0.266	1.716	0.284
14	0.235	1.671	0.329
16	0.212	1.636	0.364
18	0.194	1.608	0.392
20	0.180	1.586	0.414
25	0.153	1.541	0.459

Table 16.2

Box Filling Example

Excel QM Solution for Box-Filling Example

	A	B	C	D	E	F
1	**Box Filling Example**					
2						
3	**Quality Control**		**x bar chart**			
4						
5	Number of samples	1				
6	Sample size	36				
7	Population standard deviation	2				
8	Data				**Results**	
9		Mean				
10	Sample 1	16			x-bar value	16
11	Average	16			z value	3
12					Sigma x bar	0.33333
13						
14					**Upper control limit**	**17**
15					**Center line**	**16**
16					**Lower control limit**	**15**

Enter the population standard deviation

Enter the sample size, the standard deviation, and the mean.

The LCL and UCL are displayed here.

Press Ctrl` to see the formulas used in Excel.

Program 16.1

Super Cola

- Super Cola bottles are labeled "net weight 16 ounces."
- The overall process mean is 16.01 ounces and the average range is 0.25 ounces in a sample of size n = 5.
- What are the upper and lower control limits for this process?

$$UCL_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R}$$
$$= 16.01 + (0.577)(0.25)$$
$$= 16.01 + 0.144$$
$$= 16.154$$

$$LCL_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R}$$
$$= 16.01 - (0.577)(0.25)$$
$$= 16.01 - 0.144$$
$$= 15.866$$

Super Cola

Excel QM Solution for Super Cola Example

	A	B	C	D	E	F	G
1	**Super Cola Example**						
2							
3	**Quality Control**		**x bar chart**				
4							
5	Number of samples	1					
6	Sample size	5					
7							
8	Data				**Results**		
9		Mean	Range			Xbar	Range
10	Sample 1	16.01	0.25		x-bar value	16.01	
11	Average	16.01	0.25				
12					R bar		0.25
13							
14					**Upper control limit**	**16.1543**	**0.52875**
15					**Center line**	**16.01**	**0.25**
16					**Lower control limit**	**15.8658**	**0**

Enter the mean and range from each

Program 16.2

Setting Range Chart Limits

- We have determined upper and lower control limits for the process ***average.***
- We are also interested in the ***dispersion*** or ***variability*** of the process.
- Averages can remain the same even if variability changes.
- A control chart for ***ranges*** is commonly used to monitor process variability.
- Limits are set at $\pm 3\sigma$ for the average range $\bar{R}$

Setting Range Chart Limits

We can set the upper and lower controls using:

$$UCL_R = D_4 \overline{R}$$

$$LCL_R = D_3 \overline{R}$$

where

UCL_R = upper control chart limit for the range
LCL_R = lower control chart limit for the range
D_4 and D_3 = values from Table 16.2

Range Example

- A process has an average range of 53 pounds.
- If the sample size is 5, what are the upper and lower control limits?
- From Table 16.2, D_4 = 2.114 and D_3 = 0.

$$
\begin{aligned}
UCL_R &= D_4 \overline{R} \\
&= (2.114)(53 \text{ pounds}) \\
&= 112.042 \text{ pounds}
\end{aligned}
$$

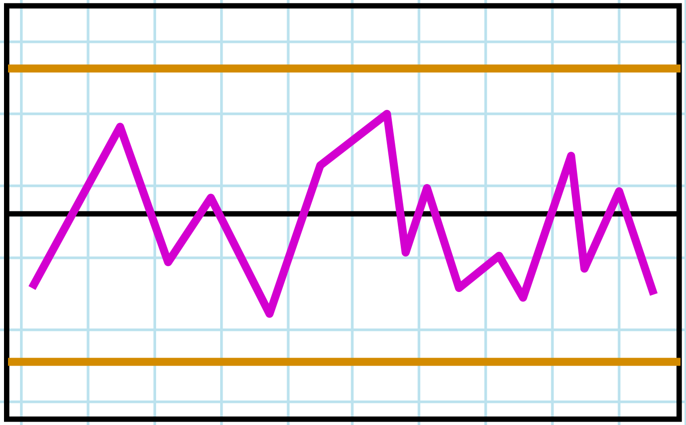
$$
\begin{aligned}
LCL_R &= D_3 \overline{R} \\
&= (0)(53 \text{ pounds}) \\
&= 0
\end{aligned}
$$

Five Steps to Follow in Using $\bar{x}$ and R-Charts

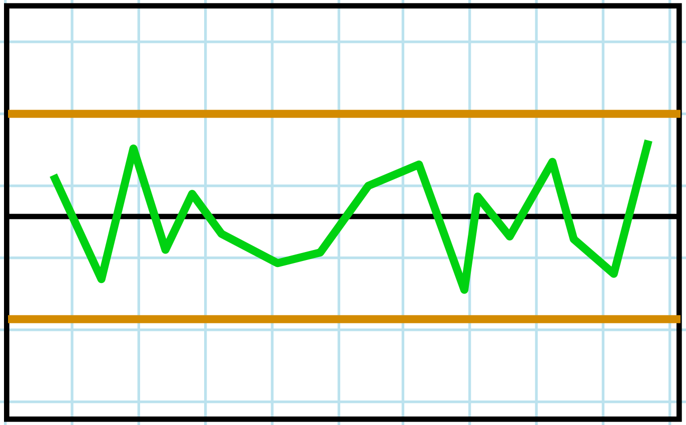
1. Collect 20 to 25 samples of n = 4 or n = 5 from a stable process and compute the mean and range of each.
2. Compute the overall means ($\bar{\bar{x}}$ and $\bar{R}$), set appropriate control limits, usually at 99.7% level and calculate the preliminary upper and lower control limits. If process not currently stable, use the desired mean, μ, instead of $\bar{\bar{x}}$ to calculate limits.
3. Graph the sample means and ranges on their respective control charts and determine whether they fall outside the acceptable limits.

Five Steps to Follow in Using $\bar{x}$ and R-Charts

4. Investigate points or patterns that indicate the process is out of control. Try to assign causes for the variation and then resume the process.
5. Collect additional samples and, if necessary, revalidate the control limits using the new data.



$\bar{x}$ chart



R-chart

Control Charts for Attributes

- We need a different type of chart to measure ***attributes.***
- These attributes are often classified as defective or nondefective.
- There are two kinds of attribute control charts:
  1. Charts that measure the percent defective in a sample are called ***p-charts.***
  2. Charts that count the number of defects in a sample are called ***c-charts.***

p-Charts

- Attributes that are good or bad typically follow the binomial distribution.
- If the sample size is large enough a normal distribution can be used to calculate the control limits:

$$UCL_p = \bar{p} + z\sigma_p$$

$$LCL_p = \bar{p} - z\sigma_p$$

where

$\bar{p}$ = mean proportion or fraction defective in the sample

z = number of standard deviations

σ_p = standard deviation of the sampling distribution which is estimated by $\hat{\sigma}_p$ where n is the size of each sample

$$\hat{\sigma}_p = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

ARCO p-Chart Example

Performance of data-entry clerks at ARCO (n = 100)

SAMPLE NUMBER	NUMBER OF ERRORS	FRACTION DEFECTIVE	SAMPLE NUMBER	NUMBER OF ERRORS	FRACTION DEFECTIVE
1	6	0.06	11	6	0.06
2	5	0.05	12	1	0.01
3	0	0.00	13	8	0.08
4	1	0.01	14	7	0.07
5	4	0.04	15	5	0.05
6	2	0.02	16	4	0.04
7	5	0.05	17	11	0.11
8	3	0.03	18	3	0.03
9	3	0.03	19	0	0.00
10	2	0.02	20	4	0.04
				80	

ARCO p-Chart Example

We want to set the control limits at 99.7% of the random variation present when the process is in control so z = 3.

$$\bar{p} = \frac{\text{Total number of errors}}{\text{Total number of records examined}} = \frac{80}{(100)(20)} = 0.04$$

$$\hat{\sigma}_p = \sqrt{\frac{(0.04)(1-0.04)}{100}} = 0.02$$

$$UCL_p = \bar{p} + z\hat{\sigma}_p = 0.04 + 3(0.02) = 0.10$$

$$LCL_p = \bar{p} - z\hat{\sigma}_p = 0.04 - 3(0.02) = 0$$

Percentage can't be negative.

ARCO p-Chart Example

p-chart for Data Entry for ARCO

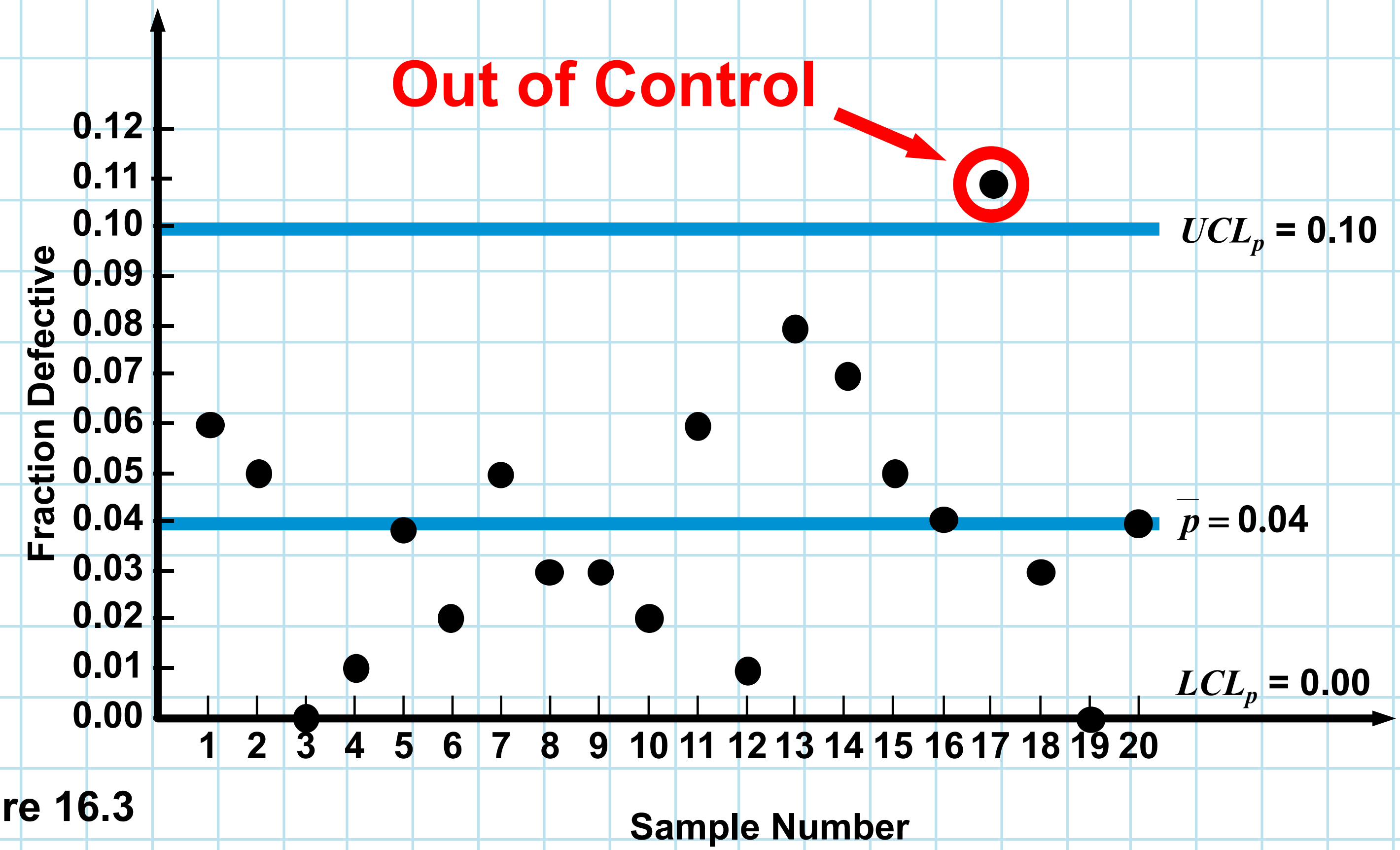


Figure 16.3

Excel QM Solution for ARCO p-chart Example

ARCO

Quality Control — **p chart**

Enter the sample size then enter the number of defects in each sample.

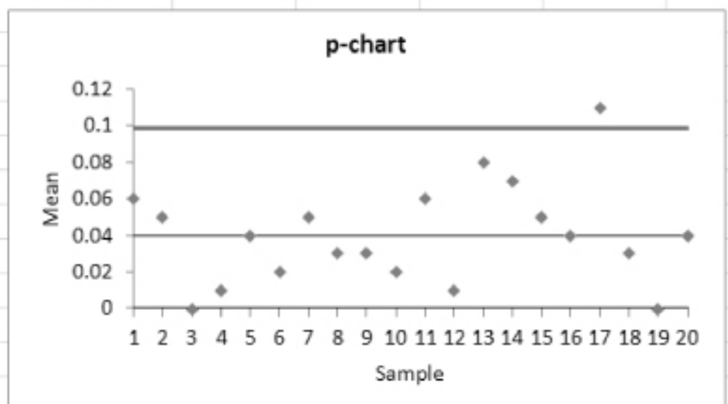
Number of samples	20
Sample size	100

Data

	# Defects	% Defects	
Sample 1	6	0.06	
Sample 2	5	0.05	
Sample 3	0	0	
Sample 4	1	0.01	
Sample 5	4	0.04	
Sample 6	2	0.02	
Sample 7	5	0.05	
Sample 8	3	0.03	
Sample 9	3	0.03	
Sample 10	2	0.02	
Sample 11	6	0.06	
Sample 12	1	0.01	
Sample 13	8	0.08	
Sample 14	7	0.07	
Sample 15	5	0.05	
Sample 16	4	0.04	
Sample 17	11	0.11	Above UCL
Sample 18	3	0.03	
Sample 19	0	0	
Sample 20	4	0.04	

Results

Total Sample Size	2000
Total Defects	80
Percentage defects	0.04
Std dev of p-bar	0.019596
z value	3
Upper Control Limit	**0.098788**
Center Line	**0.04**
Lower Control Limit	**0**



Program 16.3

c-Charts

- In the previous example we counted the number of defective records entered in the database.
- But records may contain more than one defect.
- We use ***c-charts*** to control the ***number*** of defects per unit of output.
- *c-charts* are based on the Poisson distribution which has its variance equal to its mean.
- The mean is $\bar{c}$ and the standard deviation is equal to $\sqrt{\bar{c}}$
- To compute the control limits we use:

$$\bar{c} \pm 3\sqrt{\bar{c}}$$

Red Top Cab Company
c-Chart Example

- The company receives several complaints each day about the behavior of its drivers.
- Over a nine-day period the owner received 3, 0, 8, 9, 6, 7, 4, 9, 8 calls from irate passengers, for a total of 54 complaints.
- To compute the control limits:

$$\bar{c} = \frac{54}{9} = 6 \text{ complaints per day}$$

Thus:

$$UCL_c = \bar{c} + 3\sqrt{\bar{c}} = 6 + 3\sqrt{6} = 6 + 3(2.45) = 13.35$$

$$LCL_c = \bar{c} - 3\sqrt{\bar{c}} = 6 - 3\sqrt{6} = 6 - 3(2.45) = 0$$

Excel QM Solution for Red Top Cab Company c-Chart Example

Red Top Cab Company

Quality Control — **c chart**

Number of samples: 9

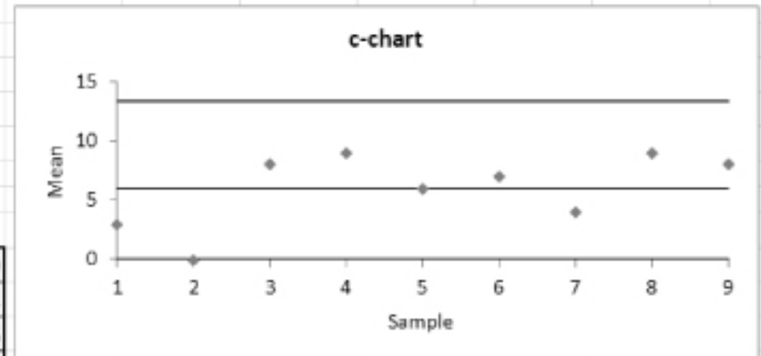
Enter the number of defects for each of the samples/items.

Data

	# Defects
Sample 1	3
Sample 2	0
Sample 3	8
Sample 4	9
Sample 5	6
Sample 6	7
Sample 7	4
Sample 8	9
Sample 9	8

Results

Total units sampled	9
Total Defects	54
Defect rate, λ	6
Standard deviation	2.44949
z value	3
Upper Control Limit	**13.3485**
Center Line	**6**
Lower Control Limit	**0**



Program 16.4

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