

# ***Chapter 3***

## ***Decision Analysis***

To accompany  
*Quantitative Analysis for Management, Eleventh Edition,*  
by Render, Stair, and Hanna  
Power Point slides created by Brian Peterson

# ***Learning Objectives***

**After completing this chapter, students will be able to:**

- 1. List the steps of the decision-making process.**
- 2. Describe the types of decision-making environments.**
- 3. Make decisions under uncertainty.**
- 4. Use probability values to make decisions under risk.**

# ***Learning Objectives***

**After completing this chapter, students will be able to:**

- 5. Develop accurate and useful decision trees.**
- 6. Revise probabilities using Bayesian analysis.**
- 7. Use computers to solve basic decision-making problems.**
- 8. Understand the importance and use of utility theory in decision making.**

# ***Chapter Outline***

- 3.1 Introduction**
- 3.2 The Six Steps in Decision Making**
- 3.3 Types of Decision-Making Environments**
- 3.4 Decision Making under Uncertainty**
- 3.5 Decision Making under Risk**
- 3.6 Decision Trees**
- 3.7 How Probability Values Are Estimated by Bayesian Analysis**
- 3.8 Utility Theory**

# ***Introduction***

- **What is involved in making a good decision?**
- **Decision theory is an analytic and systematic approach to the study of decision making.**
- **A good decision is one that is based on logic, considers all available data and possible alternatives, and the quantitative approach described here.**

# ***The Six Steps in Decision Making***

- 1. Clearly define the problem at hand.**
- 2. List the possible alternatives.**
- 3. Identify the possible outcomes or states of nature.**
- 4. List the payoff (typically profit) of each combination of alternatives and outcomes.**
- 5. Select one of the mathematical decision theory models.**
- 6. Apply the model and make your decision.**

# ***Thompson Lumber Company***

## **Step 1 – Define the problem.**

- The company is considering expanding by manufacturing and marketing a new product – backyard storage sheds.

## **Step 2 – List alternatives.**

- Construct a large new plant.
- Construct a small new plant.
- Do not develop the new product line at all.

## **Step 3 – Identify possible outcomes.**

- The market could be favorable or unfavorable.

# ***Thompson Lumber Company***

## **Step 4 – List the payoffs.**

- Identify **conditional values** for the profits for large plant, small plant, and no development for the two possible market conditions.

## **Step 5 – Select the decision model.**

- This depends on the environment and amount of risk and uncertainty.

## **Step 6 – Apply the model to the data.**

- Solution and analysis are then used to aid in decision-making.



# ***Thompson Lumber Company***

## **Decision Table with Conditional Values for Thompson Lumber**

<b>ALTERNATIVE</b>	<b>STATE OF NATURE</b>	
	<b>FAVORABLE MARKET (\$)</b>	<b>UNFAVORABLE MARKET (\$)</b>
<b>Construct a large plant</b>	<b>200,000</b>	<b>-180,000</b>
<b>Construct a small plant</b>	<b>100,000</b>	<b>-20,000</b>
<b>Do nothing</b>	<b>0</b>	<b>0</b>

**Table 3.1**

# ***Types of Decision-Making Environments***

## **Type 1: Decision making under certainty**

- The decision maker ***knows with certainty*** the consequences of every alternative or decision choice.

## **Type 2: Decision making under uncertainty**

- The decision maker ***does not know*** the probabilities of the various outcomes.

## **Type 3: Decision making under risk**

- The decision maker ***knows the probabilities*** of the various outcomes.

# ***Decision Making Under Uncertainty***

**There are several criteria for making decisions under uncertainty:**

- 1. Maximax (optimistic)**
- 2. Maximin (pessimistic)**
- 3. Criterion of realism (Hurwicz)**
- 4. Equally likely (Laplace)**
- 5. Minimax regret**

# Maximax

Used to find the alternative that maximizes the maximum payoff.

- Locate the maximum payoff for each alternative.
- Select the alternative with the maximum number.

ALTERNATIVE	STATE OF NATURE		MAXIMUM IN A ROW (\$)
	FAVORABLE MARKET (\$)	UNFAVORABLE MARKET (\$)	
Construct a large plant	200,000	-180,000	200,000
Construct a small plant	100,000	-20,000	100,000
Do nothing	0	0	0

**Maximax**

Table 3.2

# Maximin

Used to find the alternative that maximizes the minimum payoff.

- Locate the minimum payoff for each alternative.
- Select the alternative with the maximum number.

ALTERNATIVE	STATE OF NATURE		MINIMUM IN A ROW (\$)
	FAVORABLE MARKET (\$)	UNFAVORABLE MARKET (\$)	
Construct a large plant	200,000	-180,000	-180,000
Construct a small plant	100,000	-20,000	-20,000
Do nothing	0	0	0

Table 3.3

**Maximin**



# ***Criterion of Realism (Hurwicz)***

This is a **weighted average** compromise between optimism and pessimism.

- Select a coefficient of realism  $\alpha$ , with  $0 \leq \alpha \leq 1$ .
- A value of 1 is perfectly optimistic, while a value of 0 is perfectly pessimistic.
- Compute the weighted averages for each alternative.
- Select the alternative with the highest value.

$$\text{Weighted average} = \alpha(\text{maximum in row}) + (1 - \alpha)(\text{minimum in row})$$

# Criterion of Realism (Hurwicz)

- For the large plant alternative using  $\alpha = 0.8$ :  
 $(0.8)(200,000) + (1 - 0.8)(-180,000) = 124,000$
- For the small plant alternative using  $\alpha = 0.8$ :  
 $(0.8)(100,000) + (1 - 0.8)(-20,000) = 76,000$

ALTERNATIVE	STATE OF NATURE		CRITERION OF REALISM ( $\alpha = 0.8$ ) \$
	FAVORABLE MARKET (\$)	UNFAVORABLE MARKET (\$)	
Construct a large plant	200,000	-180,000	124,000
Construct a small plant	100,000	-20,000	76,000
Do nothing	0	0	0

124,000  
Realism

Table 3.4

# ***Equally Likely (Laplace)***

**Considers all the payoffs for each alternative**

- **Find the average payoff for each alternative.**
- **Select the alternative with the highest average.**

ALTERNATIVE	STATE OF NATURE		ROW AVERAGE (\$)
	FAVORABLE MARKET (\$)	UNFAVORABLE MARKET (\$)	
Construct a large plant	200,000	-180,000	10,000
Construct a small plant	100,000	-20,000	40,000
Do nothing	0	0	0

**Equally likely**

**Table 3.5**



# *Minimax Regret*

Based on *opportunity loss* or *regret*, this is the difference between the optimal profit and actual payoff for a decision.

- Create an opportunity loss table by determining the opportunity loss from not choosing the best alternative.
- Opportunity loss is calculated by subtracting each payoff in the column from the best payoff in the column.
- Find the maximum opportunity loss for each alternative and pick the alternative with the minimum number.

# *Minimax Regret*

## Determining Opportunity Losses for Thompson Lumber

STATE OF NATURE	
FAVORABLE MARKET (\$)	UNFAVORABLE MARKET (\$)
<b>200,000</b> – 200,000	<b>0</b> – (–180,000)
<b>200,000</b> – 100,000	<b>0</b> – (–20,000)
<b>200,000</b> – 0	<b>0</b> – 0

Table 3.6

# *Minimax Regret*

## Opportunity Loss Table for Thompson Lumber

ALTERNATIVE	STATE OF NATURE	
	FAVORABLE MARKET (\$)	UNFAVORABLE MARKET (\$)
Construct a large plant	0	180,000
Construct a small plant	100,000	20,000
Do nothing	200,000	0

Table 3.7

# Minimax Regret

## Thompson's Minimax Decision Using Opportunity Loss

ALTERNATIVE	STATE OF NATURE		MAXIMUM IN A ROW (\$)
	FAVORABLE MARKET (\$)	UNFAVORABLE MARKET (\$)	
Construct a large plant	0	180,000	180,000
Construct a small plant	100,000	20,000	100,000
Do nothing	200,000	0	200,000

**Minimax**

Table 3.8

# ***Decision Making Under Risk***

- This is decision making when there are several possible states of nature, and the probabilities associated with each possible state are known.
- The most popular method is to choose the alternative with the highest *expected monetary value (EMV)*.
  - This is very similar to the *expected value* calculated in the last chapter.

EMV (alternative  $i$ ) = (payoff of first state of nature)  
x (probability of first state of nature)  
+ (payoff of second state of nature)  
x (probability of second state of nature)  
+ ... + (payoff of last state of nature)  
x (probability of last state of nature)

# ***EMV for Thompson Lumber***

- Suppose each market outcome has a probability of occurrence of 0.50.
- Which alternative would give the highest EMV?
- The calculations are:

$$\begin{aligned}\text{EMV (large plant)} &= (\$200,000)(0.5) + (-\$180,000)(0.5) \\ &= \$10,000\end{aligned}$$

$$\begin{aligned}\text{EMV (small plant)} &= (\$100,000)(0.5) + (-\$20,000)(0.5) \\ &= \$40,000\end{aligned}$$

$$\begin{aligned}\text{EMV (do nothing)} &= (\$0)(0.5) + (\$0)(0.5) \\ &= \$0\end{aligned}$$

# ***EMV for Thompson Lumber***

ALTERNATIVE	STATE OF NATURE		EMV (\$)
	FAVORABLE MARKET (\$)	UNFAVORABLE MARKET (\$)	
Construct a large plant	200,000	-180,000	10,000
Construct a small plant	100,000	-20,000	40,000
Do nothing	0	0	0
Probabilities	0.50	0.50	

Table 3.9

**Largest EMV**



# ***Expected Value of Perfect Information (EVPI)***

- **EVPI places an upper bound on what you should pay for additional information.**

$$\text{EVPI} = \text{EV}_{\text{wPI}} - \text{Maximum EMV}$$

- **EV<sub>wPI</sub> is the long run average return if we have perfect information before a decision is made.**

$$\begin{aligned} \text{EV}_{\text{wPI}} = & (\text{best payoff for first state of nature}) \\ & \times (\text{probability of first state of nature}) \\ & + (\text{best payoff for second state of nature}) \\ & \times (\text{probability of second state of nature}) \\ & + \dots + (\text{best payoff for last state of nature}) \\ & \times (\text{probability of last state of nature}) \end{aligned}$$



# ***Expected Value of Perfect Information (EVPI)***

- **Suppose Scientific Marketing, Inc. offers analysis that will provide certainty about market conditions (favorable).**
- **Additional information will cost \$65,000.**
- **Should Thompson Lumber purchase the information?**

# ***Expected Value of Perfect Information (EVPI)***

## **Decision Table with Perfect Information**

<b>ALTERNATIVE</b>	<b>STATE OF NATURE</b>		<b>EMV (\$)</b>
	<b>FAVORABLE MARKET (\$)</b>	<b>UNFAVORABLE MARKET (\$)</b>	
<b>Construct a large plant</b>	<b>200,000</b>	<b>-180,000</b>	<b>10,000</b>
<b>Construct a small plant</b>	<b>100,000</b>	<b>-20,000</b>	<b>40,000</b>
<b>Do nothing</b>	<b>0</b>	<b>0</b>	<b>0</b>
<b>With perfect information</b>	<b>200,000</b>	<b>0</b>	<b>100,000</b>
<b>Probabilities</b>	<b>0.5</b>	<b>0.5</b>	<b>EVwPI</b>

**Table 3.10**

# ***Expected Value of Perfect Information (EVPI)***

**The maximum EMV without additional information is \$40,000.**

$$\begin{aligned}\text{EVPI} &= \text{EV}_{\text{wPI}} - \text{Maximum EMV} \\ &= \$100,000 - \$40,000 \\ &= \$60,000\end{aligned}$$

**So the maximum Thompson should pay for the additional information is \$60,000.**

# ***Expected Value of Perfect Information (EVPI)***

**The maximum EMV without additional information is \$40,000.**

$$\begin{aligned}\text{EVPI} &= \text{EV}_{\text{wPI}} - \text{Maximum EMV} \\ &= \$100,000 - \$40,000 \\ &= \$60,000\end{aligned}$$

**So the maximum Thompson should pay for the additional information is \$60,000.**

**Therefore, Thompson should not pay \$65,000 for this information.**

# ***Expected Opportunity Loss***

- ***Expected opportunity loss*** (EOL) is the cost of not picking the best solution.
- First construct an opportunity loss table.
- For each alternative, multiply the opportunity loss by the probability of that loss for each possible outcome and add these together.
- Minimum EOL will always result in the same decision as maximum EMV.
- Minimum EOL will always equal EVPI.

# Expected Opportunity Loss

ALTERNATIVE	STATE OF NATURE		EOL
	FAVORABLE MARKET (\$)	UNFAVORABLE MARKET (\$)	
Construct a large plant	0	180,000	90,000
Construct a small plant	100,000	20,000	60,000
Do nothing	200,000	0	100,000
Probabilities	0.50	0.50	

Table 3.11

**Minimum EOL**

$$\begin{aligned}\text{EOL (large plant)} &= (0.50)(\$0) + (0.50)(\$180,000) \\ &= \$90,000\end{aligned}$$

$$\begin{aligned}\text{EOL (small plant)} &= (0.50)(\$100,000) + (0.50)(\$20,000) \\ &= \$60,000\end{aligned}$$

$$\begin{aligned}\text{EOL (do nothing)} &= (0.50)(\$200,000) + (0.50)(\$0) \\ &= \$100,000\end{aligned}$$

# ***Sensitivity Analysis***

- **Sensitivity analysis examines how the decision might change with different input data.**
- **For the Thompson Lumber example:**

**$P$  = probability of a favorable market**

**$(1 - P)$  = probability of an unfavorable market**

# ***Sensitivity Analysis***

$$\begin{aligned}\text{EMV}(\text{Large Plant}) &= \$200,000P - \$180,000(1 - P) \\ &= \$200,000P - \$180,000 + \$180,000P \\ &= \$380,000P - \$180,000\end{aligned}$$

$$\begin{aligned}\text{EMV}(\text{Small Plant}) &= \$100,000P - \$20,000(1 - P) \\ &= \$100,000P - \$20,000 + \$20,000P \\ &= \$120,000P - \$20,000\end{aligned}$$

$$\begin{aligned}\text{EMV}(\text{Do Nothing}) &= \$0P + 0(1 - P) \\ &= \$0\end{aligned}$$



# Sensitivity Analysis

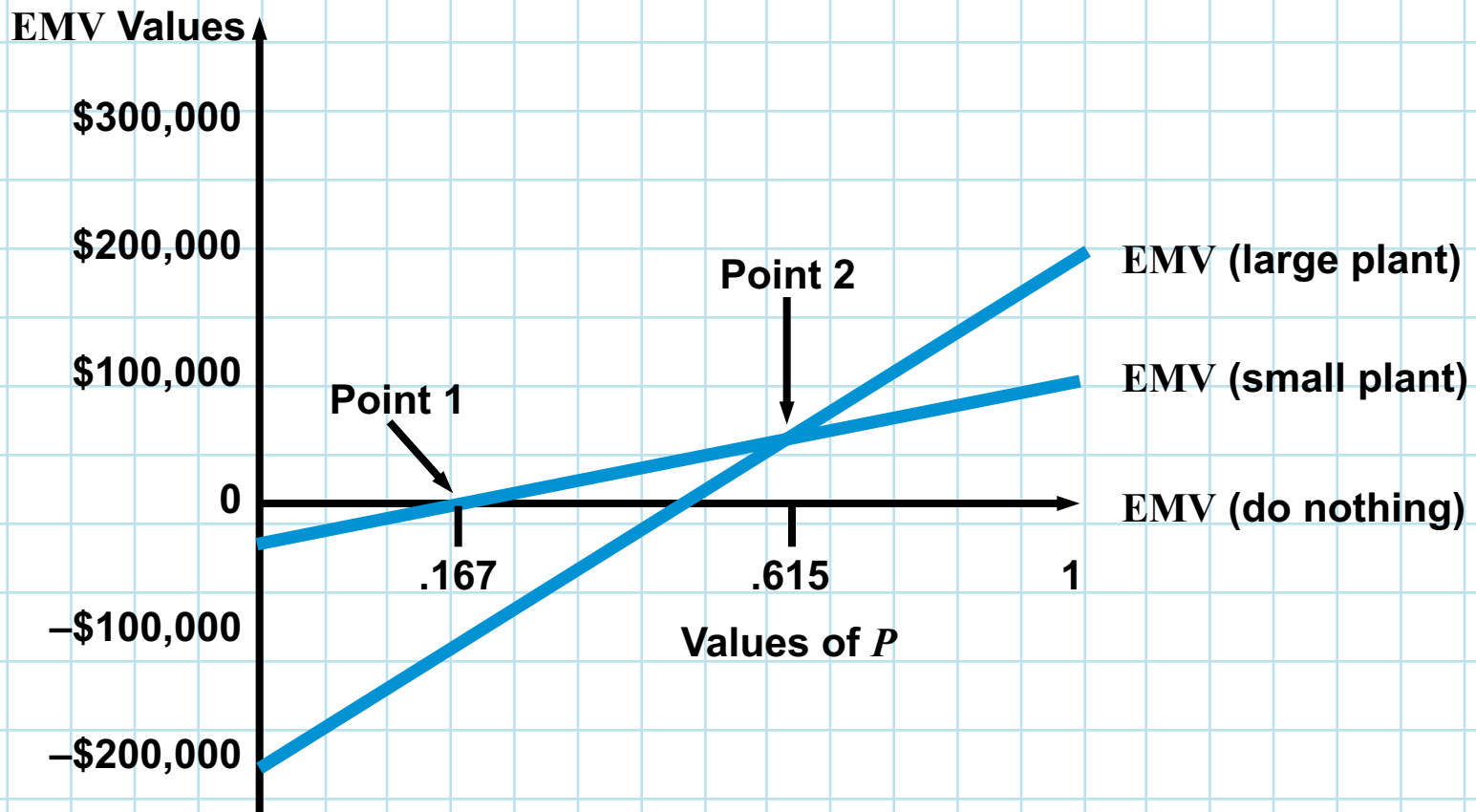


Figure 3.1

# ***Sensitivity Analysis***

## **Point 1:**

**EMV(do nothing) = EMV(small plant)**

$$0 = \$120,000P - \$20,000 \quad P = \frac{20,000}{120,000} = 0.167$$

## **Point 2:**

**EMV(small plant) = EMV(large plant)**

$$\$120,000P - \$20,000 = \$380,000P - \$180,000$$

$$P = \frac{160,000}{260,000} = 0.615$$

# Sensitivity Analysis

BEST ALTERNATIVE	RANGE OF $P$ VALUES
Do nothing	Less than 0.167
Construct a small plant	0.167 – 0.615
Construct a large plant	Greater than 0.615

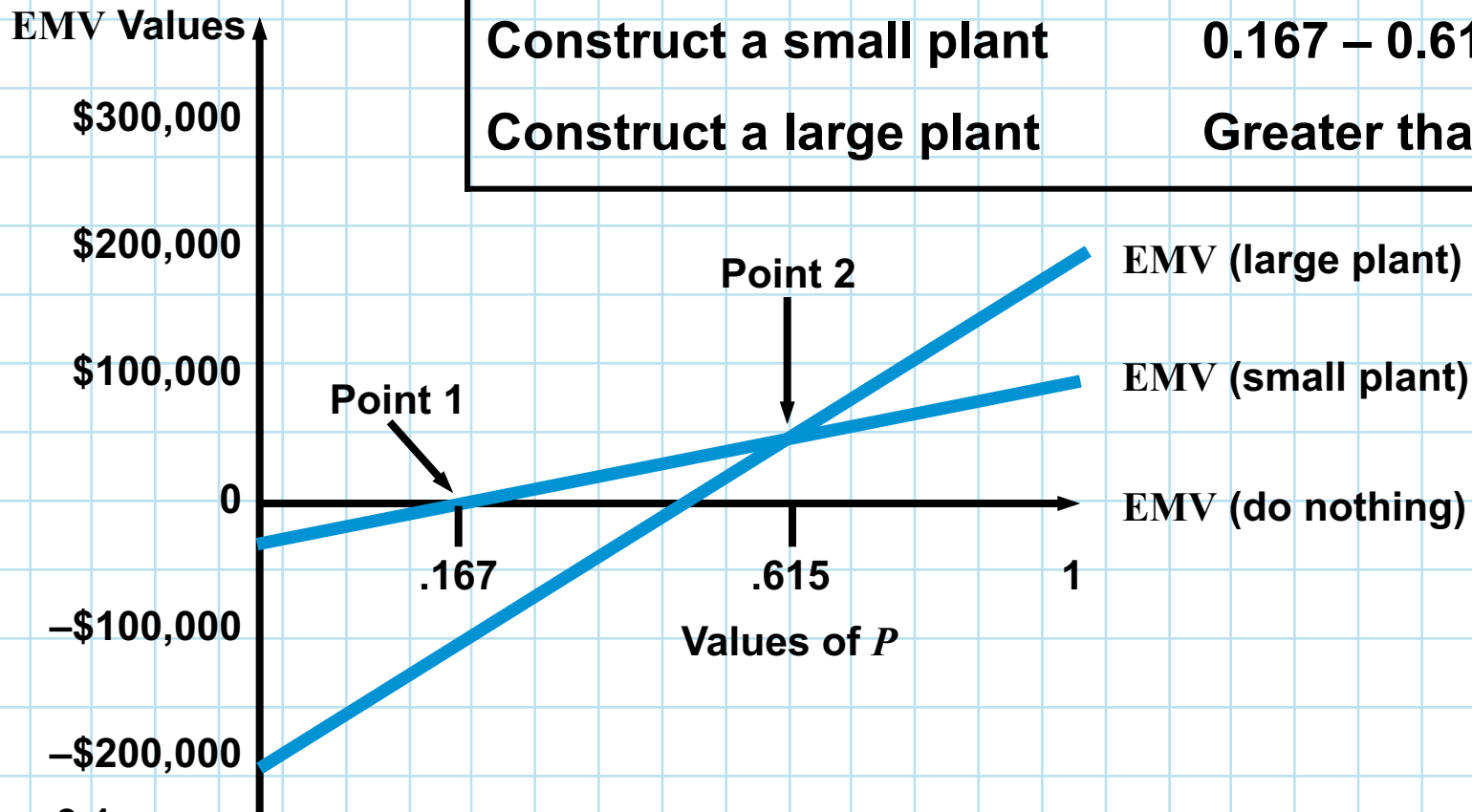


Figure 3.1

# Using Excel

## Input Data for the Thompson Lumber Problem Using Excel QM

	A	B	C	D	E	F
1	<b>Thompson Lumber</b>					
2						
3	<b>Decision Tables</b>					
4	Enter the profits or costs in the main body of the data table. Enter probabilities in the first row if you want to compute the expected value.					
5						
6	<b>Data</b>			<b>Results</b>		
7	Profit	Favorable Market	Unfavorable Market	EMV	Minimum	Maximum
8	Probability	0.5	0.5			
9	Large Plant	200000	-180000	=SUMPRODUCT(B8:C8,B9:C9)	=MIN(B9:C9)	=MAX(B9:C9)
10	Small plant	100000	-20000	=SUMPRODUCT(B8:C8,B10:C10)	=MIN(B10:C10)	=MAX(B10:C10)
11	Do nothing	0	0	=SUMPRODUCT(B8:C8,B11:C11)	=MIN(B11:C11)	=MAX(B11:C11)
12				=MAX(E9:E11)	=MAX(F9:F11)	=MAX(G9:G11)
13						
14	<b>Expected Value of Perfect Information</b>					
15	Column best	=MAX(B9:B11)	=MAX(C9:C11)	=SUMPRODUCT(B8:C8,B15:C15)	<-Expected value under certainty	
16				=E12	<-Best expected value	
17				=E15-E12	<-Expected value of perfect information	
18						
19	<b>Regret</b>					
20		=B7	=C7	Expected	Maximum	
21	=A8	=B8	=C8			
22	=A9	=B15 - B9	=C15 - C9	=SUMPRODUCT(B8:C8,B22:C22)	=MAX(B22:C22)	
23	=A10	=B15 - B10	=C15 - C10	=SUMPRODUCT(B8:C8,B23:C23)	=MAX(B23:C23)	
24	=A11	=B15 - B11	=C15 - C11	=SUMPRODUCT(B8:C8,B24:C24)	=MAX(B24:C24)	
25				=MIN(E22:E24)	=MIN(F22:F24)	
26						
27						
28						
29						

Compute the EMV for each alternative using the SUMPRODUCT function, the worst case using the MIN function, and the best case using the MAX function.

To calculate the EVPI, find the best outcome for each scenario.

Find the best outcome for each measure using the MAX function.

Use SUMPRODUCT to compute the product of the best outcomes by the probabilities and find the difference between this and the best expected value yielding the EVPI.

Program 3.1A

# Using Excel

## Output Results for the Thompson Lumber Problem Using Excel QM

	A	B	C	D	E	F	G	H	I	J	K
1	<b>Thompson Lumber</b>										
2											
3	<b>Decision Tables</b>										
4	Enter the profits or costs in the main body of the data table. Enter probabilities in the										
5	first row if you want to compute the expected value.										
6	Data			Results							
7	Profit	Favorable Market	Unfavorable Market		EMV	Minimum	Maximum		Hurwicz		
8	Probability	0.5	0.5					coefficient	0.8		
9	Large Plant	200000	-180000		10000	-180000	200000		124000		
10	Small plant	100000	-20000		40000	-20000	100000		76000		
11	Do nothing	0	0		0	0	0				
12				Maximum	40000	0	200000		124000		
13											
14	<b>Expected Value of Perfect Information</b>										
15	Column best	200000	0		100000	<-Expected value under certainty					
16					40000	<-Best expected value					
17					60000	<-Expected value of perfect information					
18											
19	<b>Regret</b>										
20		Favorable Market	Unfavorable Market		Expected	Maximum					
21	Probability	0.5	0.5								
22	Large Plant	0	180000		90000	180000					
23	Small plant	100000	20000		60000	100000					
24	Do nothing	200000	0		100000	200000					
25				Minimum	60000	100000					

Program 3.1B

# ***Decision Trees***

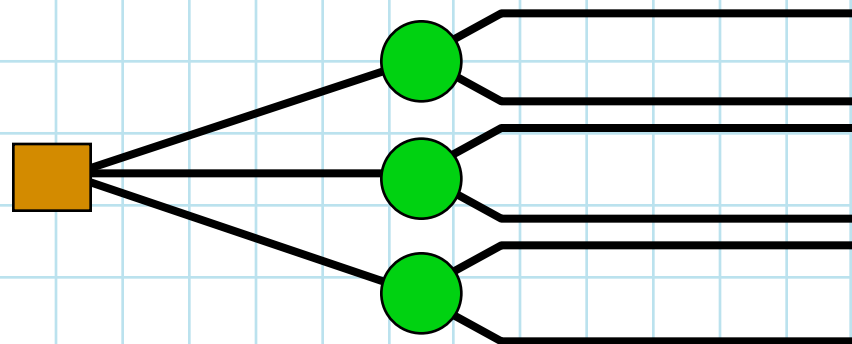
- Any problem that can be presented in a decision table can also be graphically represented in a ***decision tree***.
- Decision trees are most beneficial when a sequence of decisions must be made.
- All decision trees contain ***decision points*** or ***nodes***, from which one of several alternatives may be chosen.
- All decision trees contain ***state-of-nature points*** or ***nodes***, out of which one state of nature will occur.

# ***Five Steps of Decision Tree Analysis***

- 1. Define the problem.**
- 2. Structure or draw the decision tree.**
- 3. Assign probabilities to the states of nature.**
- 4. Estimate payoffs for each possible combination of alternatives and states of nature.**
- 5. Solve the problem by computing expected monetary values (EMVs) for each state of nature node.**

# ***Structure of Decision Trees***

- **Trees start from left to right.**
- **Trees represent decisions and outcomes in sequential order.**
  - **Squares represent decision nodes.**
  - **Circles represent states of nature nodes.**
  - **Lines or branches connect the decisions nodes and the states of nature.**





# *Thompson's Decision Tree*

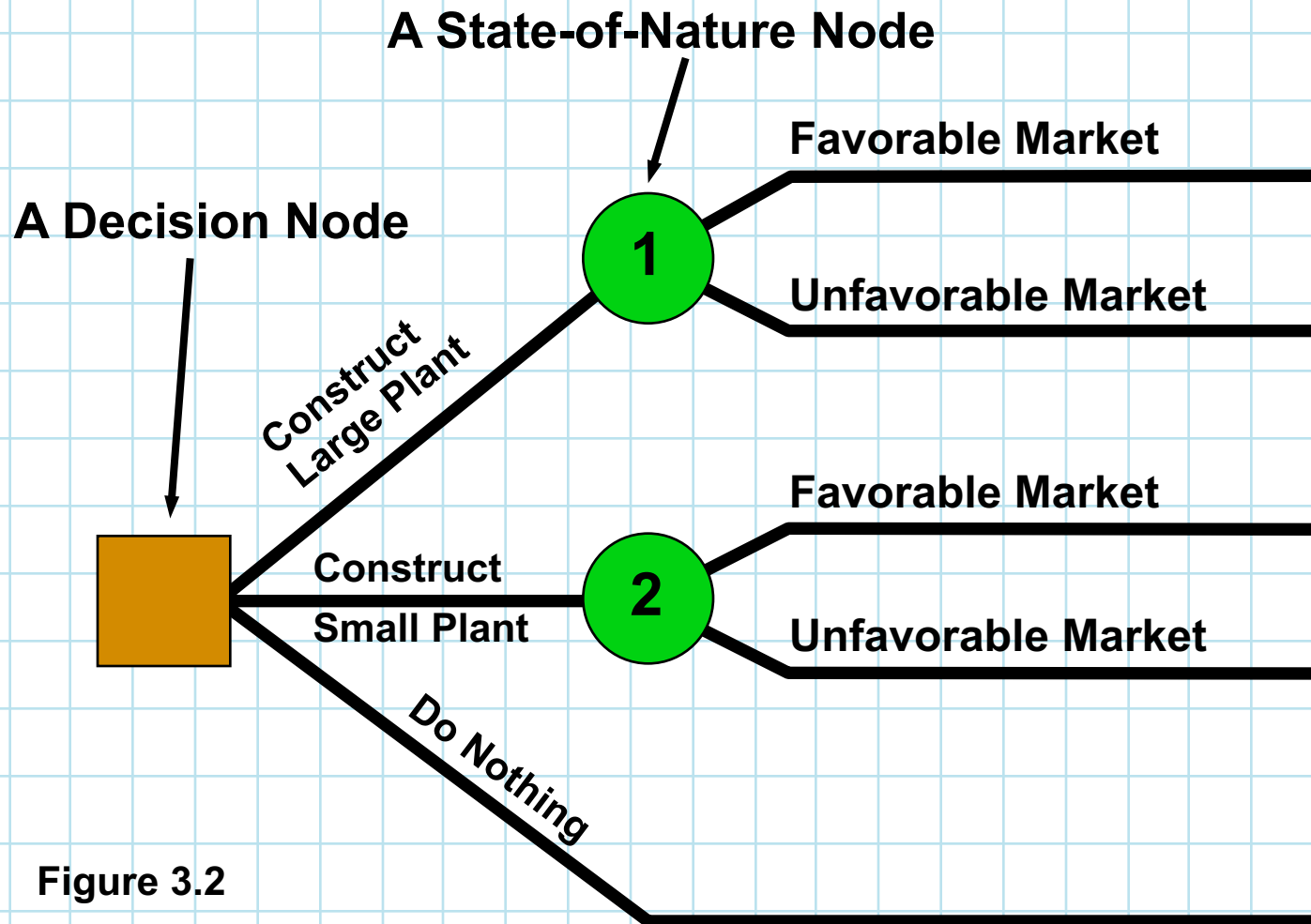


Figure 3.2

# Thompson's Decision Tree

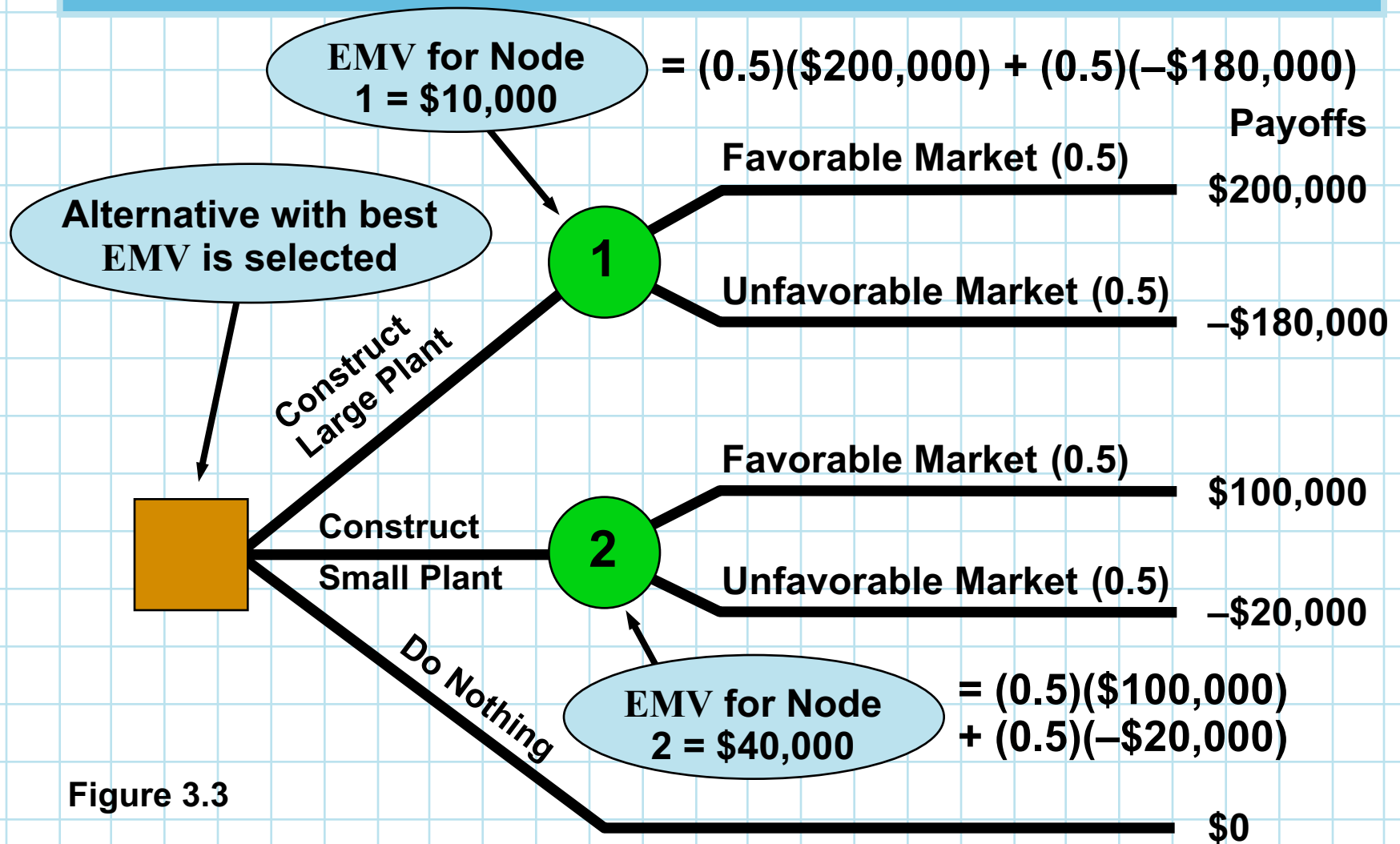


Figure 3.3

# Thompson's Complex Decision Tree

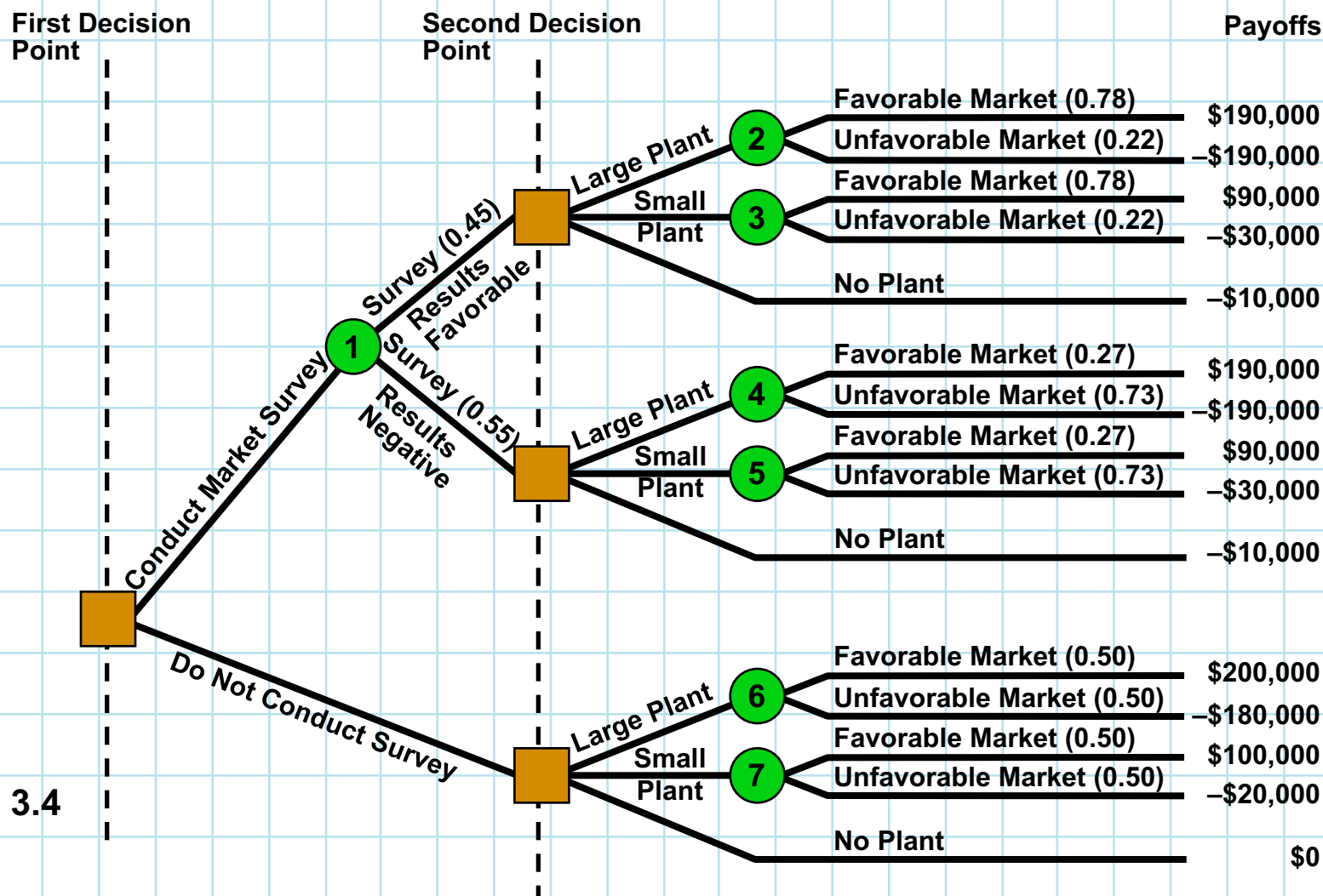


Figure 3.4

# ***Thompson's Complex Decision Tree***

## **1. Given favorable survey results,**

$$\begin{aligned}\text{EMV}(\text{node 2}) &= \text{EMV}(\text{large plant} \mid \text{positive survey}) \\ &= (0.78)(\$190,000) + (0.22)(-\$190,000) = \$106,400\end{aligned}$$

$$\begin{aligned}\text{EMV}(\text{node 3}) &= \text{EMV}(\text{small plant} \mid \text{positive survey}) \\ &= (0.78)(\$90,000) + (0.22)(-\$30,000) = \$63,600\end{aligned}$$

$$\text{EMV for no plant} = -\$10,000$$

## **2. Given negative survey results,**

$$\begin{aligned}\text{EMV}(\text{node 4}) &= \text{EMV}(\text{large plant} \mid \text{negative survey}) \\ &= (0.27)(\$190,000) + (0.73)(-\$190,000) = -\$87,400\end{aligned}$$

$$\begin{aligned}\text{EMV}(\text{node 5}) &= \text{EMV}(\text{small plant} \mid \text{negative survey}) \\ &= (0.27)(\$90,000) + (0.73)(-\$30,000) = \$2,400\end{aligned}$$

$$\text{EMV for no plant} = -\$10,000$$

# ***Thompson's Complex Decision Tree***

## **3. Compute the expected value of the market survey,**

$$\begin{aligned}\text{EMV}(\text{node 1}) &= \text{EMV}(\text{conduct survey}) \\ &= (0.45)(\$106,400) + (0.55)(\$2,400) \\ &= \$47,880 + \$1,320 = \$49,200\end{aligned}$$

## **4. If the market survey is not conducted,**

$$\begin{aligned}\text{EMV}(\text{node 6}) &= \text{EMV}(\text{large plant}) \\ &= (0.50)(\$200,000) + (0.50)(-\$180,000) = \$10,000\end{aligned}$$

$$\begin{aligned}\text{EMV}(\text{node 7}) &= \text{EMV}(\text{small plant}) \\ &= (0.50)(\$100,000) + (0.50)(-\$20,000) = \$40,000\end{aligned}$$

$$\text{EMV for no plant} = \$0$$

## **5. The best choice is to seek marketing information.**

# Thompson's Complex Decision Tree

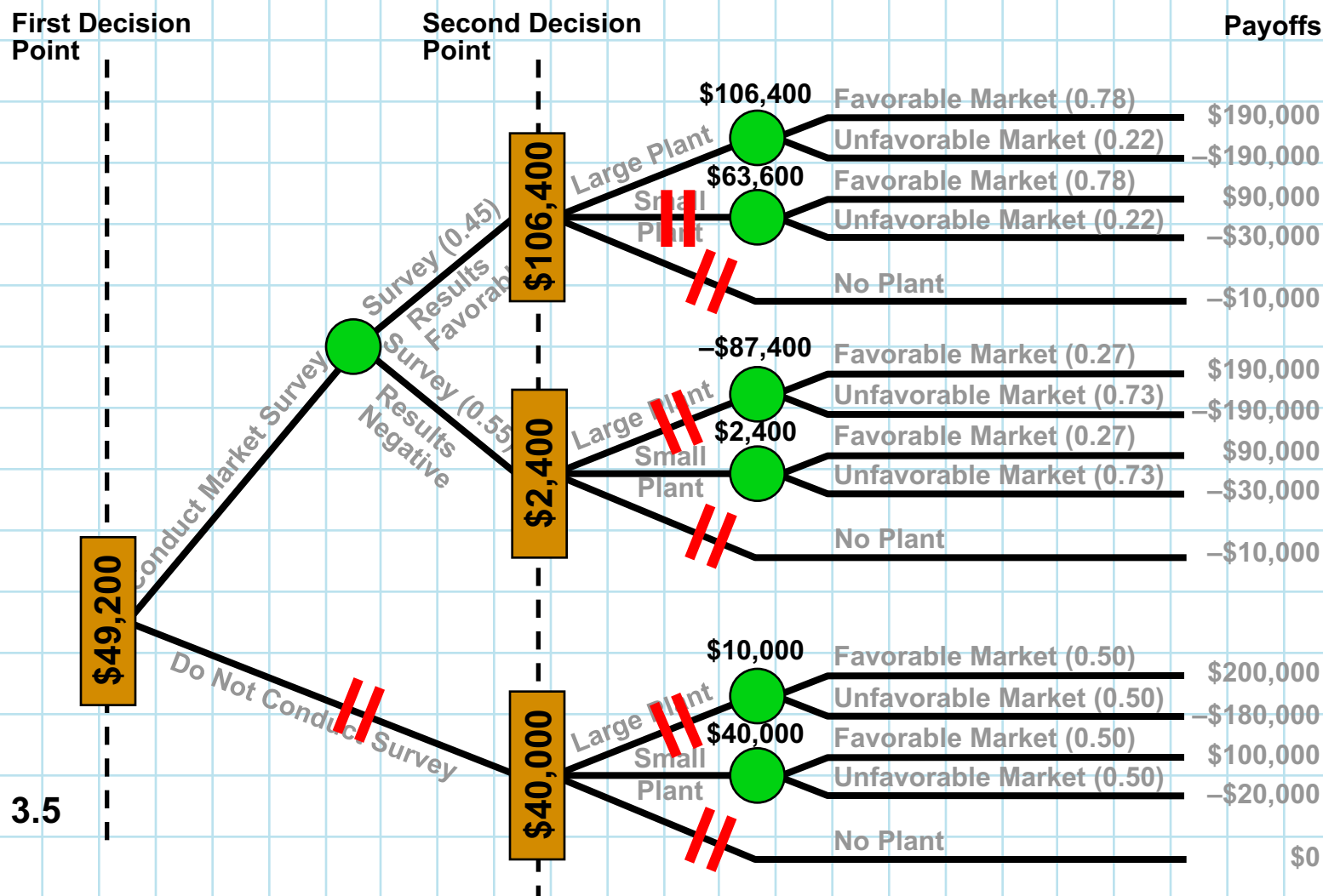


Figure 3.5

# ***Expected Value of Sample Information***

- **Suppose Thompson wants to know the actual value of doing the survey.**

$$\text{EVSI} = \left( \begin{array}{c} \text{Expected value} \\ \text{with sample} \\ \text{information, assuming} \\ \text{no cost to gather it} \end{array} \right) - \left( \begin{array}{c} \text{Expected value} \\ \text{of best decision} \\ \text{without sample} \\ \text{information} \end{array} \right)$$

$$= (\text{EV with sample information} + \text{cost}) \\ - (\text{EV without sample information})$$

$$\text{EVSI} = (\$49,200 + \$10,000) - \$40,000 = \$19,200$$

# ***Sensitivity Analysis***

- **How sensitive are the decisions to changes in the probabilities?**
  - **How sensitive is our decision to the probability of a favorable survey result?**
  - **That is, if the probability of a favorable result ( $p = .45$ ) were to change, would we make the same decision?**
  - **How much could it change before we would make a different decision?**



# ***Sensitivity Analysis***

**$p$  = probability of a favorable survey result**

**$(1 - p)$  = probability of a negative survey result**

$$\begin{aligned}\text{EMV}(\text{node 1}) &= (\$106,400)p + (\$2,400)(1 - p) \\ &= \$104,000p + \$2,400\end{aligned}$$

**We are indifferent when the EMV of node 1 is the same as the EMV of not conducting the survey, \$40,000**

$$\$104,000p + \$2,400 = \$40,000$$

$$\$104,000p = \$37,600$$

$$p = \$37,600 / \$104,000 = 0.36$$

**If  $p < 0.36$ , do not conduct the survey. If  $p > 0.36$ , conduct the survey.**

# ***Bayesian Analysis***

- **There are many ways of getting probability data. It can be based on:**
  - **Management's experience and intuition.**
  - **Historical data.**
  - **Computed from other data using Bayes' theorem.**
- **Bayes' theorem incorporates initial estimates and information about the accuracy of the sources.**
- **It also allows the revision of initial estimates based on new information.**

# ***Calculating Revised Probabilities***

- In the Thompson Lumber case we used these four conditional probabilities:

$P(\text{favorable market(FM)} \mid \text{survey results positive}) = 0.78$

$P(\text{unfavorable market(UM)} \mid \text{survey results positive}) = 0.22$

$P(\text{favorable market(FM)} \mid \text{survey results negative}) = 0.27$

$P(\text{unfavorable market(UM)} \mid \text{survey results negative}) = 0.73$

- But how were these calculated?
- The prior probabilities of these markets are:

$P(\text{FM}) = 0.50$

$P(\text{UM}) = 0.50$

# Calculating Revised Probabilities

- Through discussions with experts Thompson has learned the information in the table below.
- He can use this information and Bayes' theorem to calculate posterior probabilities.

RESULT OF SURVEY	STATE OF NATURE	
	FAVORABLE MARKET (FM)	UNFAVORABLE MARKET (UM)
Positive (predicts favorable market for product)	$P(\text{survey positive} \mid \text{FM}) = 0.70$	$P(\text{survey positive} \mid \text{UM}) = 0.20$
Negative (predicts unfavorable market for product)	$P(\text{survey negative} \mid \text{FM}) = 0.30$	$P(\text{survey negative} \mid \text{UM}) = 0.80$

**Table 3.12**

# Calculating Revised Probabilities

- Recall Bayes' theorem:

$$P(A | B) = \frac{P(B | A) \times P(A)}{P(B | A) \times P(A) + P(B | A') \times P(A')}$$

where

**$A, B$  = any two events**

**$A'$  = complement of  $A$**

**For this example,  $A$  will represent a favorable market and  $B$  will represent a positive survey.**

# Calculating Revised Probabilities

■  $P(FM \mid \text{survey positive})$

$$\begin{aligned} &= \frac{P(\text{survey positive} \mid FM) \times P(FM)}{P(\text{survey positive} \mid FM) \times P(FM) + P(\text{survey positive} \mid UM) \times P(UM)} \\ &= \frac{(0.70)(0.50)}{(0.70)(0.50) + (0.20)(0.50)} = \frac{0.35}{0.45} = 0.78 \end{aligned}$$

■  $P(UM \mid \text{survey positive})$

$$\begin{aligned} &= \frac{P(\text{survey positive} \mid UM) \times P(UM)}{P(\text{survey positive} \mid UM) \times P(UM) + P(\text{survey positive} \mid FM) \times P(FM)} \\ &= \frac{(0.20)(0.50)}{(0.20)(0.50) + (0.70)(0.50)} = \frac{0.10}{0.45} = 0.22 \end{aligned}$$

# Calculating Revised Probabilities

## Probability Revisions Given a Positive Survey

STATE OF NATURE	CONDITIONAL PROBABILITY $P(\text{SURVEY POSITIVE} \mid \text{STATE OF NATURE})$	PRIOR PROBABILITY	POSTERIOR PROBABILITY	
			JOINT PROBABILITY	$P(\text{STATE OF NATURE} \mid \text{SURVEY POSITIVE})$
FM	0.70	$X \ 0.50$	$= \ 0.35$	$0.35/0.45 = \ 0.78$
UM	0.20	$X \ 0.50$	$= \ 0.10$	$0.10/0.45 = \ 0.22$
$P(\text{survey results positive}) =$			<u>0.45</u>	<u>1.00</u>

Table 3.13

# Calculating Revised Probabilities

■  $P(FM \mid \text{survey negative})$

$$\begin{aligned} &= \frac{P(\text{survey negative} \mid FM) \times P(FM)}{P(\text{survey negative} \mid FM) \times P(FM) + P(\text{survey negative} \mid UM) \times P(UM)} \\ &= \frac{(0.30)(0.50)}{(0.30)(0.50) + (0.80)(0.50)} = \frac{0.15}{0.55} = 0.27 \end{aligned}$$

■  $P(UM \mid \text{survey negative})$

$$\begin{aligned} &= \frac{P(\text{survey negative} \mid UM) \times P(UM)}{P(\text{survey negative} \mid UM) \times P(UM) + P(\text{survey negative} \mid FM) \times P(FM)} \\ &= \frac{(0.80)(0.50)}{(0.80)(0.50) + (0.30)(0.50)} = \frac{0.40}{0.55} = 0.73 \end{aligned}$$



# Calculating Revised Probabilities

## Probability Revisions Given a Negative Survey

STATE OF NATURE	CONDITIONAL PROBABILITY $P(\text{SURVEY NEGATIVE} \mid \text{STATE OF NATURE})$	PRIOR PROBABILITY	POSTERIOR PROBABILITY	
			JOINT PROBABILITY	$P(\text{STATE OF NATURE} \mid \text{SURVEY NEGATIVE})$
FM	0.30	$\times 0.50$	$= 0.15$	$0.15/0.55 = 0.27$
UM	0.80	$\times 0.50$	$= 0.40$	$0.40/0.55 = 0.73$
$P(\text{survey results positive}) =$			<u>0.55</u>	<u>1.00</u>

Table 3.14

# Using Excel

## Formulas Used for Bayes' Calculations in Excel

A1	=	Bayes Theorem for Thompson Lumber Example			
	A	B	C	D	E
1	Bayes Theorem for Thompson Lumber Example				
2					
3	Fill in cells B7, B8, and C7.				
4					
5	Probability Revisions Given a Positive Survey				
6	State of Nature	P(Sur.Pos. state of nature)	Prior Prob.	Joint Prob.	Posterior Probability
7	FM	0.7	0.5	=B7*C7	=D7/\$D\$9
8	UM	0.2	=1-C7	=B8*C8	=D8/\$D\$9
9			P(Sur.pos.)=	=SUM(D7:D8)	
10					
11	Probability Revisions Given a Negative Survey				
12	State of Nature	P(Sur.Pos. state of nature)	Prior Prob.	Joint Prob.	Posterior Probability
13	FM	=1-B7	=C7	=B13*C13	=D13/\$D\$15
14	UM	=1-B8	=C8	=B14*C14	=D14/\$D\$15
15			P(Sur.neg.)=	=SUM(D13:D14)	
16					

### Program 3.2A

# Using Excel

## Results of Bayes' Calculations in Excel

A1    = Bayes Theorem for Thompson Lumber Example									
	A	B	C	D	E	F	G	H	I
1	<b>Bayes Theorem for Thompson Lumber Example</b>								
2									
3	<b>Fill in cells B7, B8, and C7.</b>								
4									
5	<b>Probability Revisions Given a Positive Survey</b>								
6	State of Nature	P(Sur.Pos. state of nature)	Prior Prob.	Joint Prob.	Posterior Probability				
7	FM	0.70	0.50	0.35	0.78				
8	UM	0.20	0.50	0.10	0.22				
9			P(Sur.pos.)=	0.45					
10									
11	<b>Probability Revisions Given a Negative Survey</b>								
12	State of Nature	P(Sur.Pos. state of nature)	Prior Prob.	Joint Prob.	Posterior Probability				
13	FM	0.30	0.50	0.15	0.27				
14	UM	0.80	0.50	0.40	0.73				
15			P(Sur.neg.)=	0.55					
16									

### Program 3.2B

# ***Potential Problems Using Survey Results***

- **We can not always get the necessary data for analysis.**
- **Survey results may be based on cases where an action was taken.**
- **Conditional probability information may not be as accurate as we would like.**

# *Utility Theory*

- Monetary value is not always a true indicator of the overall value of the result of a decision.
- The overall value of a decision is called *utility*.
- Economists assume that rational people make decisions to maximize their utility.

# Utility Theory

## Your Decision Tree for the Lottery Ticket

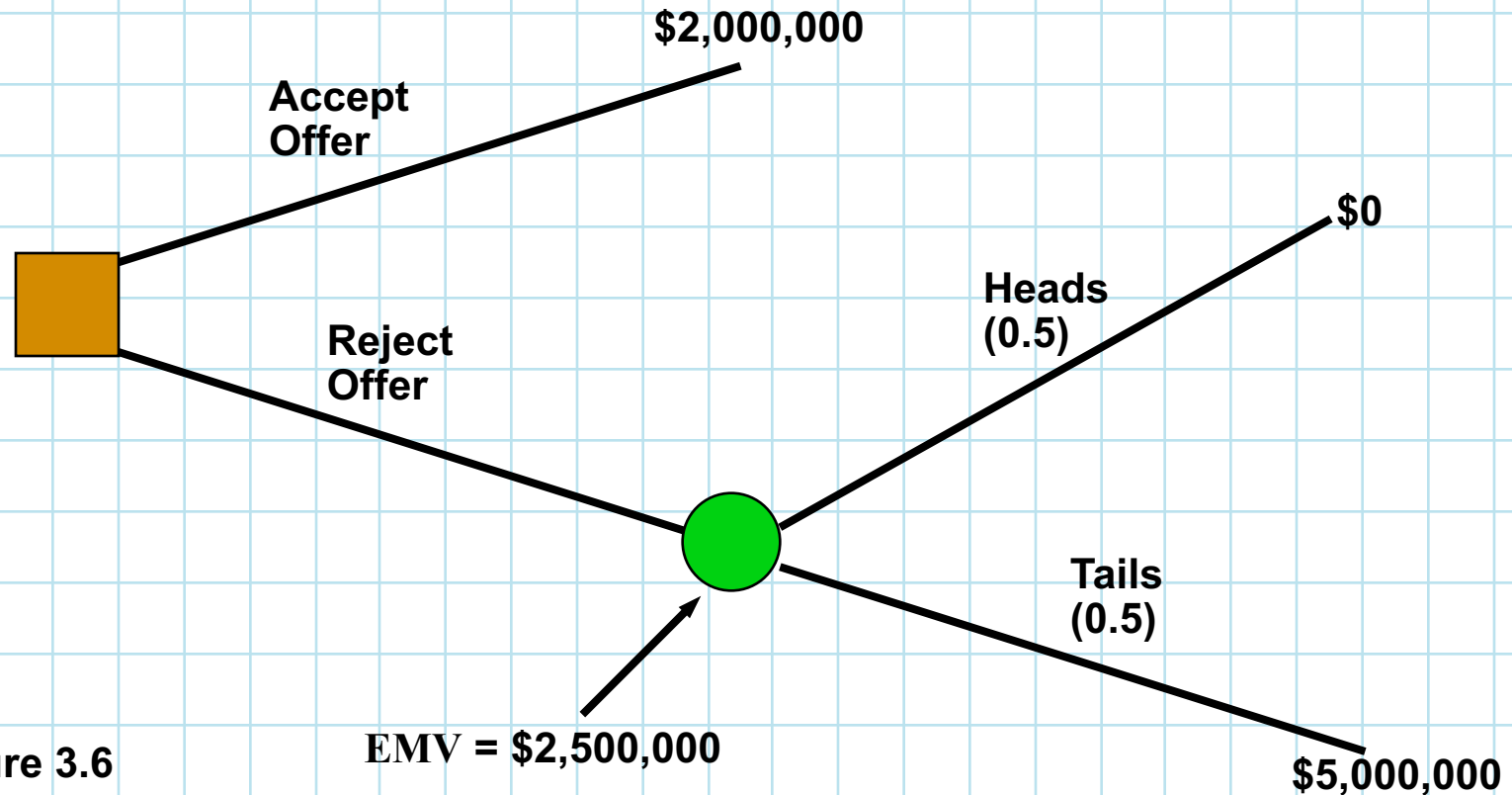


Figure 3.6

EMV = \$2,500,000

# Utility Theory

- **Utility assessment** assigns the worst outcome a utility of 0, and the best outcome, a utility of 1.
- A **standard gamble** is used to determine utility values.
- When you are indifferent, your utility values are equal.

Expected utility of alternative 2 = Expected utility of alternative 1

Utility of other outcome =  $(p)(\text{utility of best outcome, which is 1})$   
+  $(1 - p)(\text{utility of the worst outcome, which is 0})$

Utility of other outcome =  $(p)(1) + (1 - p)(0) = p$

# Standard Gamble for Utility Assessment

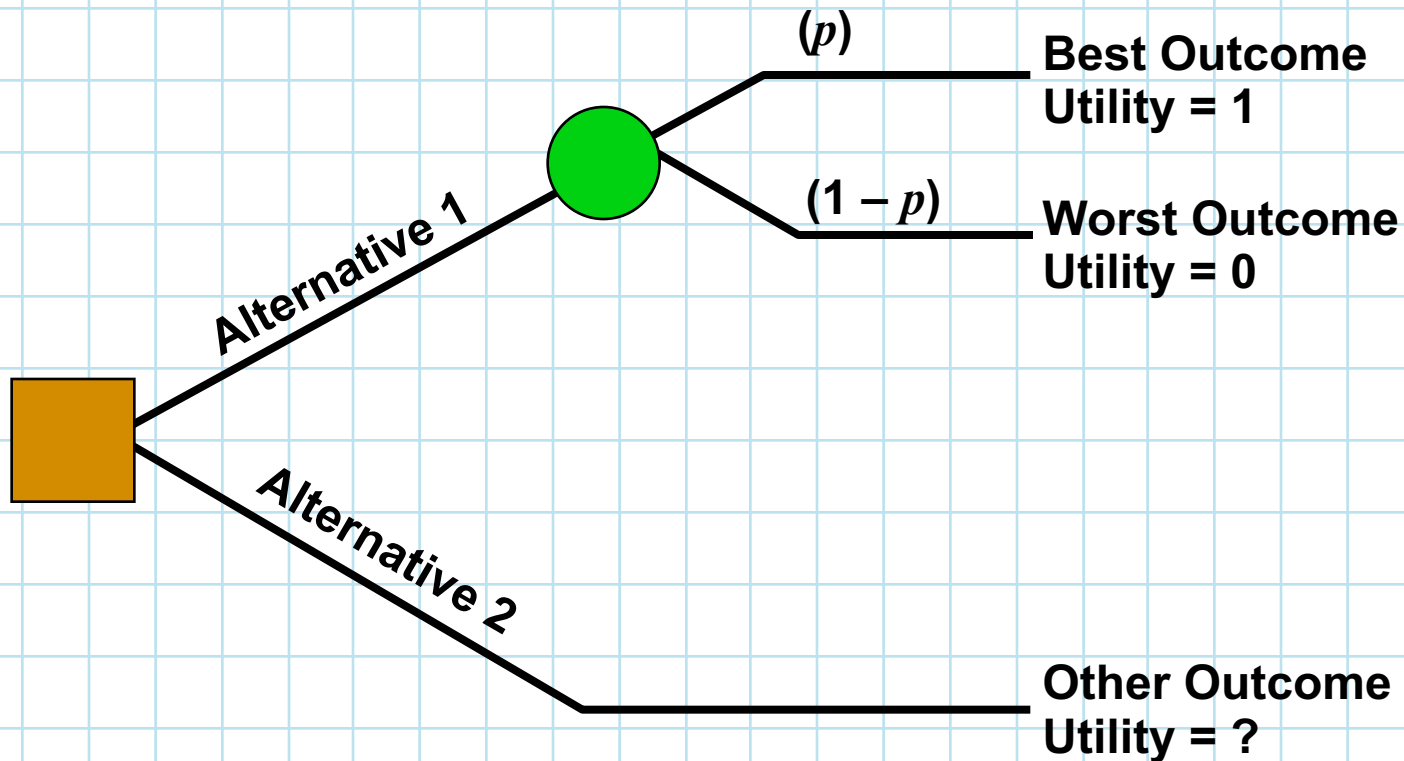


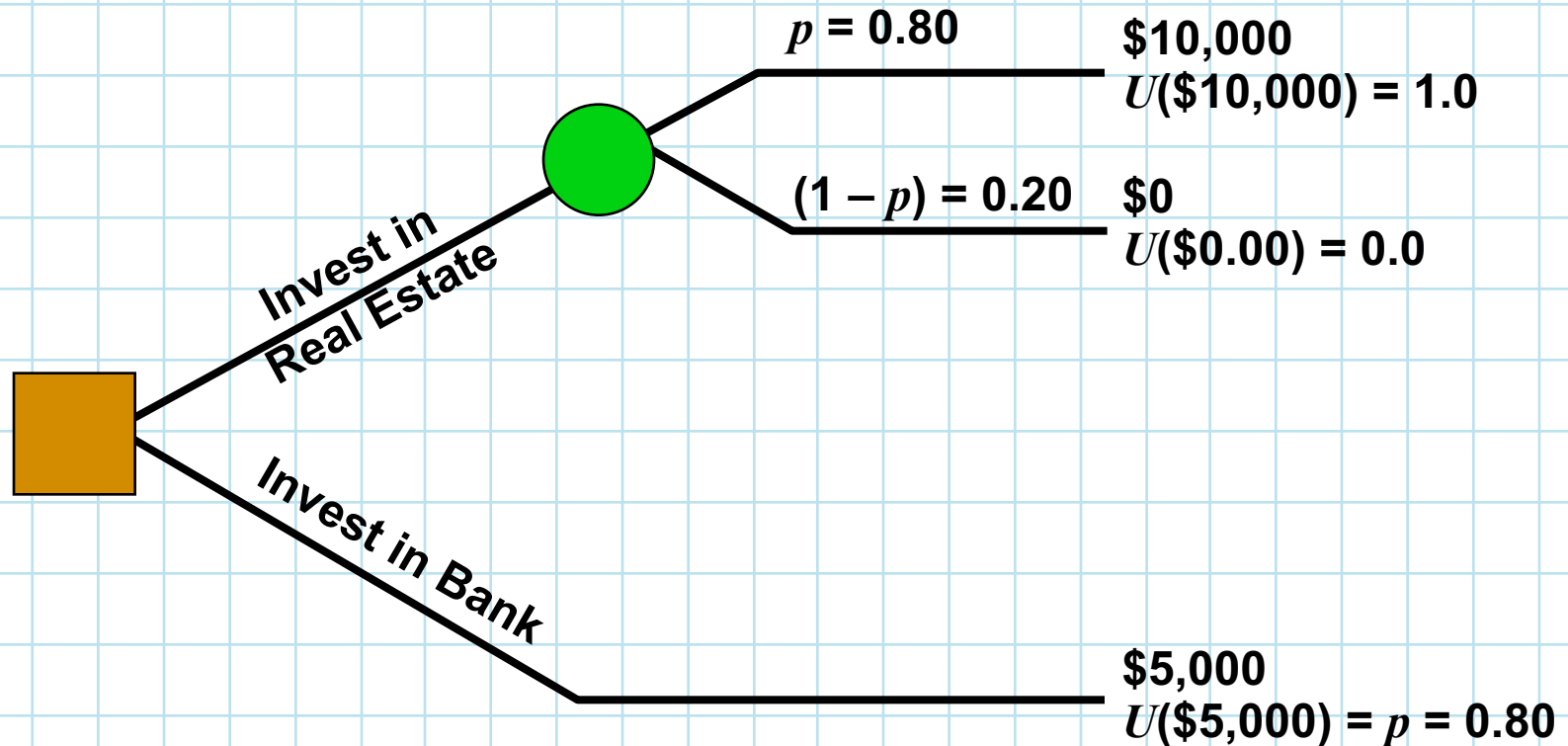
Figure 3.7



# ***Investment Example***

- Jane Dickson wants to construct a utility curve revealing her preference for money between \$0 and \$10,000.
- A utility curve plots the utility value versus the monetary value.
- An investment in a bank will result in \$5,000.
- An investment in real estate will result in \$0 or \$10,000.
- Unless there is an 80% chance of getting \$10,000 from the real estate deal, Jane would prefer to have her money in the bank.
- So if  $p = 0.80$ , Jane is indifferent between the bank or the real estate investment.

# Investment Example



$$\begin{aligned}\text{Utility for \$5,000} &= U(\$5,000) = pU(\$10,000) + (1 - p)U(\$0) \\ &= (0.8)(1) + (0.2)(0) = 0.8\end{aligned}$$

Figure 3.8

# ***Investment Example***

- **We can assess other utility values in the same way.**
- **For Jane these are:**

**Utility for \$7,000 = 0.90**

**Utility for \$3,000 = 0.50**

- **Using the three utilities for different dollar amounts, she can construct a utility curve.**

# Utility Curve

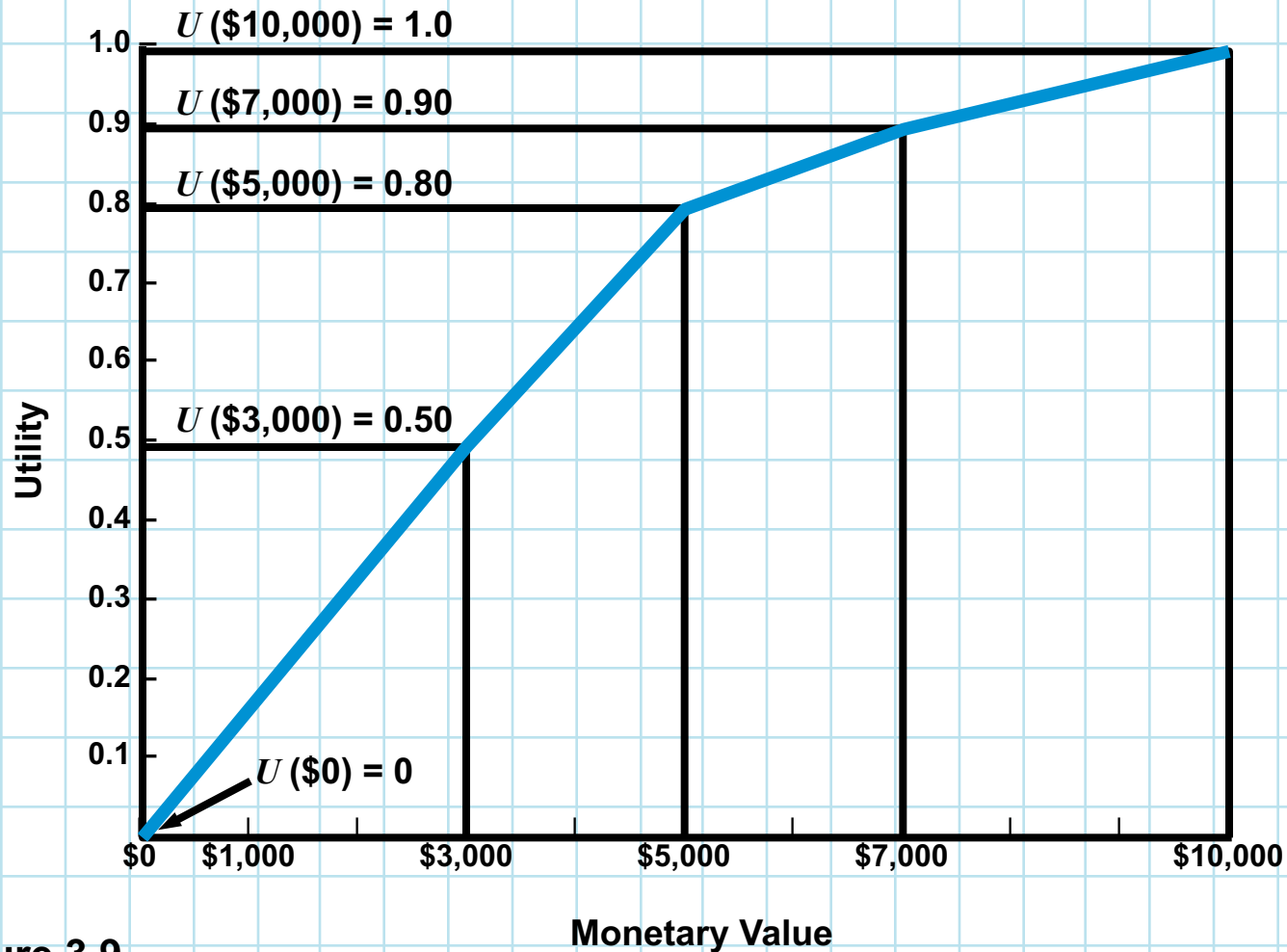


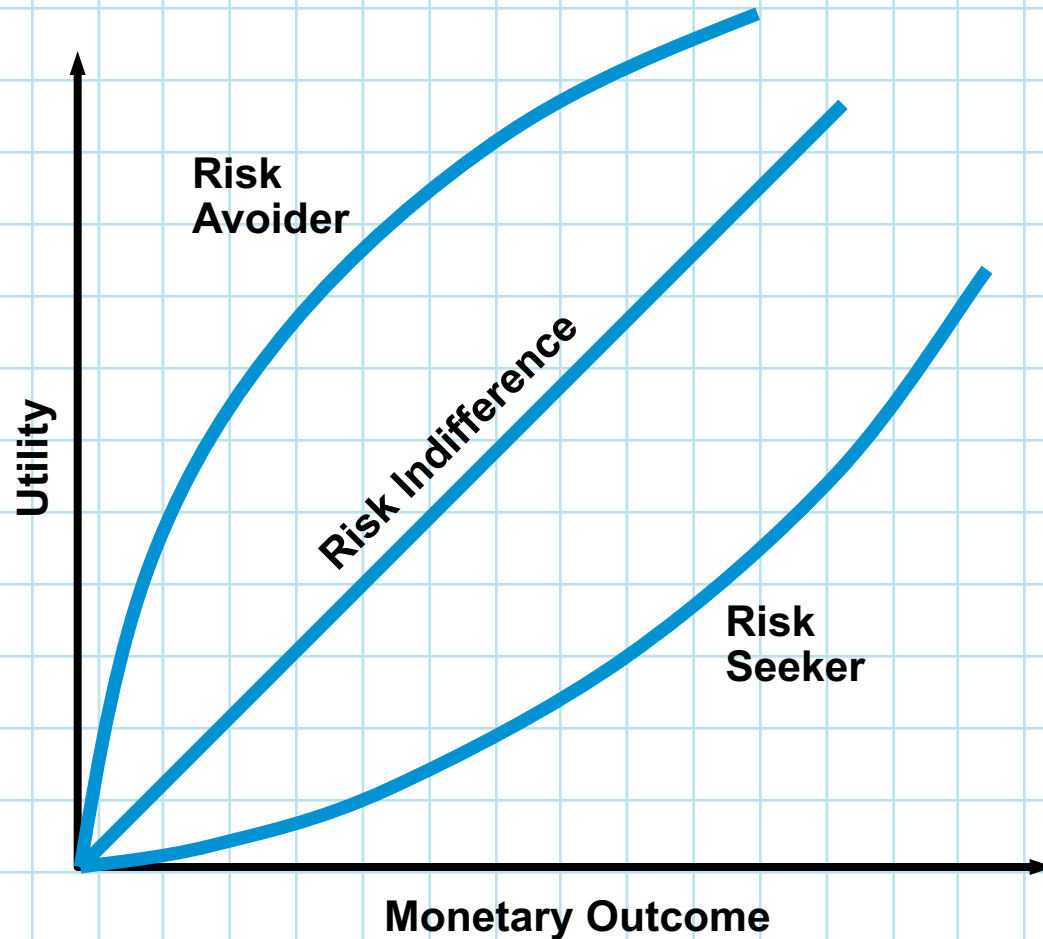
Figure 3.9

Monetary Value

# *Utility Curve*

- Jane's utility curve is typical of a **risk avoider**.
  - She gets less utility from greater risk.
  - She avoids situations where high losses might occur.
  - As monetary value increases, her utility curve increases at a slower rate.
- A **risk seeker** gets more utility from greater risk
  - As monetary value increases, the utility curve increases at a faster rate.
- Someone with **risk indifference** will have a linear utility curve.

# *Preferences for Risk*



**Figure 3.10**

# ***Utility as a Decision-Making Criteria***

- **Once a utility curve has been developed it can be used in making decisions.**
- **This replaces monetary outcomes with utility values.**
- **The expected utility is computed instead of the EMV.**

## *Utility as a Decision-Making Criteria*

- **Mark Simkin loves to gamble.**
- **He plays a game tossing thumbtacks in the air.**
- **If the thumbtack lands point up, Mark wins \$10,000.**
- **If the thumbtack lands point down, Mark loses \$10,000.**
- **Mark believes that there is a 45% chance the thumbtack will land point up.**
- **Should Mark play the game (alternative 1)?**



# *Utility as a Decision-Making Criteria*

## Decision Facing Mark Simkin

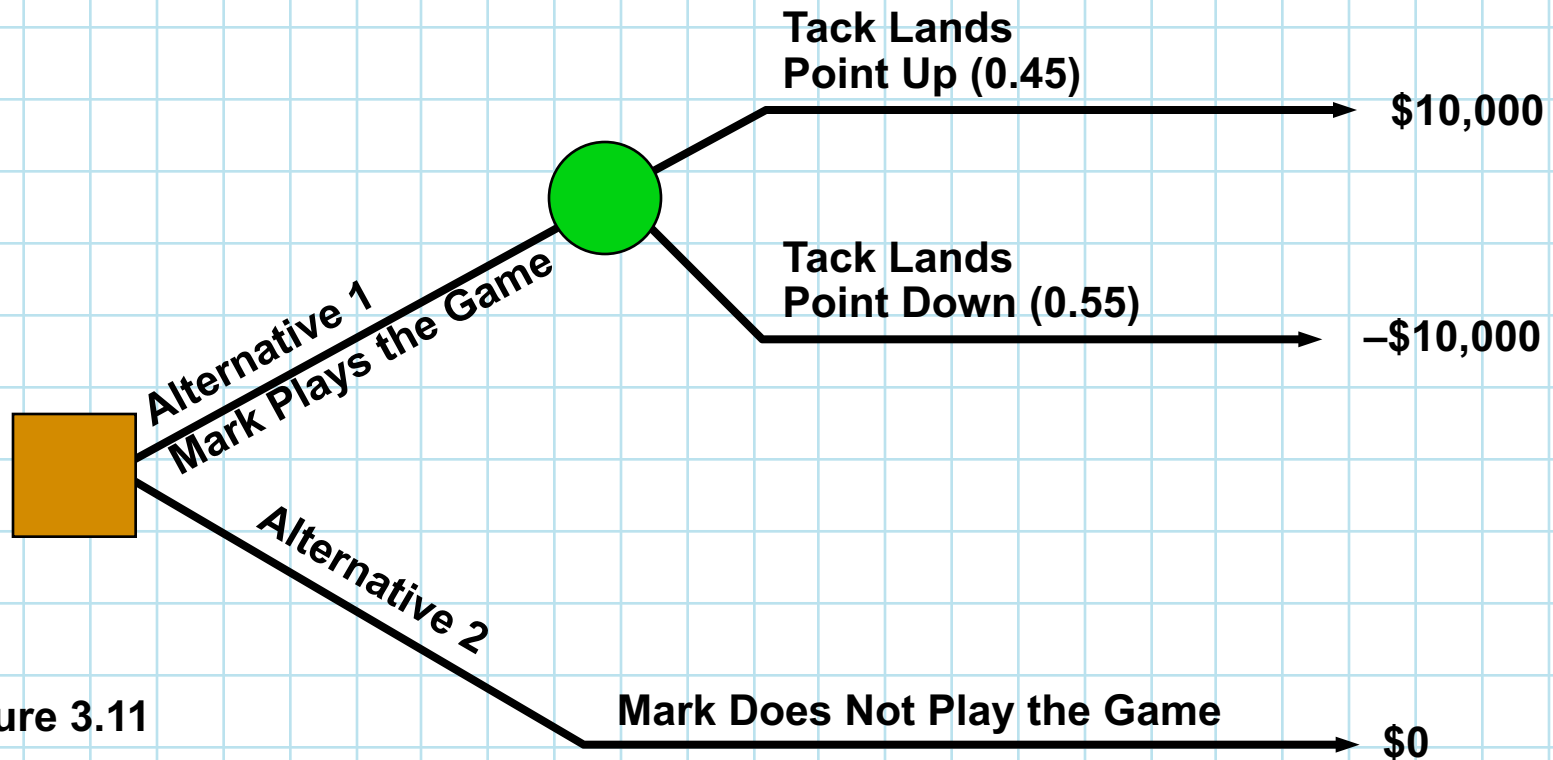


Figure 3.11

# ***Utility as a Decision-Making Criteria***

## ■ **Step 1– Define Mark’s utilities.**

$$U(-\$10,000) = 0.05$$

$$U(\$0) = 0.15$$

$$U(\$10,000) = 0.30$$

## ■ **Step 2 – Replace monetary values with utility values.**

$$\begin{aligned} E(\text{alternative 1: play the game}) &= (0.45)(0.30) + (0.55)(0.05) \\ &= 0.135 + 0.027 = 0.162 \end{aligned}$$

$$E(\text{alternative 2: don't play the game}) = 0.15$$

# *Utility Curve for Mark Simkin*

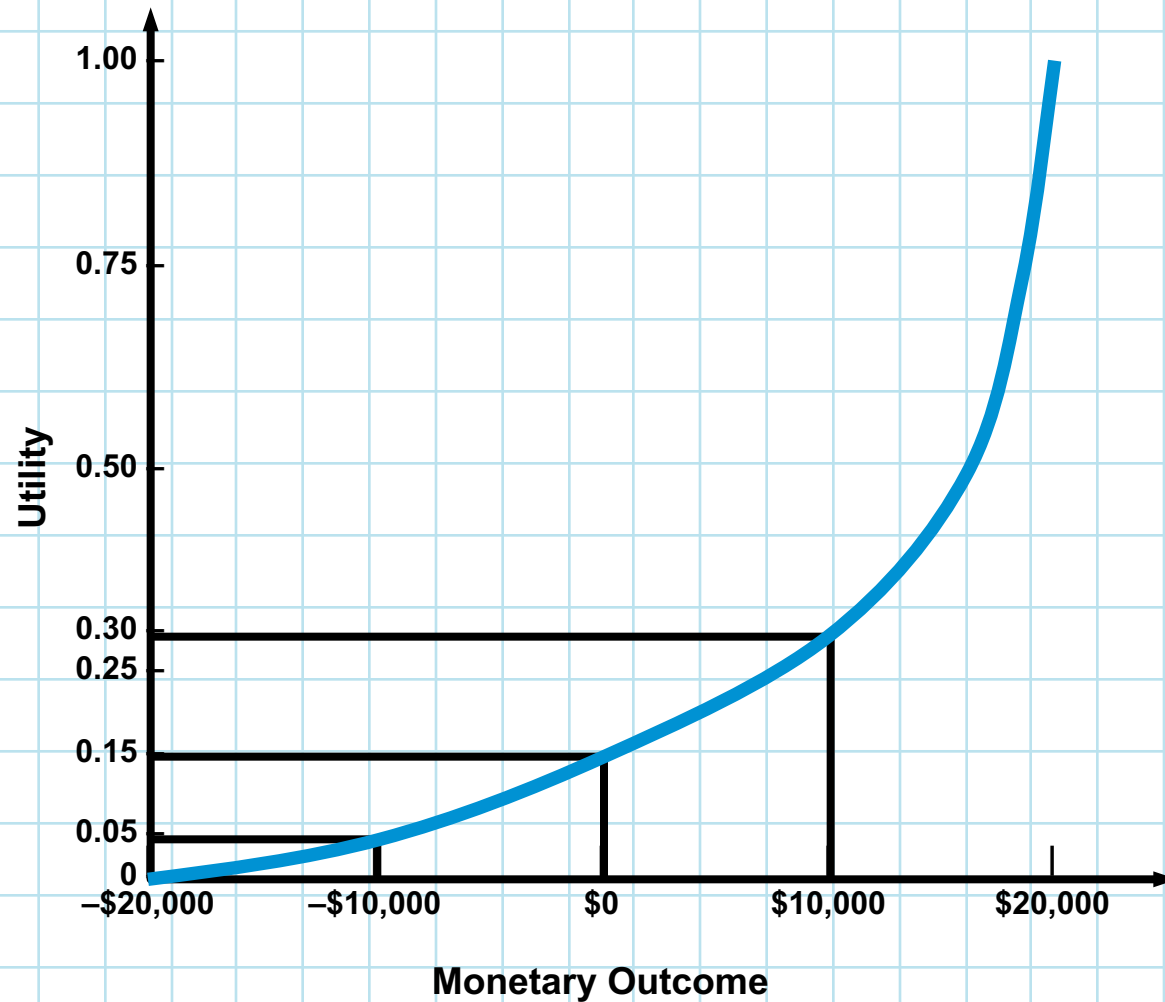


Figure 3.12

# Utility as a Decision-Making Criteria

## Using Expected Utilities in Decision Making

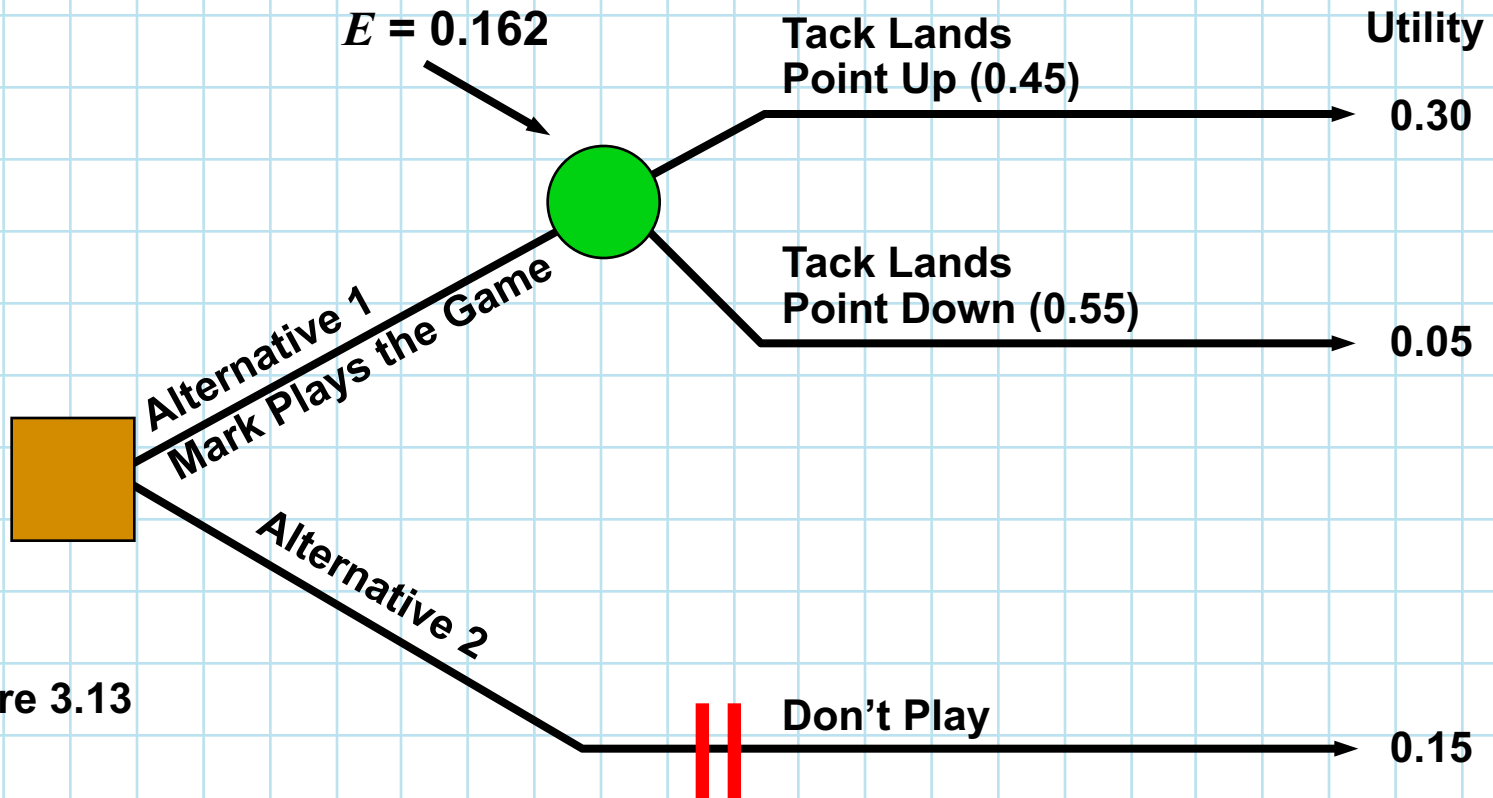
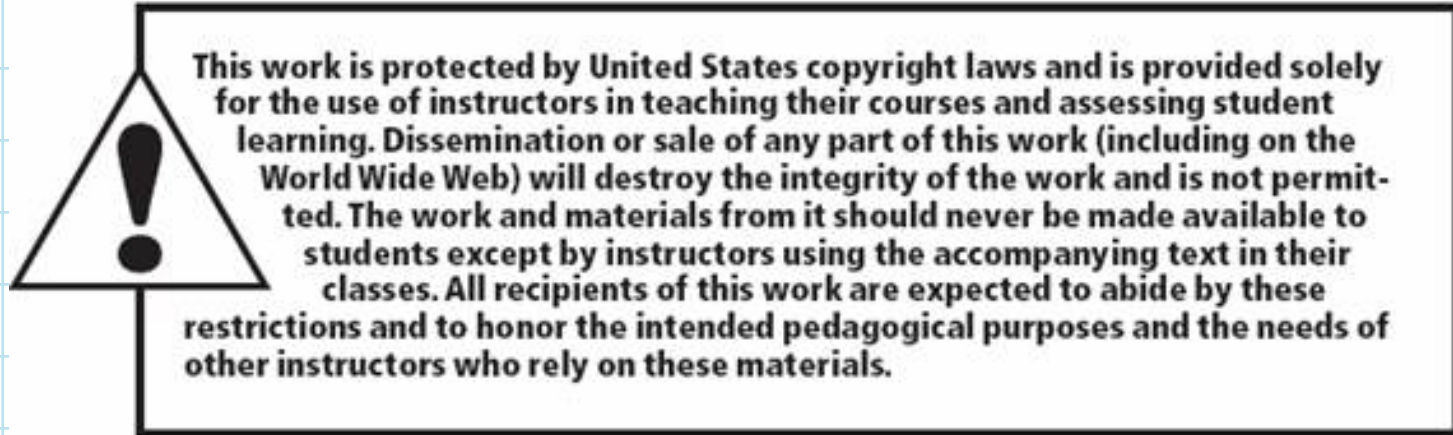


Figure 3.13

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