Chapter 10

Integer Programming, Goal Programming, and Nonlinear Programming

To accompany
Quantitative Analysis for Management, Eleventh Edition,
by Render, Stair, and Hanna
Power Point slides created by Brian Peterson

Learning Objectives

After completing this chapter, students will be able to:

1. Understand the difference between LP and integer programming.
2. Understand and solve the three types of integer programming problems.
3. Formulate and solve goal programming problems using Excel and QM for Windows.
4. Formulate nonlinear programming problems and solve using Excel.

Chapter Outline

Introduction

- Not every problem faced by businesses can easily fit into a neat linear programming context.
- A large number of business problems can be solved only if variables have integer values.
- Many business problems have multiple objectives, and goal programming is an extension to LP that can permit multiple objectives
- Linear programming requires linear models, and nonlinear programming allows objectives and constraints to be nonlinear.

Integer Programming

- An integer programming model is one where one or more of the decision variables has to take on an integer value in the final solution.
- There are three types of integer programming problems:
  1. Pure integer programming where all variables have integer values .
  2. Mixed-integer programming where some but not all of the variables will have integer values.
  3. Zero-one integer programming are special cases in which all the decision variables must have integer solution values of 0 or 1.

Harrison Electric Company Example of Integer Programming

- The Company produces two products popular with home renovators, old-fashioned chandeliers and ceiling fans.
- Both the chandeliers and fans require a two-step production process involving wiring and assembly.
- It takes about 2 hours to wire each chandelier and 3 hours to wire a ceiling fan.
- Final assembly of the chandeliers and fans requires 6 and 5 hours, respectively.
- The production capability is such that only 12 hours of wiring time and 30 hours of assembly time are available.

Harrison Electric Company Example of Integer Programming

- Each chandelier produced nets the firm \$7 and each fan \$6.
- Harrison's production mix decision can be formulated using LP as follows:

Maximize profit = $\$7X_1 + \$6X_2$

subject to

$$2X_1 + 3X_2 \leq 12 \quad \text{(wiring hours)}$$

$$6X_1 + 5X_2 \leq 30 \quad \text{(assembly hours)}$$

$$X_1, X_2 \geq 0 \quad \text{(nonnegativity)}$$

where

X_1 = number of chandeliers produced

X_2 = number of ceiling fans produced

Harrison Electric Problem

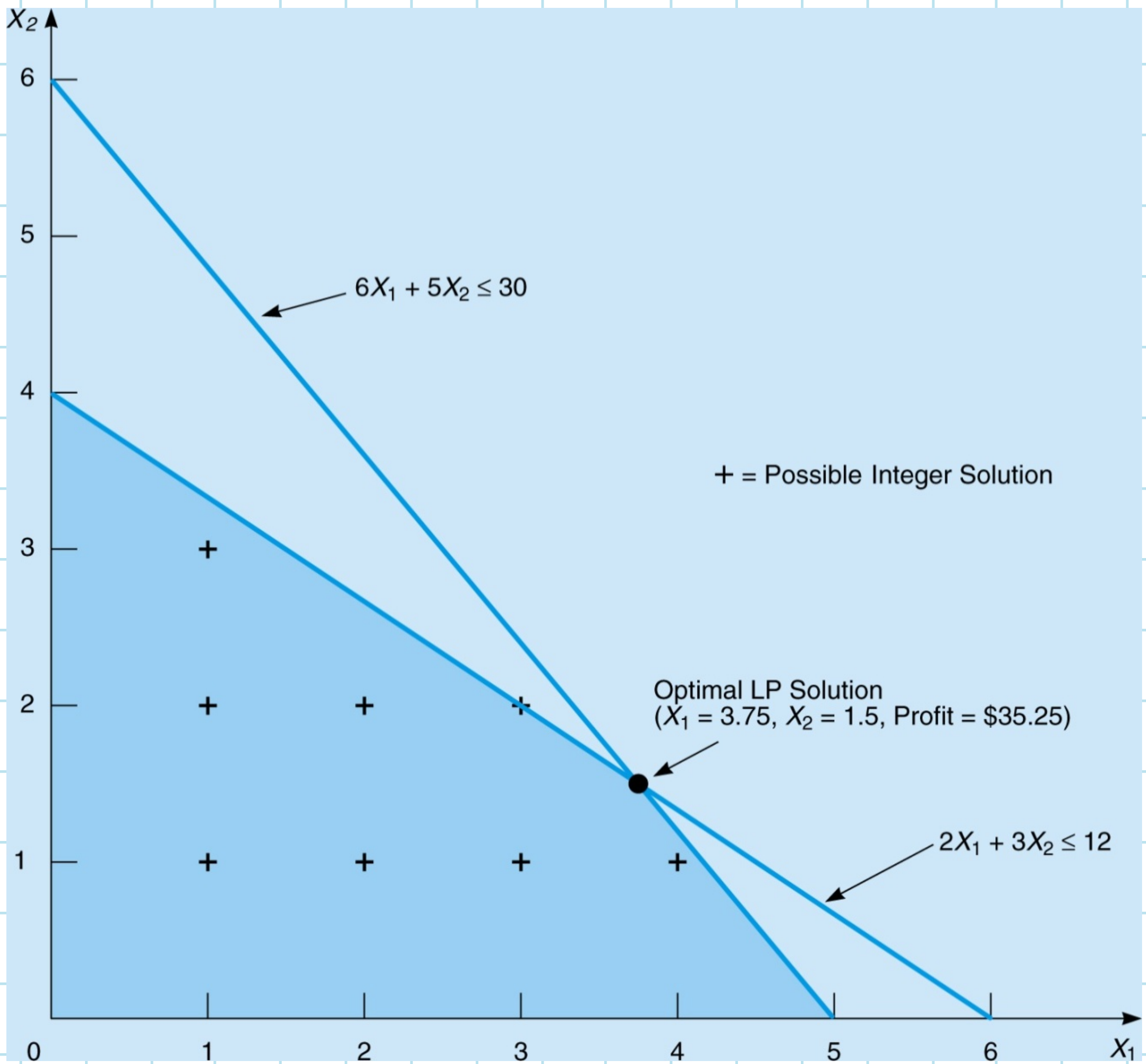


Figure 10.1

Harrison Electric Company

- The production planner recognizes this is an integer problem.
- His first attempt at solving it is to round the values to $X_1 = 4$ and $X_2 = 2$.
- However, this is not feasible.
- Rounding X_2 down to 1 gives a feasible solution, but it may not be *optimal.*
- This could be solved using the *enumeration* method, but enumeration is generally not possible for large problems.

Integer Solutions to the Harrison Electric Company Problem

CHANDELIERS (X_1)	CEILING FANS (X_2)	PROFIT ($\$7X_1 + \$6X_2$)
0	0	$0
1	0	7
2	0	14
3	0	21
4	0	28
5	0	35 ← Optimal solution to integer programming problem
0	1	6
1	1	13
2	1	20
3	1	27
4	1	34 ← Solution if rounding is used
0	2	12
1	2	19
2	2	26
3	2	33
0	3	18
1	3	25
0	4	24

Table 10.1

Harrison Electric Company

- The rounding solution of $X_1 = 4$, $X_2 = 1$ gives a profit of \$34.
- The optimal solution of $X_1 = 5$, $X_2 = 0$ gives a profit of \$35.
- The optimal integer solution is less than the optimal LP solution.
- An integer solution can ***never*** be better than the LP solution and is ***usually*** a lesser solution.

Using Software to Solve the Harrison Integer Programming Problem

QM for Windows Input Screen for Harrison Electric Problem

Objective
- (•) Maximize
- ( ) Minimize

Maximum number of iterations: 1000

Maximum level (depth) in procedure: 50

Harrison Electric Integer Programming Problem

	X1	X2		RHS	Equation form
Maximize	7	6			Max 7X1 + 6X2
Constraint 1	2	3	<=	12	2X1 + 3X2 <= 12
Constraint 2	6	5	<=	30	6X1 + 5X2 <= 30
Variable type	Integ	Integer			

Integer
Real
0/1

Program 10.1A

Using Software to Solve the Harrison Integer Programming Problem

QM for Windows Solution Screen for Harrison Electric Problem

Objective
Maximize
Minimize

Maximum number of iterations
1000

Integer & Mixed Integer Programming Results

Variable	Type	Value
X1	Integer	5
X2	Integer	0
Solution value		35

Program 10.1B

Using Software to Solve the Harrison Integer Programming Problem

Excel 2010 Solver Solution for Harrison Electric Problem

	A	B	C	D	E	F
1	**Harrison Electric Integer Programming Analysis**					
2		Chandeliers	Fans			
3	**Variables**	X1	X2			
4	**Values**	**5**	**0**	**Total Profit**		
5	**Profit**	7	6	**35**		
6						
7	**Constraints**			**LHS**	**Sign**	**RHS**
8	**Wiring hours**	2	3	10	≤	12
9	**Assembly hours**	6	5	30	≤	30

	D
5	=SUMPRODUCT($B$4:$C$4,B5:C5)

Program 10.2

Mixed-Integer Programming Problem Example

- There are many situations in which some of the variables are restricted to be integers and some are not.
- Bagwell Chemical Company produces two industrial chemicals.
- Xyline must be produced in 50-pound bags.
- Hexall is sold by the pound and can be produced in any quantity.
- Both xyline and hexall are composed of three ingredients – *A*, *B*, and *C.*
- Bagwell sells xyline for $85 a bag and hexall for $1.50 per pound.

Mixed-Integer Programming Problem Example

AMOUNT PER 50-POUND BAG OF XYLINE (LB)	AMOUNT PER POUND OF HEXALL (LB)	AMOUNT OF INGREDIENTS AVAILABLE
30	0.5	2,000 lb–ingredient *A*
18	0.4	800 lb–ingredient *B*
2	0.1	200 lb–ingredient *C*

- **Bagwell wants to maximize profit.**
- **Let X = number of 50-pound bags of xyline.**
- **Let Y = number of pounds of hexall.**
- **This is a mixed-integer programming problem as Y is not required to be an integer.**

Mixed-Integer Programming Problem Example

The model is:

Maximize profit = $85X + $1.50Y

$$
\begin{aligned}
\text{subject to} \quad 30X + 0.5Y &\le 2{,}000 \\
18X + 0.4Y &\le 800 \\
2X + 0.1Y &\le 200 \\
X,\ Y &\ge 0 \text{ and } X \text{ integer}
\end{aligned}
$$

Mixed-Integer Programming Problem Example

QM for Windows Solution for Bagwell Chemical Problem

Objective
Maximize
Minimize

Maximum number of iterations: 1000

Maximum level (depth) in procedure: 50

Limits are used, and the best solution available after a certain time is presented.

Notice that only X must be integer, while Y may be any real number.

Bagwell Chemical Company Solution

	X	Y		RHS
	85	1.5		
C	30	0.5	<=	2000
Constraint 2	18	0.4	<=	800
Constraint 3	2	0.1	<=	200
Variable type	Integer	Real		
Solution->	44	20	Optimal Z->	3770

Program 10.3

Mixed-Integer Programming Problem Example

Excel 2010 Solver Solution for Bagwell Chemical Problem

	A	B	C	D	E	F
1	Bagwell Chemical Company					
2		Xyline (bags)	Hexall (lbs)			
3	Variables	X	Y			
4	Values	44	20	Total Profit		
5	Profit	85	1.5	3770		
6						
7	Constraints			LHS	sign	RHS
8	Ingredient A	30	0.5	1330	≤	2000
9	Ingredient B	18	0.4	800	≤	800
10	Ingredient C	2	0.1	90	≤	200

Program 10.4

	E
5	=SUMPRODUCT($B$4:$D$4,B5:D5)

Modeling With 0-1 (Binary) Variables

- **We can demonstrate how 0-1 variables can be used to model several diverse situations.**
- **Typically a 0-1 variable is assigned a value of 0 if a certain condition is not met and a 1 if the condition is met.**
- **This is also called a *binary variable.***

Capital Budgeting Example

- A common capital budgeting problem is selecting from a set of possible projects when budget limitations make it impossible to select them all.
- A 0-1 variable is defined for each project.
- Quemo Chemical Company is considering three possible improvement projects for its plant:
  - A new catalytic converter.
  - A new software program for controlling operations.
  - Expanding the storage warehouse.
- It can not do them all
- It wants to maximize net present value of projects undertaken.

Quemo Chemical Capital Budgeting

Quemo Chemical Company information

PROJECT	NET PRESENT VALUE	YEAR 1	YEAR 2
Catalytic Converter	$25,000	$8,000	$7,000
Software	$18,000	$6,000	$4,000
Warehouse expansion	$32,000	$12,000	$8,000
Available funds		$20,000	$16,000

Table 10.2

The basic model is:

Maximize net present value of projects undertaken
subject to

Total funds used in year 1 ≤ $20,000
Total funds used in year 2 ≤ $16,000

Quemo Chemical Capital Budgeting

The decision variables are:

$$X_1 = \begin{cases} 1 \text{ if catalytic converter project is funded} \\ 0 \text{ otherwise} \end{cases}$$

$$X_2 = \begin{cases} 1 \text{ if software project is funded} \\ 0 \text{ otherwise} \end{cases}$$

$$X_3 = \begin{cases} 1 \text{ if warehouse expansion project is funded} \\ 0 \text{ otherwise} \end{cases}$$

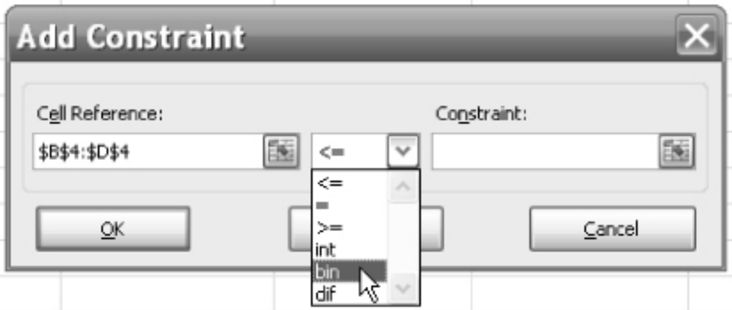
The mathematical statement of the integer programming problem becomes:

$$\begin{array}{lrcl} \text{Maximize NPV} = & 25{,}000X_1 + 18{,}000X_2 + 32{,}000X_3 & & \\ \text{subject to} & 8{,}000X_1 + 6{,}000X_2 + 12{,}000X_3 & \leq & 20{,}000 \\ & 7{,}000X_1 + 4{,}000X_2 + 8{,}000X_3 & \leq & 16{,}000 \\ & X_1, X_2, X_3 & = & 0 \text{ or } 1 \end{array}$$

Quemo Chemical Capital Budgeting

Excel 2010 Solver Solution for Quemo Chemical Problem

	A	B	C	D	E	F	G
1	Quemo Chemical Company						
2		Catalytic Conv.	Software	Warehouse Expan.			
3	Variables	X1	X2	X3			
4	Values	1	0	1	NPV		
5	Net Present Value	25000	18000	32000	57000		
6							
7	Constraints				LHS	sign	RHS
8	Year 1	8000	6000	12000	20000	≤	20000
9	Year 2	7000	4000	8000	15000	≤	16000



Program 10.5

	E
5	=SUMPRODUCT($B$4:$D$4,B5:D5)

Quemo Chemical Budgeting Capital

- This is solved with computer software, and the optimal solution is $X_1 = 1$, $X_2 = 0$, and $X_3 = 1$ with an objective function value of 57,000.
- This means that Quemo Chemical should fund the catalytic converter and warehouse expansion projects only.
- The net present value of these investments will be $57,000.

Limiting the Number of Alternatives Selected

- One common use of 0-1 variables involves limiting the number of projects or items that are selected from a group.
- Suppose Quemo Chemical is required to select no more than two of the three projects ***regardless*** of the funds available.
- This would require adding a constraint:

$$X_1 + X_2 + X_3 \leq 2$$

- If they had to fund ***exactly*** two projects the constraint would be:

$$X_1 + X_2 + X_3 = 2$$

Dependent Selections

- At times the selection of one project depends on the selection of another project.
- Suppose Quemo's catalytic converter could only be purchased if the software was purchased.
- The following constraint would force this to occur:

$$X_1 \leq X_2 \quad \text{or} \quad X_1 - X_2 \leq 0$$

- If we wished for the catalytic converter and software projects to either both be selected or both not be selected, the constraint would be:

$$X_1 = X_2 \quad \text{or} \quad X_1 - X_2 = 0$$

Fixed-Charge Problem Example

- Often businesses are faced with decisions involving a fixed charge that will affect the cost of future operations.
- Sitka Manufacturing is planning to build at least one new plant and three cities are being considered in:
  - Baytown, Texas
  - Lake Charles, Louisiana
  - Mobile, Alabama
- Once the plant or plants are built, the company wants to have capacity to produce at least 38,000 units each year.

Fixed-Charge Problem

Fixed and variable costs for Sitka Manufacturing

SITE	ANNUAL FIXED COST	VARIABLE COST PER UNIT	ANNUAL CAPACITY
Baytown, TX	$340,000	$32	21,000
Lake Charles, LA	$270,000	$33	20,000
Mobile, AL	$290,000	$30	19,000

Table 10.3

Fixed-Charge Problem

Define the decision variables as:

$$X_1 = \begin{cases} 1 \text{ if factory is built in Baytown} \\ 0 \text{ otherwise} \end{cases}$$

$$X_2 = \begin{cases} 1 \text{ factory is built in Lake Charles} \\ 0 \text{ otherwise} \end{cases}$$

$$X_3 = \begin{cases} 1 \text{ if factory is built in Mobile} \\ 0 \text{ otherwise} \end{cases}$$

X_4 = number of units produced at Baytown plant

X_5 = number of units produced at Lake Charles plant

X_6 = number of units produced at Mobile plant

Fixed-Charge Problem

The integer programming formulation becomes

$$\text{Minimize cost} = 340{,}000X_1 + 270{,}000X_2 + 290{,}000X_3 + 32X_4 + 33X_5 + 30X_6$$

$$
\begin{aligned}
\text{subject to} \quad X_4 + X_5 + X_6 &\geq 38{,}000 \\
X_4 &\leq 21{,}000X_1 \\
X_5 &\leq 20{,}000X_2 \\
X_6 &\leq 19{,}000X_3 \\
X_1, X_2, X_3 &= 0 \text{ or } 1; \\
X_4, X_5, X_6 &\geq 0 \text{ and integer}
\end{aligned}
$$

The optimal solution is

$X_1 = 0$, $X_2 = 1$, $X_3 = 1$, $X_4 = 0$, $X_5 = 19{,}000$, $X_6 = 19{,}000$

Objective function value = $1,757,000

Fixed-Charge Problem

Excel 2010 Solver Solution for Sitka Manufacturing Problem

	A	B	C	D	E	F	G	H	I	J
1	Sitka Manufacturing Company									
2		Baytown	Lake Charles	Mobile	Baytown units	L. Charles units	Mobile units			
3	Variables	X1	X2	X3	X4	X5	X6			
4	Values	0	1	1	0	19000	19000	Cost		
5	Cost	340000	270000	290000	32	33	30	1757000		
6										
7	Constraints							LHS	Sign	RHS
8	Minimum capacity				1	1	1	38000	≥	38000
9	Maximum in Baytown	-21000			1			0	≤	0
10	Maximum in L. C.		-20000			1		-1000	≤	0
11	Maximum in Mobile			-19000			1	0	≤	0

Program 10.6

	H
5	=SUMPRODUCT($B$4:$G$4,B5:G5)

Financial Investment Example

- Simkin, Simkin, and Steinberg specialize in recommending oil stock portfolios for wealthy clients.
- One client has the following specifications:
  - At least two Texas firms must be in the portfolio.
  - No more than one investment can be made in a foreign oil company.
  - One of the two California oil stocks must be purchased.
- The client has $3 million to invest and wants to buy large blocks of shares.

Financial Investment

Oil investment opportunities

STOCK	COMPANY NAME	EXPECTED ANNUAL RETURN ($1,000s)	COST FOR BLOCK OF SHARES ($1,000s)
1	Trans-Texas Oil	50	480
2	British Petroleum	80	540
3	Dutch Shell	90	680
4	Houston Drilling	120	1,000
5	Texas Petroleum	110	700
6	San Diego Oil	40	510
7	California Petro	75	900

Table 10.4

Financial Investment

Model formulation:

Maximize return = $50X_1 + 80X_2 + 90X_3 + 120X_4 + 110X_5 + 40X_6 + 75X_7$

subject to

$X_1 + X_4 + X_5 \geq 2$ (Texas constraint)

$X_2 + X_3 \leq 1$ (foreign oil constraint)

$X_6 + X_7 = 1$ (California constraint)

$480X_1 + 540X_2 + 680X_3 + 1{,}000X_4 + 700X_5$
$+ 510X_6 + 900X_7 \leq 3{,}000$ ($3 million limit)

All variables must be 0 or 1

Financial Investment

Excel 2010 Solver Solution for Financial Investment Problem

	A	B	C	D	E	F	G	H	I	J	K
1	Simkin, Simkin and Steinberg										
2											
3	Variables	X1	X2	X3	X4	X5	X6	X7			
4	Values	0	0	1	1	1	1	0	Return		
5	Return ($1,000s)	50	80	90	120	110	40	75	360		
6	Constraints								LHS	Sign	RHS
7	Texas	1			1	1			2	≥	2
8	Foreigh Oil		1	1					1	≤	1
9	California						1	1	1	=	1
10	$3 Million	480	540	680	1000	700	510	900	2890	≤	3000

Program 10.7

	I
5	=SUMPRODUCT($B$4:$H$4,B5:H5)

Goal Programming

- Firms often have more than one goal.
- In linear and integer programming methods the objective function is measured in one dimension only.
- It is not possible for LP to have ***multiple goals*** unless they are all measured in the same units, and this is a highly unusual situation.
- An important technique that has been developed to supplement LP is called ***goal programming.***

Goal Programming

- Typically goals set by management can be achieved only at the expense of other goals.
- A hierarchy of importance needs to be established so that higher-priority goals are satisfied before lower-priority goals are addressed.
- It is not always possible to satisfy every goal so goal programming attempts to reach a satisfactory level of multiple objectives.
- The main difference is in the objective function where goal programming tries to minimize the *deviations* between goals and what we can actually achieve within the given constraints.

Example of Goal Programming: Harrison Electric Company Revisited

The LP formulation for the Harrison Electric problem is:

Maximize profit = $\$7X_1 + \$6X_2$

subject to

$$2X_1 + 3X_2 \leq 12 \quad \text{(wiring hours)}$$
$$6X_1 + 5X_2 \leq 30 \quad \text{(assembly hours)}$$
$$X_1, X_2 \geq 0$$

where

X_1 = number of chandeliers produced

X_2 = number of ceiling fans produced

Example of Goal Programming: Harrison Electric Company Revisited

- Harrison is moving to a new location and feels that maximizing profit is not a realistic objective.
- Management sets a profit level of $30 that would be satisfactory during this period.
- The goal programming problem is to find the production mix that achieves this goal as closely as possible given the production time constraints.
- We need to define two deviational variables:

  d_1^- = underachievement of the profit target

  d_1^+ = overachievement of the profit target

Example of Goal Programming: Harrison Electric Company Revisited

We can now state the Harrison Electric problem as a single-goal programming model:

Minimize under or overachievement of profit target $= d_1^- + d_1^+$

$$
\begin{array}{llll}
\text{subject to} & \$7X_1 + \$6X_2 + d_1^- - d_1^+ = \$30 & & \text{(profit goal constraint)} \\
& 2X_1 + 3X_2 & \leq 12 & \text{(wiring hours)} \\
& 6X_1 + 5X_2 & \leq 30 & \text{(assembly hours)} \\
& X_1, X_2, d_1^-, d_1^+ \geq 0 & &
\end{array}
$$

Extension to Equally Important Multiple Goals

- Suppose Harrison's management wants to achieve several goals that are equal in priority:

  Goal 1: to produce a profit of $30 if possible during the production period.

  Goal 2: to fully utilize the available wiring department hours.

  Goal 3: to avoid overtime in the assembly department.

  Goal 4: to meet a contract requirement to produce at least seven ceiling fans.

Extension to Equally Important Multiple Goals

The deviational variables are:

d_1^- = underachievement of the profit target

d_1^+ = overachievement of the profit target

d_2^- = idle time in the wiring department (underutilization)

d_2^+ = overtime in the wiring department (overutilization)

d_3^- = idle time in the assembly department (underutilization)

d_3^+ = overtime in the assembly department (overutilization)

d_4^- = underachievement of the ceiling fan goal

d_4^+ = overachievement of the ceiling fan goal

Extension to Equally Important Multiple Goals

Because management is unconcerned about d_1^+, d_2^+, d_3^-, and d_4^+ these may be omitted from the objective function.

- The new objective function and constraints are:

Minimize total deviation = $d_1^- + d_2^- + d_3^+ + d_4^-$

subject to

$7X_1 + 6X_2 + d_1^- - d_1^+$	$= 30$	(profit constraint)
$2X_1 + 3X_2 + d_2^- - d_2^+$	$= 12$	(wiring hours)
$6X_1 + 5X_2 + d_3^- - d_3^+$	$= 30$	(assembly hours)
$X_2 + d_4^- - d_4^+$	$= 7$	(ceiling fan constraint)

All X_i, d_i variables ≥ 0

Ranking Goals with Priority Levels

- In most goal programming problems, one goal will be more important than another, which will in turn be more important than a third.
- Higher-order goals are satisfied before lower-order goals.
- Priorities (*P_i's*) are assigned to each deviational variable with the ranking so that P_1 is the most important goal, P_2 the next most important, P_3 the third, and so on.

Ranking Goals with Priority Levels

Harrison Electric has set the following priorities for their four goals:

GOAL	PRIORITY
Reach a profit as much above $30 as possible	P_1
Fully use wiring department hours available	P_2
Avoid assembly department overtime	P_3
Produce at least seven ceiling fans	P_4

Ranking Goals with Priority Levels

- **This effectively means that each goal is infinitely more important than the next lower goal.**
- **With the ranking of goals considered, the new objective function is:**

Minimize total deviation = $P_1d_1^- + P_2d_2^- + P_3d_3^+ + P_4d_4^-$

Constraints remain identical to the previous formulation.

Goal Programming with Weighted Goals

- Normally priority levels in goal programming assume that each level is infinitely more important than the level below it.
- Sometimes a goal may be only two or three times more important than another.
- Instead of placing these goals on different levels, we place them on the same level but with different weights.
- The coefficients of the deviation variables in the objective function include both the priority level and the weight.

Goal Programming with Weighted Goals

- Suppose Harrison decides to add another goal of producing at least two chandeliers.
- The goal of producing seven ceiling fans is considered twice as important as this goal.
- The goal of two chandeliers is assigned a weight of 1 and the goal of seven ceiling fans is assigned a weight of 2 and both of these will be priority level 4.
- The new constraint and objective function are:

$$X_1 + d_5^- - d_5^+ = 2 \quad \text{(chandeliers)}$$

$$\text{Minimize} = P_1 d_1^- + P_2 d_2^- + P_3 d_3^+ + P_4(2d_4^-) + P_4 d_5^-$$

Using QM for Windows to Solve Harrison's Problem

Harrison Electric's Goal Programming Analysis Using QM for Windows: Inputs

Harrison Electric Company								
	Wt(d+)	Prty(d+)	Wt(d-)	Prty(d-)	X1	X2		RHS
Constraint 1	0	0	1	1	7	6	=	30
Constraint 2	0	0	1	2	2	3	=	12
Constraint 3	1	3	0	0	6	5	=	30
Constraint 4	0	0	1	4	0	1	=	7

Program 10.8A

Using QM for Windows to Solve Harrison's Problem

Summary Screen for Harrison Electric's Goal Programming Analysis Using QM for Windows

Summary

Harrison Electric Company Solution			
Item			
Decision variable analysis	Value		
X1	0.		
X2	6.		
Priority analysis	Nonachievement		
Priority 1	0.		
Priority 2	0.		
Priority 3	0.		
Priority 4	1.		
Constraint Analysis	RHS	d+ (row i)	d- (row i)
Constraint 1	30.	6.	0.
Constraint 2	12.	6.	0.
Constraint 3	30.	0.	0.
Constraint 4	7.	0.	1.

Program 10.8B

Nonlinear Programming

- The methods seen so far have assumed that the objective function and constraints are linear.
- Terms such as X_1^3, $1/X_2$, $\log X_3$, or $5X_1X_2$ are not allowed.
- But there are many nonlinear relationships in the real world that would require the objective function, constraint equations, or both to be nonlinear.
- Excel can be used to solve these ***nonlinear programming*** (***NLP***) problems.
- One disadvantage of NLP is that the solution yielded may only be a local optimum, rather than a global optimum.
  - In other words, it may be an optimum over a particular range, but not overall.

Nonlinear Objective Function and Linear Constraints

- The Great Western Appliance Company sells two models of toaster ovens, the Microtoaster (X1) and the Self-Clean Toaster Oven (X2).
- They earn a profit of $28 for each Microtoaster no matter the number of units sold.
- For the Self-Clean oven, profits increase as more units are sold due to a fixed overhead.
  - The profit function for the Self-Clean over may be expressed as:

$$21X_2 + 0.25X_2^2$$

Nonlinear Objective Function and Linear Constraints

The objective function is nonlinear and there are two linear constraints on production capacity and sales time available.

Maximize profit = $28X_1 + 21X_2 + 0.25X_2^2$

subject to

$X_1 + 21X_2 \le 1{,}000$ (units of production capacity)

$0.5X_1 + 0.4X_2 \le 500$ (hours of sales time available)

$X_1, X_2 \ge 0$

When an objective function contains a squared term and the problem constraints are linear, it is called a *quadratic programming* problem.

Nonlinear Objective Function and Linear Constraints

Excel 2010 Solver Solution for Great Western Appliance NLP Problem

	A	B	C	D	E	F	G
1	Great Western Appliance						
2		Micro	Self-Clean				
3	Variables	X1	X2				
4	Values	0	1000				
5							
6	Terms	X1	X2	$X2^2$			
7	Calculated Values	0	1000	1000000	Profit		
8	Profit	28	21	0.25	21000		
9							
10	Constraints				LHS	Sign	RHS
11	Capacity	1	1		1000	≤	1000
12	Hours Available	0.5	0.4		400	≤	500

	E
8	=SUMPRODUCT($B$7:$D$7,B8:D8)
9	
10	LHS
11	=SUMPRODUCT($B$4:$C$4,B11:C11)
12	=SUMPRODUCT($B$4:$C$4,B12:C12)

	B	C	D
7	=B4	=C4	=C4^2

Program 10.9

Both Nonlinear Objective Function and Nonlinear Constraints

- The annual profit at a medium-sized (200-400 beds) Hospicare Corporation hospital depends on the number of medical patients admitted (X_1) and the number of surgical patients admitted (X_2).
- The objective function for the hospital is nonlinear.
- They have identified three constraints, two of which are nonlinear.
  - Nursing capacity - nonlinear
  - X-ray capacity - nonlinear
  - Marketing budget required

Both Nonlinear Objective Function and Nonlinear Constraints

The objective function and constraint equations for this problem are:

Maximize profit = $\$13X_1 + \$6X_1X_2 + \$5X_2 + \$1/X_2$

subject to

$2X_1^2 + 4X_2 \leq 90$ (nursing capacity in thousands of labor-days)

$X_1 + X_2^3 \leq 75$ (x-ray capacity in thousands)

$8X_1 - 2X_2 \leq 61$ (marketing budget required in thousands of $)

Both Nonlinear Objective Function and Nonlinear Constraints

Excel 2010 Solution for Hospicare's NLP Problem

	A	B	C	D	E	F	G	H	I	J
1	Hospicare Corp									
2										
3	Variables	X1	X2							
4	Values	6.0663	4.1003							
5										
6	Terms	X1	$X1^2$	X1*X2	X2	$X2^3$	1/X2			
7	Calculated Values	6.0663	36.7995	24.8732	4.1003	68.9337	0.2439	Total Profit		
8	Profit	13	0	6	5		1	248.8457		
9										
10	Constraints							LHS	Sign	RHS
11	Nursing		2		4			90.00	≤	90
12	X-Ray	1				1		75.00	≤	75
13	Budget	8			-2			40.33	≤	61

	H
8	=SUMPRODUCT($B$7:$G$7,B8:G8)

	B	C	D	E	F	G
7	=B4	=B4^2	=B4*C4	=C4	=C4^3	=1/C4

Program 10.10

Linear Objective Function and Nonlinear Constraints

- Thermlock Corp. produces massive rubber washers and gaskets like the type used to seal joints on the NASA Space Shuttles.
- It combines two ingredients, rubber (X_1) and oil (X_2).
- The cost of the industrial quality rubber is $5 per pound and the cost of high viscosity oil is $7 per pound.
- Two of the three constraints are nonlinear.

Linear Objective Function and Nonlinear Constraints

The firm's objective function and constraints are:

Minimize costs = $\$5X_1 + \$7X_2$

subject to

$$3X_1 + 0.25X_1^2 + 4X_2 + 0.3X_2^2 \geq 125 \quad \text{(hardness constraint)}$$

$$13X_1 + X_1^3 \geq 80 \quad \text{(tensile strength)}$$

$$0.7X_1 + X_2 \geq 17 \quad \text{(elasticity)}$$

Linear Objective Function and Nonlinear Constraints

Excel 2010 Solution for Thermlock NLP Problem

	A	B	C	D	E	F	G	H	I
1	Thermlock Gaskets								
2									
3	Variables	X1	X2						
4	Values	3.325	14.672	Total Cost					
5	Cost	5	7	119.333					
6									
7		X1	$X1^2$	$X1^3$	X2	$X2^2$			
8	Value	3.325	11.058	36.771	14.672	215.276			
9	Constraints						LHS	Sign	RHS
10	Hardness	3	0.25		4	0.3	136.012	≥	125
11	Tensile Strengt	13		1			80	≥	80
12	Elasticity	0.7			1		17	≥	17

	D
5	=SUMPRODUCT($B$4:$C$4,B5:C5)

	G
10	=SUMPRODUCT($B$8:$F$8,B10:F10)

Program 10.11

	B	C	D	E	F
8	=B4	=B4^2	=B4^3	=C4	=C4^2

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