Chapter 8

Linear Programming Applications

To accompany
Quantitative Analysis for Management, Eleventh Edition,
by Render, Stair, and Hanna
Power Point slides created by Brian Peterson

Learning Objectives

After completing this chapter, students will be able to:

1. **Model a wide variety of medium to large LP problems.**
2. **Understand major application areas, including marketing, production, labor scheduling, fuel blending, transportation, and finance.**
3. **Gain experience in solving LP problems with QM for Windows and Excel Solver software.**

Chapter Outline

Introduction

- The graphical method of LP is useful for understanding how to formulate and solve small LP problems.
- There are many types of problems that can be solved using LP.
- The principles developed here are applicable to larger problems.

Marketing Applications

- Linear programming models have been used in the advertising field as a decision aid in selecting an effective media mix.
- Media selection problems can be approached with LP from two perspectives:
  - Maximize audience exposure.
  - Minimize advertising costs.

Win Big Gambling Club

- The Win Big Gambling Club promotes gambling junkets to the Bahamas.
- It has $8,000 per week to spend on advertising.
- Its goal is to reach the largest possible high-potential audience.
- Media types and audience figures are shown in the following table.
- It needs to place at least five radio spots per week.
- No more than $1,800 can be spent on radio advertising each week.

Win Big Gambling Club

Advertising options

MEDIUM	AUDIENCE REACHED PER AD	COST PER AD ($)	MAXIMUM ADS PER WEEK
TV spot (1 minute)	5,000	800	12
Daily newspaper (full-page ad)	8,500	925	5
Radio spot (30 seconds, prime time)	2,400	290	25
Radio spot (1 minute, afternoon)	2,800	380	20

Win Big Gambling Club

The problem formulation is

X_1 = number of 1-minute TV spots each week
X_2 = number of daily paper ads each week
X_3 = number of 30-second radio spots each week
X_4 = number of 1-minute radio spots each week

Objective:

Maximize audience coverage $= 5{,}000X_1 + 8{,}500X_2 + 2{,}400X_3 + 2{,}800X_4$

Subject to

X_1	≤ 12	(max TV spots/wk)
X_2	≤ 5	(max newspaper ads/wk)
X_3	≤ 25	(max 30-sec radio spots ads/wk)
X_4	≤ 20	(max newspaper ads/wk)
$800X_1 + 925X_2 + 290X_3 + 380X_4$	$\leq \$8{,}000$	(weekly advertising budget)
$X_3 + X_4$	≥ 5	(min radio spots contracted)
$290X_3 + 380X_4$	$\leq \$1{,}800$	(max dollars spent on radio)
X_1, X_2, X_3, X_4	≥ 0	

Win Big Gambling Club Solution in Excel 2010

	A	B	C	D	E	F	G	H
1	Win Big Gambling Club							
2				Radio	Radio			
3		TV	Newspaper	30 sec.	1 min.			
4	Variables	X1	X2	X3	X4			
5	Solution	1.9688	5	6.2069	0	Total Audience		
6	Audience per ad	5000	8500	2400	2800	67240.3017		
7								
8	Constraints					LHS		RHS
9	Max. TV	1				1.9688	≤	12
10	Max. Newspaper		1			5	≤	5
11	Max. 30-sec. radio			1		6.2069	≤	25
12	Max. 1 min. radio				1	0	≤	20
13	Cost	800	925	290	380	8000	≤	8000
14	Radio dollars			290	380	1800	≤	1800
15	Radio spots			1	1	6.2069	≥	5

Program 8.1

Marketing Research

- Linear programming has also been applied to marketing research problems and the area of consumer research.
- Statistical pollsters can use LP to help make strategy decisions.

Management Sciences Association

- **Management Sciences Associates (MSA) is a marketing research firm.**
- **MSA determines that it must fulfill several requirements in order to draw statistically valid conclusions:**
  - **Survey at least 2,300 U.S. households.**
  - **Survey at least 1,000 households whose heads are 30 years of age or younger.**
  - **Survey at least 600 households whose heads are between 31 and 50 years of age.**
  - **Ensure that at least 15% of those surveyed live in a state that borders on Mexico.**
  - **Ensure that no more than 20% of those surveyed who are 51 years of age or over live in a state that borders on Mexico.**

Management Sciences Association

- MSA decides that all surveys should be conducted in person.
- It estimates the costs of reaching people in each age and region category are as follows:

	COST PER PERSON SURVEYED ($)		
REGION	AGE ≤ 30	AGE 31-50	AGE ≥ 51
State bordering Mexico	$7.50	$6.80	$5.50
State not bordering Mexico	$6.90	$7.25	$6.10

Management Sciences Association

- MSA's goal is to meet the sampling requirements at the least possible cost.
- The decision variables are:

X_1 = number of 30 or younger and in a border state
X_2 = number of 31-50 and in a border state
X_3 = number 51 or older and in a border state
X_4 = number 30 or younger and not in a border state
X_5 = number of 31-50 and not in a border state
X_6 = number 51 or older and not in a border state

Management Sciences Association

Objective function

Minimize total interview costs $= \$7.50X_1 + \$6.80X_2 + \$5.50X_3 + \$6.90X_4 + \$7.25X_5 + \$6.10X_6$

subject to

$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 \geq 2{,}300$ (total households)

$X_1 + X_4 \geq 1{,}000$ (households 30 or younger)

$X_2 + X_5 \geq 600$ (households 31-50)

$X_1 + X_2 + X_3 \geq 0.15(X_1 + X_2 + X_3 + X_4 + X_5 + X_6)$ (border states)

$X_3 \leq 0.20(X_3 + X_6)$ (limit on age group 51+ who can live in border state)

$X_1, X_2, X_3, X_4, X_5, X_6 \geq 0$

MSA Solution in Excel 2010

	A	B	C	D	E	F	G	H	I	J
1	**Management Science Associates**									
2										
3	**Variable**	**X1**	**X2**	**X3**	**X4**	**X5**	**X6**			
4	**Solution**	**0**	**600**	**140**	**1000**	**0**	**560**	**Total Cost**		
5	**Min. Cost**	7.5	6.8	5.5	6.9	7.25	6.1	**15166**		
6										
7	**Constraints**							**LHS**		**RHS**
8	**Total Households**	1	1	1	1	1	1	2300	≥	2,300
9	**30 and Younger**	1	0	0	1	0	0	1000	≥	1,000
10	**31-50**	0	1	0	0	1	0	600	≥	600
11	**Border States**	0.85	0.85	0.85	-0.15	-0.15	-0.15	395	≥	0
12	**51+ Border States**	0	0	0.8	0	0	-0.2	0	≤	0

Program 8.2

Management Sciences Association

- The following table summarizes the results of the MSA analysis.
- It will cost MSA $15,166 to conduct this research.

REGION	AGE ≤ 30	AGE 31-50	AGE ≥ 51
State bordering Mexico	0	600	140
State not bordering Mexico	1,000	0	560

Manufacturing Applications

- Production Mix
  - LP can be used to plan the optimal mix of products to manufacture.
  - Company must meet a myriad of constraints, ranging from financial concerns to sales demand to material contracts to union labor demands.
  - Its primary goal is to generate the largest profit possible.

Fifth Avenue Industries

- Fifth Avenue Industries produces four varieties of ties:
  - One is expensive all-silk
  - One is all-polyester
  - Two are polyester and cotton blends
- The table on the below shows the cost and availability of the three materials used in the production process:

MATERIAL	COST PER YARD ($)	MATERIAL AVAILABLE PER MONTH (YARDS)
Silk	24	1,200
Polyester	6	3,000
Cotton	9	1,600

Fifth Avenue Industries

- The firm has contracts with several major department store chains to supply ties.
- Contracts require a minimum number of ties but may be increased if demand increases.
- Fifth Avenue's goal is to maximize monthly profit given the following decision variables.

X_1 = number of all-silk ties produced per month
X_2 = number all-polyester ties
X_3 = number of blend 1 polyester-cotton ties
X_4 = number of blend 2 silk-cotton ties

Fifth Avenue Industries Data

VARIETY OF TIE	SELLING PRICE PER TIE ($)	MONTHLY CONTRACT MINIMUM	MONTHLY DEMAND	MATERIAL REQUIRED PER TIE (YARDS)	MATERIAL REQUIREMENTS
All silk	19.24	5,000	7,000	0.125	100% silk
All polyester	8.70	10,000	14,000	0.08	100% polyester
Poly – cotton blend 1	9.52	13,000	16,000	0.10	50% polyester – 50% cotton
Silk-cotton blend 2	10.64	5,000	8,500	0.11	60% silk - 40% cotton

Table 8.1

Fifth Avenue Industries

- Fifth Avenue also has to calculate profit per tie for the objective function.

VARIETY OF TIE	SELLING PRICE PER TIE ($)	MATERIAL REQUIRED PER TIE (YARDS)	MATERIAL COST PER YARD ($)	COST PER TIE ($)	PROFIT PER TIE ($)
All silk	$19.24	0.125	$24	$3.00	$16.24
All polyester	$8.70	0.08	$6	$0.48	$8.22
Poly-cotton blend 1	$9.52	0.05	$6	$0.30	
		0.05	$9	$0.45	$8.77
Silk – cotton blend 2	$10.64	0.06	$24	$1.44	
		0.06	$9	$0.54	$8.66

Fifth Avenue Industries

The complete Fifth Avenue Industries Model

Objective function

Maximize profit = \$16.24X_1 + \$8.22X_2 + \$8.77X_3 + \$8.66X_4

Subject to

$0.125X_1 + 0.066X_4 \leq$	1200	(yds of silk)
$0.08X_2 + 0.05X_3 \leq$	3,000	(yds of polyester)
$0.05X_3 + 0.44X_4 \leq$	1,600	(yds of cotton)
$X_1 \geq$	5,000	(contract min for silk)
$X_1 \leq$	7,000	(contract min)
$X_2 \geq$	10,000	(contract min for all polyester)
$X_2 \leq$	14,000	(contract max)
$X_3 \geq$	13,000	(contract mini for blend 1)
$X_3 \leq$	16,000	(contract max)
$X_4 \geq$	5,000	(contract mini for blend 2)
$X_4 \leq$	8,500	(contract max)
$X_1, X_2, X_3, X_4 \geq$	0	

Fifth Avenue Solution in Excel 2010

	A	B	C	D	E	F	G	H
1	**Fifth Avenue Industries**							
2								
3		All silk	All poly.	Blend 1	Blend 2			
4	**Variables**	**X1**	**X2**	**X3**	**X4**			
5	**Values**	5112	14000	16000	8500	**Total Profit**		
6	**Profit**	16.24	8.22	8.77	8.66	412028.88		
7								
8	**Constraints**					**LHS**		**RHS**
9	**Silk available**	0.125			0.066	1200	≤	1200
10	**Polyester available**		0.08	0.05		1920	≤	3000
11	**Cotton available**			0.05	0.044	1174	≤	1600
12	**Maximum silk**	1				5112	≤	7000
13	**Maximum polyester**		1			14000	≤	14000
14	**Maximum blend 1**			1		16000	≤	16000
15	**Maximum blend 2**				1	8500	≤	8500
16	**Minimum silk**	1				5112	≥	5000
17	**Minimum polyester**		1			14000	≥	10000
18	**Minimum blend 1**			1		16000	≥	13000
19	**Minimum blend 2**				1	8500	≥	5000

Program 8.3

Manufacturing Applications

- Production Scheduling
  - Setting a low-cost production schedule over a period of weeks or months is a difficult and important management task.
  - Important factors include labor capacity, inventory and storage costs, space limitations, product demand, and labor relations.
  - When more than one product is produced, the scheduling process can be quite complex.
  - The problem resembles the product mix model for each time period in the future.

Greenberg Motors

- Greenberg Motors, Inc. manufactures two different electric motors for sale under contract to Drexel Corp.
- Drexel places orders three times a year for four months at a time.
- Demand varies month to month as shown below.
- Greenberg wants to develop its production plan for the next four months.

MODEL	JANUARY	FEBRUARY	MARCH	APRIL
GM3A	800	700	1,000	1,100
GM3B	1,000	1,200	1,400	1,400

Table 8.2

Greenberg Motors

- **Production planning at Greenberg must consider four factors:**
  - **Desirability of producing the same number of motors each month to simplify planning and scheduling.**
  - **Necessity to keep inventory carrying costs down.**
  - **Warehouse limitations.**
  - **Its no-lay-off policy.**
- **LP is a useful tool for creating a minimum total cost schedule the resolves conflicts between these factors.**

Greenberg Motors

A_i = Number of model GM3A motors produced in month i (i = 1, 2, 3, 4 for January – April)

B_i = Number of model GM3B motors produced in month i

- It costs $20 to produce a GM3A and $15 to produce a GM3B
- Both costs increase by 10% on March 1, thus

$$\text{Cost of production} = \$20A_1 + \$20A_2 + \$22A_3 + \$22A_4 + \$15B_1 + \$15B_2 + \$16.50B_3 + \$16.50B_4$$

Greenberg Motors

- We can use the same approach to create the portion of the objective function dealing with inventory carrying costs.

IA_i = Units of GM3A left in inventory at the end of month i (i = 1, 2, 3, 4 for January – April)

IB_i = Units of GM3B left in inventory at the end of month i (i = 1, 2, 3, 4 for January – April)

- **The carrying cost for GM3A motors is $0.36 per unit per month and the GM3B costs $0.26 per unit per month.**
- **Monthly ending inventory levels are used for the average inventory level.**

Cost of carrying inventory = $\$0.36A_1 + \$0.36A_2 + \$0.36A_3 + 0.36A_4 + \$0.26B_1 + \$0.26B_2 + \$0.26B_3 + \$0.26B_4$

Greenberg Motors

We combine these two for the objective function:

$$
\begin{aligned}
\text{Minimize total cost} = {} & \$20A_1 + \$20A_2 + \$22A_3 + 22A_4 \\
& + \$15B_1 + \$15B_2 + \$16.50B_3 + \$16.50B_4 \\
& + \$0.36IA_1 + \$0.36IA_2 + \$0.36IA_3 + 0.36IA_4 \\
& + \$0.26IB_1 + \$0.26IB_2 + \$0.26IB_3 + \$0.26IB_4
\end{aligned}
$$

End of month inventory is calculated using this relationship:

$$
\left(\begin{array}{c}\text{Inventory}\\ \text{at the end}\\ \text{of last}\\ \text{month}\end{array}\right) + \left(\begin{array}{c}\text{Current}\\ \text{month's}\\ \text{production}\end{array}\right) - \left(\begin{array}{c}\text{Inventory at}\\ \text{the end of}\\ \text{this month}\end{array}\right) = \left(\begin{array}{c}\text{Sales to}\\ \text{Drexel this}\\ \text{month}\end{array}\right)
$$

Greenberg Motors

- Greenberg is starting a new four-month production cycle with a change in design specification that left no old motors in stock on January 1.
- Given January demand for both motors:

$$IA_1 = 0 + A_1 - 800$$
$$IB_1 = 0 + B_1 - 1{,}000$$

- Rewritten as January's constraints:

$$A_1 - IA_1 = 800$$
$$B_1 - IB_1 = 1{,}000$$

Greenberg Motors

Constraints for February, March, and April:

$A_2 + IA_1 - IA_2 =$	700	February GM3A demand
$B_2 + IB_1 - IB_2 =$	1,200	February GM3B demand
$A_3 + IA_2 - IA_3 =$	1,000	March GM3A demand
$B_3 + IB_2 - IB_3 =$	1,400	March GM3B demand
$A_4 + IA_3 - IA_4 =$	1,100	April GM3A demand
$B_4 + IB_3 - IB_4 =$	1,400	April GM3B demand

And constraints for April's ending inventory:

$$IA_4 = 450$$
$$IB_4 = 300$$

Greenberg Motors

- We also need constraints for warehouse space:

$$IA_1 + IB_1 \leq 3{,}300$$
$$IA_2 + IB_2 \leq 3{,}300$$
$$IA_3 + IB_3 \leq 3{,}300$$
$$IA_4 + IB_4 \leq 3{,}300$$

- No worker is ever laid off so Greenberg has a base employment level of 2,240 labor hours per month.
- By adding temporary workers, available labor hours can be increased to 2,560 hours per month.
- Each GM3A motor requires 1.3 labor hours and each GM3B requires 0.9 hours.

Greenberg Motors

Labor hour constraints:

$1.3A_1 + 0.9B_1$	$\geq 2{,}240$	(January min hrs/month)
$1.3A_1 + 0.9B_1$	$\leq 2{,}560$	(January max hrs/month)
$1.3A_2 + 0.9B_2$	$\geq 2{,}240$	(February labor min)
$1.3A_2 + 0.9B_2$	$\leq 2{,}560$	(February labor max)
$1.3A_3 + 0.9B_3$	$\geq 2{,}240$	(March labor min)
$1.3A_3 + 0.9B_3$	$\leq 2{,}560$	(March labor max)
$1.3A_4 + 0.9B_4$	$\geq 2{,}240$	(April labor min)
$1.3A_4 + 0.9B_4$	$\leq 2{,}560$	(April labor max)
All variables	≥ 0	Nonnegativity constraints

Greenberg Motors Solution in Excel 2010

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
1	Greenberg Motors																			
2																				
3	Variable	A1	A2	A3	A4	B1	B2	B3	B4	IA1	IA2	IA3	IA4	IB1	IB2	IB3	IB4			
4	Solution	1276.9	223.1	1757.7	792.3	1000	2522.2	77.8	1700	476.9	0	757.7	450	0	1322.2	0	300	Total Cost		
5	Min. Cost	20	20	22	22	15	15	16.5	16.5	0.36	0.36	0.36	0.36	0.26	0.26	0.26	0.26	169294.9		
6																				
7	Demand Constraints																	LHS	Sign	RHS
8	Jan. GM3A	1								-1								800	=	800
9	Feb. GM3A		1							1	-1							700	=	700
10	Mar. GM3A			1							1	-1						1000	=	1000
11	Apr. GM3A				1							1	-1					1100	=	1100
12	Jan. GM3B					1								-1				1000	=	1000
13	Feb. GM3B						1							1	-1			1200	=	1200
14	Mar. GM3B							1							1	-1		1400	=	1400
15	Apr. GM3B								1							1	-1	1400	=	1400
16	Inv.GM3A Apr.												1					450	=	450
17	Inv.GM3B Apr.																1	300	=	300
18	Labor Hour Constraints																			
19	Hrs Min. Jan.	1.3				0.9												2560	≥	2240
20	Hrs Min. Feb.		1.3				0.9											2560	≥	2240
21	Hrs Min. Mar.			1.3				0.9										2355	≥	2240
22	Hrs Min. Apr.				1.3				0.9									2560	≥	2240
23	Hrs Max. Jan.	1.3				0.9												2560	≤	2560
24	Hrs Max. Feb.		1.3				0.9											2560	≤	2560
25	Hrs Max.Mar.			1.3				0.9										2355	≤	2560
26	Hrs Max. Apr.				1.3				0.9									2560	≤	2560
27	Storage Constraints																			
28	Jan. Inv. Limit									1				1				476.92	≤	3300
29	Feb. Inv. Limit										1				1			1322.22	≤	3300
30	Mar. Inv. Limit											1				1		757.69	≤	3300
31	Apr. Inv. Limit												1				1	750	≤	3300

Program 8.4

Greenberg Motors

Solution to Greenberg Motors Problem

PRODUCTION SCHEDULE	JANUARY	FEBRUARY	MARCH	APRIL
Units GM3A produced	1,277	223	1,758	792
Units GM3B produced	1,000	2,522	78	1,700
Inventory GM3A carried	477	0	758	450
Inventory GM3B carried	0	1,322	0	300
Labor hours required	2,560	2,560	2,355	2,560

Table 8.3

- Total cost for this four month period is $169,294.90.
- Complete model has 16 variables and 22 constraints.

Employee Scheduling Applications

- Labor Planning
  - These problems address staffing needs over a particular time.
  - They are especially useful when there is some flexibility in assigning workers that require overlapping or interchangeable talents.

Hong Kong Bank of Commerce and Industry

- Hong Kong Bank of Commerce and Industry has requirements for between 10 and 18 tellers depending on the time of day.
- Lunch time from noon to 2 pm is generally the busiest.
- The bank employs 12 full-time tellers but has many part-time workers available.
- Part-time workers must put in exactly four hours per day, can start anytime between 9 am and 1 pm, and are inexpensive.
- Full-time workers work from 9 am to 3 pm and have 1 hour for lunch.

Hong Kong Bank of Commerce and Industry

Labor requirements for Hong Kong Bank of Commerce and Industry

TIME PERIOD	NUMBER OF TELLERS REQUIRED
9 am – 10 am	10
10 am – 11 am	12
11 am – Noon	14
Noon – 1 pm	16
1 pm – 2 pm	18
2 pm – 3 pm	17
3 pm – 4 pm	15
4 pm – 5 pm	10

Table 8.4

Hong Kong Bank of Commerce and Industry

- Part-time hours are limited to a maximum of 50% of the day's total requirements.
- Part-timers earn $8 per hour on average.
- Full-timers earn $100 per day on average.
- The bank wants a schedule that will minimize total personnel costs.
- It will release one or more of its part-time tellers if it is profitable to do so.

Hong Kong Bank of Commerce and Industry

Let

F = full-time tellers
P_1 = part-timers starting at 9 am (leaving at 1 pm)
P_2 = part-timers starting at 10 am (leaving at 2 pm)
P_3 = part-timers starting at 11 am (leaving at 3 pm)
P_4 = part-timers starting at noon (leaving at 4 pm)
P_5 = part-timers starting at 1 pm (leaving at 5 pm)

Hong Kong Bank of Commerce and Industry

Objective:

Minimize total daily personnel cost $= \$100F + \$32(P_1 + P_2 + P_3 + P_4 + P_5)$

subject to:

$F + P_1 \geq 10$		(9 am – 10 am needs)
$F + P_1 + P_2 \geq 12$		(10 am – 11 am needs)
$0.5F + P_1 + P_2 + P_3 \geq 14$		(11 am – noon needs)
$0.5F + P_1 + P_2 + P_3 + P_4 \geq 16$		(noon – 1 pm needs)
$F + P_2 + P_3 + P_4 + P_5 \geq 18$		(1 pm – 2 pm needs)
$F + P_3 + P_4 + P_5 \geq 17$		(2 pm – 3 pm needs)
$F + P_4 + P_5 \geq 15$		(3 pm – 4 pm needs)
$F + P_5 \geq 10$		(4 pm – 5 pm needs)
$F \leq 12$		(12 full-time tellers)
$4P_1 + 4P_2 + 4P_3 + 4P_4 + 4P_5 \leq 0.50(112)$		(max 50% part-timers)

$$P_1, P_2, P_3, P_4, P_5 \geq 0$$

Hong Kong Bank of Commerce and Industry

- There are several alternate optimal schedules Hong Kong Bank can follow:
- $F = 10$, $P_2 = 2$, $P_3 = 7$, $P_4 = 5$, P_1, $P_5 = 0$
- $F = 10$, $P_1 = 6$, $P_2 = 1$, $P_3 = 2$, $P_4 = 5$, $P_5 = 0$
- The cost of either of these two policies is $1,448 per day.

Labor Planning Solution in Excel 2010

	A	B	C	D	E	F	G	H	I	J
1	Labor Planning Example									
2										
3										
4	Variables	F	P1	P2	P3	P4	P5			
5	Values	10	0	7	2	5	0	Total Cost		
6	Cost	100	32	32	32	32	32	1448		
7										
8	Constraints							LHS	Sign	RHS
9	9 a.m. - 10 a.m.	1	1					10	≥	10
10	10 a.m. - 11 a.m.	1	1	1				17	≥	12
11	11 a.m. - noon	0.5	1	1	1			14	≥	14
12	noon - 1 p.m.	0.5	1	1	1	1		19	≥	16
13	1 p.m. - 2 p.m.	1		1	1	1	1	24	≥	18
14	2 p.m. - 3 p.m.	1			1	1	1	17	≥	17
15	3 p.m. - 4 p.m.	1				1	1	15	≥	15
16	4 p.m. - 5 p.m.	1					1	10	≥	10
17	Max. Full time	1						10	≤	12
18	Total PT hours		4	4	4	4	4	56	≤	56

Program 8.5

Financial Applications

- Portfolio Selection
  - Bank, investment funds, and insurance companies often have to select specific investments from a variety of alternatives.
  - The manager's overall objective is generally to maximize the potential return on the investment given a set of legal, policy, or risk restraints.

International City Trust

- International City Trust (ICT) invests in short-term trade credits, corporate bonds, gold stocks, and construction loans.
- The board of directors has placed limits on how much can be invested in each area:

INVESTMENT	INTEREST EARNED (%)	MAXIMUM INVESTMENT ($ MILLIONS)
Trade credit	7	1.0
Corporate bonds	11	2.5
Gold stocks	19	1.5
Construction loans	15	1.8

International City Trust

- ICT has $5 million to invest and wants to accomplish two things:
  - Maximize the return on investment over the next six months.
  - Satisfy the diversification requirements set by the board.
- The board has also decided that at least 55% of the funds must be invested in gold stocks and construction loans and no less than 15% be invested in trade credit.

International City Trust

The variables in the model are:

X_1 = dollars invested in trade credit
X_2 = dollars invested in corporate bonds
X_3 = dollars invested in gold stocks
X_4 = dollars invested in construction loans

International City Trust

Objective:

Maximize dollars of interest earned $= 0.07X_1 + 0.11X_2 + 0.19X_3 + 0.15X_4$

subject to:

$$
\begin{aligned}
X_1 &\le 1{,}000{,}000 \\
X_2 &\le 2{,}500{,}000 \\
X_3 &\le 1{,}500{,}000 \\
X_4 &\le 1{,}800{,}000 \\
X_3 + X_4 &\ge 0.55(X_1 + X_2 + X_3 + X_4) \\
X_1 &\ge 0.15(X_1 + X_2 + X_3 + X_4) \\
X_1 + X_2 + X_3 + X_4 &\le 5{,}000{,}000 \\
X_1, X_2, X_3, X_4 &\ge 0
\end{aligned}
$$

International City Trust

- The optimal solution to the ICT is to make the following investments:

  X_1 = $750,000

  X_2 = $950,000

  X_3 = $1,500,000

  X_4 = $1,800,000
- The total interest earned with this plan is $712,000.

ICT Portfolio Solution in Excel 2010

	A	B	C	D	E	F	G	H
1	ICT Portfolio Selection							
2								
3	Variable	X1	X2	X3	X4			
4	Solution	750000	950000	1500000	1800000	Total Return		
5	Max. Return	0.07	0.11	0.19	0.15	712000		
6								
7						LHS		RHS
8	Trade	1				750000	≤	1,000,000
9	Bonds		1			950000	≤	2,500,000
10	Gold			1		1500000	≤	1,500,000
11	Construction				1	1800000	≤	1,800,000
12	Min. Gold+Const	-0.55	-0.55	0.45	0.45	550000	≥	0
13	Min. Trade	0.85	-0.15	-0.15	-0.15	0	≥	0
14	Total Invested	1	1	1	1	5000000	≤	5000000

Program 8.6

Truck Loading Problem

- Truck Loading Problem
  - The truck loading problem involves deciding which items to load on a truck so as to maximize the value of a load shipped.
  - Goodman Shipping has to ship the following six items:

ITEM	VALUE ($)	WEIGHT (POUNDS)
1	22,500	7,500
2	24,000	7,500
3	8,000	3,000
4	9,500	3,500
5	11,500	4,000
6	9,750	3,500

Goodman Shipping

- The objective is to maximize the value of items loaded into the truck.
- The truck has a capacity of 10,000 pounds.
- The decision variable is:

  X_i = proportion of each item i loaded on the truck

Goodman Shipping

Objective:

Maximize load value $= \$22{,}500X_1 + \$24{,}000X_2 + \$8{,}000X_3 + \$9{,}500X_4 + \$11{,}500X_5 + \$9{,}750X_6$

subject to

$$7{,}500X_1 + 7{,}500X_2 + 3{,}000X_3 + 3{,}500X_4 + 4{,}000X_5 + 3{,}500X_6 \le 10{,}000 \text{ lb capacity}$$

$$X_1 \le 1$$
$$X_2 \le 1$$
$$X_3 \le 1$$
$$X_4 \le 1$$
$$X_5 \le 1$$
$$X_6 \le 1$$
$$X_1, X_2, X_3, X_4, X_5, X_6 \ge 0$$

Goodman Truck Loading Solution in Excel

	A	B	C	D	E	F	G	H	I	J
1	Goodman Shipping									
2										
3	Variables	X1	X2	X3	X4	X5	X6			
4	Values	0.3333	1	0	0	0	0	Total Value		
5	Load Value $	22500	24000	8000	9500	11500	9750	31500		
6										
7	Constraints							LHS	Sign	RHS
8	Total weight	7500	7500	3000	3500	4000	3500	10000	≤	10000
9	% Item 1	1						0.3333333	≤	1
10	% Item 2		1					1	≤	1
11	% Item 3			1				0	≤	1
12	% Item 4				1			0	≤	1
13	% Item 5					1		0	≤	1
14	% Item 6						1	0	≤	1

Program 8.7

Goodman Shipping

- The Goodman Shipping problem raises an interesting issue:
  - The solution calls for one third of Item 1 to be loaded on the truck.
  - What if Item 1 cannot be divided into smaller pieces?
- Rounding down leaves unused capacity on the truck and results in a value of $24,000.
- Rounding up is not possible since this would exceed the capacity of the truck.
- Using ***integer programming***, in which the solution is required to contain only integers, the solution is to load one unit of Items 3, 4, and 6 for a value of $27,250.

Ingredient Blending Applications

- Diet Problems
  - This is one of the earliest LP applications, and is used to determine the most economical diet for hospital patients.
  - This is also known as the feed mix problem.

Whole Food Nutrition Center

- The Whole Food Nutrition Center uses three bulk grains to blend a natural cereal.
- It advertises that the cereal meets the U.S. Recommended Daily Allowance (USRDA) for four key nutrients.
- It wants to select the blend that will meet the requirements at the minimum cost.

NUTRIENT	USRDA
Protein	3 units
Riboflavin	2 units
Phosphorus	1 unit
Magnesium	0.425 unit

Whole Food Nutrition Center

Let

X_A = pounds of grain A in one 2-ounce serving of cereal
X_B = pounds of grain B in one 2-ounce serving of cereal
X_C = pounds of grain C in one 2-ounce serving of cereal

Whole Food's Natural Cereal requirements:

GRAIN	COST PER POUND (CENTS)	PROTEIN (UNITS/LB)	RIBOFLAVIN (UNITS/LB)	PHOSPHOROUS (UNITS/LB)	MAGNESIUM (UNITS/LB)
A	33	22	16	8	5
B	47	28	14	7	0
C	38	21	25	9	6

Table 8.5

Whole Food Nutrition Center

The objective is:

Minimize total cost of mixing a 2-ounce serving $= \$0.33X_A + \$0.47X_B + \$0.38X_C$

subject to

$$
\begin{array}{rcrcrcll}
22X_A & + & 28X_B & + & 21X_C & \geq & 3 & \text{(protein units)} \\
16X_A & + & 14X_B & + & 25X_C & \geq & 2 & \text{(riboflavin units)} \\
8X_A & + & 7X_B & + & 9X_C & \geq & 1 & \text{(phosphorous units)} \\
5X_A & + & 0X_B & + & 6X_C & \geq & 0.425 & \text{(magnesium units)} \\
X_A & + & X_B & + & X_C & = & 0.125 & \text{(total mix)} \\
 & & X_A, X_B, X_C & & & \geq & 0 &
\end{array}
$$

Whole Food Diet Solution in Excel 2010

	A	B	C	D	E	F	G
1	Whole Foods Nutrition Problem						
2							
3		**Grain A**	**Grain B**	**Grain C**			
4	**Variable**	**Xa**	**Xb**	**Xc**			
5	**Solution**	0.025	0.05	0.05	**Total Cost**		
6	**Minimize**	0.33	0.47	0.38	0.05075		
7							
8	**Constraints**				**LHS**	**Sign**	**RHS**
9	**Protein**	22	28	21	3	≥	3
10	**Riboflavin**	16	14	25	2.35	≥	2
11	**Phosphorus**	8	7	9	1	≥	1
12	**Magnesium**	5	0	6	0.425	≥	0.425
13	**Total Weight**	1	1	1	0.125	=	0.125

Program 8.8

Ingredient Blending Applications

- Ingredient Mix and Blending Problems
  - Diet and feed mix problems are special cases of a more general class of problems known as ***ingredient*** or ***blending problems.***
  - Blending problems arise when decisions must be made regarding the blending of two or more resources to produce one or more product.
  - Resources may contain essential ingredients that must be blended so that a specified percentage is in the final mix.

Low Knock Oil Company

- The Low Knock Oil Company produces two grades of cut-rate gasoline for industrial distribution.
- The two grades, regular and economy, are created by blending two different types of crude oil.
- The crude oil differs in cost and in its content of crucial ingredients.

CRUDE OIL TYPE	INGREDIENT A (%)	INGREDIENT B (%)	COST/BARREL ($)
X100	35	55	30.00
X220	60	25	34.80

Low Knock Oil Company

The firm lets

X_1 = barrels of crude X100 blended to produce the refined regular

X_2 = barrels of crude X100 blended to produce the refined economy

X_3 = barrels of crude X220 blended to produce the refined regular

X_4 = barrels of crude X220 blended to produce the refined economy

The objective function is

$$\text{Minimize cost} = \$30X_1 + \$30X_2 + \$34.80X_3 + \$34.80X_4$$

Low Knock Oil Company

Problem formulation

At least 45% of each barrel of regular must be ingredient A

$(X_1 + X_3)$ = total amount of crude blended to produce the refined regular gasoline demand

Thus,

$0.45(X_1 + X_3)$ = amount of ingredient A required

But:

$0.35X_1 + 0.60X_3$ = amount of ingredient A in refined regular gas

So

$$0.35X_1 + 0.60X_3 \geq 0.45X_1 + 0.45X_3$$

or

$-0.10X_1 + 0.15X_3 \geq 0$ (ingredient A in regular constraint)

Low Knock Oil Company

Problem formulation

$$
\begin{array}{lrrrrl}
\text{Minimize cost} = & 30X_1 & + 30X_2 & + 34.80X_3 & + 34.80X_4 & \\
\text{subject to} & X_1 & & + X_3 & & \geq 25{,}000 \\
& & X_2 & & + X_4 & \geq 32{,}000 \\
& -0.10X_1 & & + 0.15X_3 & & \geq 0 \\
& & 0.05X_2 & & -0.25X_4 & \leq 0 \\
& & & & X_1, X_2, X_3, X_4 & \geq 0
\end{array}
$$

Low Knock Oil Solution in Excel 2010

	A	B	C	D	E	F	G	H
1	Low Knock Oil Company							
2								
3		X100 Reg	X100 Econ	X220 Reg	X220 Econ			
4	Variable	X1	X2	X3	X4			
5	Solution	15000	26666.67	10000	5333.33	Total Cost		
6	Cost	30	30	34.8	34.8	1783600		
7								
8	Constraints					LHS	Sign	RHS
9	Demand Regular	1		1		25000	≥	25000
10	Demand Economy		1		1	32000	≥	32000
11	Ing. A in Regular	-0.1		0.15		0	≥	0
12	Ing. B in Economy		0.05		-0.25	0	≤	0

Program 8.9

Transportation Applications

- Shipping Problem
  - The transportation or shipping problem involves determining the amount of goods or items to be transported from a number of origins to a number of destinations.
  - The objective usually is to minimize total shipping costs or distances.
  - This is a specific case of LP and a special algorithm has been developed to solve it.

Top Speed Bicycle Company

- The Top Speed Bicycle Co. manufactures and markets a line of 10-speed bicycles.
- The firm has final assembly plants in two cities where labor costs are low.
- It has three major warehouses near large markets.
- The sales requirements for the next year are:
  - New York – 10,000 bicycles
  - Chicago – 8,000 bicycles
  - Los Angeles – 15,000 bicycles
- The factory capacities are:
  - New Orleans – 20,000 bicycles
  - Omaha – 15,000 bicycles

Top Speed Bicycle Company

The cost of shipping bicycles from the plants to the warehouses is different for each plant and warehouse:

FROM \ TO	NEW YORK	CHICAGO	LOS ANGELES
New Orleans	$2	$3	$5
Omaha	$3	$1	$4

The company wants to develop a shipping schedule that will minimize its total annual cost.

Top Speed Bicycle Company

Network Representation of the Transportation Problem with Costs, Demands, and Supplies

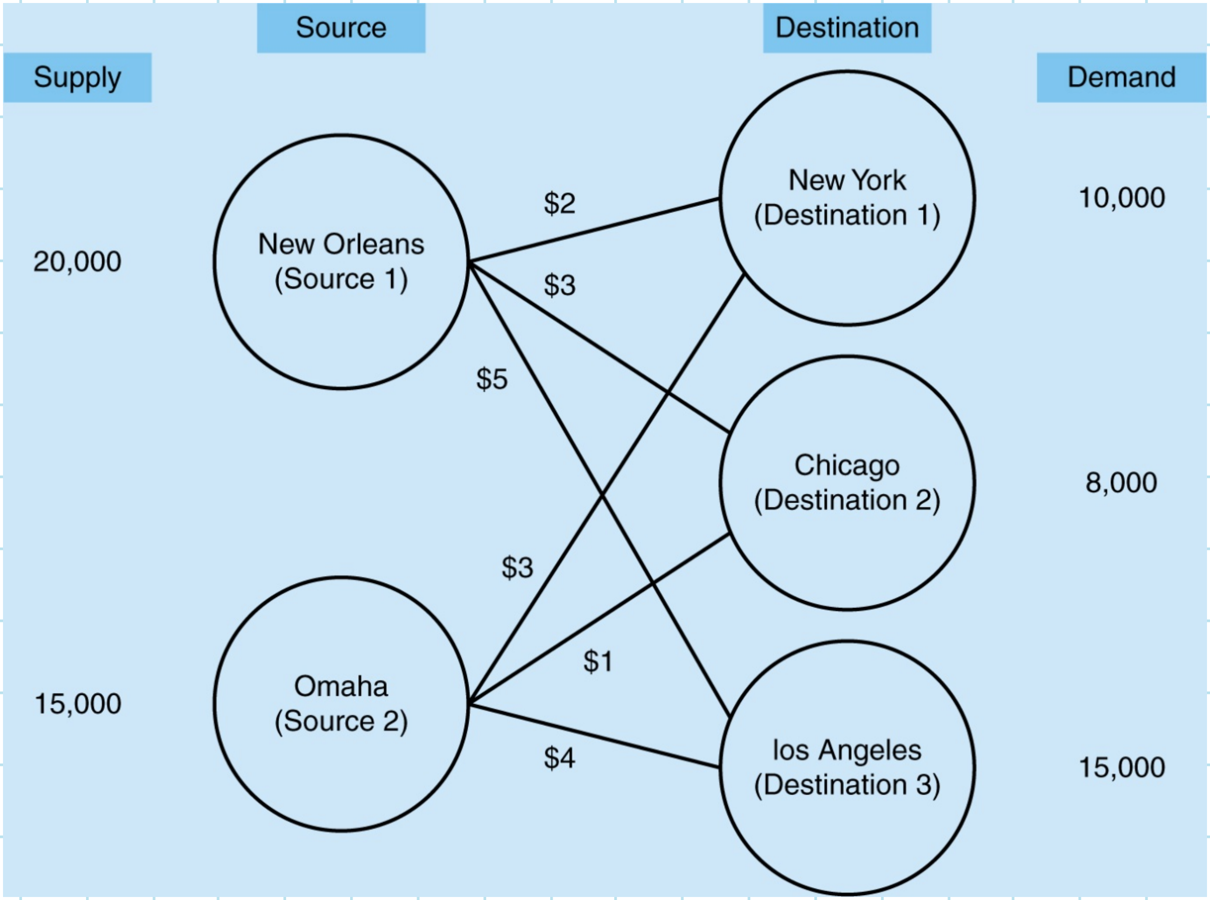

Figure 8.1

Top Speed Bicycle Company

The double subscript variables will represent the origin factory and the destination warehouse:

X_{ij} **= bicycles shipped from factory** i **to warehouse** j

So:

X_{11} **= number of bicycles shipped from New Orleans to New York**
X_{12} **= number of bicycles shipped from New Orleans to Chicago**
X_{13} **= number of bicycles shipped from New Orleans to Los Angeles**
X_{21} **= number of bicycles shipped from Omaha to New York**
X_{22} **= number of bicycles shipped from Omaha to Chicago**
X_{23} **= number of bicycles shipped from Omaha to Los Angeles**

Top Speed Bicycle Company

Objective:

Minimize total shipping costs $= 2X_{11} + 3X_{12} + 5X_{13} + 3X_{21} + 1X_{22} + 4X_{23}$

subject to

$X_{11} + X_{21} = 10{,}000$	(New York demand)
$X_{12} + X_{22} = 8{,}000$	(Chicago demand)
$X_{13} + X_{23} = 15{,}000$	(Los Angeles demand)
$X_{11} + X_{12} + X_{13} \leq 20{,}000$	(New Orleans factory supply)
$X_{21} + X_{22} + X_{23} \leq 15{,}000$	(Omaha factory supply)
All variables ≥ 0	

Top Speed Bicycle Company Solution in Excel 2010

	A	B	C	D	E	F	G	H	I	J
1	Top Speed Bicycle Company									
2		N.O. to	N.O. to	N.O. to	Omaha to	Omaha to	Omaha to			
3		NY	Chicago	LA	NY	Chicago	LA			
4	**Variables**	X11	X12	X13	X21	X22	X23			
5	**Values**	10000	0	8000	0	8000	7000	Total Cost		
6	**Cost**	2	3	5	3	1	4	96000		
7										
8	Constraints							LHS	Sign	RHS
9	NY Demand	1			1			10000	=	10000
10	Chi. Demand		1			1		8000	=	8000
11	LA Demand			1			1	15000	=	15000
12	N.O. Supply	1	1	1				18000	≤	20000
13	Omaha Supply				1	1	1	15000	≤	15000

Program 8.10

Top Speed Bicycle Company

Top Speed Bicycle solution:

FROM \ TO	NEW YORK	CHICAGO	LOS ANGELES
New Orleans	10,000	0	8,000
Omaha	0	8,000	7,000

- Total shipping cost equals $96,000.
- Transportation problems are a special case of LP as the coefficients for every variable in the constraint equations equal 1.

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