Chapter 9

Transportation and Assignment Models

To accompany
Quantitative Analysis for Management, Eleventh Edition,
by Render, Stair, and Hanna
Power Point slides created by Brian Peterson

Learning Objectives

After completing this chapter, students will be able to:

1. Structure LP problems using the transportation, transshipment and assignment models.
2. Use the northwest corner and stepping-stone methods.
3. Solve facility location and other application problems with transportation models.
4. Solve assignment problems with the Hungarian (matrix reduction) method.

Chapter Outline

9.1 Introduction
9.2 The Transportation Problem
9.3 The Assignment Problem
9.4 The Transshipment Problem
9.5 The Transportation Algorithm
9.6 Special Situations with the Transportation Algorithm
9.7 Facility Location Analysis
9.8 The Assignment Algorithm
9.9 Special Situations with the Assignment Algorithm

Introduction

- **In this chapter we will explore three special linear programming models:**
  - **The transportation problem.**
  - **The assignment problem.**
  - **The transshipment problem.**
- **These problems are members of a category of LP techniques called *network flow problems.***

The Transportation Problem

- The ***transportation problem*** deals with the distribution of goods from several points of supply (***sources***) to a number of points of demand (***destinations***).
- Usually we are given the capacity of goods at each source and the requirements at each destination.
- Typically the objective is to minimize total transportation and production costs.

The Transportation Problem

- The Executive Furniture Corporation manufactures office desks at three locations: Des Moines, Evansville, and Fort Lauderdale.
- The firm distributes the desks through regional warehouses located in Boston, Albuquerque, and Cleveland.

The Transportation Problem

Network Representation of a Transportation Problem, with Costs, Demands and Supplies

Executive Furniture Company

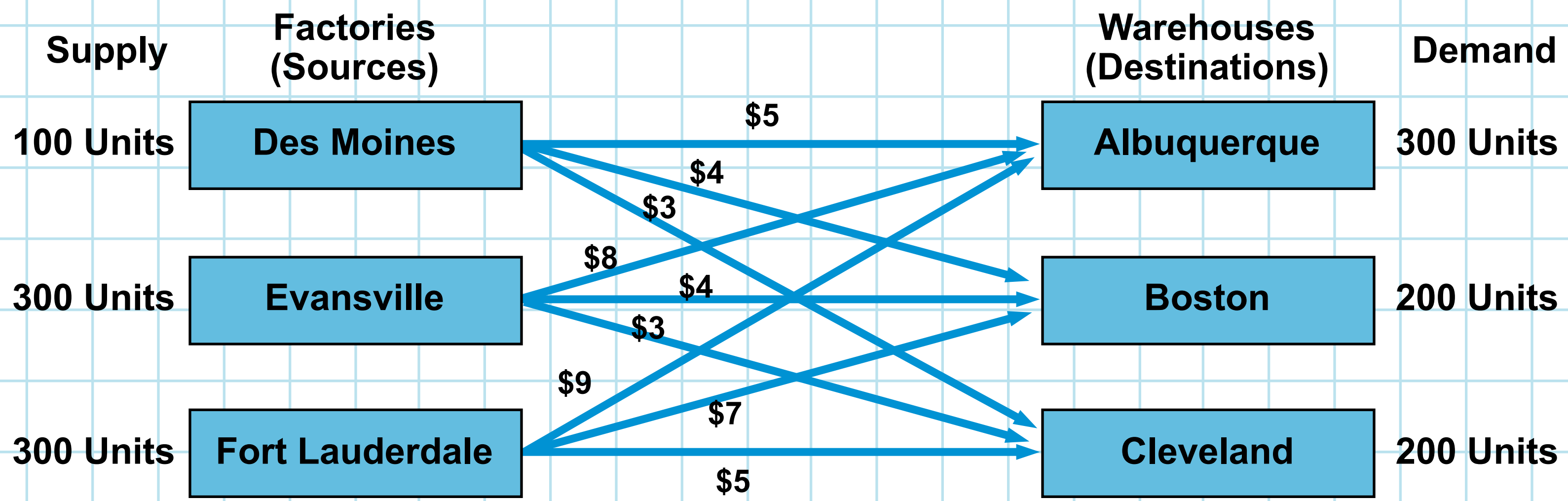


Figure 9.1

Linear Programming for the Transportation Example

- Let X_{ij} = number of units shipped from source *i* to destination *j*,
  - Where:
    - *i* = 1, 2, 3, with 1 = Des Moines, 2 = Evansville, and 3 = Fort Lauderdale
    - *j* = 1, 2, 3, with 1 = Albuquerque, 2 = Boston, and 3 = Cleveland.

Linear Programming for the Transportation Example

- Minimize total cost = $5X_{11} + 4X_{12} + 3X_{13} +$
  $8X_{21} + 4X_{22} + 3X_{23}$
  $+ 9X_{31} + 7X_{32} + 5X_{33}$
- Subject to:
  - $X_{11} + X_{12} + X_{13} \leq 100$ (Des Moines supply)
  - $X_{21} + X_{22} + X_{23} \leq 300$ (Evansville supply)
  - $X_{31} + X_{32} + X_{33} \leq 300$ (Fort Lauderdale supply)
  - $X_{11} + X_{21} + X_{31} = 300$ (Albuquerque demand)
  - $X_{12} + X_{22} + X_{32} = 200$ (Boston demand)
  - $X_{13} + X_{23} + X_{33} = 200$ (Cleveland demand)
  - $X_{ij} \geq 0$ for all *i* and *j*.

Executive Furniture Corporation Solution in Excel 2010

	A	B	C	D	E	F
1			Shipping Cost Per Unit			
2	From\To	Albuquerque	Boston	Cleveland		
3	Des Moines	5	4	3		
4	Evansville	8	4	3		
5	Fort Lauderdale	9	7	5		
6						
7						
8			Solution - Number of units shipped			
9		Albuquerque	Boston	Cleveland	Total shipped	Supply
10	Des Moines	100	0	0	100	100
11	Evansville	0	200	100	300	300
12	Fort Lauderdale	200	0	100	300	300
13	**Total received**	300	200	200		
14	**Demand**	300	200	200		
15						
16	**Total cost =**	3900				

	E
9	**Total shipped**
10	=SUM(B10:D10)

	B
13	=SUM(B10:B12)

Program 9.1

	B
16	=SUMPRODUCT(B3:D5,B10:D12)

A General LP Model for Transportation Problems

- **Let:**
  - **X_{ij} = number of units shipped from source *i* to destination *j*.**
  - **c_{ij} = cost of one unit from source *i* to destination *j*.**
  - **s_i = supply at source *i*.**
  - **d_j = demand at destination *j*.**

A General LP Model for Transportation Problems

Minimize cost = $$\sum_{j=1}^{n}\sum_{i=1}^{m} c_{ij}x_{ij}$$

Subject to:

$$\sum_{j=1}^{n} x_{ij} \le s_i$$ *i* = 1, 2,…, *m*.

$$\sum_{i=1}^{m} x_{ij} \le d_j$$ *j* = 1, 2, …, *n*.

$x_{ij} \ge 0$ *for all i and j.*

The Assignment Problem

- This type of problem determines the most efficient assignment of people to particular tasks, etc.
- Objective is typically to minimize total cost or total task time.

Linear Program for Assignment Example

- The Fix-it Shop has just received three new repair projects that must be repaired quickly: a radio, a toaster oven, and a coffee table.
- Three workers with different talents are able to do the jobs.
- The owner estimates the cost in wages if the workers are assigned to each of the three jobs.
- Objective: minimize total cost.

Example of an Assignment Problem in a Transportation Network Format

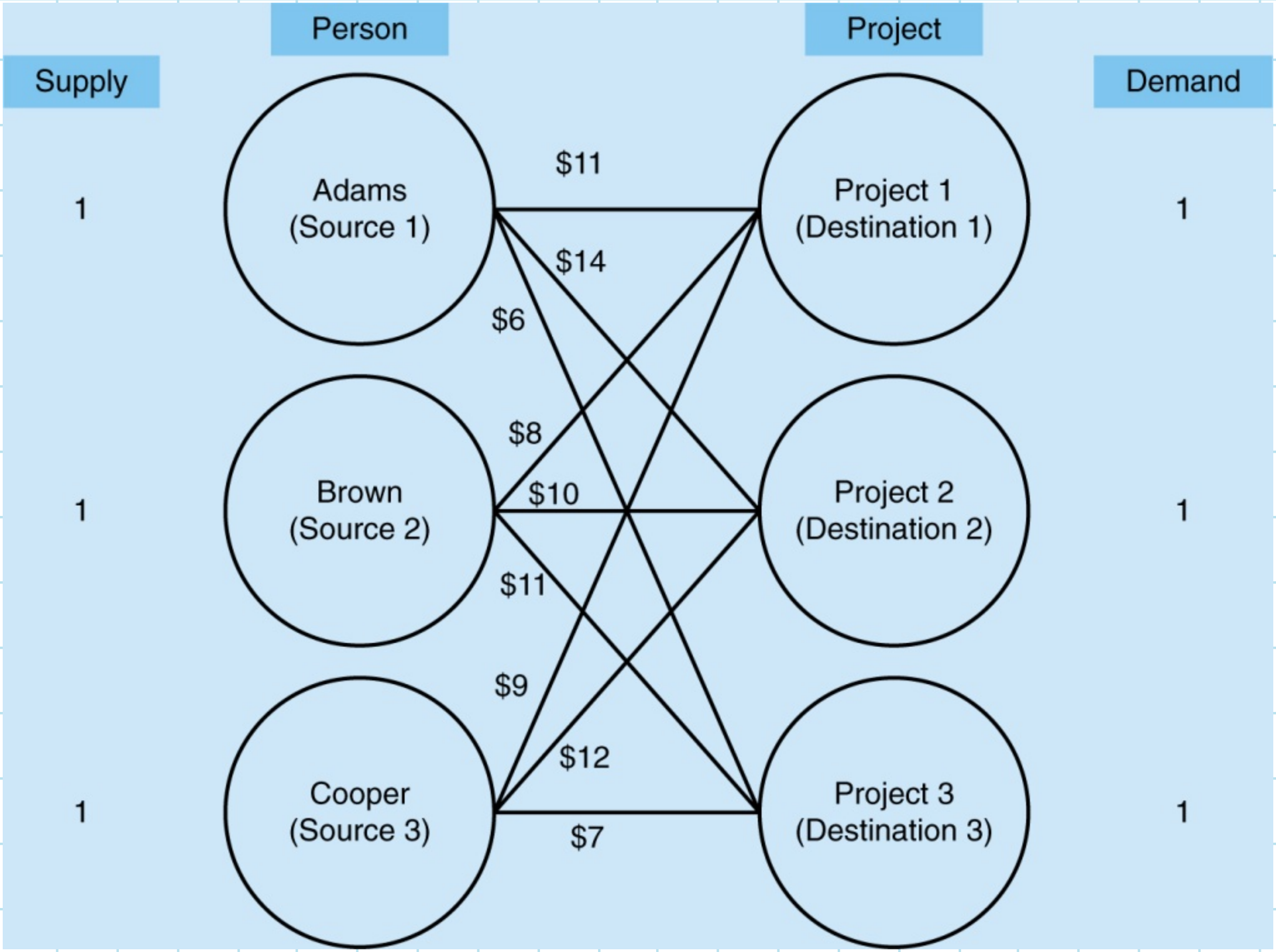


Figure 9.2

Linear Program for Assignment Example

Let:

- X_{ij} = 1 if person *i* is assigned to project *j*, or 0 otherwise.

Where:

- *i* = 1,2,3 with 1 = Adams, 2 = Brown, and 3 = Cooper
- *j* = 1,2,3, with 1 = Project 1, 2 = Project 2, and 3 = Project 3.

Linear Program for Assignment Example

Minimize total cost = $11X_{11} + 14X_{12} + 6X_{13} + 8X_{21} + 10X_{22} + 11X_{23} + 9X_{31} + 12X_{32} + 7X_{33}$

Subject to:

- $X_{11} + X_{12} + X_{13} \leq 1$
- $X_{21} + X_{22} + X_{23} \leq 1$
- $X_{31} + X_{32} + X_{33} \leq 1$
- $X_{11} + X_{21} + X_{31} = 1$
- $X_{12} + X_{22} + X_{32} = 1$
- $X_{13} + X_{23} + X_{33} = 1$
- $X_{ij} = 0$ or 1 for all *i* and *j*

Fix-it Shop Solution in Excel 2010

	A	B	C	D	E	F
1		Cost for Assignments				
2	Person\Project	Project 1	Project 2	Project 3		
3	Adams	11	14	6		
4	Brown	8	10	11		
5	Cooper	9	12	7		
6						
7						
8			Made			
9		Project 1	Project 2	Project 3	Total projects	Supply
10	Adams	0	0	1	1	1
11	Brown	0	1	0	1	1
12	Cooper	1	0	0	1	1
13	Total assigned	1	1	1		
14	Total workers	1	1	1		
15						
16	Total cost =	25				

	E
10	=SUM(B10:D10)

	B
13	=SUM(B10:B12)

	B
16	=SUMPRODUCT(B3:D5,B10:D12)

Program 9.2

Linear Program for Assignment Example

- X_{13} = 1, so Adams is assigned to project 3.
- X_{22} = 1, so Brown is assigned to project 2.
- X_{31} = 1, so Cooper is assigned to project 3.
- Total cost of the repairs is $25.

Transshipment Applications

When the items are being moved from a source to a destination through an intermediate point (a *transshipment point*), the problem is called a *transshipment problem.*

Distribution Centers

- **Frosty Machines manufactures snow blowers in Toronto and Detroit.**
- **These are shipped to regional distribution centers in Chicago and Buffalo.**
- **From there they are shipped to supply houses in New York, Philadelphia, and St Louis.**
- **Shipping costs vary by location and destination.**
- **Snow blowers cannot be shipped directly from the factories to the supply houses.**

Network Representation of Transshipment Example

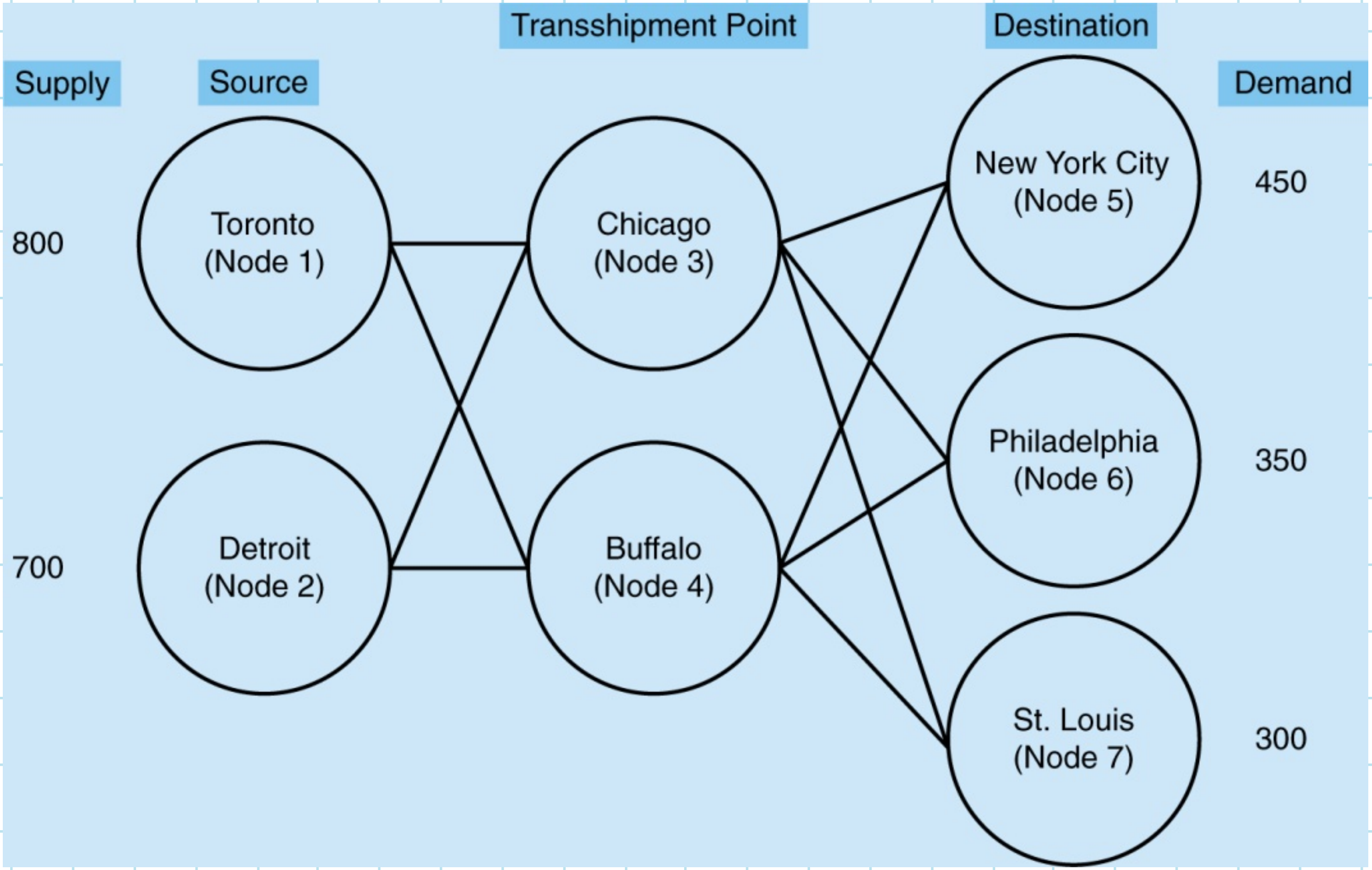


Figure 9.3

Transshipment Applications

Frosty Machines Transshipment Data

	TO					
FROM	CHICAGO	BUFFALO	NEW YORK CITY	PHILADELPHIA	ST LOUIS	SUPPLY
Toronto	$4	$7	—	—	—	800
Detroit	$5	$7	—	—	—	700
Chicago	—	—	$6	$4	$5	—
Buffalo	—	—	$2	$3	$4	—
Demand	—	—	450	350	300	

Table 9.1

Frosty would like to minimize the transportation costs associated with shipping snow blowers to meet the demands at the supply centers given the supplies available.

Transshipment Applications

A description of the problem would be to minimize cost subject to:

1. The number of units shipped from Toronto is not more than 800.
2. The number of units shipped from Detroit is not more than 700.
3. The number of units shipped to New York is 450.
4. The number of units shipped to Philadelphia is 350.
5. The number of units shipped to St Louis is 300.
6. The number of units shipped out of Chicago is equal to the number of units shipped into Chicago.
7. The number of units shipped out of Buffalo is equal to the number of units shipped into Buffalo.

Transshipment Applications

The decision variables should represent the number of units shipped from each source to the transshipment points and from there to the final destinations.

X_{13} = the number of units shipped from Toronto to Chicago
X_{14} = the number of units shipped from Toronto to Buffalo
X_{23} = the number of units shipped from Detroit to Chicago
X_{24} = the number of units shipped from Detroit to Buffalo
X_{35} = the number of units shipped from Chicago to New York
X_{36} = the number of units shipped from Chicago to Philadelphia
X_{37} = the number of units shipped from Chicago to St Louis
X_{45} = the number of units shipped from Buffalo to New York
X_{46} = the number of units shipped from Buffalo to Philadelphia
X_{47} = the number of units shipped from Buffalo to St Louis

Transshipment Applications

The linear program is:

Minimize cost = $4X_{13} + 7X_{14} + 5X_{23} + 7X_{24} + 6X_{35} + 4X_{36} + 5X_{37} + 2X_{45} + 3X_{46} + 4X_{47}$

subject to

$X_{13} + X_{14} \le 800$	(supply at Toronto)
$X_{23} + X_{24} \le 700$	(supply at Detroit)
$X_{35} + X_{45} = 450$	(demand at New York)
$X_{36} + X_{46} = 350$	(demand at Philadelphia)
$X_{37} + X_{47} = 300$	(demand at St Louis)
$X_{13} + X_{23} = X_{35} + X_{36} + X_{37}$	(shipping through Chicago)
$X_{14} + X_{24} = X_{45} + X_{46} + X_{47}$	(shipping through Buffalo)

$X_{ij} \ge 0$ for all *i* and *j* (nonnegativity)

Solution to Frosty Machines Transshipment Problem

	A	B	C	D	E	F	G	H
1	Frosty Machines Transshipment Problem							
2								
3				Shipping Cost Per Unit				
4	From\To	Chicago	Buffalo	NYC	Phil.	St.Louis		
5	Toronto	4	7					
6	Detroit	5	7					
7	Chicago			6	4	5		
8	Buffalo			2	3	4		
9								
10				Solution - Number of units shipped				
11		Chicago	Buffalo	NYC	Phil.	St.Louis	Total shipped	Supply
12	Toronto	650	150				800	800
13	Detroit	0	300				300	700
14	Chicago			0	350	300	650	
15	Buffalo			450	0	0	450	
16	Total received	650	450	450	350	300		
17	Demand			450	350	300		
18								
19	Total cost =	9550						

	G
11	Total shipped
12	=SUM(B12:C12)

	G
14	=SUM(D14:F14)

	B
16	=SUM(B12:B13)

	D
16	=SUM(D14:D15)

Program 9.3

	B
19	=SUMPRODUCT(B5:F8,B12:F15)

The Transportation Algorithm

- This is an iterative procedure in which a solution to a transportation problem is found and evaluated using a special procedure to determine whether the solution is optimal.
  - When the solution is optimal, the process stops.
  - If not, then a new solution is generated.

Transportation Table for Executive Furniture Corporation

Des Moines capacity constraint

FROM \ TO	WAREHOUSE AT ALBUQUERQUE	WAREHOUSE AT BOSTON	WAREHOUSE AT CLEVELAND	FACTORY CAPACITY
DES MOINES FACTORY	$5	$4	$3	100
EVANSVILLE FACTORY	$8	$4	$3	300
FORT LAUDERDALE FACTORY	$9	$7	$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Table 9.2

Cost of shipping 1 unit from Fort Lauderdale factory to Boston warehouse

Cleveland warehouse demand

Total supply and demand

Cell representing a source-to-destination (Evansville to Cleveland) shipping assignment that could be made

Developing an Initial Solution: Northwest Corner Rule

- Once we have arranged the data in a table, we must establish an initial feasible solution.
- One systematic approach is known as the *northwest corner rule.*
- Start in the upper left-hand cell and allocate units to shipping routes as follows:
  1. Exhaust the supply (factory capacity) of each row before moving down to the next row.
  2. Exhaust the demand (warehouse) requirements of each column before moving to the right to the next column.
  3. Check that all supply and demand requirements are met.
- This problem takes five steps to make the initial shipping assignments.

Developing an Initial Solution: Northwest Corner Rule

1. **Beginning in the upper left hand corner, we assign 100 units from Des Moines to Albuquerque. This exhaust the supply from Des Moines but leaves Albuquerque 200 desks short. We move to the second row in the same column.**

FROM \ TO	ALBUQUERQUE (*A*)	BOSTON (*B*)	CLEVELAND (*C*)	FACTORY CAPACITY
DES MOINES (*D*)	100 / $5	$4	$3	100
EVANSVILLE (*E*)	$8	$4	$3	300
FORT LAUDERDALE (*F*)	$9	$7	$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Developing an Initial Solution: Northwest Corner Rule

2. Assign 200 units from Evansville to Albuquerque. This meets Albuquerque's demand. Evansville has 100 units remaining so we move to the right to the next column of the second row.

FROM \ TO	ALBUQUERQUE (*A*)	BOSTON (*B*)	CLEVELAND (*C*)	FACTORY CAPACITY
DES MOINES (*D*)	100 \$5	\$4	\$3	100
EVANSVILLE (*E*)	200 \$8	\$4	\$3	300
FORT LAUDERDALE (*F*)	\$9	\$7	\$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Developing an Initial Solution: Northwest Corner Rule

3. Assign 100 units from Evansville to Boston. The Evansville supply has now been exhausted but Boston is still 100 units short. We move down vertically to the next row in the Boston column.

FROM \ TO	ALBUQUERQUE (*A*)	BOSTON (*B*)	CLEVELAND (*C*)	FACTORY CAPACITY
DES MOINES (*D*)	100 $5	$4	$3	100
EVANSVILLE (*E*)	200 $8	100 $4	$3	300
FORT LAUDERDALE (*F*)	$9	$7	$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Developing an Initial Solution: Northwest Corner Rule

4. **Assign 100 units from Fort Lauderdale to Boston. This fulfills Boston's demand and Fort Lauderdale still has 200 units available.**

FROM \ TO	ALBUQUERQUE (*A*)	BOSTON (*B*)	CLEVELAND (*C*)	FACTORY CAPACITY
DES MOINES (*D*)	100 ($5)	($4)	($3)	100
EVANSVILLE (*E*)	200 ($8)	100 ($4)	($3)	300
FORT LAUDERDALE (*F*)	($9)	100 ($7)	($5)	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Developing an Initial Solution: Northwest Corner Rule

5. **Assign 200 units from Fort Lauderdale to Cleveland. This exhausts Fort Lauderdale's supply and Cleveland's demand. The initial shipment schedule is now complete.**

FROM \ TO	ALBUQUERQUE (*A*)	BOSTON (*B*)	CLEVELAND (*C*)	FACTORY CAPACITY
DES MOINES (*D*)	100 \$5	\$4	\$3	100
EVANSVILLE (*E*)	200 \$8	100 \$4	\$3	300
FORT LAUDERDALE (*F*)	\$9	100 \$7	200 \$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Table 9.3

Developing an Initial Solution: Northwest Corner Rule

The cost of this shipping assignment:

ROUTE						
FROM	TO	UNITS SHIPPED	x	PER UNIT COST ($)	=	TOTAL COST ($)
D	*A*	100		5		500
E	*A*	200		8		1,600
E	*B*	100		4		400
F	*B*	100		7		700
F	*C*	200		5		1,000
						4,200

This solution is feasible but we need to check to see if it is optimal.

Stepping-Stone Method: Finding a Least Cost Solution

- The ***stepping-stone method*** is an iterative technique for moving from an initial feasible solution to an optimal feasible solution.
- There are two distinct parts to the process:
  - Testing the current solution to determine if improvement is possible.
  - Making changes to the current solution to obtain an improved solution.
- This process continues until the optimal solution is reached.

Stepping-Stone Method: Finding a Least Cost Solution

- There is one very important rule: ***The number of occupied routes (or squares) must always be equal to one less than the sum of the number of rows plus the number of columns***
  - In the Executive Furniture problem this means the initial solution must have 3 + 3 – 1 = 5 squares used.

$$\text{Occupied shipping routes (squares)} = \text{Number of rows} + \text{Number of columns} - 1$$

- When the number of occupied rows is less than this, the solution is called ***degenerate.***

Testing the Solution for Possible Improvement

- The stepping-stone method works by testing each unused square in the transportation table to see what would happen to total shipping costs if one unit of the product were tentatively shipped on an unused route.
- There are five steps in the process.

Five Steps to Test Unused Squares with the Stepping-Stone Method

1. Select an unused square to evaluate.
2. Beginning at this square, trace a closed path back to the original square via squares that are currently being used with only horizontal or vertical moves allowed.
3. Beginning with a plus (+) sign at the unused square, place alternate minus (–) signs and plus signs on each corner square of the closed path just traced.

Five Steps to Test Unused Squares with the Stepping-Stone Method

4. Calculate an *improvement index* by adding together the unit cost figures found in each square containing a plus sign and then subtracting the unit costs in each square containing a minus sign.
5. Repeat steps 1 to 4 until an improvement index has been calculated for all unused squares. If all indices computed are greater than or equal to zero, an optimal solution has been reached. If not, it is possible to improve the current solution and decrease total shipping costs.

Five Steps to Test Unused Squares with the Stepping-Stone Method

For the Executive Furniture Corporation data:

***Steps 1 and 2*. Beginning with Des Moines–Boston route we trace a closed path using only currently occupied squares, alternately placing plus and minus signs in the corners of the path.**

- **In a *closed path*, only squares currently used for shipping can be used in turning corners.**
- ***Only one* closed route is possible for each square we wish to test.**

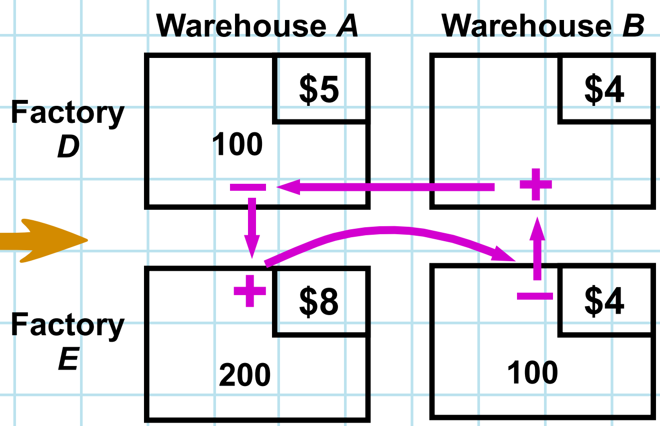
Five Steps to Test Unused Squares with the Stepping-Stone Method

***Step 3*. Test the cost-effectiveness of the Des Moines–Boston shipping route by pretending that we are shipping one desk from Des Moines to Boston. Put a plus in that box.**

- **But if we ship one *more* unit out of Des Moines we will be sending out 101 units.**
- **Since the Des Moines factory capacity is only 100, we must ship *fewer* desks from Des Moines to Albuquerque so place a minus sign in that box.**
- **But that leaves Albuquerque one unit short so increase the shipment from Evansville to Albuquerque by one unit and so on until the entire closed path is completed.**

Five Steps to Test Unused Squares with the Stepping-Stone Method

Evaluating the unused Des Moines–Boston shipping route

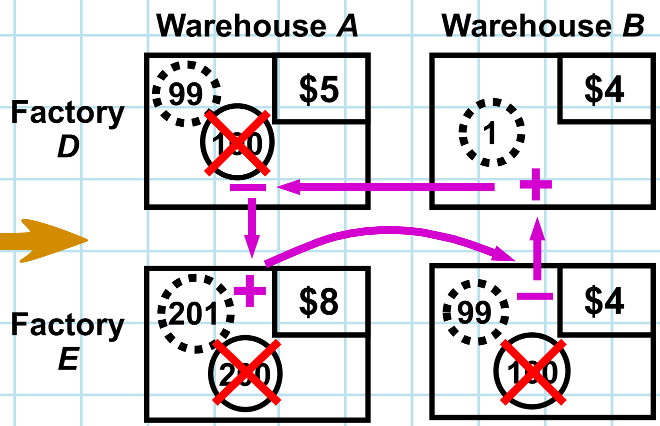


FROM \ TO	ALBUQUERQUE	BOSTON	CLEVELAND	FACTORY CAPACITY
DES MOINES	100 ($5)	($4)	($3)	100
EVANSVILLE	200 ($8)	100 ($4)	($3)	300
FORT LAUDERDALE	($9)	100 ($7)	200 ($5)	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Table 9.3

Five Steps to Test Unused Squares with the Stepping-Stone Method

Evaluating the unused Des Moines–Boston shipping route

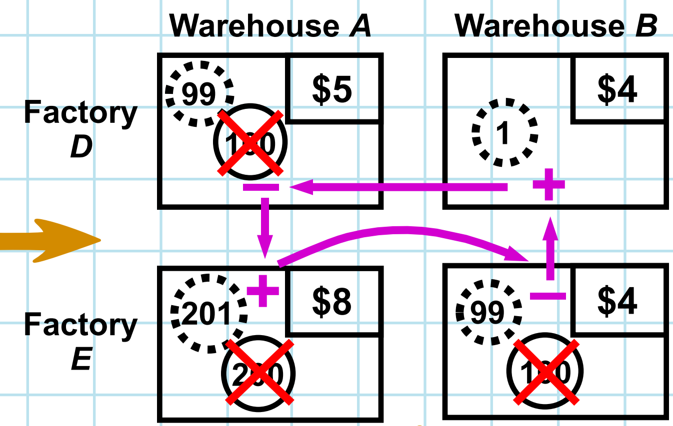


FROM \ TO	ALBUQUERQUE	BOSTON	CLEVELAND	FACTORY CAPACITY
DES MOINES	100 ($5)	($4)	($3)	100
EVANSVILLE	200 ($8)	100 ($4)	($3)	300
FORT LAUDERDALE	($9)	100 ($7)	200 ($5)	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Table 9.4

Five Steps to Test Unused Squares with the Stepping-Stone Method

Evaluating the unused Des Moines–Boston shipping route



FROM \ TO	ALBUQUERQUE	BOSTON	CLEVELAND	
DES MOINES	100 $5	$4	$3	
EVANSVILLE	200 $8	100 $4	$3	
FORT LAUDERDALE	$9	100 $7	200 $5	
WAREHOUSE REQUIREMENTS	300	200	200	700

Result of Proposed Shift in Allocation

= 1 x $4
– 1 x $5
+ 1 x $8
– 1 x $4 = +$3

Table 9.4

Five Steps to Test Unused Squares with the Stepping-Stone Method

Step 4. Now compute an ***improvement index*** (I_{ij}) for the Des Moines–Boston route.

Add the costs in the squares with plus signs and subtract the costs in the squares with minus signs:

Des Moines–Boston index $= I_{DB} = +\$4 - \$5 + \$5 - \$4 = +\$3$

This means for every desk shipped via the Des Moines–Boston route, total transportation cost will ***increase*** by $3 over their current level.

Five Steps to Test Unused Squares with the Stepping-Stone Method

Step 5. **Now examine the Des Moines–Cleveland unused route which is slightly more difficult to draw.**

- **Again, only turn corners at squares that represent existing routes.**
- **Pass through the Evansville–Cleveland square but we can not turn there or put a + or – sign.**
- **The closed path we will use is:**

$$+ DC - DA + EA - EB + FB - FC$$

Five Steps to Test Unused Squares with the Stepping-Stone Method

Evaluating the Des Moines–Cleveland Shipping Route

FROM \ TO	ALBUQUERQUE	BOSTON	CLEVELAND	FACTORY CAPACITY
DES MOINES	100 ($5) –	($4)	Start ($3) +	100
EVANSVILLE	200 ($8) +	100 ($4) –	($3)	300
FORT LAUDERDALE	($9)	100 ($7) +	200 ($5) –	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Table 9.5

Des Moines–Cleveland improvement index $= I_{DC} = + \$3 - \$5 + \$8 - \$4 + \$7 - \$5 = + \$4$

Five Steps to Test Unused Squares with the Stepping-Stone Method

Opening the Des Moines–Cleveland route will not lower our total shipping costs.

Evaluating the other two routes we find:

Evansville-Cleveland index $= I_{EC} = +\$3 - \$4 + \$7 - \$5 = +\$1$

The closed path is

$+EC - EB + FB - FC$

Fort Lauderdale–Albuquerque index $= I_{FA} = +\$9 - \$7 + \$4 - \$8 = -\$2$

The closed path is

$+FA - FB + EB - EA$

Opening the Fort Lauderdale-Albuquerque route ***will*** lower our total transportation costs.

Obtaining an Improved Solution

- In the Executive Furniture problem there is only one unused route with a negative index (Fort Lauderdale-Albuquerque).
  - If there was more than one route with a negative index, we would choose the one with the largest improvement
- We now want to ship the maximum allowable number of units on the new route
- The quantity to ship is found by referring to the closed path of plus and minus signs for the new route and selecting the ***smallest number*** found in those squares containing minus signs.

Obtaining an Improved Solution

- To obtain a new solution, that number is added to all squares on the closed path with plus signs and subtracted from all squares the closed path with minus signs.
- All other squares are unchanged.
- In this case, the maximum number that can be shipped is 100 desks as this is the smallest value in a box with a negative sign (*FB* route).
- We add 100 units to the *FA* and *EB* routes and subtract 100 from *FB* and *EA* routes.
- This leaves balanced rows and columns and an improved solution.

Obtaining an Improved Solution

Stepping-Stone Path Used to Evaluate Route *F-A*

FROM \ TO	*A*	*B*	*C*	FACTORY CAPACITY
D	100 | $5	| $4	| $3	100
E	200 | $8 + ...	100 | $4	| $3	300
F	| $9	100 | $7	200 | $5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Table 9.6

Obtaining an Improved Solution

Second Solution to the Executive Furniture Problem

FROM \ TO	*A*	*B*	*C*	FACTORY CAPACITY
D	100 \$5	\$4	\$3	100
E	100 \$8	200 \$4	\$3	300
F	100 \$9	\$7	200 \$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Table 9.7

Total shipping costs have been reduced by (100 units) x (\$2 saved per unit) and now equals \$4,000.

Obtaining an Improved Solution

- This second solution may or may not be optimal.
- To determine whether further improvement is possible, we return to the first five steps to test each square that is *now* unused.
- The four new improvement indices are:

D to *B* = I_{DB} = + \$4 – \$5 + \$8 – \$4 = + \$3
(closed path: + *DB* – *DA* + *EA* – *EB*)

D to *C* = I_{DC} = + \$3 – \$5 + \$9 – \$5 = + \$2
(closed path: + *DC* – *DA* + *FA* – *FC*)

E to *C* = I_{EC} = + \$3 – \$8 + \$9 – \$5 = – \$1
(closed path: + *EC* – *EA* + *FA* – *FC*)

F to *B* = I_{FB} = + \$7 – \$4 + \$8 – \$9 = + \$2
(closed path: + *FB* – *EB* + *EA* – *FA*)

Obtaining an Improved Solution

Path to Evaluate the *E-C* Route

TO / FROM	*A*	*B*	*C*	FACTORY CAPACITY
D	100 $5	$4	$3	100
E	100 $8 −	200 $4	Start $3 +	300
F	100 $9 +	$7	200 $5 −	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Table 9.8

An improvement can be made by shipping the maximum allowable number of units from *E* to *C*.

Obtaining an Improved Solution

Total cost of third solution:

ROUTE FROM	ROUTE TO	DESKS SHIPPED	x	PER UNIT COST ($)	=	TOTAL COST ($)
D	*A*	100		5		500
E	*B*	200		4		800
E	*C*	100		3		300
F	*A*	200		9		1,800
F	*C*	100		5		500
						3,900

Obtaining an Improved Solution

Third and optimal solution:

FROM \ TO	A	B	C	FACTORY CAPACITY
D	100 ($5)	($4)	($3)	100
E	($8)	200 ($4)	100 ($3)	300
F	200 ($9)	($7)	100 ($5)	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Table 9.9

Obtaining an Improved Solution

This solution is optimal as the improvement indices that can be computed are all greater than or equal to zero.

D to *B* = I_{DB} = + \$4 – \$5 + \$9 – \$5 + \$3 – \$4 = + \$2
(closed path: + *DB* – *DA* + *FA* – *FC* + *EC* – *EB*)

D to *C* = I_{DC} = + \$3 – \$5 + \$9 – \$5 = + \$2
(closed path: + *DC* – *DA* + *FA* – *FC*)

E to *A* = I_{EA} = + \$8 – \$9 + \$5 – \$3 = + \$1
(closed path: + *EA* – *FA* + *FC* – *EC*)

F to *B* = I_{FB} = + \$7 – \$5 + \$3 – \$4 = + \$1
(closed path: + *FB* – *FC* + *EC* – *EB*)

Summary of Steps in Transportation Algorithm (Minimization)

1. **Set up a balanced transportation table.**
2. **Develop initial solution using the northwest corner method method.**
3. **Calculate an improvement index for each empty cell using the stepping-stone method. If improvement indices are all nonnegative, stop as the optimal solution has been found. If any index is negative, continue to step 4.**
4. **Select the cell with the improvement index indicating the greatest decrease in cost. Fill this cell using the stepping-stone path and go to step 3.**

Unbalanced Transportation Problems

- **In real-life problems, total demand is frequently not equal to total supply.**
- **These *unbalanced problems* can be handled easily by introducing *dummy sources* or *dummy destinations.***
- **If total supply is greater than total demand, a dummy destination (warehouse), with demand exactly equal to the surplus, is created.**
- **If total demand is greater than total supply, we introduce a dummy source (factory) with a supply equal to the excess of demand over supply.**

Special Situations with the Transportation Algorithm

- **Unbalanced Transportation Problems**
  - **In either case, shipping cost coefficients of zero are assigned to each dummy location or route as no goods will actually be shipped.**
  - **Any units assigned to a dummy destination represent excess capacity.**
  - **Any units assigned to a dummy source represent unmet demand.**

Demand Less Than Supply

- Suppose that the Des Moines factory increases its rate of production from 100 to 250 desks.
- The firm is now able to supply a total of 850 desks each period.
- Warehouse requirements remain the same (700) so the row and column totals do not balance.
- We add a dummy column that will represent a fake warehouse requiring 150 desks.
- This is somewhat analogous to adding a slack variable.
- We use the stepping-stone method to find the optimal solution.

Demand Less Than Supply

Initial Solution to an Unbalanced Problem Where Demand is Less Than Supply

FROM \ TO	A	B	C	DUMMY WAREHOUSE	TOTAL AVAILABLE
D	250 ($5)	($4)	($3)	(0)	250
E	50 ($8)	200 ($4)	50 ($3)	(0)	300
F	($9)	($7)	150 ($5)	150 (0)	300
WAREHOUSE REQUIREMENTS	300	200	200	150	850

Total cost = 250($5) + 50($8) + 200($4) + 50($3) + 150($5) + 150(0) = $3,350

Table 9.10

New Des Moines capacity

Demand Greater than Supply

- The second type of unbalanced condition occurs when total demand is greater than total supply.
- In this case we need to add a dummy row representing a fake factory.
- The new factory will have a supply exactly equal to the difference between total demand and total real supply.
- The shipping costs from the dummy factory to each destination will be zero.

Demand Greater than Supply

Unbalanced Transportation Table for Happy Sound Stereo Company

FROM \ TO	WAREHOUSE *A*	WAREHOUSE *B*	WAREHOUSE *C*	PLANT SUPPLY
PLANT *W*	$6	$4	$9	200
PLANT *X*	$10	$5	$8	175
PLANT *Y*	$12	$7	$6	75
WAREHOUSE DEMAND	250	100	150	500 \ 450

Totals do not balance

Table 9.11

Demand Greater than Supply

Initial Solution to an Unbalanced Problem in Which Demand is Greater Than Supply

FROM \ TO	WAREHOUSE *A*	WAREHOUSE *B*	WAREHOUSE *C*	PLANT SUPPLY
PLANT *W*	200 | $6	| $4	| $9	200
PLANT *X*	50 | $10	100 | $5	25 | $8	175
PLANT *Y*	| $12	| $7	75 | $6	75
PLANT *Y*	| 0	| 0	50 | 0	50
WAREHOUSE DEMAND	250	100	150	500

Total cost of initial solution = 200($6) + 50($10) + 100($5) + 25($8) + 75($6) + $50(0) = $2,850

Table 9.12

Degeneracy in Transportation Problems

- *Degeneracy* occurs when the number of occupied squares or routes in a transportation table solution is less than the number of rows plus the number of columns minus 1.
- Such a situation may arise in the initial solution or in any subsequent solution.
- Degeneracy requires a special procedure to correct the problem since there are not enough occupied squares to trace a closed path for each unused route and it would be impossible to apply the stepping-stone method.

Degeneracy in Transportation Problems

- To handle degenerate problems, create an artificially occupied cell.
- That is, place a zero (representing a fake shipment) in one of the unused squares and then treat that square as if it were occupied.
- The square chosen must be in such a position as to allow all stepping-stone paths to be closed.
- There is usually a good deal of flexibility in selecting the unused square that will receive the zero.

Degeneracy in an Initial Solution

- The Martin Shipping Company example illustrates degeneracy in an initial solution.
- It has three warehouses which supply three major retail customers.
- Applying the northwest corner rule the initial solution has only four occupied squares
- To correct this problem, place a zero in an unused square, typically one adjacent to the last filled cell.

Degeneracy in an Initial Solution

Initial Solution of a Degenerate Problem

FROM \ TO	CUSTOMER 1	CUSTOMER 2	CUSTOMER 3	WAREHOUSE SUPPLY
WAREHOUSE 1	100 | $8	0 | $2	| $6	100
WAREHOUSE 2	0 | $10	100 | $9	20 | $9	120
WAREHOUSE 3	| $7	| $10	80 | $7	80
CUSTOMER DEMAND	100	100	100	300

Table 9.13

Possible choices of cells to address the degenerate solution

Degeneracy During Later Solution Stages

- A transportation problem can become degenerate after the initial solution stage if the filling of an empty square results in two or more cells becoming empty simultaneously.
- This problem can occur when two or more cells with minus signs tie for the lowest quantity.
- To correct this problem, place a zero in one of the previously filled cells so that only one cell becomes empty.

Degeneracy During Later Solution Stages

Bagwell Paint Example

- After one iteration, the cost analysis at Bagwell Paint produced a transportation table that was not degenerate but was not optimal.
- The improvement indices are:

factory *A* – warehouse 2 index = +2
factory *A* – warehouse 3 index = +1
factory *B* – warehouse 3 index = –15
factory *C* – warehouse 2 index = +11

Only route with a negative index

Degeneracy During Later Solution Stages

Bagwell Paint Transportation Table

FROM \ TO	WAREHOUSE 1	WAREHOUSE 2	WAREHOUSE 3	FACTORY CAPACITY
FACTORY *A*	70 ($8)	($5)	($16)	70
FACTORY *B*	50 ($15)	80 ($10)	($7)	130
FACTORY *C*	30 ($3)	($9)	50 ($10)	80
WAREHOUSE REQUIREMENT	150	80	50	280

Table 9.14

Degeneracy During Later Solution Stages

Tracing a Closed Path for the Factory B – Warehouse 3 Route

FROM \ TO	WAREHOUSE 1	WAREHOUSE 3
FACTORY *B*	50 ($15) –	+ ($7)
FACTORY *C*	30 ($3) +	50 ($10) –

Table 9.15

- This would cause two cells to drop to zero.
- We need to place an artificial zero in one of these cells to avoid degeneracy.

More Than One Optimal Solution

- It is possible for a transportation problem to have multiple optimal solutions.
- This happens when one or more of the improvement indices is zero in the optimal solution.
  - This means that it is possible to design alternative shipping routes with the same total shipping cost.
  - The alternate optimal solution can be found by shipping the most to this unused square using a stepping-stone path.
- In the real world, alternate optimal solutions provide management with greater flexibility in selecting and using resources.

Maximization Transportation Problems

- If the objective in a transportation problem is to maximize profit, a minor change is required in the transportation algorithm.
- Now the optimal solution is reached when all the improvement indices are negative or zero.
- The cell with the largest positive improvement index is selected to be filled using a stepping-stone path.
- This new solution is evaluated and the process continues until there are no positive improvement indices.

Unacceptable Or Prohibited Routes

- At times there are transportation problems in which one of the sources is unable to ship to one or more of the destinations.
  - The problem is said to have an ***unacceptable*** or ***prohibited route.***
- In a minimization problem, such a prohibited route is assigned a very high cost to prevent this route from ever being used in the optimal solution.
- In a maximization problem, the very high cost used in minimization problems is given a negative sign, turning it into a very bad profit.

Facility Location Analysis

- **The transportation method is especially useful in helping a firm to decide where to locate a new factory or warehouse.**
- **Each alternative location should be analyzed within the framework of one *overall* distribution system.**
- **The new location that yields the minimum cost for the *entire system* is the one that should be chosen.**

Locating a New Factory for Hardgrave Machine Company

- **Hardgrave Machine produces computer components at three plants and ships to four warehouses.**
- **The plants have not been able to keep up with demand so the firm wants to build a new plant.**
- **Two sites are being considered, Seattle and Birmingham.**
- **Data has been collected for each possible location. Which new location will yield the lowest cost for the firm in combination with the existing plants and warehouses?**

Locating a New Factory for Hardgrave Machine Company

Hardgrave's Demand and Supply Data

WAREHOUSE	MONTHLY DEMAND (UNITS)	PRODUCTION PLANT	MONTHLY SUPPLY	COST TO PRODUCE ONE UNIT ($)
Detroit	10,000	Cincinnati	15,000	48
Dallas	12,000	Salt Lake	6,000	50
New York	15,000	Pittsburgh	14,000	52
Los Angeles	9,000		35,000	
	46,000			

Supply needed from new plant = 46,000 – 35,000 = 11,000 units per month

ESTIMATED PRODUCTION COST PER UNIT AT PROPOSED PLANTS	
Seattle	$53
Birmingham	$49

Table 9.16

Locating a New Factory for Hardgrave Machine Company

Hardgrave's Shipping Costs

FROM \ TO	DETROIT	DALLAS	NEW YORK	LOS ANGELES
CINCINNATI	$25	$55	$40	$60
SALT LAKE	35	30	50	40
PITTSBURGH	36	45	26	66
SEATTLE	60	38	65	27
BIRMINGHAM	35	30	41	50

Table 9.17

Locating a New Factory for Hardgrave Machine Company

Birmingham Plant Optimal Solution: Total Hardgrave Cost is $3,741,000

FROM \ TO	DETROIT	DALLAS	NEW YORK	LOS ANGELES	FACTORY CAPACITY
CINCINNATI	10,000 (73)	(103)	1,000 (88)	4,000 (108)	15,000
SALT LAKE	(85)	1,000 (80)	(100)	5,000 (90)	6,000
PITTSBURGH	(88)	(97)	14,000 (78)	(118)	14,000
BIRMINGHAM	(84)	11,000 (79)	(90)	(99)	11,000
WAREHOUSE REQUIREMENT	10,000	12,000	15,000	9,000	46,000

Table 9.18

Locating a New Factory for Hardgrave Machine Company

Seattle Plant Optimal Solution: Total Hardgrave Cost is $3,704,000.

FROM \ TO	DETROIT	DALLAS	NEW YORK	LOS ANGELES	FACTORY CAPACITY
CINCINNATI	10,000 [73]	4,000 [103]	1,000 [88]	[108]	15,000
SALT LAKE	[85]	6,000 [80]	[100]	[90]	6,000
PITTSBURGH	[88]	[97]	14,000 [78]	[118]	14,000
SEATTLE	[113]	2,000 [91]	[118]	9,000 [80]	11,000
WAREHOUSE REQUIREMENT	10,000	12,000	15,000	9,000	46,000

Table 9.19

Locating a New Factory for Hardgrave Machine Company

- By comparing the total system costs of the two alternatives, Hardgrave can select the lowest cost option:
  - The Birmingham location yields a total system cost of $3,741,000.
  - The Seattle location yields a total system cost of $3,704,000.
- With the lower total system cost, the Seattle location is favored.
- Excel QM can also be used as a solution tool.

Excel QM Solution for Facility Location Example

From the Data tab, select Solver and click Solve.

Enter the transportation data in the shaded area. Then go to the DATA Tab on the ribbon, click on Solver in the Data Analysis Group and then click SOLVE.
If SOLVER is not on the Data Tab then please see the Help file (Solver) for instructions.

Enter the costs, supplies, and demands in this table.

Solver puts the solution here.

	A	B	C	D	E	F	G	H
1	**Birmingham**							
2								
3	**Transportation**							
4								
5								
6								
7								
8	**Data**							
9	COSTS	Dest 1	Dest 2	Dest 3	Dest 4	Supply		
10	Origin 1	73	103	88	108	15000		
11	Origin 2	85	80	100	90	6000		
12	Origin 3	88	97	78	118	14000		
13	Origin 4	84	79	90	99	11000		
14	Demand	10000	12000	15000	9000	46000 \ 46000		
15								
16	**Shipments**							
17	**Shipments**	**Dest 1**	**Dest 2**	**Dest 3**	**Dest 4**	**Row Total**		
18	**Origin 1**	**10000**		**1000**	**4000**	**15000**		
19	**Origin 2**		**1000**		**5000**	**6000**		
20	**Origin 3**			**14000**		**14000**		
21	**Origin 4**		**11000**			**11000**		
22	**Column Total**	**10000**	**12000**	**15000**	**9000**	**46000 \ 46000**		
23								
24	**Total Cost**	**3741000**						

Program 9.4

The Assignment Algorithm

- The second special-purpose LP algorithm is the assignment method.
- Each assignment problem has associated with it a table, or matrix.
- Generally, the rows contain the objects or people we wish to assign, and the columns comprise the tasks or things to which we want them assigned.
- The numbers in the table are the costs associated with each particular assignment.
- An assignment problem can be viewed as a transportation problem in which the capacity from each source is 1 and the demand at each destination is 1.

Assignment Model Approach

- The Fix-It Shop has three rush projects to repair.
- The shop has three repair persons with different talents and abilities.
- The owner has estimates of wage costs for each worker for each project.
- The owner’s objective is to assign the three project to the workers in a way that will result in the lowest cost to the shop.
- Each project will be assigned exclusively to one worker.

Assignment Model Approach

Estimated Project Repair Costs for the Fix-It Shop Assignment Problem

	PROJECT		
PERSON	1	2	3
Adams	$11	$14	$6
Brown	8	10	11
Cooper	9	12	7

Table 9.20

Assignment Model Approach

Summary of Fix-It Shop Assignment Alternatives and Costs

PRODUCT ASSIGNMENT				
1	2	3	LABOR COSTS ($)	TOTAL COSTS ($)
Adams	Brown	Cooper	11 + 10 + 7	28
Adams	Cooper	Brown	11 + 12 + 11	34
Brown	Adams	Cooper	8 + 14 + 7	29
Brown	Cooper	Adams	8 + 12 + 6	26
Cooper	Adams	Brown	9 + 14 + 11	34
Cooper	Brown	Adams	9 + 10 + 6	25

Table 9.21

The Hungarian Method (Flood's Technique)

- The ***Hungarian method*** is an efficient method of finding the optimal solution to an assignment problem without having to make direct comparisons of every option.
- It operates on the principle of ***matrix reduction.***
- By subtracting and adding appropriate numbers in the cost table or matrix, we can reduce the problem to a matrix of ***opportunity costs.***
- Opportunity costs show the relative penalty associated with assigning any person to a project as opposed to making the ***best*** assignment.
- We want to make assignment so that the opportunity cost for each assignment is zero.

Three Steps of the Assignment Method

1. ***Find the opportunity cost table by***:
   (a) Subtracting the smallest number in each row of the original cost table or matrix from every number in that row.
   (b) Then subtracting the smallest number in each column of the table obtained in part (a) from every number in that column.
2. ***Test the table resulting from step 1 to see whether an optimal assignment can be made*** by drawing the minimum number of vertical and horizontal straight lines necessary to cover all the zeros in the table. If the number of lines is less than the number of rows or columns, proceed to step 3.

Three Steps of the Assignment Method

3. *Revise the opportunity cost table* by *subtracting* the smallest number not covered by a line from all numbers not covered by a straight line. This same number is also added to every number lying at the intersection of any two lines. Return to step 2 and continue the cycle until an optimal assignment is possible.

Steps in the Assignment Method

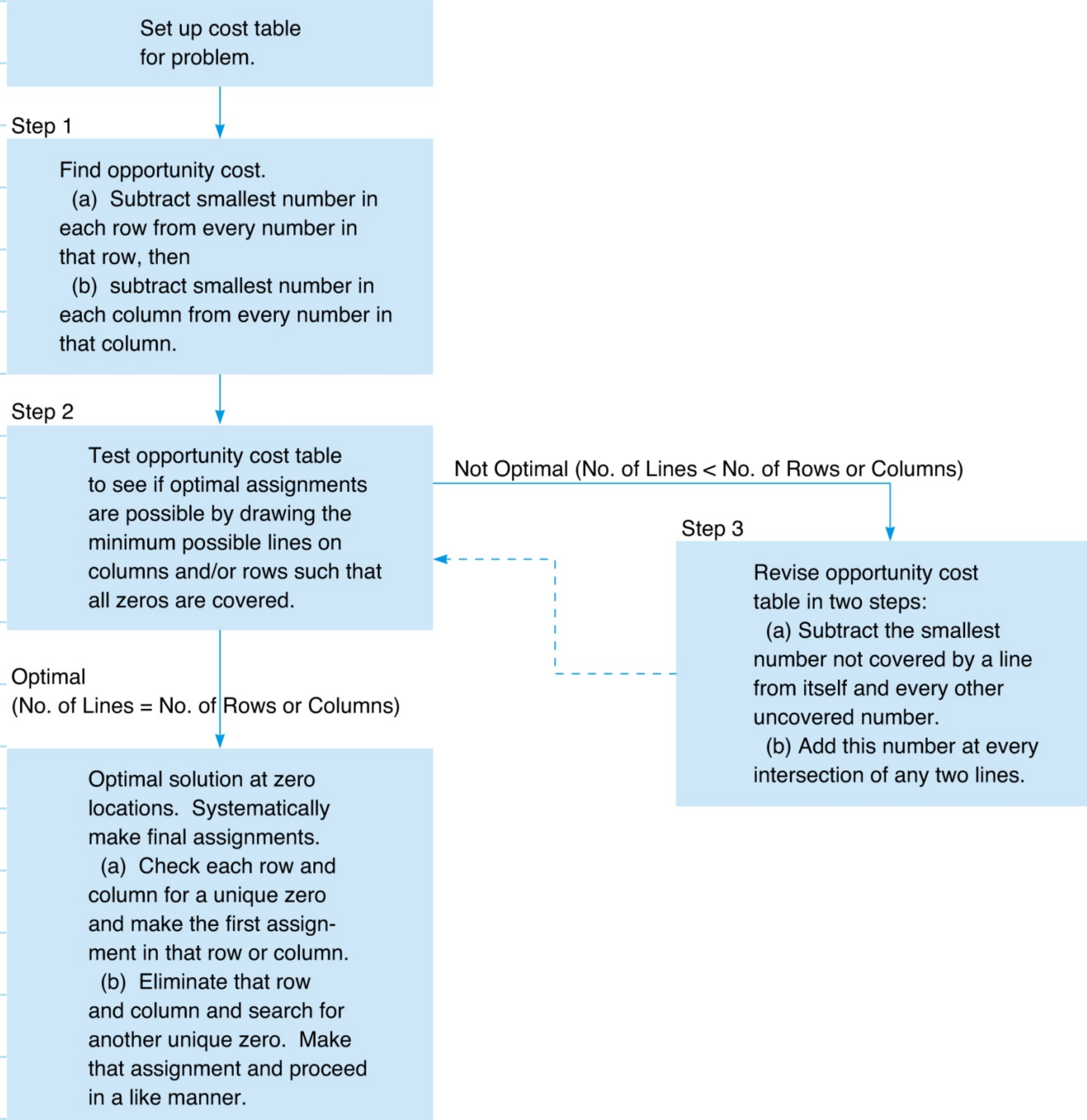


Figure 9.4

The Hungarian Method (Flood's Technique)

- **Step 1: Find the opportunity cost table.**
  - **We can compute *row* opportunity costs and *column* opportunity costs.**
  - **What we need is the *total* opportunity cost.**
  - **We derive this by taking the row opportunity costs and subtract the smallest number in that column from each number in that column.**

The Hungarian Method (Flood's Technique)

Cost of Each Person-Project Assignment for the Fix-it Shop Problem

PERSON	PROJECT 1	2	3
Adams	$11	$14	$6
Brown	8	10	11
Cooper	9	12	7

Tables 9.22-9.23

Row Opportunity Cost Table for the Fix-it Shop Step 1, Part (a)

PERSON	PROJECT 1	2	3
Adams	$5	$8	$0
Brown	0	2	3
Cooper	2	5	0

The opportunity cost of assigning Cooper to project 2 is $12 – $7 = $5.

The Hungarian Method (Flood's Technique)

Derive the total opportunity costs by taking the costs in Table 9.23 and subtract the smallest number in each column from each number in that column.

Total Opportunity Cost Table for the Fix-it Shop Step 1, Part (b)

	PROJECT		
PERSON	1	2	3
Adams	$5	$6	$0
Brown	0	0	3
Cooper	2	3	0

Table 9.24

The Hungarian Method (Flood's Technique)

- **Step 2: Test for the optimal assignment.**
  - **We want to assign workers to projects in such a way that the total labor costs are at a minimum.**
  - **We would like to have a total assigned opportunity cost of zero.**
  - **The test to determine if we have reached an optimal solution is simple.**
  - **We find the *minimum* number of straight lines necessary to cover all the zeros in the table.**
  - **If the number of lines equals the number of rows or columns, an optimal solution has been reached.**

The Hungarian Method (Flood's Technique)

Test for Optimal Solution to Fix-it Shop Problem

	PROJECT		
PERSON	1	2	3
Adams	$5	$6	$0
Brown	0	0	3
Cooper	2	3	0

Covering line 1

Covering line 2

Table 9.25

This requires only two lines to cover the zeros so the solution is not optimal.

The Hungarian Method (Flood's Technique)

- **Step 3: Revise the opportunity-cost table.**
  - **We *subtract* the smallest number not covered by a line from all numbers not covered by a straight line.**
  - **The same number is added to every number lying at the intersection of any two lines.**
  - **We then return to step 2 to test this new table.**

The Hungarian Method (Flood's Technique)

Revised Opportunity Cost Table for the Fix-it Shop Problem

	PROJECT		
PERSON	1	2	3
Adams	$3	$4	$0
Brown	0	0	5
Cooper	0	1	0

Table 9.26

The Hungarian Method (Flood's Technique)

Optimality Test on the Revised Fix-it Shop Opportunity Cost Table

PERSON	PROJECT 1	PROJECT 2	PROJECT 3
Adams	$3	$4	$0
Brown	0	0	5
Cooper	0	1	0

Table 9.27

Covering line 1

Covering line 2

Covering line 3

This requires three lines to cover the zeros so the solution is optimal.

Making the Final Assignment

- The optimal assignment is Adams to project 3, Brown to project 2, and Cooper to project 1.
- For larger problems one approach to making the final assignment is to select a row or column that contains only one zero.
  - Make the assignment to that cell and rule out its row and column.
  - Follow this same approach for all the remaining cells.

Making the Final Assignment

Total labor costs of this assignment are:

ASSIGNMENT	COST ($)
Adams to project 3	6
Brown to project 2	10
Cooper to project 1	9
Total cost	25

Making the Final Assignment

Making the Final Fix-it Shop Assignments

(A) FIRST ASSIGNMENT	1	2	3	(B) SECOND ASSIGNMENT	1	2	3	(C) THIRD ASSIGNMENT	1	2	3
Adams	3	4	$\boxed{0}$	Adams	3	4	$\boxed{0}$	Adams	3	4	$\boxed{0}$
Brown	0	0	5	Brown	0	0	5	Brown	0	$\boxed{0}$	5
Cooper	0	1	0	Cooper	$\boxed{0}$	1	0	Cooper	$\boxed{0}$	1	0

Table 9.28

Excel QM Solution for Fix-It Shop Assignment Problem

Fix-It Shop Assignment

From the Data tab, select Solver and click Solve.

Assignment

Enter the assignment costs in the shaded area. Then go to the DATA Tab on the ribbon, click on Solver in the Data Analysis Group and then click SOLVE.
If SOLVER is not on the Data Tab then please see the Help file (Solver) for instructions.

Enter the costs.

Data

COSTS	Project 1	Project 2	Project 3
Adams	11	14	6
Brown	8	10	11
Cooper	9	12	7

Solver puts the optimal assignments here.

Assignments

Shipments	Project 1	Project 2	Project 3	Row Total
Adams			1	1
Brown		1		1
Cooper	1			1
Column Tota	1	1	1	3

Total Cost	25

Program 9.5

Unbalanced Assignment Problems

- Often the number of people or objects to be assigned does not equal the number of tasks or clients or machines listed in the columns, and the problem is ***unbalanced.***
- When this occurs, and there are more rows than columns, simply add a ***dummy column*** or task.
- If the number of tasks exceeds the number of people available, we add a ***dummy row***.
- Since the dummy task or person is nonexistent, we enter zeros in its row or column as the cost or time estimate.

Unbalanced Assignment Problems

- Suppose the Fix-It Shop has another worker available.
- The shop owner still has the same basic problem of assigning workers to projects, but the problem now needs a dummy column to balance the four workers and three projects.

	PROJECT			
PERSON	1	2	3	DUMMY
Adams	$11	$14	$6	$0
Brown	8	10	11	0
Cooper	9	12	7	0
Davis	10	13	8	0

Table 9.29

Maximization Assignment Problems

- Some assignment problems are phrased in terms of maximizing the payoff, profit, or effectiveness.
- It is easy to obtain an equivalent minimization problem by converting all numbers in the table to opportunity costs.
- This is brought about by subtracting every number in the original payoff table from the largest single number in that table.
- Transformed entries represent opportunity costs.
- Once the optimal assignment has been found, the total payoff is found by adding the original payoffs of those cells that are in the optimal assignment.

Maximization Assignment Problems

- The British navy wishes to assign four ships to patrol four sectors of the North Sea.
- Ships are rated for their probable efficiency in each sector.
- The commander wants to determine patrol assignments producing the greatest overall efficiencies.

Maximization Assignment Problems

Efficiencies of British Ships in Patrol Sectors

	SECTOR			
SHIP	*A*	*B*	*C*	*D*
1	20	60	50	55
2	60	30	80	75
3	80	100	90	80
4	65	80	75	70

Table 9.30

Maximization Assignment Problems

Opportunity Costs of British Ships

SHIP	SECTOR A	B	C	D
1	80	40	50	45
2	40	70	20	25
3	20	0	10	20
4	35	20	25	30

Table 9.31

Maximization Assignment Problems

- Convert the maximization efficiency table into a minimizing opportunity cost table by subtracting each rating from 100, the largest rating in the whole table.
- The smallest number in each row is subtracted from every number in that row and the smallest number in each column is subtracted from every number in that column.
- The minimum number of lines needed to cover the zeros in the table is four, so this represents an optimal solution.

Maximization Assignment Problems

Row Opportunity Costs for the British Navy Problem

	SECTOR			
SHIP	*A*	*B*	*C*	*D*
1	40	0	10	5
2	20	50	0	5
3	20	0	10	20
4	15	0	5	10

Table 9.32

Maximization Assignment Problems

Total Opportunity Costs for the British Navy Problem

	SECTOR			
SHIP	*A*	*B*	*C*	*D*
1	25	0	10	0
2	5	50	0	0
3	5	0	10	15
4	0	0	5	5

Table 9.33

Maximization Assignment Problems

The overall efficiency

ASSIGNMENT	EFFICIENCY
Ship 1 to sector *D*	55
Ship 2 to sector *C*	80
Ship 3 to sector *B*	100
Ship 4 to sector *A*	65
Total efficiency	300

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