Chapter 7

Linear Programming Models: Graphical and Computer Methods

To accompany
Quantitative Analysis for Management, Eleventh Edition,
by Render, Stair, and Hanna
Power Point slides created by Brian Peterson

Learning Objectives

After completing this chapter, students will be able to:

1. Understand the basic assumptions and properties of linear programming (LP).
2. Graphically solve any LP problem that has only two variables by both the corner point and isoprofit line methods.
3. Understand special issues in LP such as infeasibility, unboundedness, redundancy, and alternative optimal solutions.
4. Understand the role of sensitivity analysis.
5. Use Excel spreadsheets to solve LP problems.

Chapter Outline

Introduction

- **Many management decisions involve trying to make the most effective use of limited resources.**
- ***Linear programming* (*LP*) is a widely used mathematical modeling technique designed to help managers in planning and decision making relative to resource allocation.**
  - **This belongs to the broader field of *mathematical programming.***
  - **In this sense, *programming* refers to modeling and solving a problem mathematically.**

Requirements of a Linear Programming Problem

- All LP problems have 4 properties in common:
  1. All problems seek to *maximize* or *minimize* some quantity (the *objective function*).
  2. Restrictions or *constraints* that limit the degree to which we can pursue our objective are present.
  3. There must be alternative courses of action from which to choose.
  4. The objective and constraints in problems must be expressed in terms of *linear* equations or *inequalities.*

Basic Assumptions of LP

- We assume conditions of *certainty* exist and numbers in the objective and constraints are known with certainty and do not change during the period being studied.
- We assume *proportionality* exists in the objective and constraints.
- We assume *additivity* in that the total of all activities equals the sum of the individual activities.
- We assume *divisibility* in that solutions need not be whole numbers.
- All answers or variables are *nonnegative.*

LP Properties and Assumptions

PROPERTIES OF LINEAR PROGRAMS
1. One objective function
2. One or more constraints
3. Alternative courses of action
4. Objective function and constraints are linear – proportionality and divisibility
5. Certainty
6. Divisibility
7. Nonnegative variables

Table 7.1

Formulating LP Problems

- Formulating a linear program involves developing a mathematical model to represent the managerial problem.
- The steps in formulating a linear program are:
  1. Completely understand the managerial problem being faced.
  2. Identify the objective and the constraints.
  3. Define the decision variables.
  4. Use the decision variables to write mathematical expressions for the objective function and the constraints.

Formulating LP Problems

- One of the most common LP applications is the ***product mix problem.***
- Two or more products are produced using limited resources such as personnel, machines, and raw materials.
- The profit that the firm seeks to maximize is based on the profit contribution per unit of each product.
- The company would like to determine how many units of each product it should produce so as to maximize overall profit given its limited resources.

Flair Furniture Company

- The Flair Furniture Company produces inexpensive tables and chairs.
- Processes are similar in that both require a certain amount of hours of carpentry work and in the painting and varnishing department.
- Each table takes 4 hours of carpentry and 2 hours of painting and varnishing.
- Each chair requires 3 of carpentry and 1 hour of painting and varnishing.
- There are 240 hours of carpentry time available and 100 hours of painting and varnishing.
- Each table yields a profit of $70 and each chair a profit of $50.

Flair Furniture Company Data

The company wants to determine the best combination of tables and chairs to produce to reach the maximum profit.

	HOURS REQUIRED TO PRODUCE 1 UNIT		
DEPARTMENT	(T) TABLES	(C) CHAIRS	AVAILABLE HOURS THIS WEEK
Carpentry	4	3	240
Painting and varnishing	2	1	100
Profit per unit	$70	$50	

Table 7.2

Flair Furniture Company

- The objective is to:

  Maximize profit

- The constraints are:
  1. The hours of carpentry time used cannot exceed 240 hours per week.
  2. The hours of painting and varnishing time used cannot exceed 100 hours per week.
- The decision variables representing the actual decisions we will make are:

  T = number of tables to be produced per week.

  C = number of chairs to be produced per week.

Flair Furniture Company

- We create the LP objective function in terms of T and C:

  Maximize profit = \$70T + \$50C

- Develop mathematical relationships for the two constraints:
  - For carpentry, total time used is:

    (4 hours per table)(Number of tables produced)
    + (3 hours per chair)(Number of chairs produced).

  - We know that:

Carpentry time used ≤ Carpentry time available.

$4T + 3C \leq 240$ (hours of carpentry time)

Flair Furniture Company

- **Similarly,**

  Painting and varnishing time used
  ≤ Painting and varnishing time available.

  $2T + 1C \leq 100$ **(hours of painting and varnishing time)**

  This means that each table produced requires two hours of painting and varnishing time.

- **Both of these constraints restrict production capacity and affect total profit.**

Flair Furniture Company

The values for T and C must be nonnegative.

$T \geq 0$ (number of tables produced is greater than or equal to 0)

$C \geq 0$ (number of chairs produced is greater than or equal to 0)

The complete problem stated mathematically:

$$\text{Maximize profit} = \$70T + \$50C$$

subject to

$4T + 3C \leq 240$ (carpentry constraint)

$2T + 1C \leq 100$ (painting and varnishing constraint)

$T, C \geq 0$ (nonnegativity constraint)

Graphical Solution to an LP Problem

- The easiest way to solve a small LP problems is graphically.
- The graphical method only works when there are just two decision variables.
- When there are more than two variables, a more complex approach is needed as it is not possible to plot the solution on a two-dimensional graph.
- The graphical method provides valuable insight into how other approaches work.

Graphical Representation of a Constraint

Quadrant Containing All Positive Values

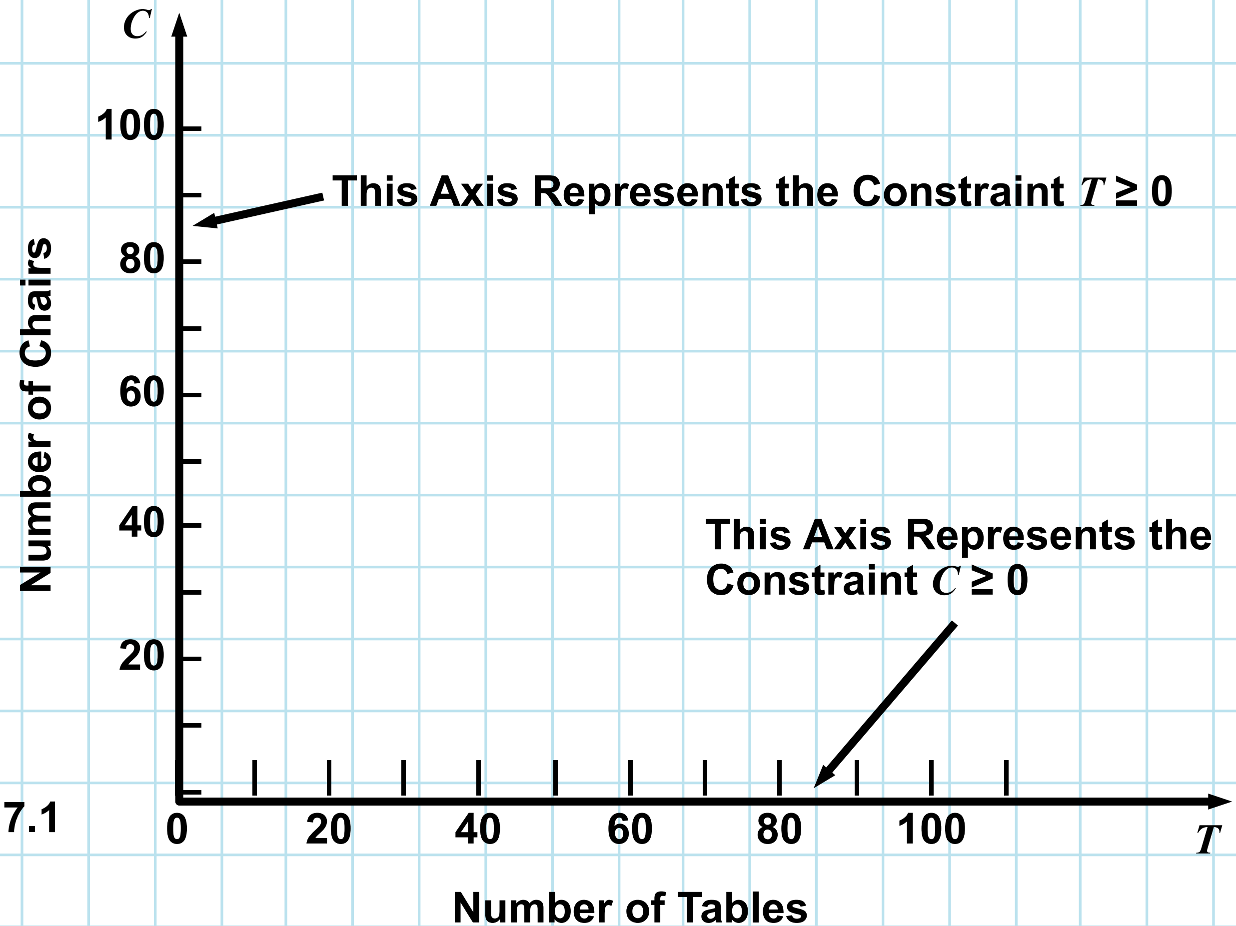


Figure 7.1

Graphical Representation of a Constraint

- The first step in solving the problem is to identify a set or region of feasible solutions.
- To do this we plot each constraint equation on a graph.
- We start by graphing the equality portion of the constraint equations:

$$4T + 3C = 240$$

- We solve for the axis intercepts and draw the line.

Graphical Representation of a Constraint

- When Flair produces no tables, the carpentry constraint is:

$$4(0) + 3C = 240$$
$$3C = 240$$
$$C = 80$$

- Similarly for no chairs:

$$4T + 3(0) = 240$$
$$4T = 240$$
$$T = 60$$

- This line is shown on the following graph:

Graphical Representation of a Constraint

Graph of carpentry constraint equation

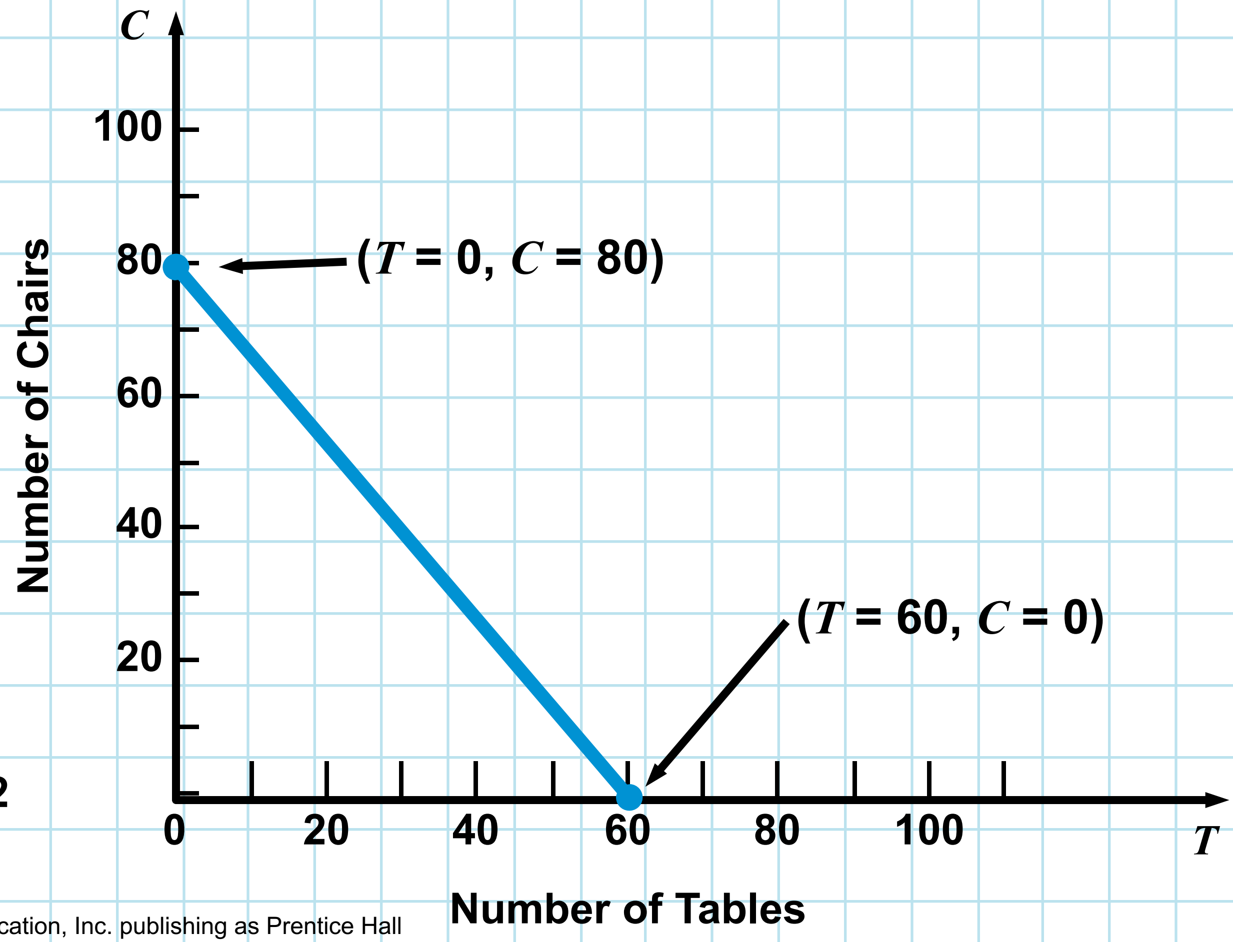


Figure 7.2

Graphical Representation of a Constraint

Region that Satisfies the Carpentry Constraint

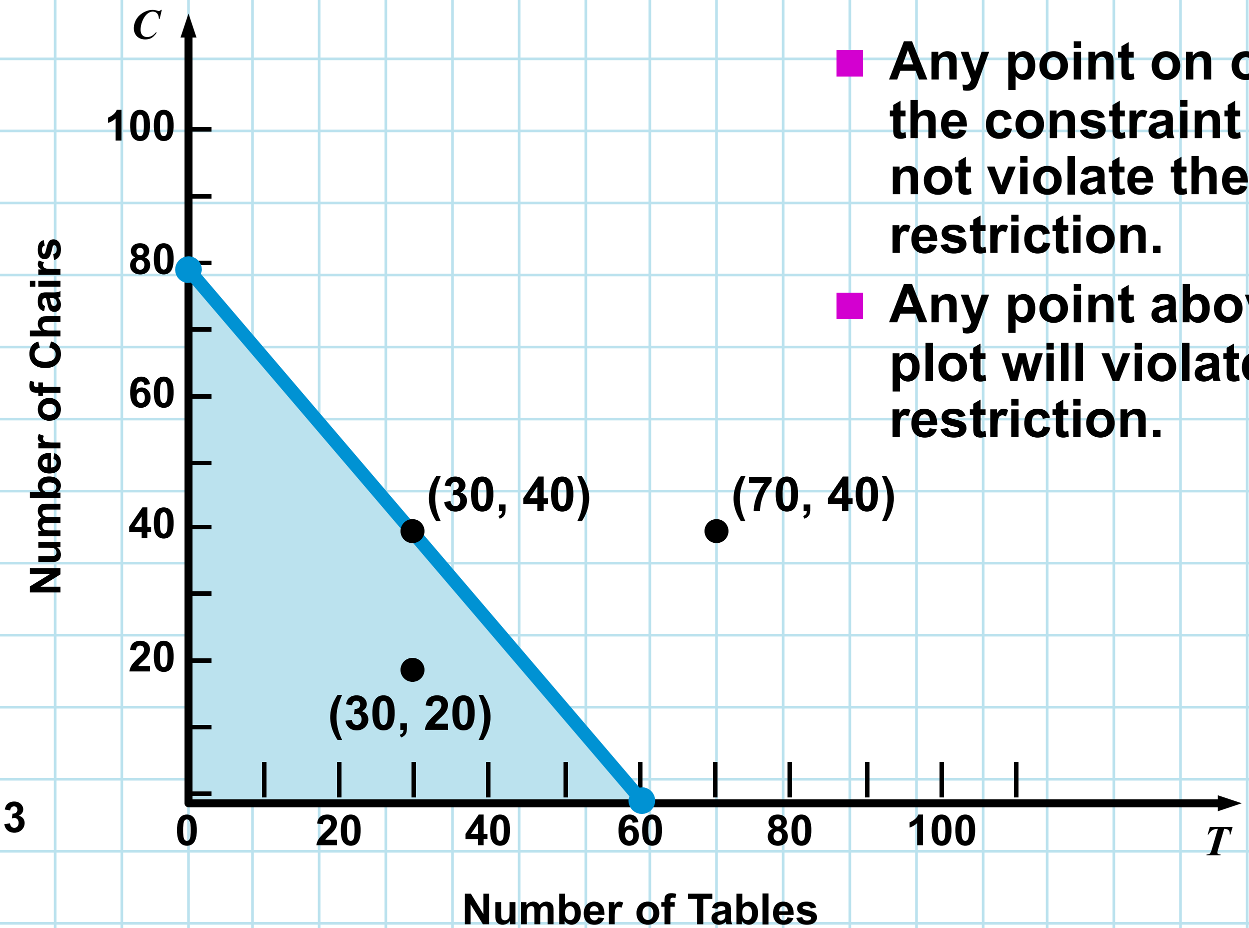


Figure 7.3

- Any point on or below the constraint plot will not violate the restriction.
- Any point above the plot will violate the restriction.

Graphical Representation of a Constraint

- The point (30, 40) lies on the plot and exactly satisfies the constraint

  $$4(30) + 3(40) = 240.$$

- The point (30, 20) lies below the plot and satisfies the constraint

  $$4(30) + 3(20) = 180.$$

- The point (70, 40) lies above the plot and does not satisfy the constraint

  $$4(70) + 3(40) = 400.$$

Graphical Representation of a Constraint

Region that Satisfies the Painting and Varnishing Constraint

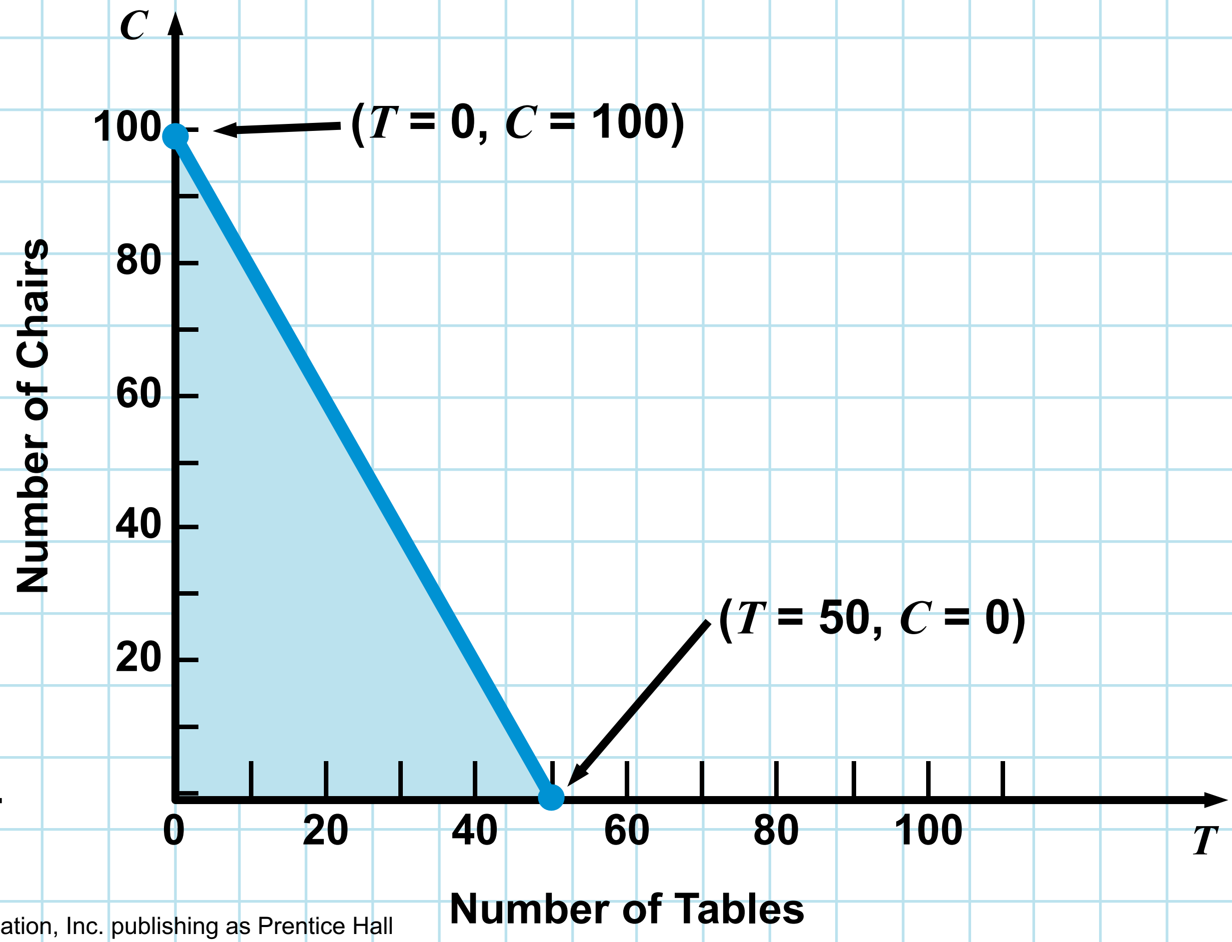


Figure 7.4

Graphical Representation of a Constraint

- To produce tables and chairs, both departments must be used.
- We need to find a solution that satisfies both constraints *simultaneously.*
- A new graph shows both constraint plots.
- The *feasible region* (or *area of feasible solutions*) is where all constraints are satisfied.
- Any point inside this region is a *feasible* solution.
- Any point outside the region is an *infeasible* solution.

Graphical Representation of a Constraint

Feasible Solution Region for the Flair Furniture Company Problem

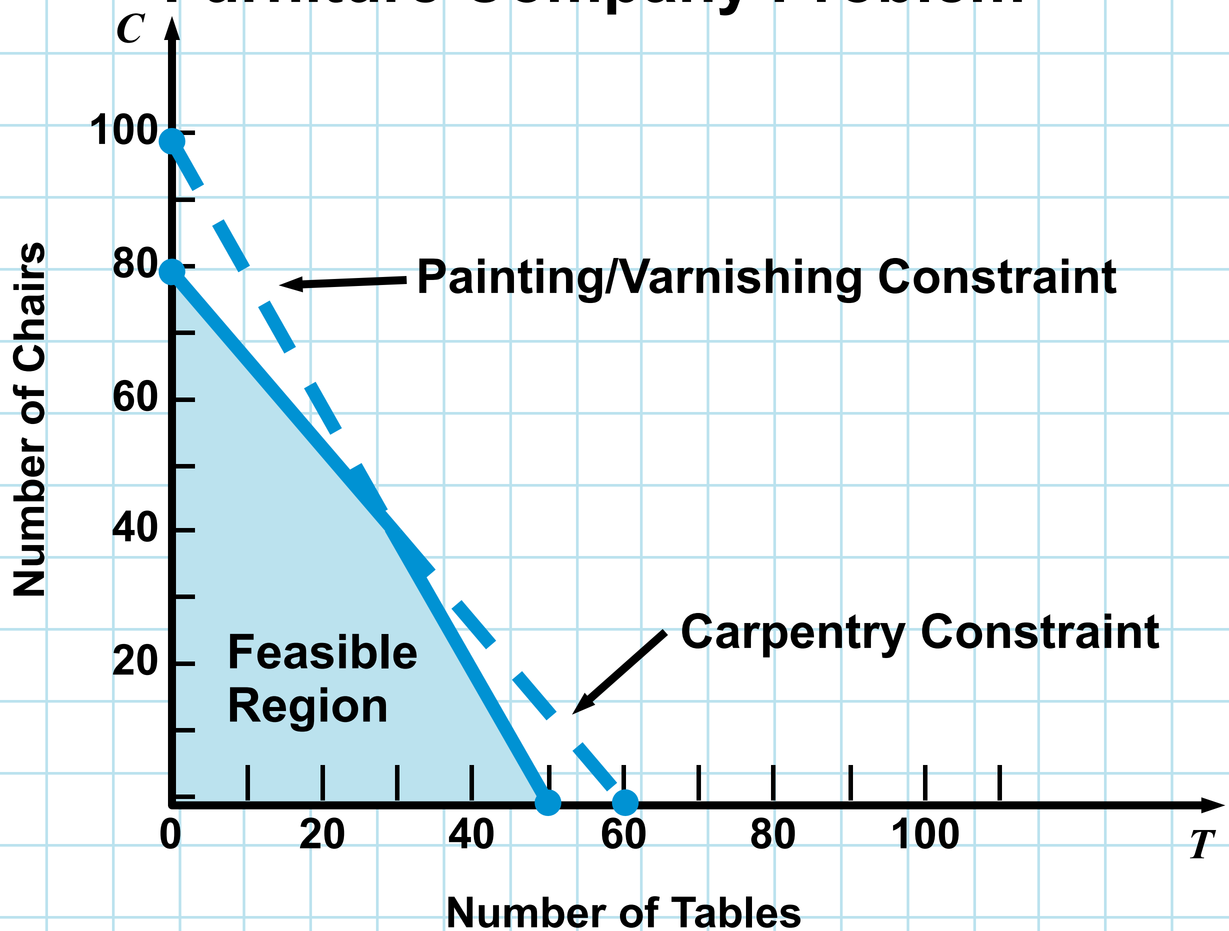


Figure 7.5

Graphical Representation of a Constraint

- **For the point (30, 20)**

Carpentry constraint	$4T + 3C \leq 240$ hours available (4)(30) + (3)(20) = 180 hours used	✓
Painting constraint	$2T + 1C \leq 100$ hours available (2)(30) + (1)(20) = 80 hours used	✓

- **For the point (70, 40)**

Carpentry constraint	$4T + 3C \leq 240$ hours available (4)(70) + (3)(40) = 400 hours used	✗
Painting constraint	$2T + 1C \leq 100$ hours available (2)(70) + (1)(40) = 180 hours used	✗

Graphical Representation of a Constraint

- For the point (50, 5)

Carpentry constraint	$4T + 3C \leq 240$ hours available (4)(50) + (3)(5) = 215 hours used	✓
Painting constraint	$2T + 1C \leq 100$ hours available (2)(50) + (1)(5) = 105 hours used	✗

Isoprofit Line Solution Method

- Once the feasible region has been graphed, we need to find the optimal solution from the many possible solutions.
- The speediest way to do this is to use the isoprofit line method.
- Starting with a small but possible profit value, we graph the objective function.
- We move the objective function line in the direction of increasing profit while maintaining the slope.
- The last point it touches in the feasible region is the optimal solution.

Isoprofit Line Solution Method

- For Flair Furniture, choose a profit of \$2,100.
- The objective function is then

$$\$2{,}100 = 70T + 50C$$

- Solving for the axis intercepts, we can draw the graph.
- This is obviously not the best possible solution.
- Further graphs can be created using larger profits.
- The further we move from the origin, the larger the profit will be.
- The highest profit (\$4,100) will be generated when the isoprofit line passes through the point (30, 40).

Isoprofit Line Solution Method

Profit line of $2,100 Plotted for the Flair Furniture Company

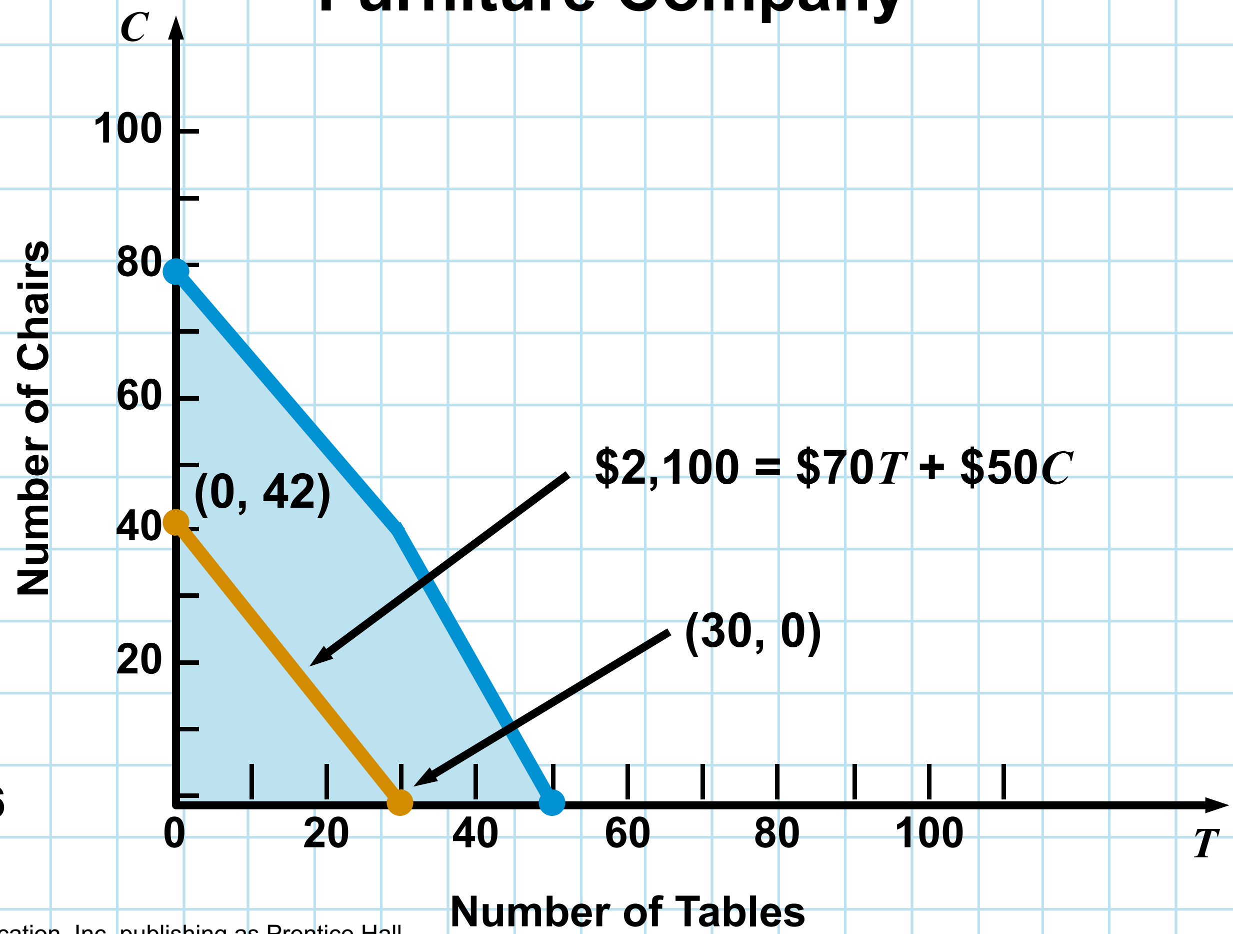


Figure 7.6

Isoprofit Line Solution Method

Four Isoprofit Lines Plotted for the Flair Furniture Company

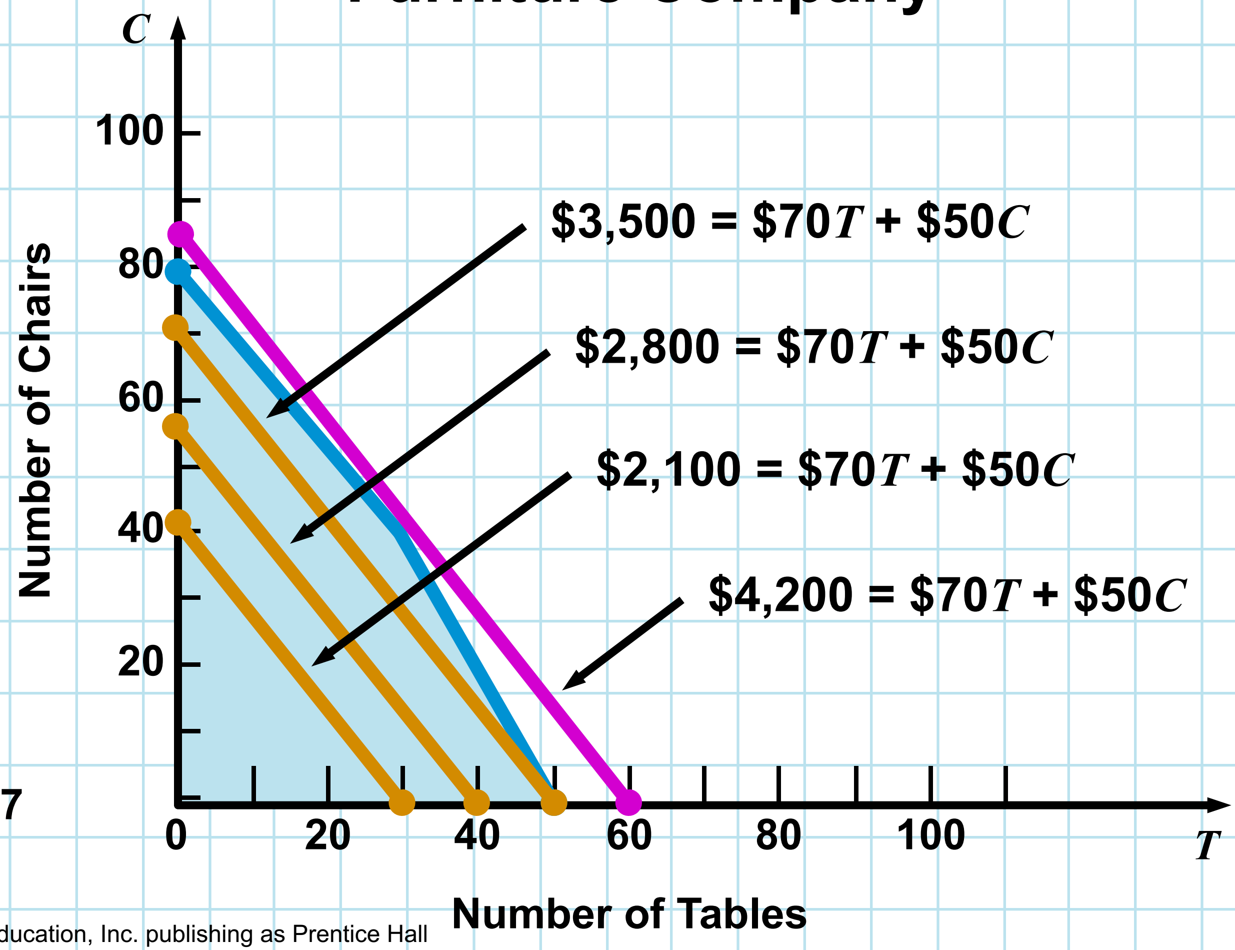


Figure 7.7

Isoprofit Line Solution Method

Optimal Solution to the Flair Furniture problem

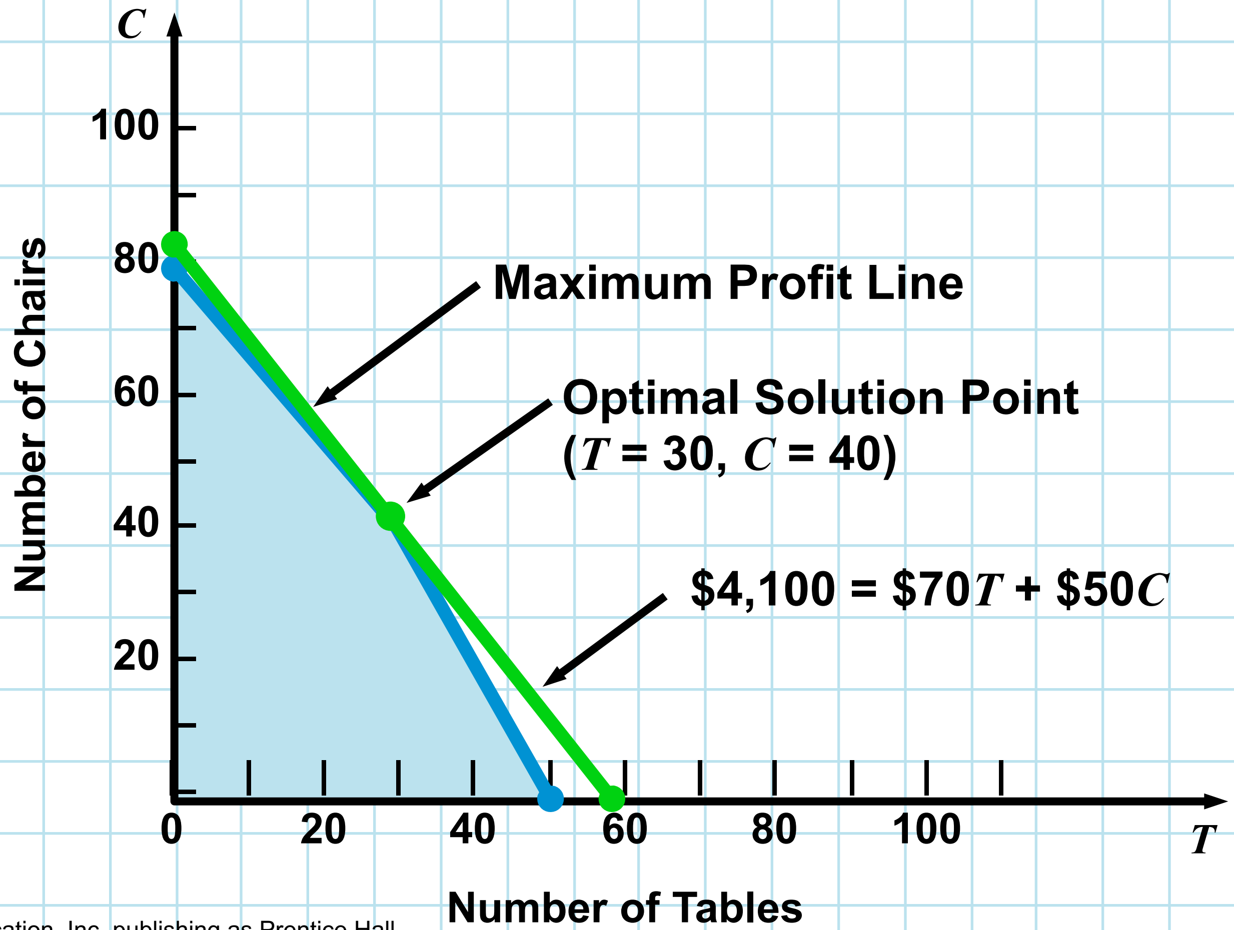


Figure 7.8

Corner Point Solution Method

- A second approach to solving LP problems employs the *corner point method.*
- It involves looking at the profit at every corner point of the feasible region.
- The mathematical theory behind LP is that the optimal solution must lie at one of the *corner points*, or *extreme point*, in the feasible region.
- For Flair Furniture, the feasible region is a four-sided polygon with four corner points labeled 1, 2, 3, and 4 on the graph.

Corner Point Solution Method

Four Corner Points of the Feasible Region

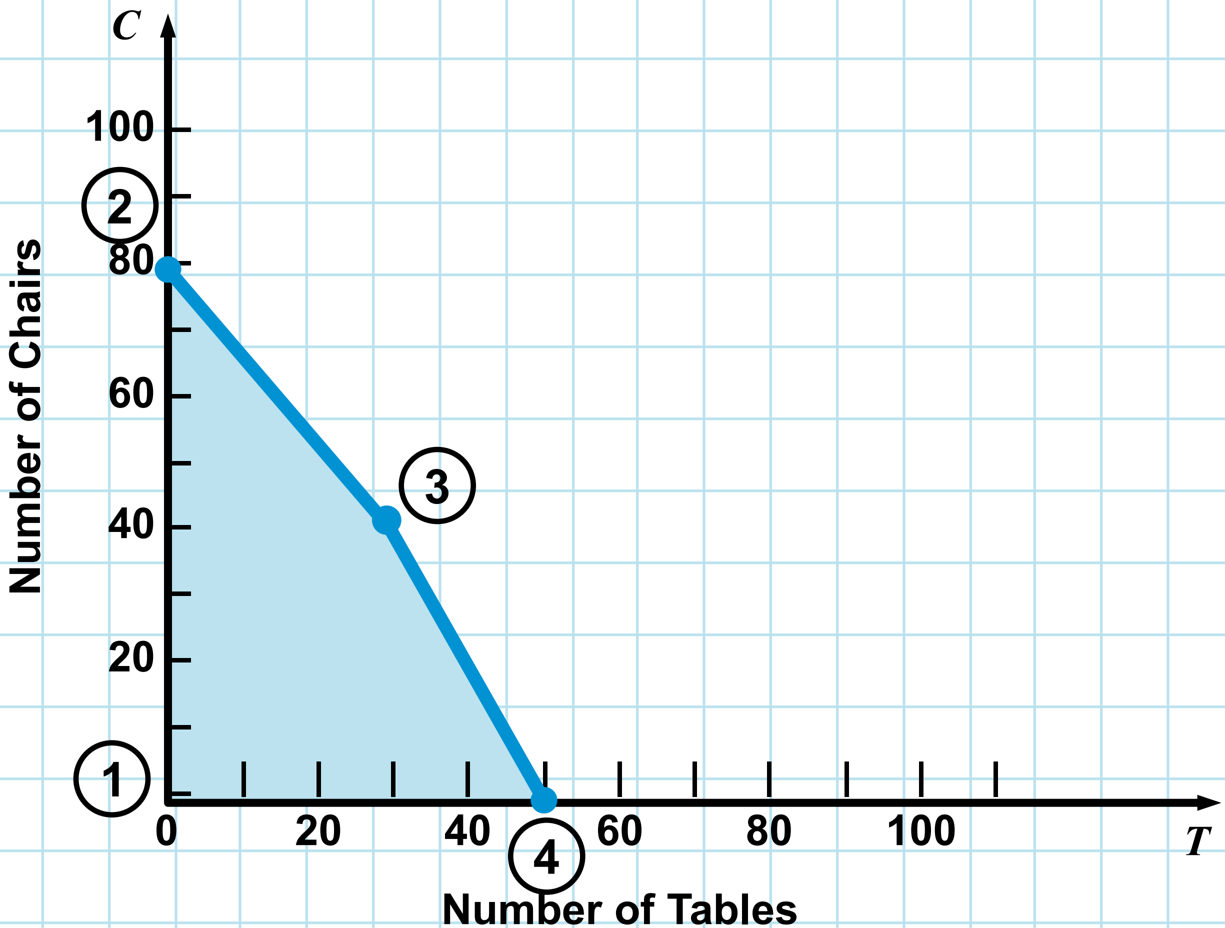


Figure 7.9

Corner Point Solution Method

- To find the coordinates for Point 3 accurately we have to solve for the intersection of the two constraint lines.
- Using the ***simultaneous equations method***, we multiply the painting equation by –2 and add it to the carpentry equation

$$\begin{aligned} 4T + 3C &= 240 \qquad \text{(carpentry line)} \\ -4T - 2C &= -200 \qquad \text{(painting line)} \\ C &= 40 \end{aligned}$$

- Substituting 40 for C in either of the original equations allows us to determine the value of T.

$$\begin{aligned} 4T + (3)(40) &= 240 \qquad \text{(carpentry line)} \\ 4T + 120 &= 240 \\ T &= 30 \end{aligned}$$

Corner Point Solution Method

Point 1: (T = 0, C = 0)	**Profit = \$70(0) + \$50(0) = \$0**
Point 2: (T = 0, C = 80)	**Profit = \$70(0) + \$50(80) = \$4,000**
Point 4: (T = 50, C = 0)	**Profit = \$70(50) + \$50(0) = \$3,500**
Point 3: (T = 30, C = 40)	**Profit = \$70(30) + \$50(40) = \$4,100**

Because Point 3 returns the highest profit, this is the optimal solution.

Slack and Surplus

- **Slack** is the amount of a resource that is not used. For a less-than-or-equal constraint:
  - **Slack** = Amount of resource available – amount of resource used.
- Surplus is used with a greater-than-or-equal constraint to indicate the amount by which the right hand side of the constraint is exceeded.
  - **Surplus** = Actual amount – minimum amount.

Summary of Graphical Solution Methods

ISOPROFIT METHOD
1. Graph all constraints and find the feasible region.
2. Select a specific profit (or cost) line and graph it to find the slope.
3. Move the objective function line in the direction of increasing profit (or decreasing cost) while maintaining the slope. The last point it touches in the feasible region is the optimal solution.
4. Find the values of the decision variables at this last point and compute the profit (or cost).
CORNER POINT METHOD
1. Graph all constraints and find the feasible region.
2. Find the corner points of the feasible reason.
3. Compute the profit (or cost) at each of the feasible corner points.
4. Select the corner point with the best value of the objective function found in Step 3. This is the optimal solution.

Table 7.4

Solving Flair Furniture's LP Problem Using QM for Windows and Excel

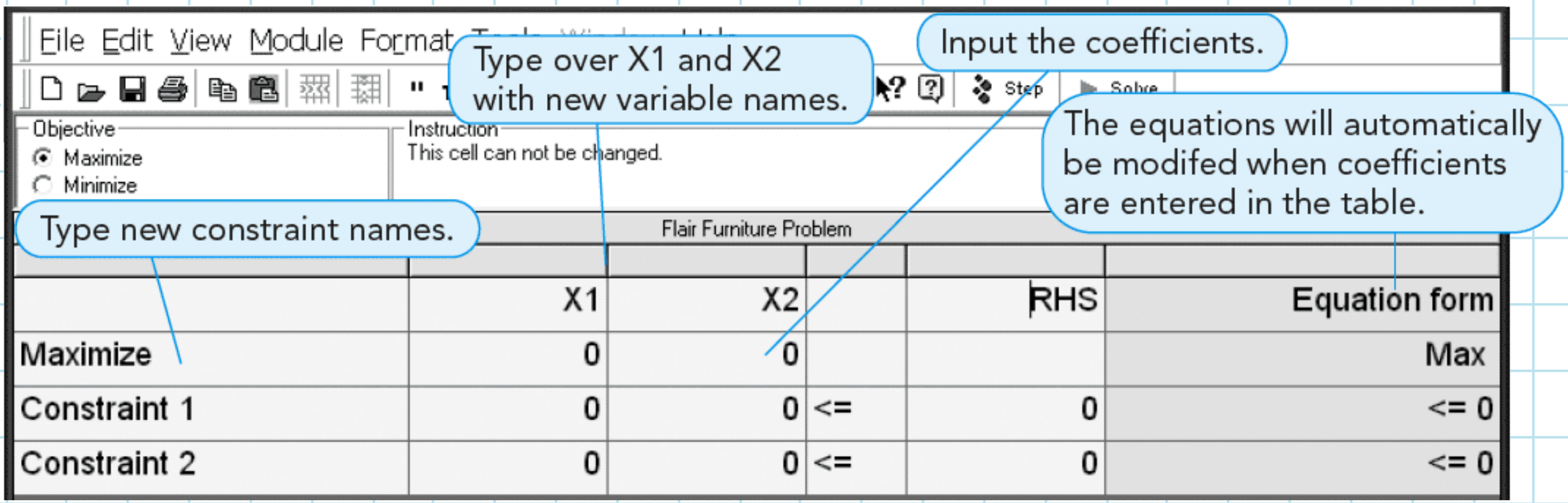
- **Most organizations have access to software to solve big LP problems.**
- **While there are differences between software implementations, the approach each takes towards handling LP is basically the same.**
- **Once you are experienced in dealing with computerized LP algorithms, you can easily adjust to minor changes.**

Using QM for Windows

- First select the Linear Programming module.
- Specify the number of constraints (non-negativity is assumed).
- Specify the number of decision variables.
- Specify whether the objective is to be maximized or minimized.
- For the Flair Furniture problem there are two constraints, two decision variables, and the objective is to maximize profit.

Using QM for Windows

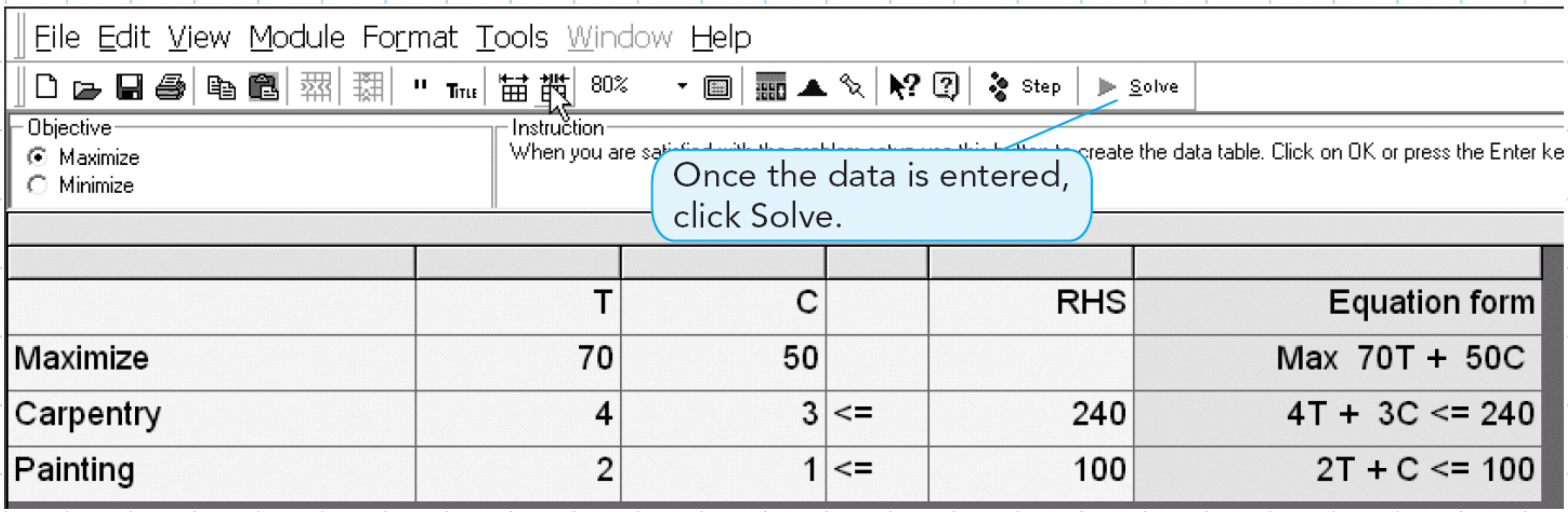
QM for Windows Linear Programming Computer screen for Input of Data



Program 7.1A

Using QM for Windows

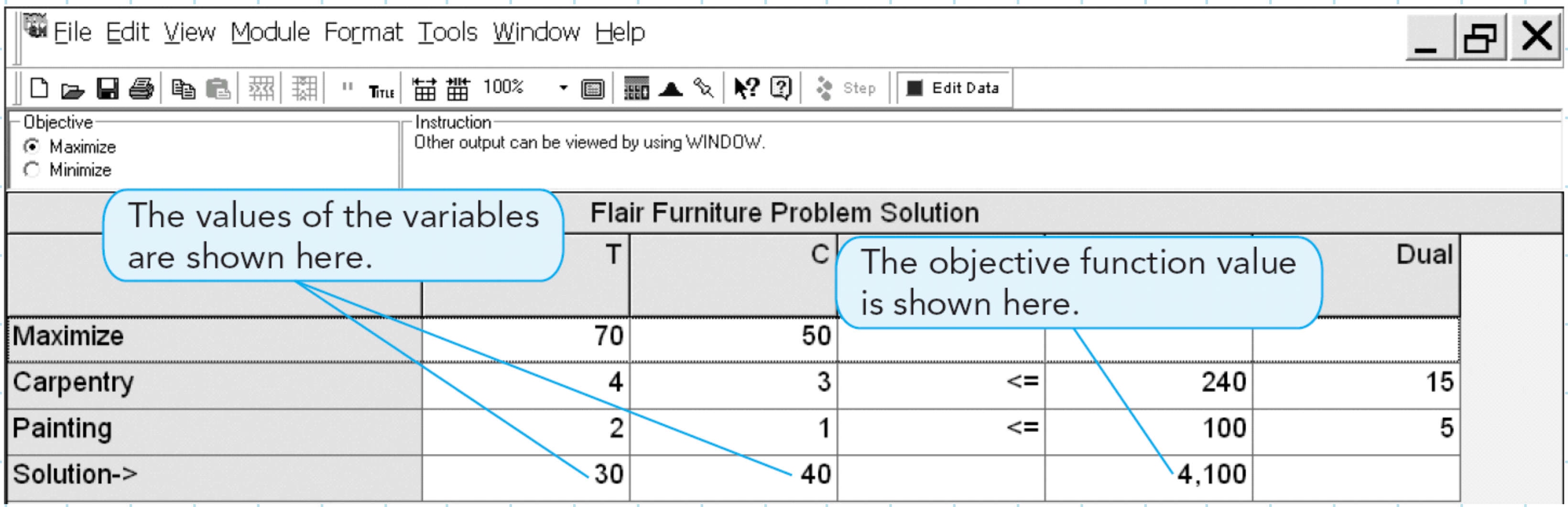
QM for Windows Data Input for Flair Furniture Problem



Program 7.1B

Using QM for Windows

QM for Windows Output for Flair Furniture Problem



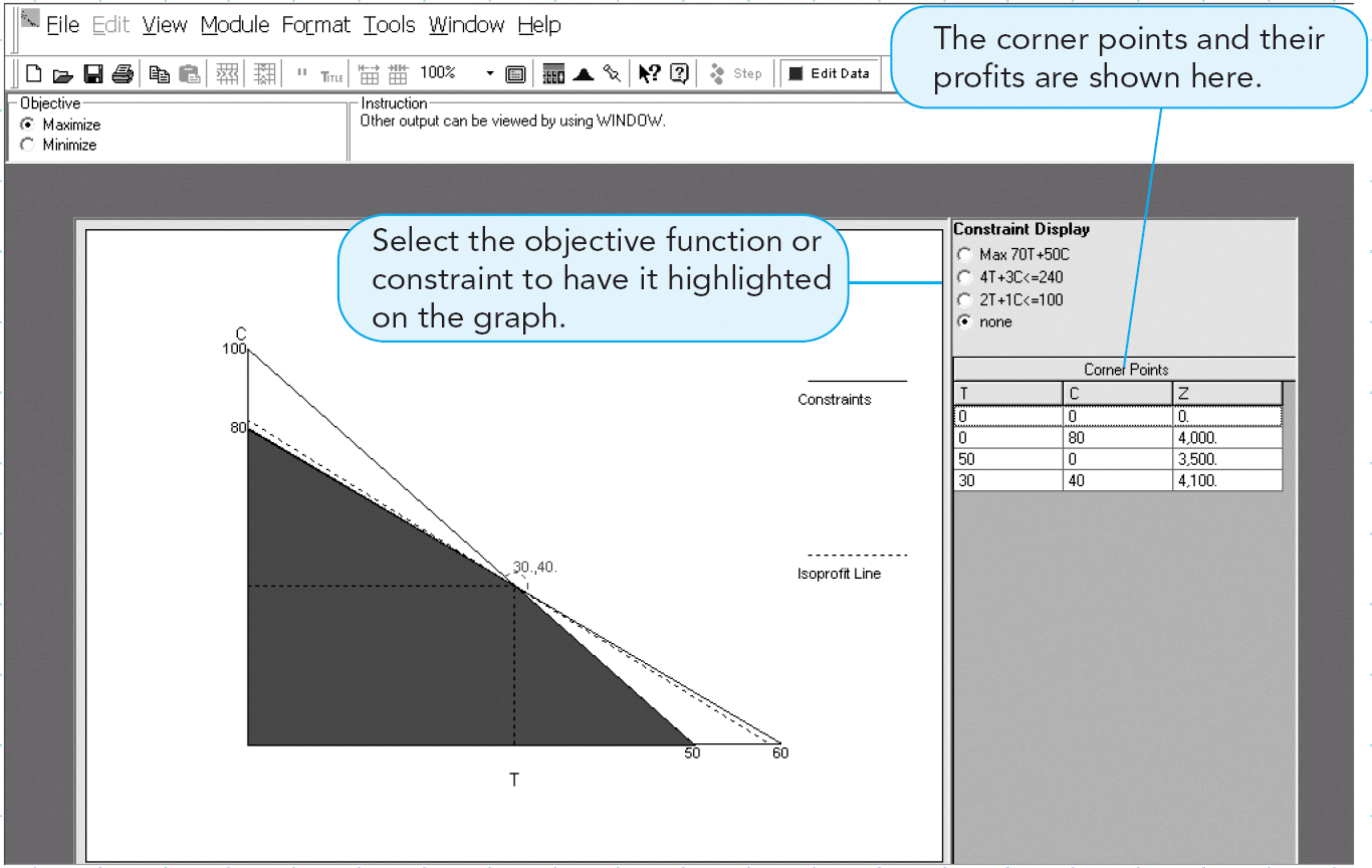
Flair Furniture Problem Solution

	T	C			Dual
Maximize	70	50			
Carpentry	4	3	<=	240	15
Painting	2	1	<=	100	5
Solution->	30	40		4,100	

Program 7.1C

Using QM for Windows

QM for Windows Graphical Output for Flair Furniture Problem



Program 7.1D

Using Excel's Solver Command to Solve LP Problems

- The Solver tool in Excel can be used to find solutions to:
  - LP problems.
  - Integer programming problems.
  - Noninteger programming problems.
- Solver is limited to 200 variables and 100 constraints.

Using Solver to Solve the Flair Furniture Problem

- Recall the model for Flair Furniture is:

Maximize profit =	$\$70T$ +	$\$50C$	
Subject to	$4T$ +	$3C$	≤ 240
	$2T$ +	$1C$	≤ 100

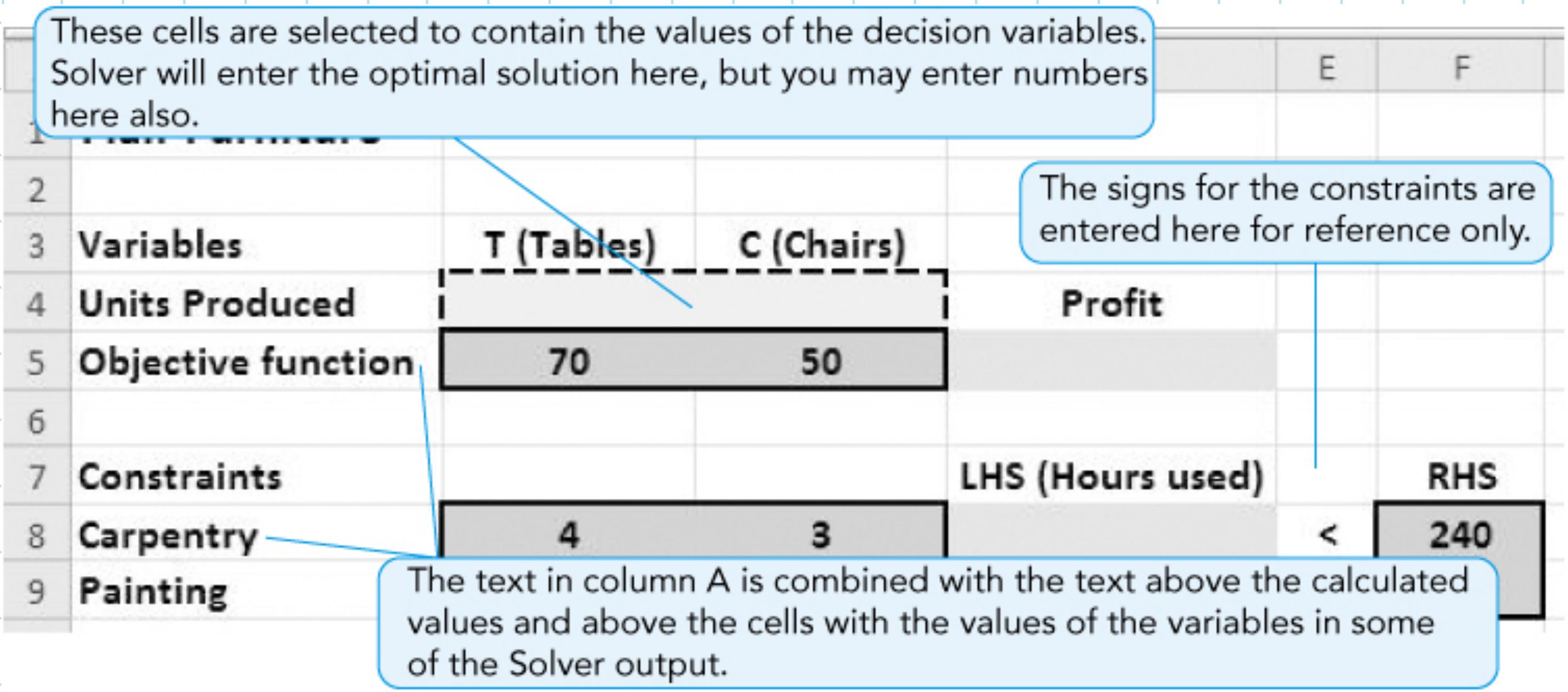
- To use Solver, it is necessary to enter formulas based on the initial model.

Using Solver to Solve the Flair Furniture Problem

1. Enter the variable names, the coefficients for the objective function and constraints, and the right-hand-side values for each of the constraints.
2. Designate specific cells for the values of the decision variables.
3. Write a formula to calculate the value of the objective function.
4. Write a formula to compute the left-hand sides of each of the constraints.

Using Solver to Solve the Flair Furniture Problem

Excel Data Input for the Flair Furniture Example



Program 7.2A

Using Solver to Solve the Flair Furniture Problem

Formulas for the Flair Furniture Example

H16

	A	B	C	D	E	F
1	Flair Furniture					
2						
3	Variables	T (Tables)	C (Chairs)			
4	Units Produced	1	1	Profit		
5	Objective function	70	50	=SUMPRODUCT($B$4:$C$4,B5:C5)		
6						
7	Constraints			LHS (Hours used)		RHS
8	Carpentry	4	3	=SUMPRODUCT($B$4:$C$4,B8:C8)	<	240
9	Painting	2	1	=SUMPRODUCT($B$4:$C$4,B9:C9)	<	100

A 1 was entered as the value of *T* and value of *C* to help find obvious errors in the formulas.

The values of the variables are in B4 and C4, and the profits for these are in cells B5 and C5. This formula will calculate B4*B5+C4*C5, or 1(70)+1(50), and return a value of 120.

The formula for the LHS of each constraint can be copied from cell D5. The $ signs cause the cell addresses to remain unchanged when the cell (D5) is copied.

Program 7.2B

Using Solver to Solve the Flair Furniture Problem

Excel Spreadsheet for the Flair Furniture Example

	A	B	C	D	E	F
1	**Flair Furniture**					
2						
3	**Variables**	**T (Tables)**	**C (Chairs)**			
4	**Units Produced**	**1**	**1**			
5	**Objective function**	**70**	**50**			
6						
7	**Constraints**			**LHS (Hours used)**		**RHS**
8	**Carpentry**	**4**	**3**	**7**	**<**	**240**
9	**Painting**	**2**	**1**	**3**	**<**	**100**

You can change these values to see how the profit and resource utilization change.

Because there is a 1 in each of these cells, the LHS values can be calculated very easily to see if a mistake hase been made.

The problem is ready to use the Solver add-in.

Program 7.2C

Using Solver to Solve the Flair Furniture Problem

- **Once the model has been entered, the following steps can be used to solve the problem.**

  In Excel 2010, select Data – Solver.

  If Solver does not appear in the indicated place, see Appendix F for instructions on how to activate this add-in.

1. **In the Set Objective box, enter the cell address for the total profit.**
2. **In the By Changing Cells box, enter the cell addresses for the variable values.**
3. **Click *Max* for a maximization problem and *Min* for a minimization problem.**

Using Solver to Solve the Flair Furniture Problem

4. Check the box for *Make Unconstrained Variables Non-negative*.
5. Click the *Select Solving Method* button and select *Simplex LP* from the menu that appears.
6. Click *Add* to add the constraints.
7. In the dialog box that appears, enter the cell references for the left-hand-side values, the type of equation, and the right-hand-side values.
8. Click *Solve*.

Using Solver to Solve the Flair Furniture Problem

Starting Solver

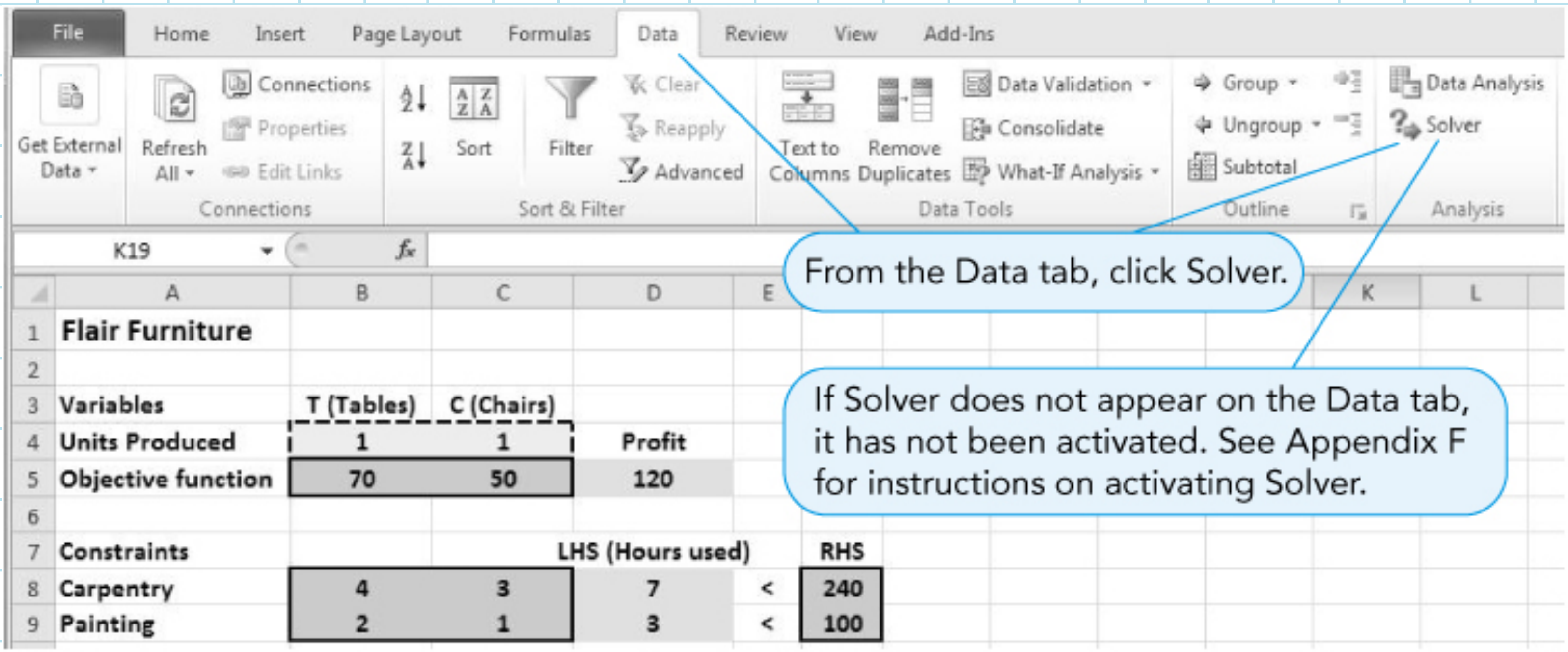


Figure 7.2D

Using Solver to Solve the Flair Furniture Problem

Solver Parameters Dialog Box

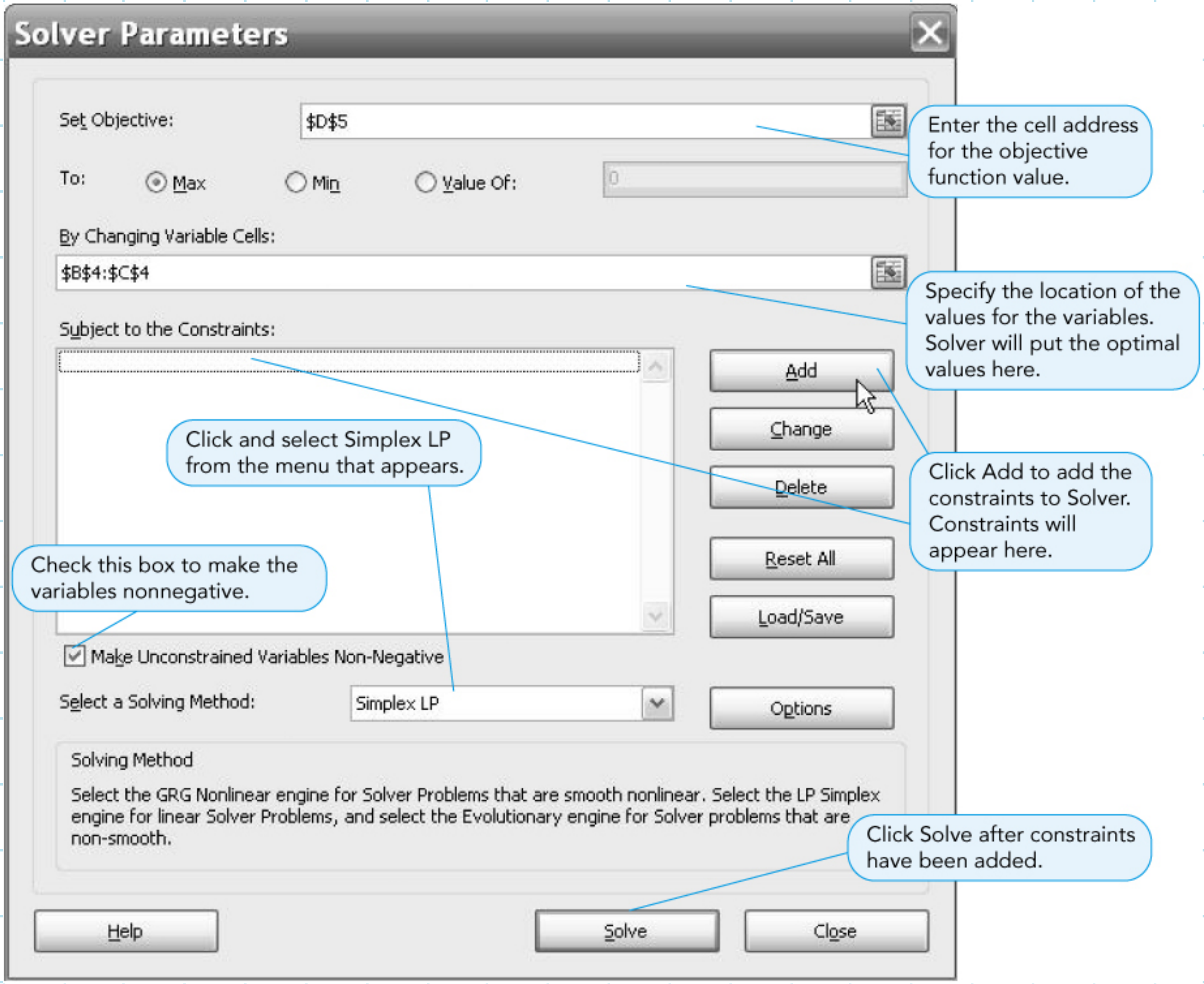


Figure 7.2E

Using Solver to Solve the Flair Furniture Problem

Solver Add Constraint Dialog Box

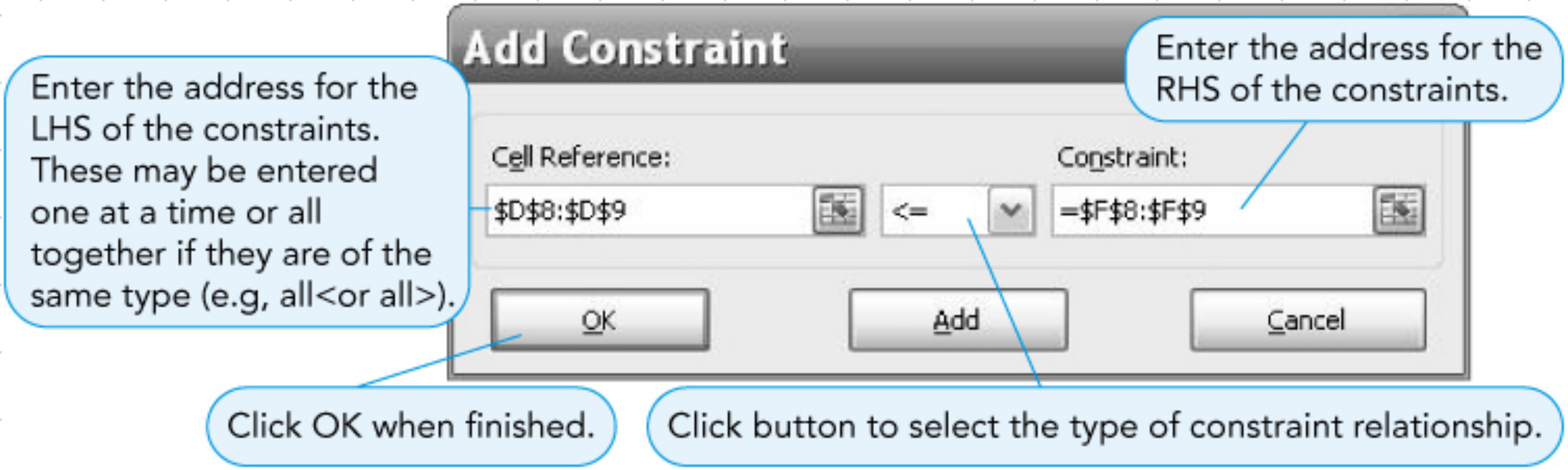


Figure 7.2F

Using Solver to Solve the Flair Furniture Problem

Solver Results Dialog Box

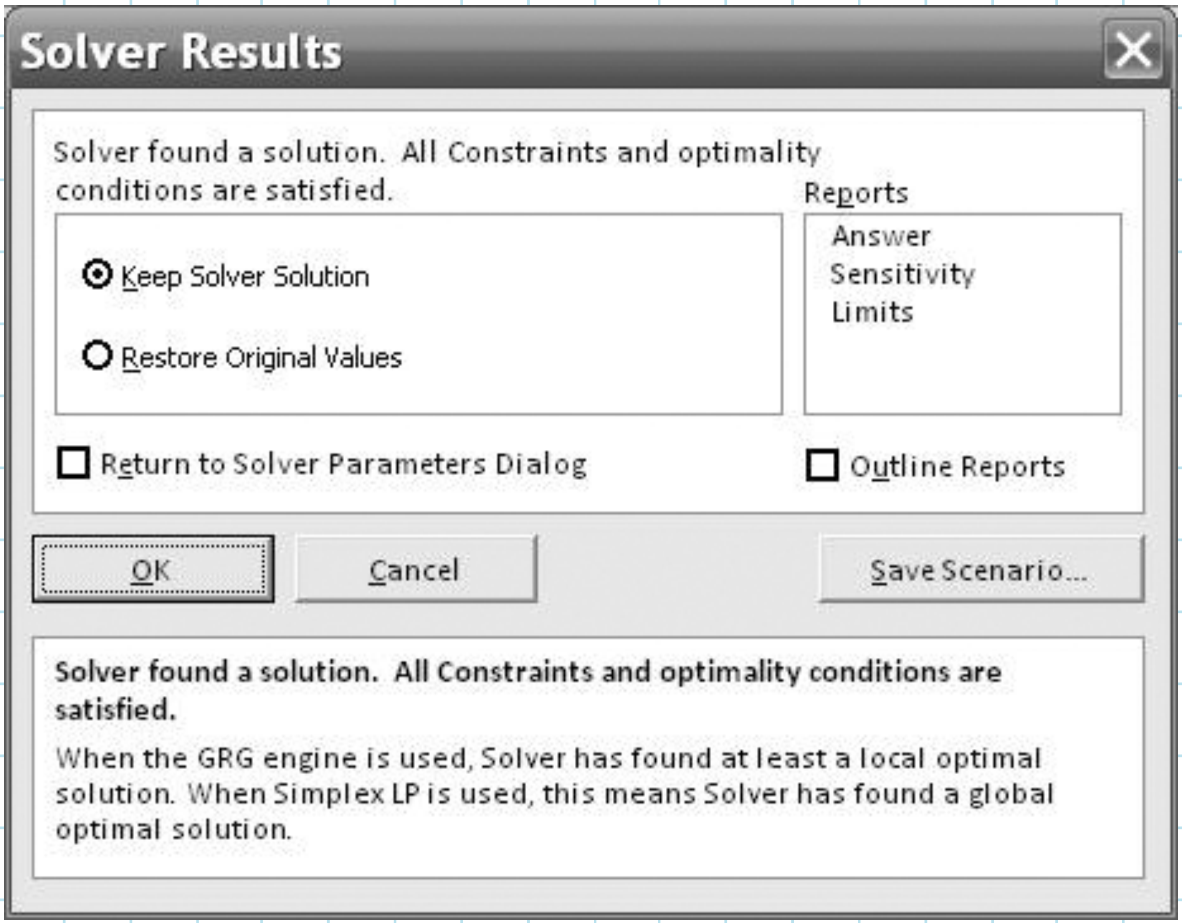


Figure 7.2G

Using Solver to Solve the Flair Furniture Problem

Solution Found by Solver

	A	B	C	D	E	F
1	Flair Furniture					
2						
3	Variables	T (Tables)	C (Chairs)			
4	Units Produced	30	40	Profit		
5	Objective function	70	50	4100		
6						
7	Constraints			LHS (Hours used)		RHS
8	Carpentry	4	3	240	<	240
9	Painting	2	1	100	<	100

The optimal solution is T=30, C=40, profit=4100.

The hours used are given here.

Figure 7.2H

Solving Minimization Problems

- Many LP problems involve minimizing an objective such as cost instead of maximizing a profit function.
- Minimization problems can be solved graphically by first setting up the feasible solution region and then using either the corner point method or an isocost line approach (which is analogous to the isoprofit approach in maximization problems) to find the values of the decision variables (e.g., X_1 and X_2) that yield the minimum cost.

Holiday Meal Turkey Ranch

The Holiday Meal Turkey Ranch is considering buying two different brands of turkey feed and blending them to provide a good, low-cost diet for its turkeys

Let

X_1 = number of pounds of brand 1 feed purchased

X_2 = number of pounds of brand 2 feed purchased

Minimize cost (in cents) = $2X_1 + 3X_2$

subject to:

$5X_1 + 10X_2 \geq 90$ ounces		(ingredient constraint A)
$4X_1 + 3X_2 \geq 48$ ounces		(ingredient constraint B)
$0.5X_1 \geq 1.5$ ounces		(ingredient constraint C)
$X_1 \geq 0$		(nonnegativity constraint)
$X_2 \geq 0$		(nonnegativity constraint)

Holiday Meal Turkey Ranch

Holiday Meal Turkey Ranch data

INGREDIENT	COMPOSITION OF EACH POUND OF FEED (OZ.) BRAND 1 FEED	BRAND 2 FEED	MINIMUM MONTHLY REQUIREMENT PER TURKEY (OZ.)
A	5	10	90
B	4	3	48
C	0.5	0	1.5
Cost per pound	2 cents	3 cents	

Table 7.5

Holiday Meal Turkey Ranch

- Use the corner point method.
- First construct the feasible solution region.
- The optimal solution will lie at one of the corners as it would in a maximization problem.

Feasible Region for the Holiday Meal Turkey Ranch Problem

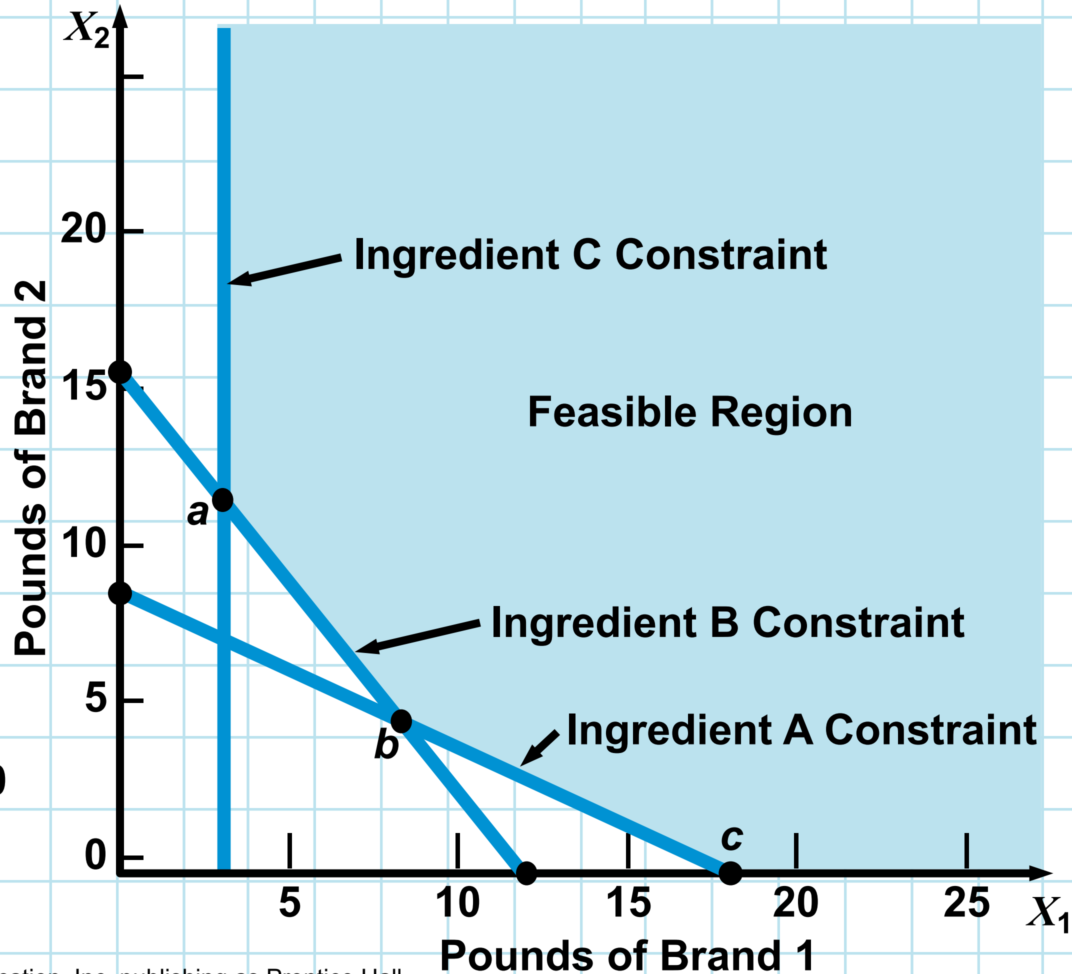


Figure 7.10

Holiday Meal Turkey Ranch

- Solve for the values of the three corner points.
- Point *a* is the intersection of ingredient constraints C and B.

$$4X_1 + 3X_2 = 48$$
$$X_1 = 3$$

- Substituting 3 in the first equation, we find $X_2 = 12$.
- Solving for point *b* with basic algebra we find $X_1 = 8.4$ and $X_2 = 4.8$.
- Solving for point *c* we find $X_1 = 18$ and $X_2 = 0$.

Holiday Meal Turkey Ranch

Substituting these value back into the objective function we find

$$\text{Cost} = 2X_1 + 3X_2$$

Cost at point *a* = 2(3) + 3(12) = 42

$\boxed{\text{Cost at point } b = 2(8.4) + 3(4.8) = 31.2}$

Cost at point *c* = 2(18) + 3(0) = 36

The lowest cost solution is to purchase 8.4 pounds of brand 1 feed and 4.8 pounds of brand 2 feed for a total cost of 31.2 cents per turkey.

Holiday Meal Turkey Ranch

Graphical Solution to the Holiday Meal Turkey Ranch Problem Using the Isocost Approach

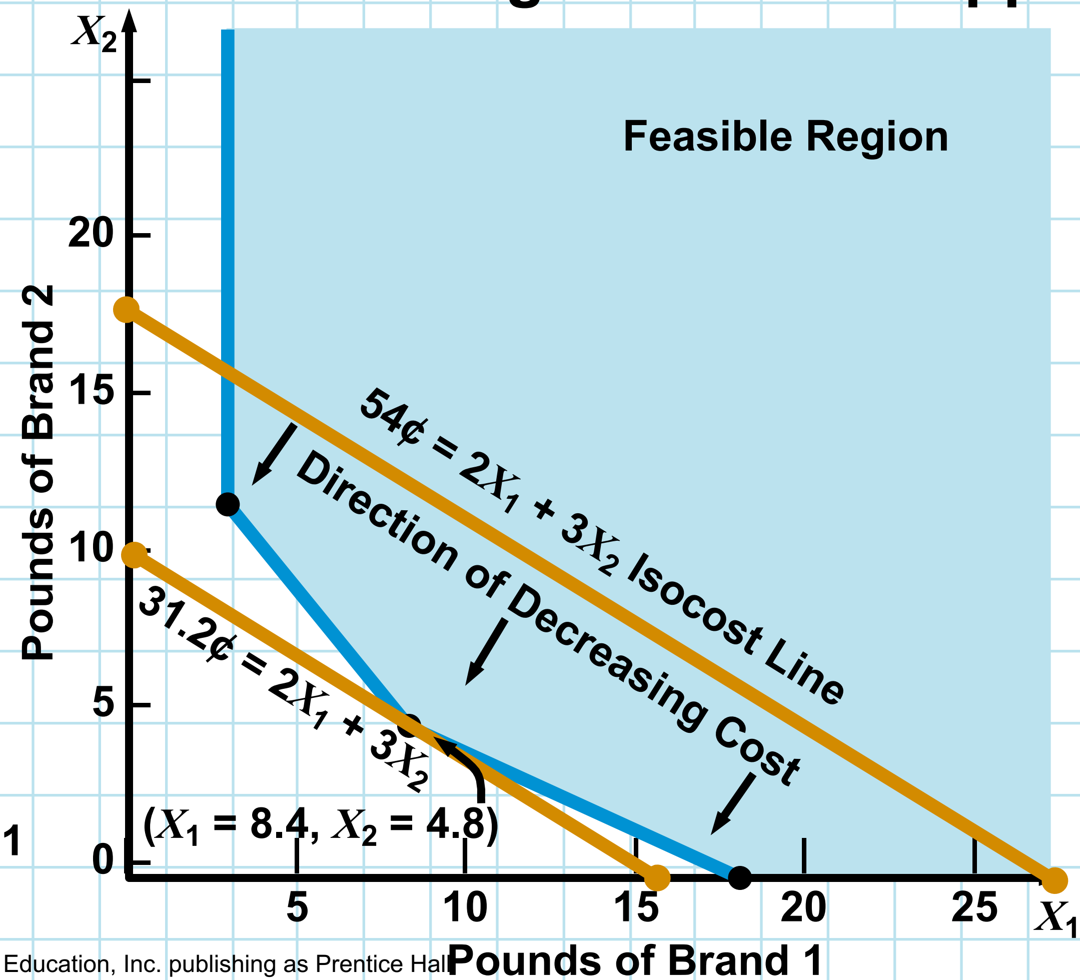


Figure 7.11

Holiday Meal Turkey Ranch

Solving the Holiday Meal Turkey Ranch Problem Using QM for Windows

QM for Windows - C:\Prentice\Data\RenderStair7\HolidayMeal.LIN

Objective
- Maximize
- Minimize (selected)

Linear Programming Results

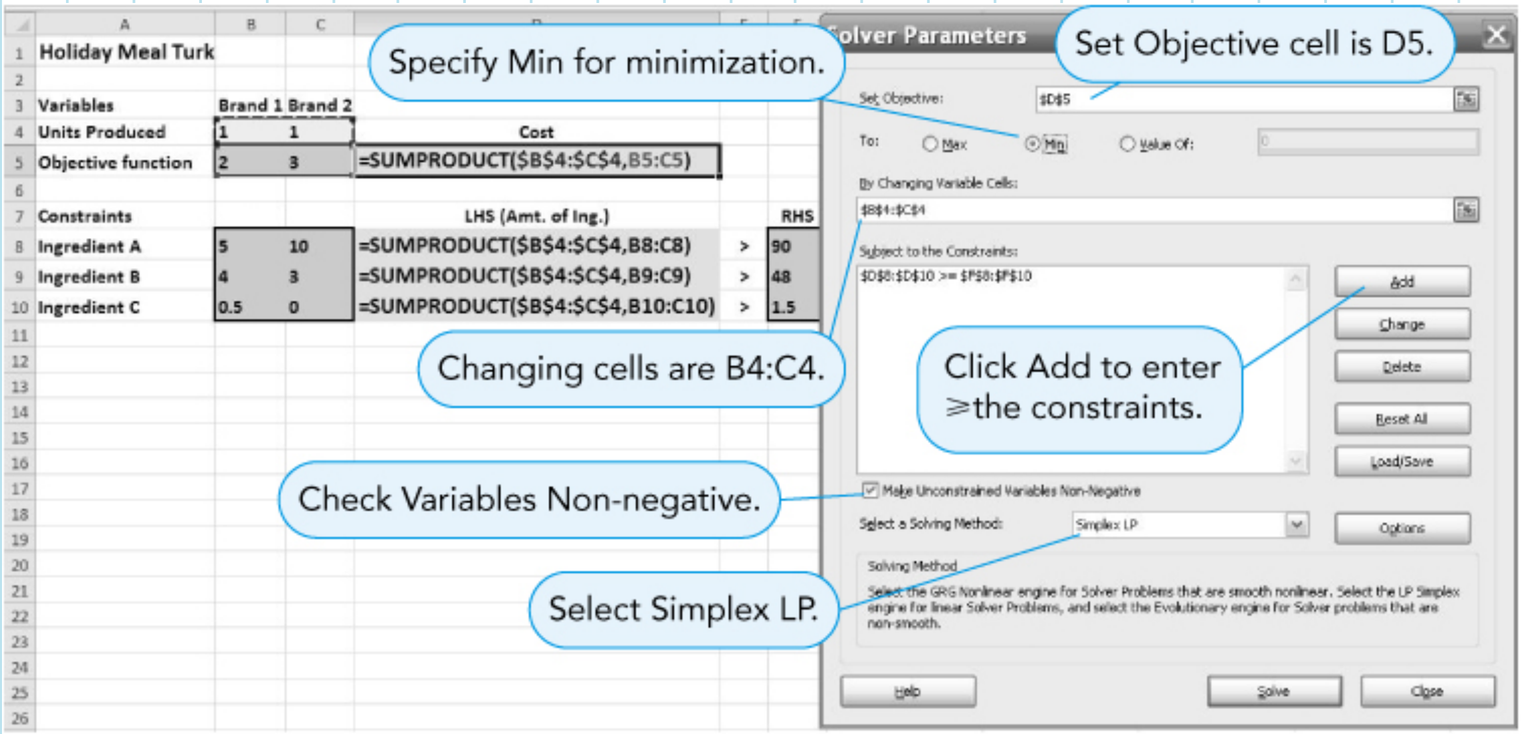
Holiday Meal Turkey Ranch Solution

	Brand 1	Brand 2		RHS	Dual
Minimize	2.	3.			
Ingredient A	5.	10.	>=	90.	-0.24
Ingredient B	4.	3.	>=	48.	-0.2
Ingredient C	0.5	0.	>=	1.5	0.
Solution->	8.4	4.8		31.2	

Program 7.3

Holiday Meal Turkey Ranch

Excel 2010 Spreadsheet for the Holiday Meal Turkey Ranch problem



Program 7.4A

Holiday Meal Turkey Ranch

Excel 2010 Solution to the Holiday Meal Turkey Ranch Problem

	A	B	C	D	E	F
1	Holiday Meal Turkey Ranch					
2						
3	Variables	Brand 1	Brand 2			
4	Units Produced	8.4	4.8	Cost		
5	Objective function	2	3	31.2		
6						
7	Constraints			LHS (Amt. of Ing.)		RHS
8	Ingredient A	5	10	90	>	90
9	Ingredient B	4	3	48	>	48
10	Ingredient C	0.5	0	4.2	>	1.5

Notice that there is a surplus for ingredient C as LHS>RHS.

Program 7.4B

Four Special Cases in LP

- Four special cases and difficulties arise at times when using the graphical approach to solving LP problems.
  - No feasible solution
  - Unboundedness
  - Redundancy
  - Alternate Optimal Solutions

Four Special Cases in LP

No feasible solution

- This exists when there is no solution to the problem that satisfies all the constraint equations.
- No feasible solution region exists.
- This is a common occurrence in the real world.
- Generally one or more constraints are relaxed until a solution is found.

Four Special Cases in LP

A problem with no feasible solution

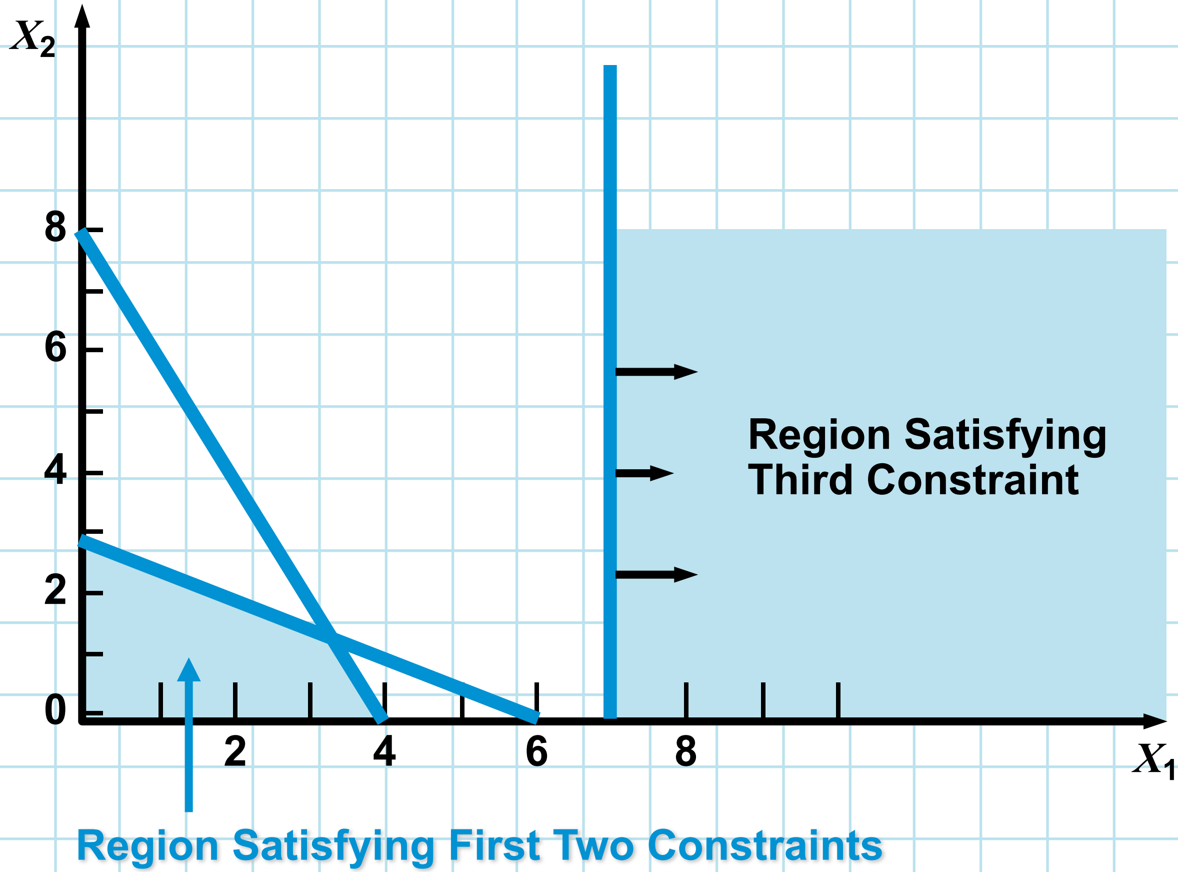


Figure 7.12

Four Special Cases in LP

Unboundedness

- Sometimes a linear program will not have a finite solution.
- In a maximization problem, one or more solution variables, and the profit, can be made infinitely large without violating any constraints.
- In a graphical solution, the feasible region will be open ended.
- This usually means the problem has been formulated improperly.

Four Special Cases in LP

A Feasible Region That is Unbounded to the Right

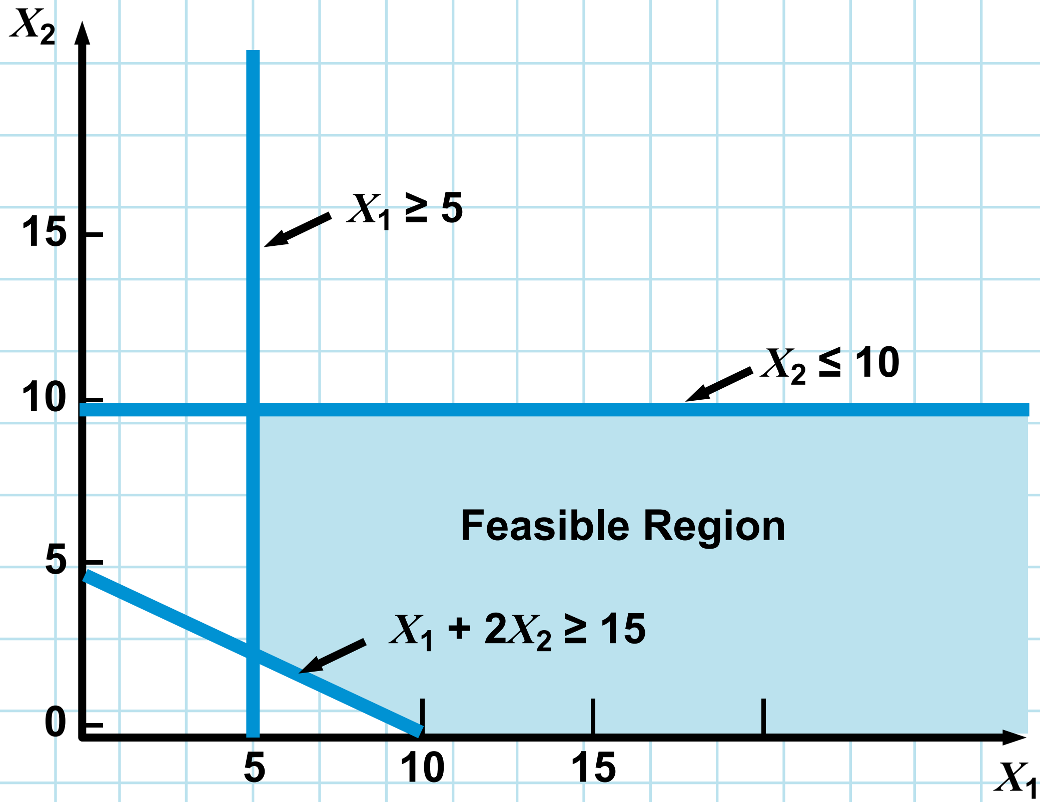


Figure 7.13

Four Special Cases in LP

Redundancy

- A redundant constraint is one that does not affect the feasible solution region.
- One or more constraints may be binding.
- This is a very common occurrence in the real world.
- It causes no particular problems, but eliminating redundant constraints simplifies the model.

Four Special Cases in LP

Problem with a Redundant Constraint

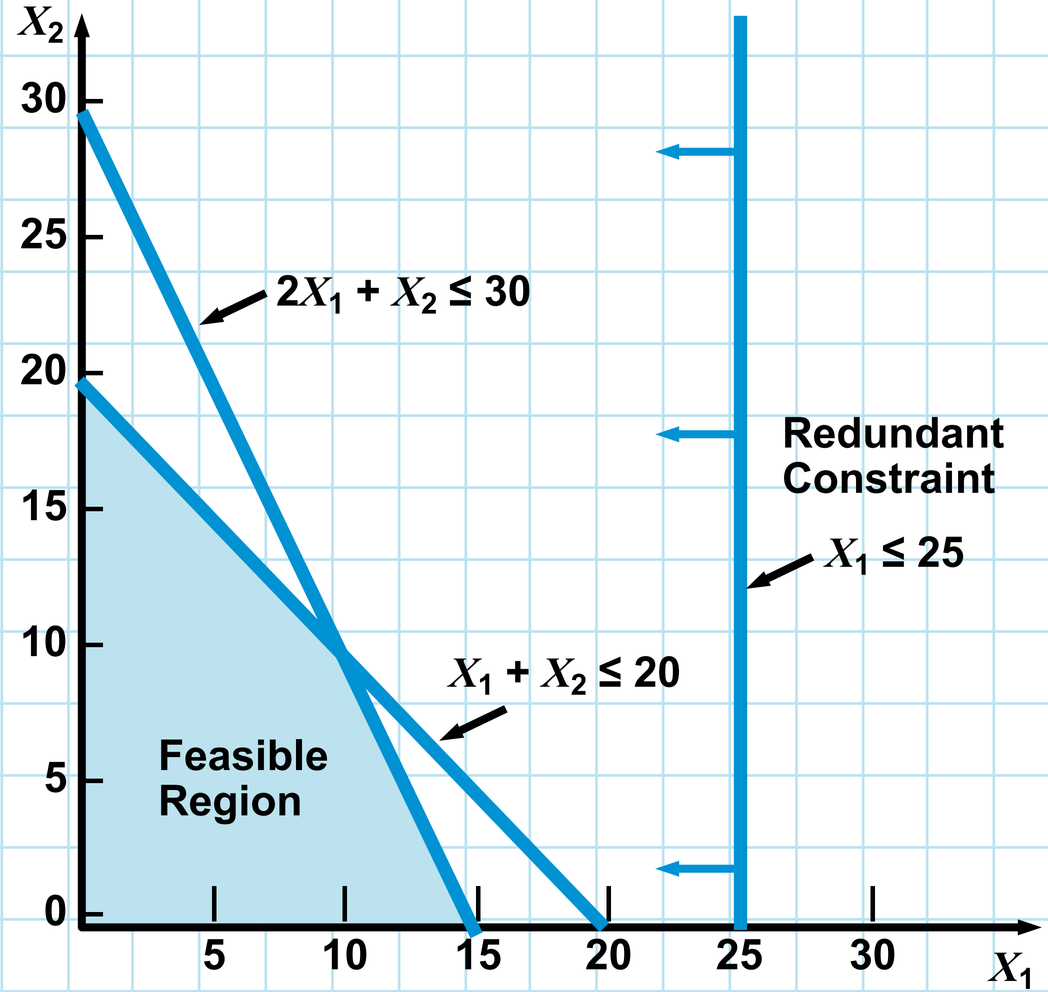


Figure 7.14

Four Special Cases in LP

Alternate Optimal Solutions

- Occasionally two or more optimal solutions may exist.
- Graphically this occurs when the objective function's isoprofit or isocost line runs perfectly parallel to one of the constraints.
- This actually allows management great flexibility in deciding which combination to select as the profit is the same at each alternate solution.

Four Special Cases in LP

Example of Alternate Optimal Solutions

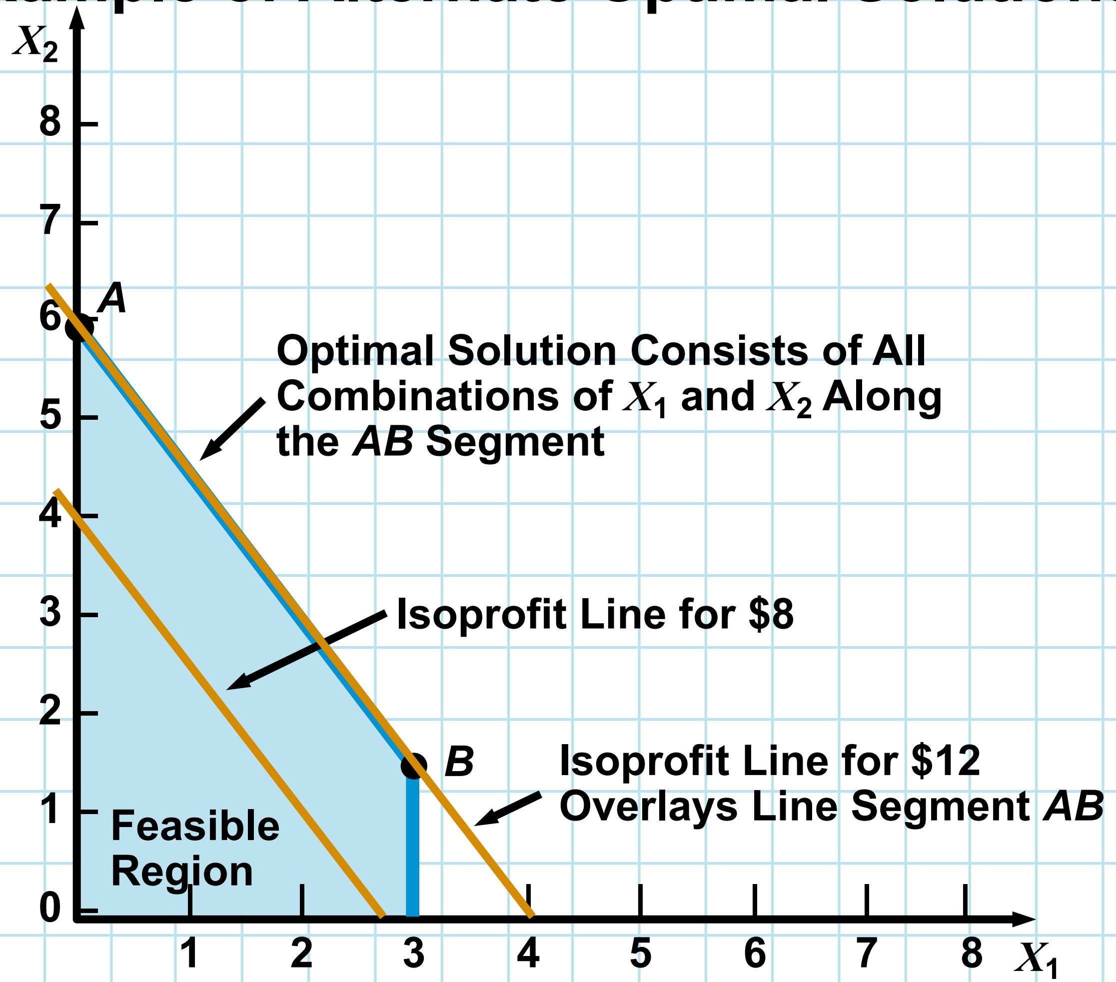


Figure 7.15

Sensitivity Analysis

- **Optimal solutions to LP problems thus far have been found under what are called *deterministic assumptions.***
- **This means that we assume complete certainty in the data and relationships of a problem.**
- **But in the real world, conditions are dynamic and changing.**
- **We can analyze how *sensitive* a deterministic solution is to changes in the assumptions of the model.**
- **This is called *sensitivity analysis*, *postoptimality analysis*, *parametric programming*, or *optimality analysis.***

Sensitivity Analysis

- Sensitivity analysis often involves a series of what-if? questions concerning constraints, variable coefficients, and the objective function.
- One way to do this is the trial-and-error method where values are changed and the entire model is resolved.
- The preferred way is to use an analytic postoptimality analysis.
- After a problem has been solved, we determine a range of changes in problem parameters that will not affect the optimal solution or change the variables in the solution.

High Note Sound Company

- The High Note Sound Company manufactures quality CD players and stereo receivers.
- Products require a certain amount of skilled artisanship which is in limited supply.
- The firm has formulated the following product mix LP model.

Maximize profit = $\$50X_1 + \$120X_2$

Subject to

$2X_1 + 4X_2 \leq 80$ (hours of electrician's time available)

$3X_1 + 1X_2 \leq 60$ (hours of audio technician's time available)

$X_1, X_2 \geq 0$

High Note Sound Company

The High Note Sound Company Graphical Solution

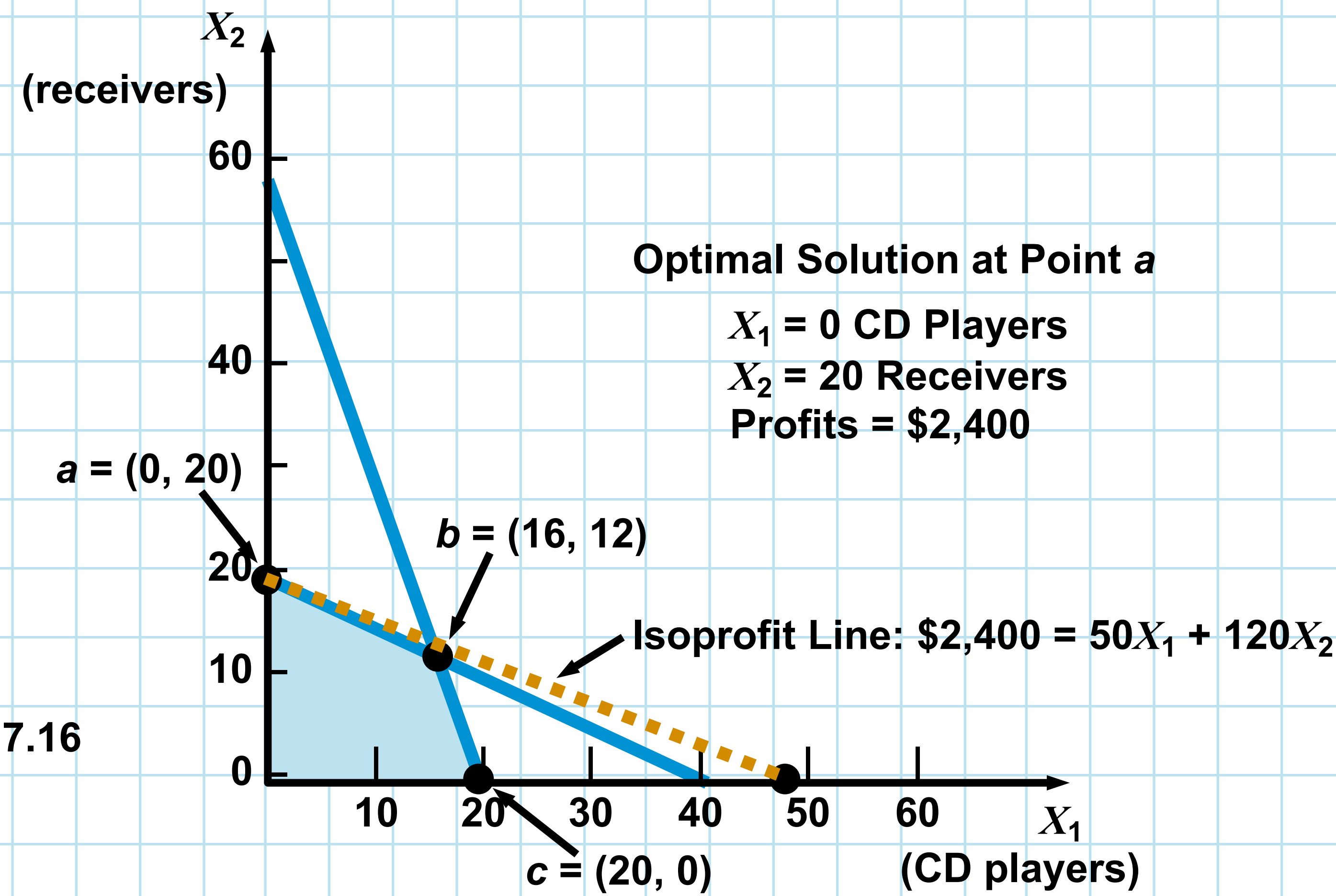


Figure 7.16

Changes in the Objective Function Coefficient

- In real-life problems, contribution rates in the objective functions fluctuate periodically.
- Graphically, this means that although the feasible solution region remains exactly the same, the slope of the isoprofit or isocost line will change.
- We can often make modest increases or decreases in the objective function coefficient of any variable without changing the current optimal corner point.
- We need to know how much an objective function coefficient can change before the optimal solution would be at a different corner point.

Changes in the Objective Function Coefficient

Changes in the Receiver Contribution Coefficients

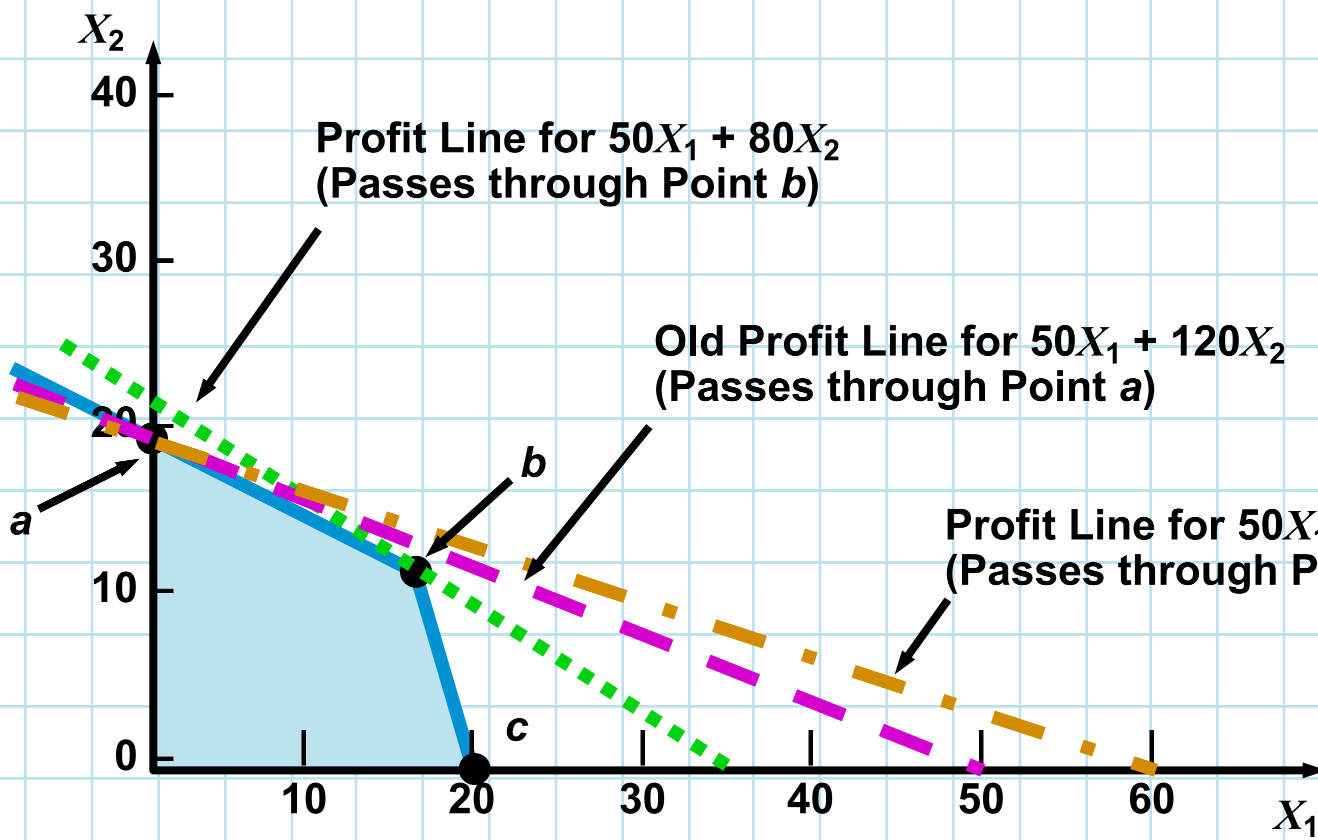


Figure 7.17

QM for Windows and Changes in Objective Function Coefficients

Input and Sensitivity Analysis for High Note Sound Data Using QM For Windows

Objective
- (•) Maximize
- ( ) Minimize

High Note Sound				
	CD players	Receivers		RHS
Maximize	50	120		
Electrician hrs	2	4	<=	80
Audio tech hrs	3	1	<=	60

Program 7.5A

Ranging

High Note Sound Solution					
Variable	Value	Reduced Cost	Original Val	Lower Bound	Upper Bound
CD players	0.	10.	50.	-Infinity	60.
Receivers	20.	0.	120.	100.	Infinity
Constraint	Dual Value	Slack/Surplus	Original Val	Lower Bound	Upper Bound
Electrician hrs	30.	0.	80.	0.	240.
Audio tech hrs	0.	40.	60.	20.	Infinity

Program 7.5B

Excel Solver and Changes in Objective Function Coefficients

Excel 2010 Spreadsheet for High Note Sound Company

The Changing Variable cells in the Solver dialog box are B4:C4.

The Set Objective cell in the Solver dialog box is D5.

	A	B	C	D	E	F
1	High Note Sound C					
2						
3	Variables	CD Player	Receivers			
4	Units Produced	1	1	Profit		
5	Objective function	50	120	=SUMPRODUCT($B$4:$C$4,B5:C5)		
6						
7	Constraints			LHS (Hrs. Used)		RHS
8	Electrician Hours	2	4	=SUMPRODUCT($B$4:$C$4,B8:C8)	<	80
9	Audio Tech Hours	3	1	=SUMPRODUCT($B$4:$C$4,B9:C9)	<	60

The constraints added into Solver will be D8:D9 <=F8:F9.

Program 7.6A

Excel Solver and Changes in Objective Function Coefficients

Excel 2010 Solution and Solver Results Window for High Note Sound Company

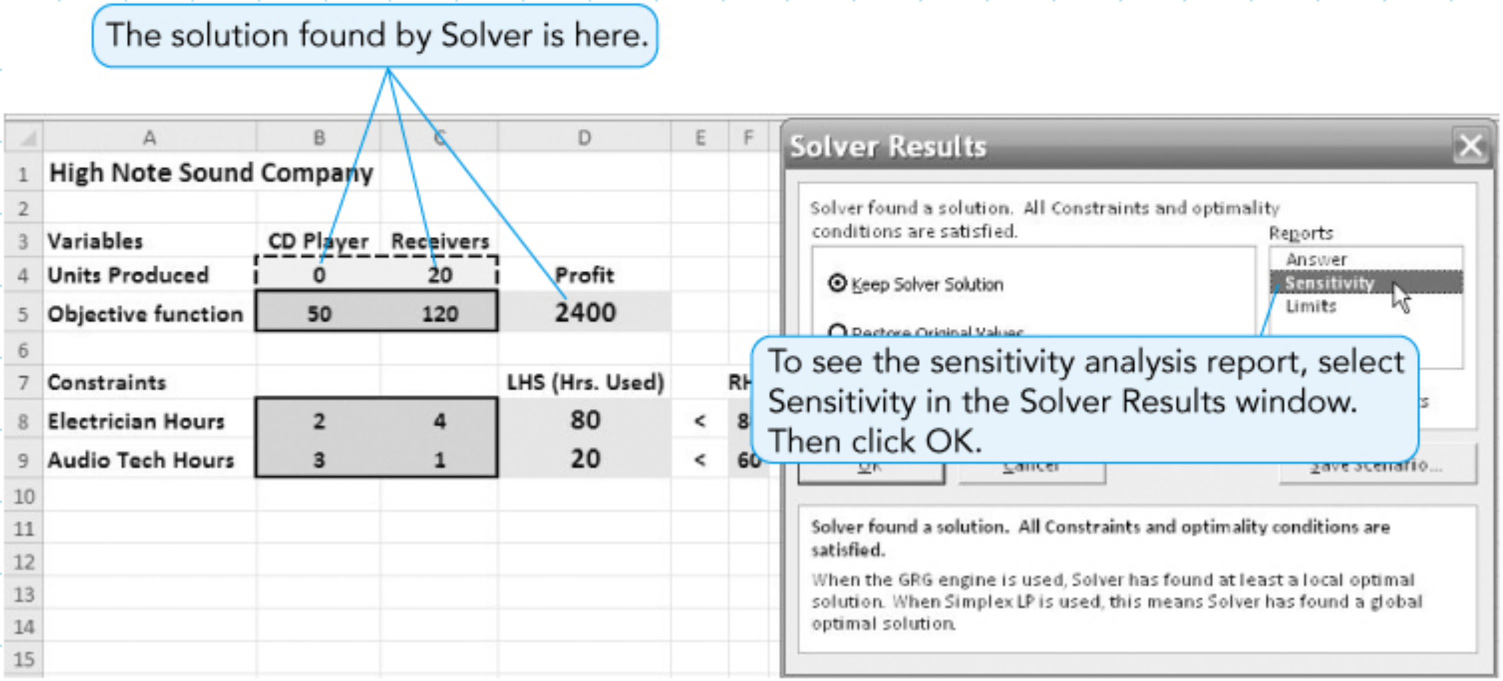


Figure 7.6B

Excel Solver and Changes in Objective Function Coefficients

Excel 2010 Sensitivity Report for High Note Sound Company

The names presented in the Sensitivity Report combine the text in column A and the text above the data, unless the cells have been named using the Name Manager from the Formulas tab.

	A	B	C	D	E	F	G	H
1	Microsoft Excel 14.0 Sensitivity Repo							
2								
3	Variable Cells							
4				Final	Reduced	Objective	Allowable	Allowable
5		Cell	Name	Value	Cost	Coefficient	Increase	Decrease
6		$B$4	Units Produced CD Player	0	-10	50	10	1E+30
7		$C$4	Units Produced Receivers	20	0	120	1E+30	20
8								
9	Constraints							
10				Final	Shadow	Constraint	Allowable	Allowable
11		Cell	Name	Value	Price	R.H. Side	Increase	Decrease
12		$D$8	Electrician Hours LHS (Hrs. Used)	80	30	80	160	80
13		$D$9	Audio Tech Hours LHS (Hrs. Used	20	0	60	1E+30	40

The profit on CD may change by these amounts, and the current corner point will remain optimal.

The resources used are here. The RHS can change by these amounts, and the shadow price will still be relevant.

Program 7.6C

Changes in the Technological Coefficients

- Changes in the ***technological coefficients*** often reflect changes in the state of technology.
- If the amount of resources needed to produce a product changes, coefficients in the constraint equations will change.
- This does not change the objective function, but it can produce a significant change in the shape of the feasible region.
- This may cause a change in the optimal solution.

Changes in the Technological Coefficients

Change in the Technological Coefficients for the High Note Sound Company

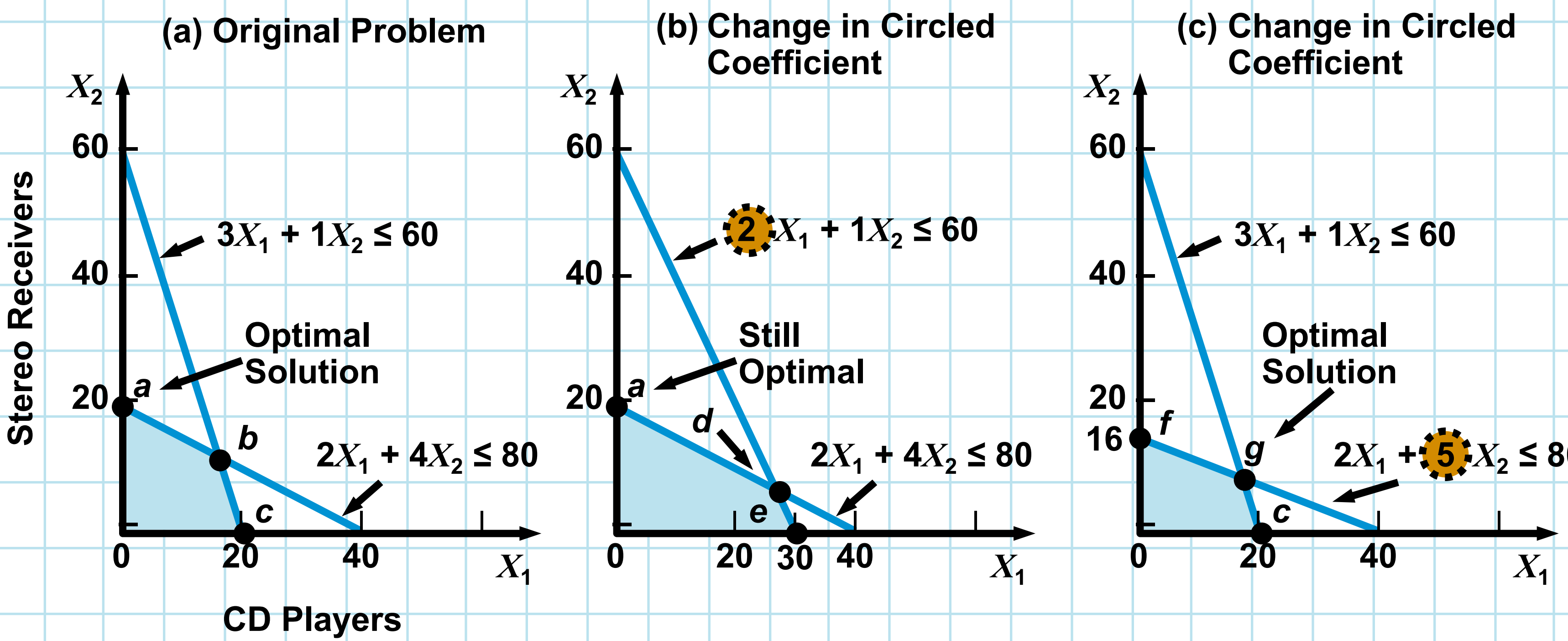


Figure 7.18

Changes in Resources or Right-Hand-Side Values

- The right-hand-side values of the constraints often represent resources available to the firm.
- If additional resources were available, a higher total profit could be realized.
- Sensitivity analysis about resources will help answer questions about how much should be paid for additional resources and how much more of a resource would be useful.

Changes in Resources or Right-Hand-Side Values

- If the right-hand side of a constraint is changed, the feasible region will change (unless the constraint is redundant).
- Often the optimal solution will change.
- The amount of change in the objective function value that results from a unit change in one of the resources available is called the ***dual price*** or ***dual value*** .
- The dual price for a constraint is the improvement in the objective function value that results from a one-unit increase in the right-hand side of the constraint.

Changes in Resources or Right-Hand-Side Values

- However, the amount of possible increase in the right-hand side of a resource is limited.
- If the number of hours increased beyond the upper bound, then the objective function would no longer increase by the dual price.
- There would simply be excess (***slack***) hours of a resource or the objective function may change by an amount different from the dual price.
- The dual price is relevant only within limits.

Changes in the Electricians' Time Resource for the High Note Sound Company

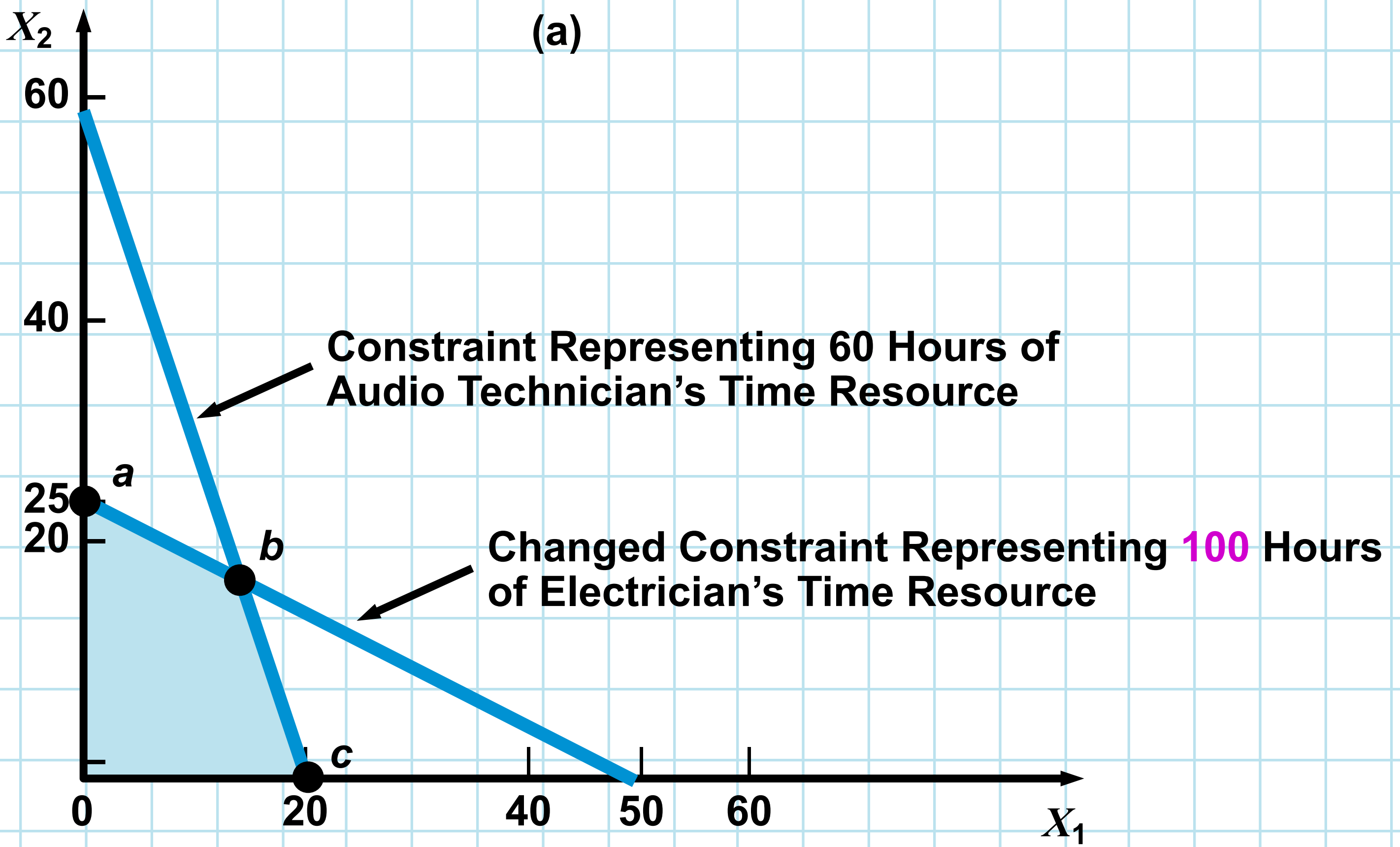


Figure 7.19

Changes in the Electricians' Time Resource for the High Note Sound Company

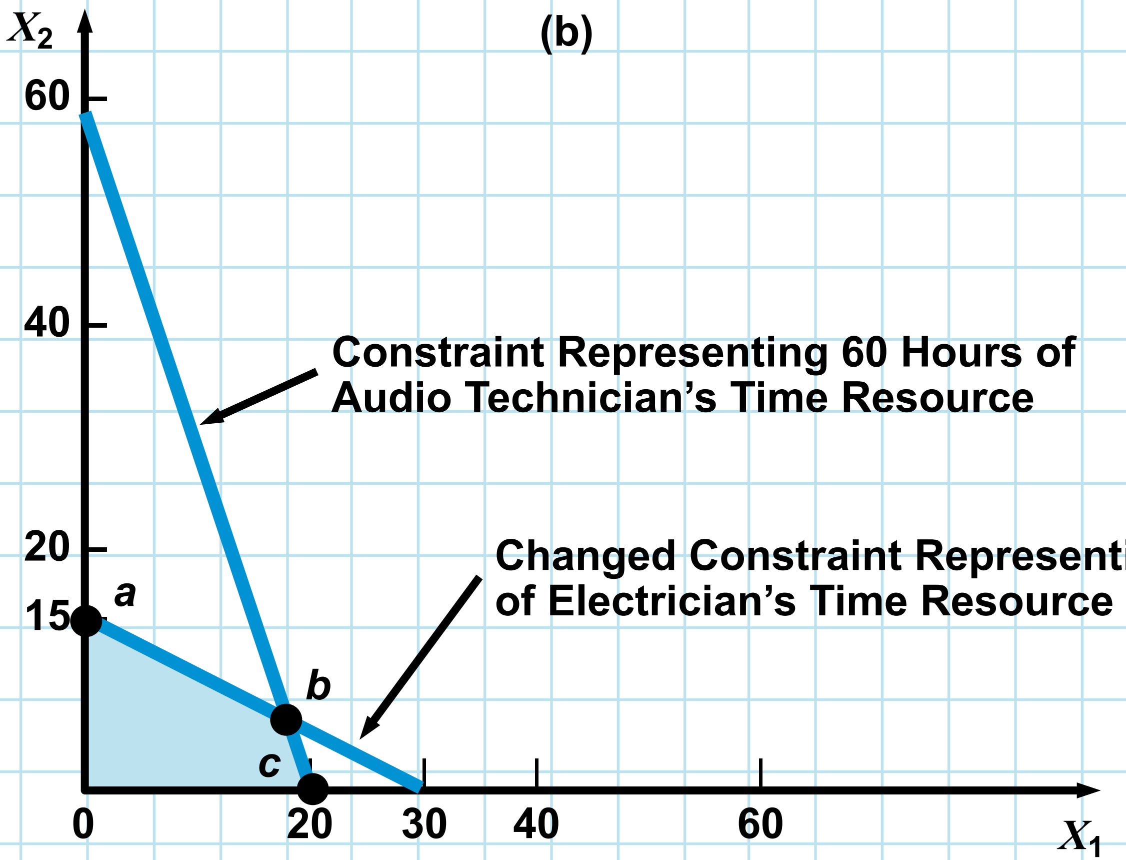


Figure 7.19

Changes in the Electricians' Time Resource for the High Note Sound Company

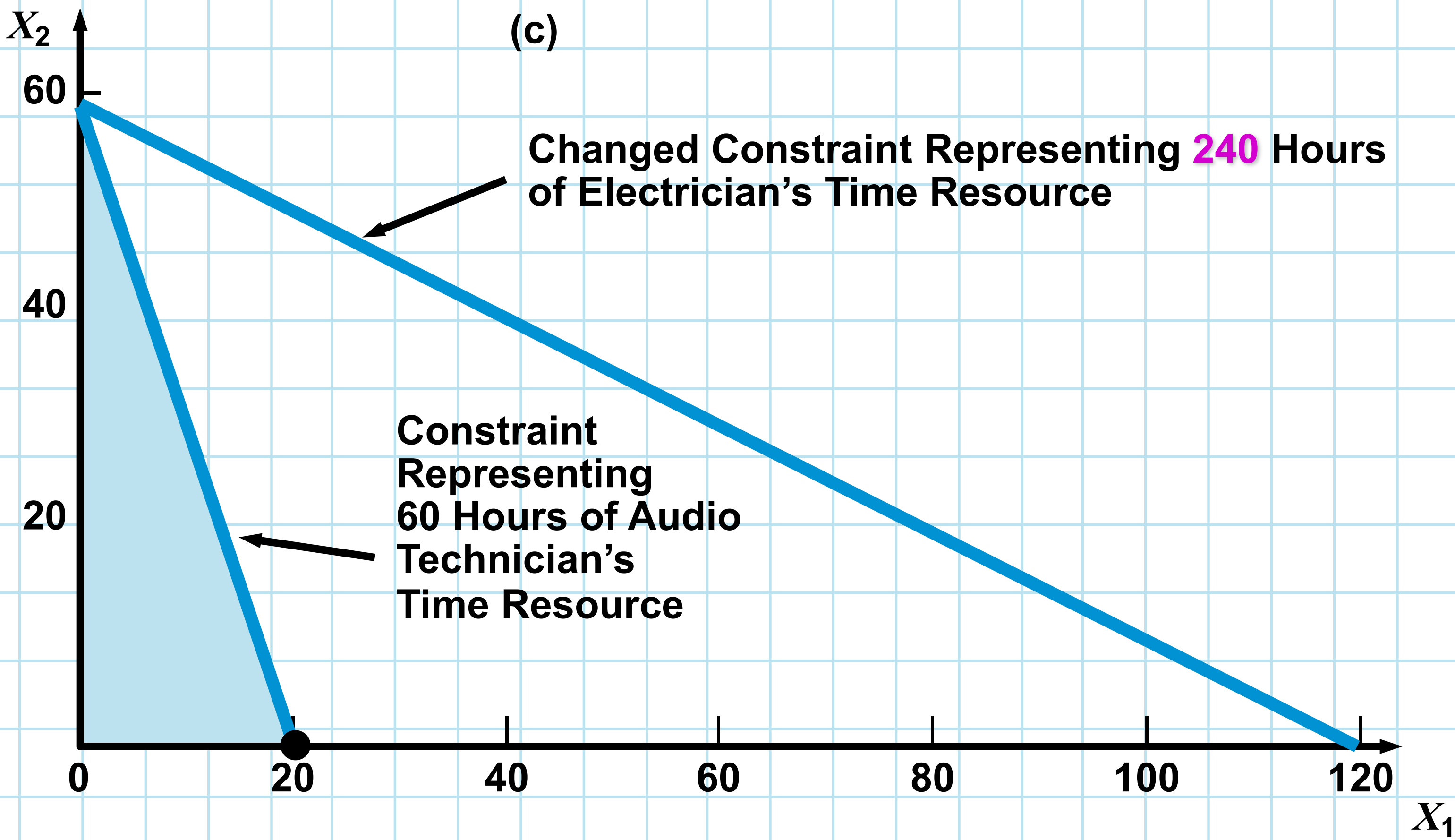


Figure 7.19

QM for Windows and Changes in Right-Hand-Side Values

Sensitivity Analysis for High Note Sound Company Using QM for Windows

Ranging

High Note Sound Solution					
Variable	Value	Reduced Cost	Original Val	Lower Bound	Upper Bound
CD players	0.	10.	50.	-Infinity	60.
Receivers	20.	0.	120.	100.	Infinity
Constraint	Dual Value	Slack/Surplus	Original Val	Lower Bound	Upper Bound
Electrician hrs	30.	0.	80.	0.	240.
Audio tech hrs	0.	40.	60.	20.	Infinity

Program 7.5B

Excel Solver and Changes in Right-Hand-Side Values

Excel 2010 Sensitivity Analysis for High Note Sound Company

Microsoft Excel 8.0 Sensitivity Report
Worksheet: [captures.xls]7.8
Report Created: 8/13/98 10:54:59 AM

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$4	Value CD Players	0	-10	50	10	1E+30
$C$4	Value Receivers	20	0	120	1E+30	20

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$9	Electrician hours Used	80	30	80	160	80
$D$10	Audio technician hours Used	20	0	60	1E+30	40

The solution values for the variables indicate that we should make 0 CD players and 20 receivers.

If we produce a CD player, our profit will fall by $10.

We will use 80 hours and 20 hours of electrician and audio technician time, respectively.

If we use 1 more electrician hour, our profit will increase by $30. This is true for up to 160 more hours. The profit will fall by $30 for each electrician hour less than 80 hours.

Program 7.6C

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