Chapter 5

Forecasting

To accompany
Quantitative Analysis for Management, Eleventh Edition,
by Render, Stair, and Hanna
Power Point slides created by Brian Peterson

Learning Objectives

After completing this chapter, students will be able to:

1. Understand and know when to use various families of forecasting models.
2. Compare moving averages, exponential smoothing, and other time-series models.
3. Seasonally adjust data.
4. Understand Delphi and other qualitative decision-making approaches.
5. Compute a variety of error measures.

Chapter Outline

5.1 Introduction
5.2 Types of Forecasts
5.3 Scatter Diagrams and Time Series
5.4 Measures of Forecast Accuracy
5.5 Time-Series Forecasting Models
5.6 Monitoring and Controlling Forecasts

Introduction

- Managers are always trying to reduce uncertainty and make better estimates of what will happen in the future.
  - This is the main purpose of forecasting.
  - Some firms use subjective methods: seat-of-the pants methods, intuition, experience.
  - There are also several quantitative techniques, including:
    - Moving averages
    - Exponential smoothing
    - Trend projections
    - Least squares regression analysis

Introduction

- Eight steps to forecasting:
  1. Determine the use of the forecast—what objective are we trying to obtain?
  2. Select the items or quantities that are to be forecasted.
  3. Determine the time horizon of the forecast.
  4. Select the forecasting model or models.
  5. Gather the data needed to make the forecast.
  6. Validate the forecasting model.
  7. Make the forecast.
  8. Implement the results.

Introduction

- These steps are a systematic way of initiating, designing, and implementing a forecasting system.
- When used regularly over time, data is collected routinely and calculations performed automatically.
- There is seldom one superior forecasting system.
  - Different organizations may use different techniques.
  - Whatever tool works best for a firm is the one that should be used.

Forecasting Models

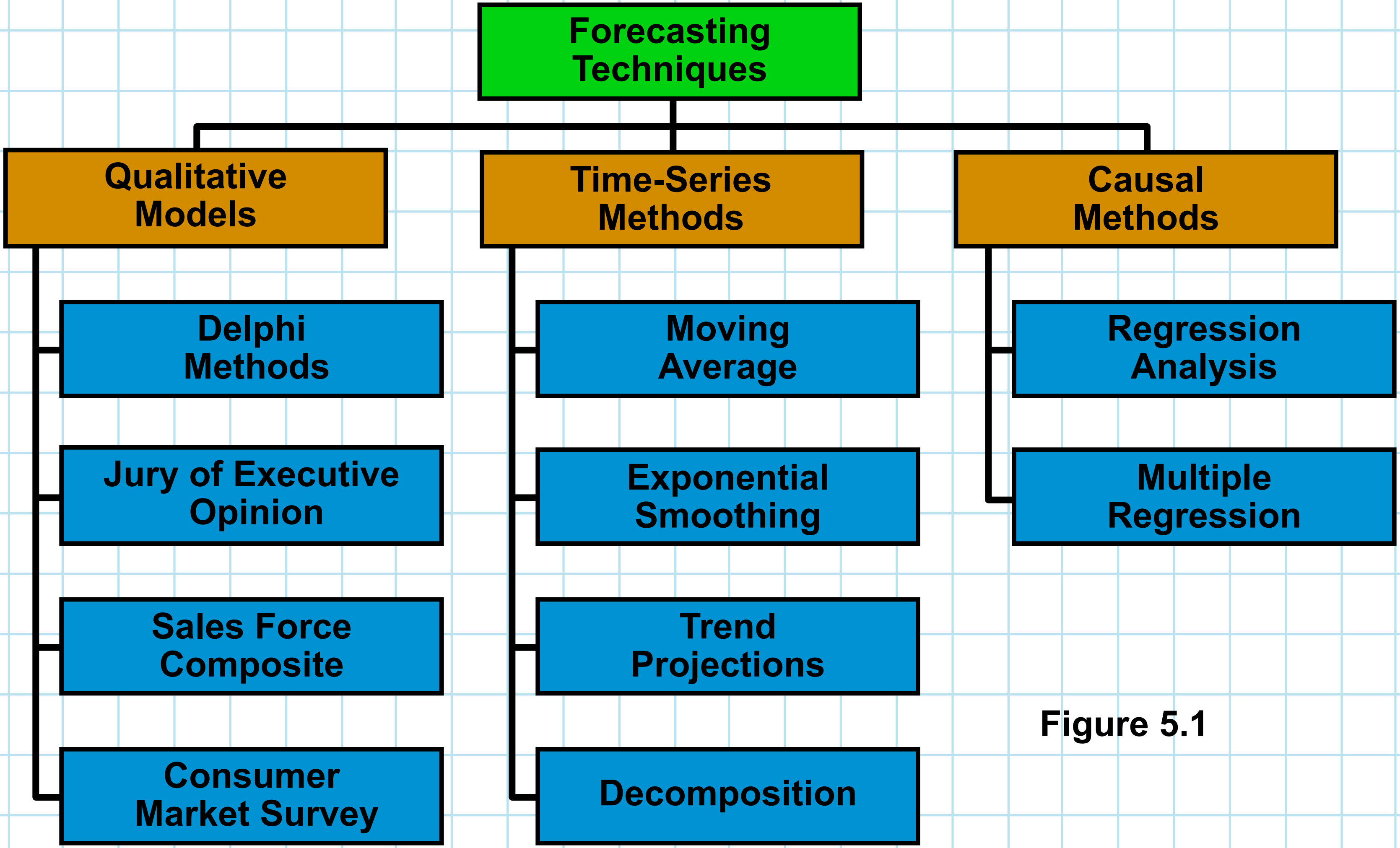


Figure 5.1

Qualitative Models

- ***Qualitative models*** incorporate judgmental or subjective factors.
- These are useful when subjective factors are thought to be important or when accurate quantitative data is difficult to obtain.
- Common qualitative techniques are:
  - Delphi method.
  - Jury of executive opinion.
  - Sales force composite.
  - Consumer market surveys.

Qualitative Models

- ***Delphi Method*** – This is an iterative group process where (possibly geographically dispersed) ***respondents*** provide input to ***decision makers.***
- ***Jury of Executive Opinion*** – This method collects opinions of a small group of high-level managers, possibly using statistical models for analysis.
- ***Sales Force Composite*** – This allows individual salespersons estimate the sales in their region and the data is compiled at a district or national level.
- ***Consumer Market Survey*** – Input is solicited from customers or potential customers regarding their purchasing plans.

Time-Series Models

- *Time-series models* attempt to predict the future based on the past.
- Common time-series models are:
  - Moving average.
  - Exponential smoothing.
  - Trend projections.
  - Decomposition.
- Regression analysis is used in trend projections and one type of decomposition model.

Causal Models

- ***Causal models*** use variables or factors that might influence the quantity being forecasted.
- The objective is to build a model with the best statistical relationship between the variable being forecast and the independent variables.
- Regression analysis is the most common technique used in causal modeling.

Scatter Diagrams

Wacker Distributors wants to forecast sales for three different products (annual sales in the table, in units):

YEAR	TELEVISION SETS	RADIOS	COMPACT DISC PLAYERS
1	250	300	110
2	250	310	100
3	250	320	120
4	250	330	140
5	250	340	170
6	250	350	150
7	250	360	160
8	250	370	190
9	250	380	200
10	250	390	190

Table 5.1

Scatter Diagram for TVs

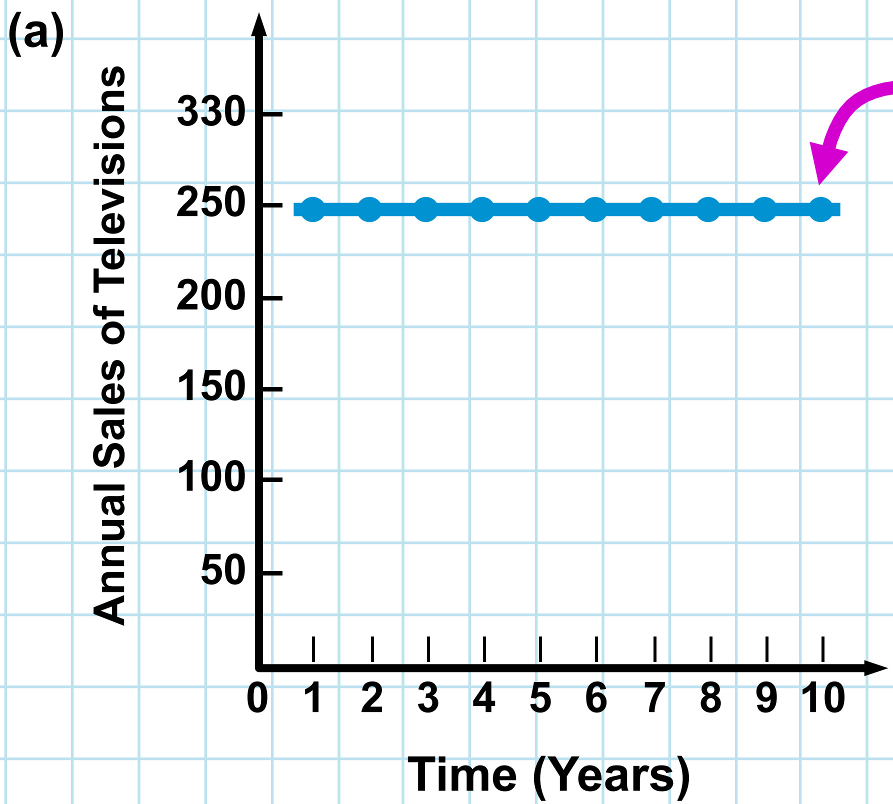


Figure 5.2a

- Sales appear to be constant over time

  Sales = 250

- A good estimate of sales in year 11 is 250 televisions

Scatter Diagram for Radios

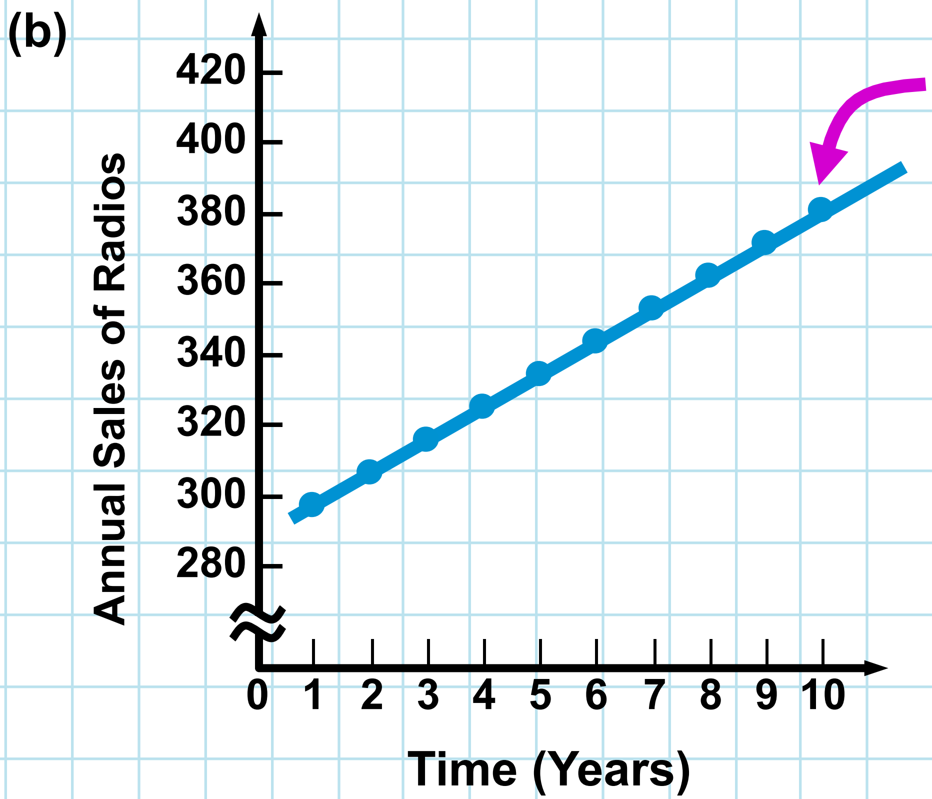


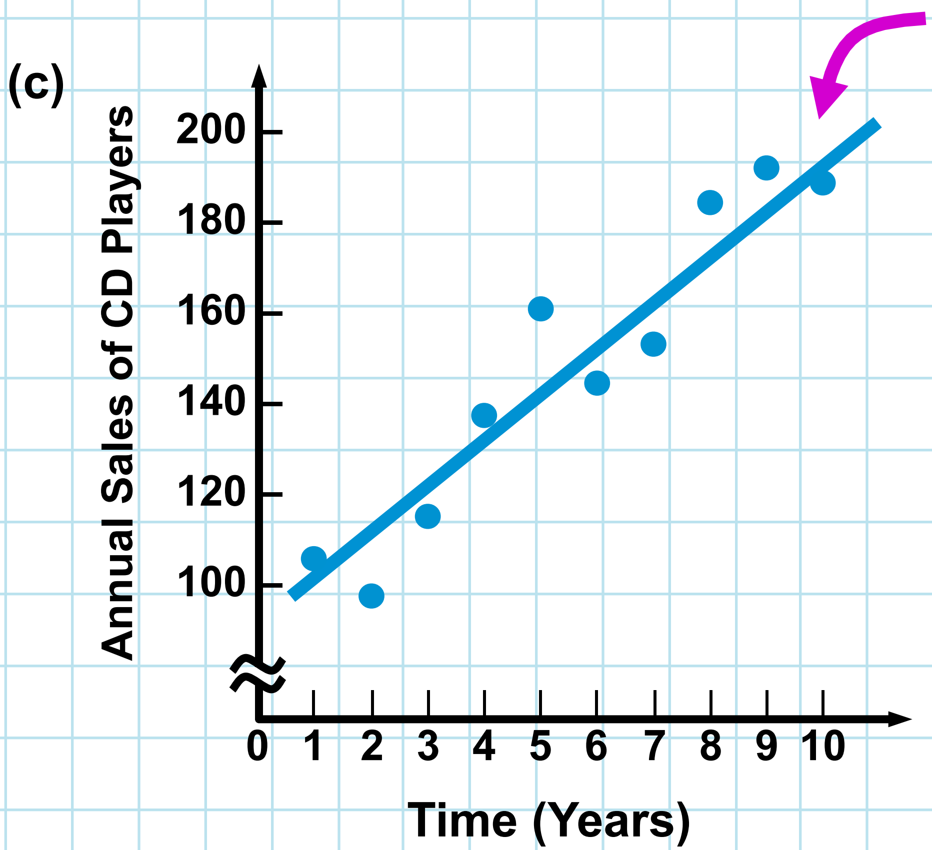
Figure 5.2b

- Sales appear to be increasing at a constant rate of 10 radios per year

Sales = 290 + 10(Year)

- A reasonable estimate of sales in year 11 is 400 radios.

Scatter Diagram for CD Players



- This trend line may not be perfectly accurate because of variation from year to year
- Sales appear to be increasing
- A forecast would probably be a larger figure each year

Figure 5.2c

Measures of Forecast Accuracy

- **We compare forecasted values with actual values to see how well one model works or to compare models.**

Forecast error = Actual value – Forecast value

- **One measure of accuracy is the *mean absolute deviation* (*MAD*):**

$$\text{MAD} = \frac{\sum |\text{forecast error}|}{n}$$

Measures of Forecast Accuracy

Using a *naïve* forecasting model we can compute the MAD:

YEAR	ACTUAL SALES OF CD PLAYERS	FORECAST SALES	ABSOLUTE VALUE OF ERRORS (DEVIATION), (ACTUAL – FORECAST)
1	110	—	—
2	100	110	|100 – 110| = 10
3	120	100	|120 – 110| = 20
4	140	120	|140 – 120| = 20
5	170	140	|170 – 140| = 30
6	150	170	|150 – 170| = 20
7	160	150	|160 – 150| = 10
8	190	160	|190 – 160| = 30
9	200	190	|200 – 190| = 10
10	190	200	|190 – 200| = 10
11	—	190	—
			Sum of |errors| = 160
			MAD = 160/9 = 17.8

Table 5.2

Measures of Forecast Accuracy

Using a *naïve* forecasting model we can compute the MAD:

$$\text{MAD} = \frac{\sum |\text{forecast error}|}{n} = \frac{160}{9} = 17.8$$

YEAR	ACTUAL SALES OF CD PLAYERS	FORECAST SALES	ABSOLUTE VALUE OF ERRORS (DEVIATION), (ACTUAL – FORECAST)
1	110	—	—
			– 110| = 10
			– 110| = 20
			– 120| = 20
			– 140| = 30
			– 170| = 20
			– 150| = 10
			|190 – 160| = 30
9	200	190	|200 – 190| = 10
10	190	200	|190 – 200| = 10
11	—	190	—
			Sum of |errors| = 160
			MAD = 160/9 = 17.8

Table 5.2

Measures of Forecast Accuracy

- There are other popular measures of forecast accuracy.
- The ***mean squared error:***

$$\text{MSE} = \frac{\sum (\text{error})^2}{n}$$

- The ***mean absolute percent error:***

$$\text{MAPE} = \frac{\sum \left| \dfrac{\text{error}}{\text{actual}} \right|}{n} 100\%$$

- And ***bias*** is the average error.

Time-Series Forecasting Models

- A time series is a sequence of evenly spaced events.
- Time-series forecasts predict the future based solely on the past values of the variable, and other variables are ignored.

Components of a Time-Series

A time series typically has four components:

1. ***Trend*** (*T*) is the gradual upward or downward movement of the data over time.
2. ***Seasonality*** (*S*) is a pattern of demand fluctuations above or below the trend line that repeats at regular intervals.
3. ***Cycles*** (*C*) are patterns in annual data that occur every several years.
4. ***Random variations*** (*R*) are "blips" in the data caused by chance or unusual situations, and follow no discernible pattern.

Decomposition of a Time-Series

Product Demand Charted over 4 Years, with Trend and Seasonality Indicated

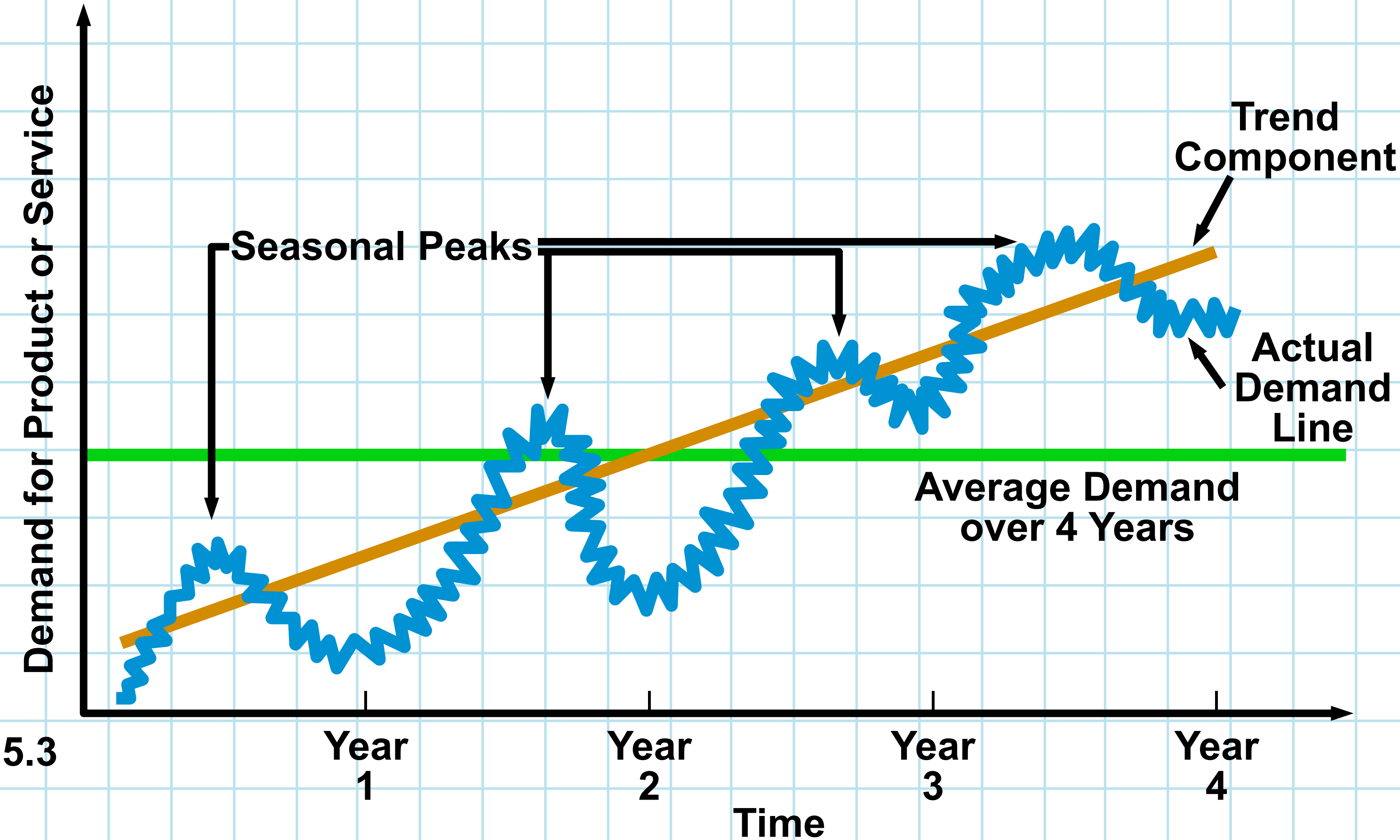


Figure 5.3

Decomposition of a Time-Series

- There are two general forms of time-series models:
  - The multiplicative model:

$$\text{Demand} = T \times S \times C \times R$$

  - The additive model:

$$\text{Demand} = T + S + C + R$$

- Models may be combinations of these two forms.
- Forecasters often assume errors are normally distributed with a mean of zero.

Moving Averages

- ***Moving averages*** **can be used when demand is relatively steady over time.**
- **The next forecast is the average of the most recent n data values from the time series.**
- **This methods tends to smooth out short-term irregularities in the data series.**

$$\text{Moving average forecast} = \frac{\text{Sum of demands in previous } n \text{ periods}}{n}$$

Moving Averages

- **Mathematically:**

$$F_{t+1} = \frac{Y_t + Y_{t-1} + \ldots + Y_{t-n+1}}{n}$$

Where:

F_{t+1} = forecast for time period $t + 1$

Y_t = actual value in time period t

n = number of periods to average

Wallace Garden Supply

- Wallace Garden Supply wants to forecast demand for its Storage Shed.
- They have collected data for the past year.
- They are using a three-month moving average to forecast demand (n = 3).

Wallace Garden Supply

MONTH	ACTUAL SHED SALES	THREE-MONTH MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	(10 + 12 + 13)/3 = 11.67
May	19	(12 + 13 + 16)/3 = 13.67
June	23	(13 + 16 + 19)/3 = 16.00
July	26	(16 + 19 + 23)/3 = 19.33
August	30	(19 + 23 + 26)/3 = 22.67
September	28	(23 + 26 + 30)/3 = 26.33
October	18	(26 + 30 + 28)/3 = 28.00
November	16	(30 + 28 + 18)/3 = 25.33
December	14	(28 + 18 + 16)/3 = 20.67
January	—	(18 + 16 + 14)/3 = 16.00

Table 5.3

Weighted Moving Averages

- ***Weighted moving averages*** use weights to put more emphasis on previous periods.
- This is often used when a trend or other pattern is emerging.

$$F_{t+1} = \frac{\sum(\text{Weight in period } i)(\text{Actual value in period})}{\sum(\text{Weights})}$$

- Mathematically:

$$F_{t+1} = \frac{w_1 Y_t + w_2 Y_{t-1} + \ldots + w_n Y_{t-n+1}}{w_1 + w_2 + \ldots + w_n}$$

where

w_i = weight for the i^{th} observation

Wallace Garden Supply

- Wallace Garden Supply decides to try a weighted moving average model to forecast demand for its Storage Shed.
- They decide on the following weighting scheme:

WEIGHTS APPLIED	PERIOD
3	Last month
2	Two months ago
1	Three months ago

$$\frac{3 \times \text{Sales last month} + 2 \times \text{Sales two months ago} + 1 \times \text{Sales three months ago}}{6}$$

Sum of the weights

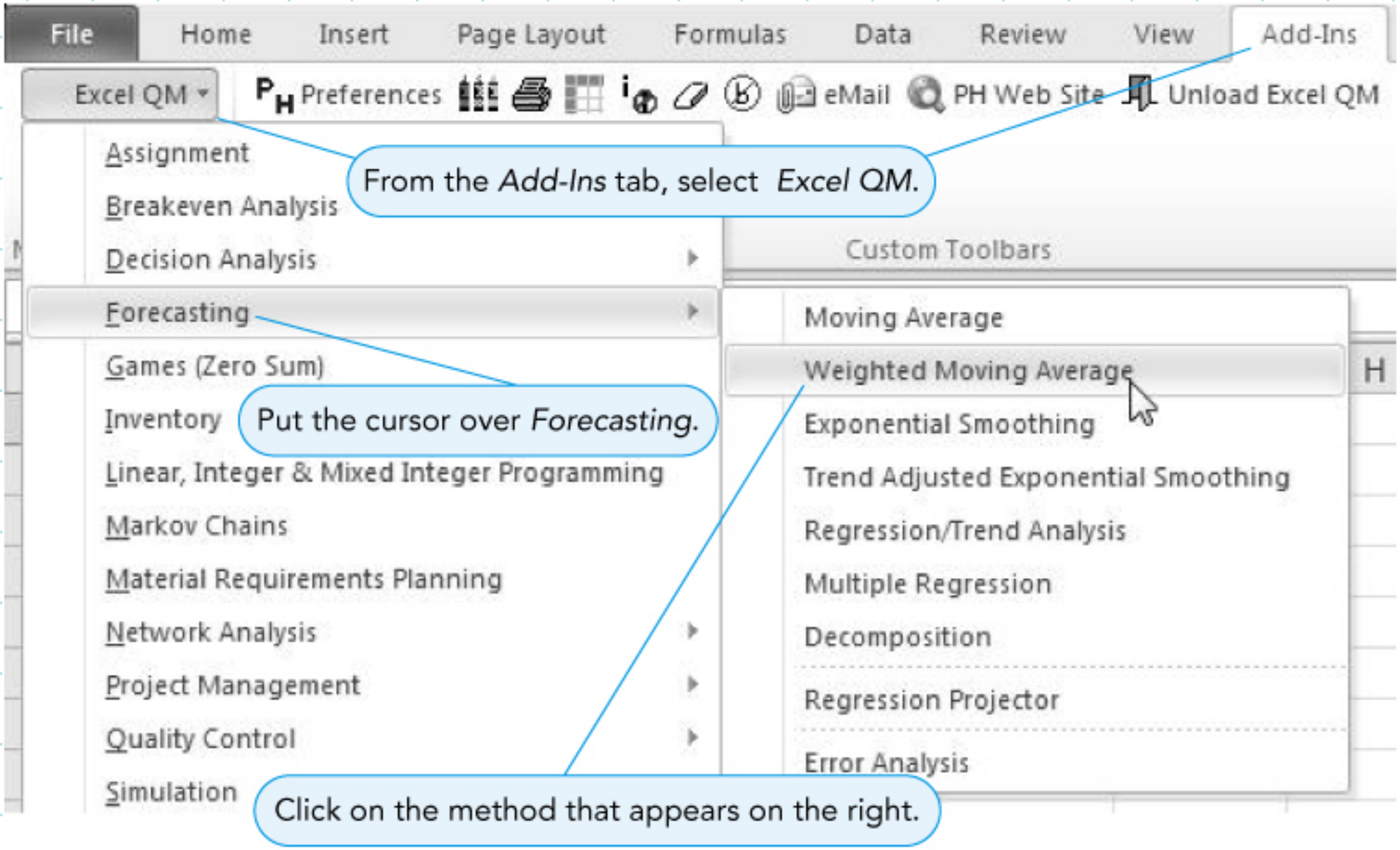
Wallace Garden Supply

MONTH	ACTUAL SHED SALES	THREE-MONTH WEIGHTED MOVING AVERAGE
January	10	
February	12	
March	13	
April	16	[(3 X 13) + (2 X 12) + (10)]/6 = 12.17
May	19	[(3 X 16) + (2 X 13) + (12)]/6 = 14.33
June	23	[(3 X 19) + (2 X 16) + (13)]/6 = 17.00
July	26	[(3 X 23) + (2 X 19) + (16)]/6 = 20.50
August	30	[(3 X 26) + (2 X 23) + (19)]/6 = 23.83
September	28	[(3 X 30) + (2 X 26) + (23)]/6 = 27.50
October	18	[(3 X 28) + (2 X 30) + (26)]/6 = 28.33
November	16	[(3 X 18) + (2 X 28) + (30)]/6 = 23.33
December	14	[(3 X 16) + (2 X 18) + (28)]/6 = 18.67
January	—	[(3 X 14) + (2 X 16) + (18)]/6 = 15.33

Table 5.4

Wallace Garden Supply

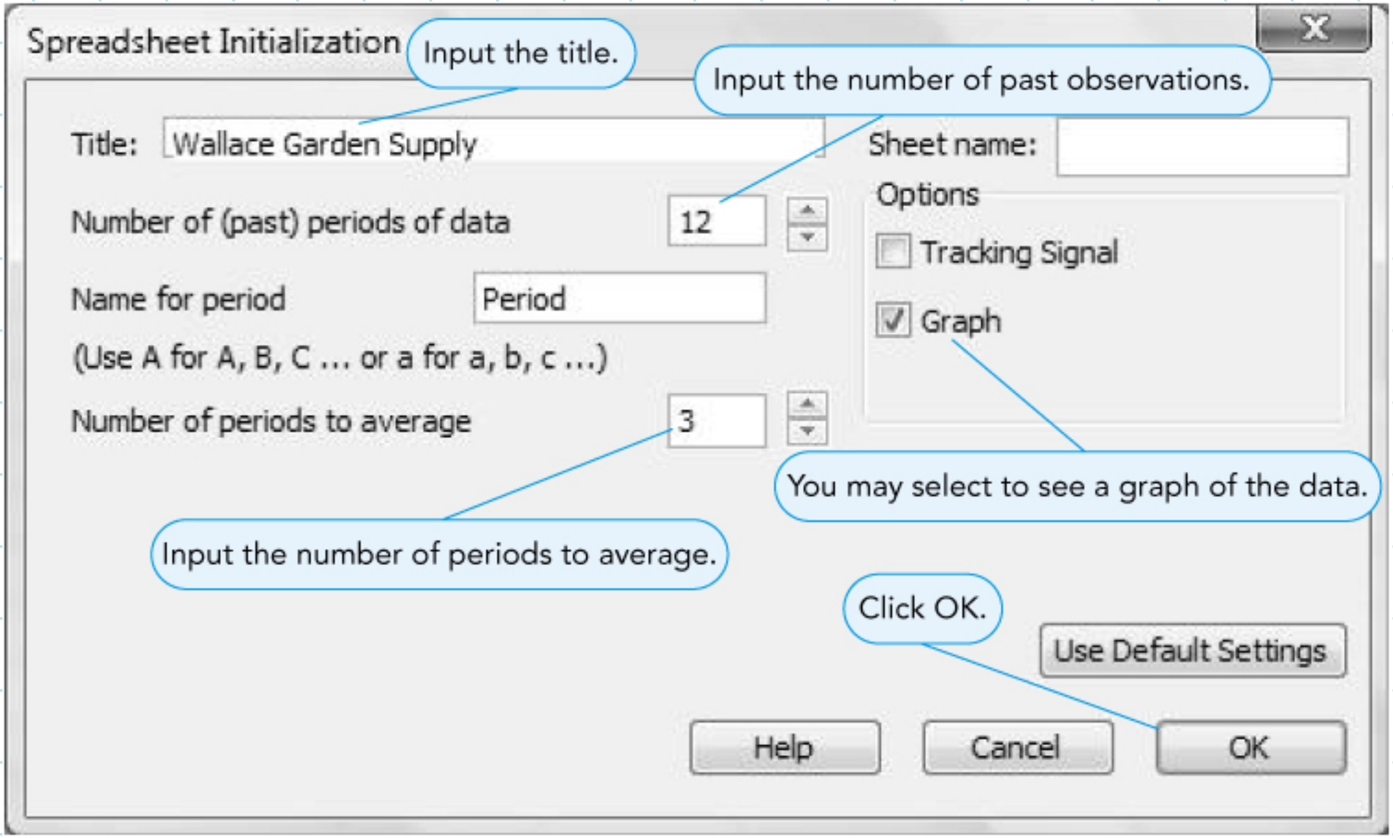
Selecting the Forecasting Module in Excel QM



Program 5.1A

Wallace Garden Supply

Initialization Screen for Weighted Moving Average



Program 5.1B

Wallace Garden Supply

Weighted Moving Average in Excel QM for Wallace Garden Supply

Wallace Garden Supply

Forecasting — **Weighted moving averages - 3 period moving average**

Input the past observations.

The names of the periods can be changed.

Past forecasts, errors, and measures of accuracy are shown.

Input the weight. Note that the highest weight is for the most recent observation.

The forecast for the next period is here.

Data

Period	Demand	Weights
January	10	1
February	12	2
March	13	3
April	16	
May	[illegible]	
June	[illegible]	
July	[illegible]	
August	[illegible]	
September	[illegible]	
October	18	
November	16	
December	14	

Next period 15.3333333

Forecasts and Error Analysis

Forecast	Error	Absolute	Squared	Abs Pct Err
12.1667	3.8333	3.8333	14.6944	23.96%
14.3333	4.6667	4.6667	21.7778	24.56%
17	6	6	36	26.09%
[illegible]	5.5	5.5	30.25	21.15%
[illegible]	6.1667	6.1667	38.0278	20.56%
27.5	0.5	0.5	0.25	01.79%
28.3333	-10.3333	10.3333	106.7778	57.41%
23.3333	-7.3333	7.3333	53.7778	45.83%
18.6667	-4.6667	4.6667	21.7778	33.33%
Total	4.3333	49.0000	323.3333	254.68%
Average	0.4815	5.4444	35.9259	28.30%
	Bias	MAD	MSE	MAPE
		SE	6.79636	

Program 5.1C

Exponential Smoothing

- ***Exponential smoothing*** **is a type of moving average that is easy to use and requires little record keeping of data.**

New forecast = Last period's forecast
+ α(Last period's actual demand
– Last period's forecast)

Here α is a weight (or *smoothing constant*) in which $0 \leq \alpha \leq 1$.

Exponential Smoothing

Mathematically:

$$F_{t+1} = F_t + \alpha(Y_t - F_t)$$

Where:

F_{t+1} = new forecast (for time period t + 1)

F_t = pervious forecast (for time period t)

α = smoothing constant ($0 \le \alpha \le 1$)

Y_t = pervious period's actual demand

The idea is simple – the new estimate is the old estimate plus some fraction of the error in the last period.

Exponential Smoothing Example

- In January, February's demand for a certain car model was predicted to be 142.
- Actual February demand was 153 autos
- Using a smoothing constant of $\alpha = 0.20$, what is the forecast for March?

New forecast (for March demand) $= 142 + 0.2(153 – 142)$
$= 144.2$ or 144 autos

- If actual demand in March was 136 autos, the April forecast would be:

New forecast (for April demand) $= 144.2 + 0.2(136 – 144.2)$
$= 142.6$ or 143 autos

Selecting the Smoothing Constant

- Selecting the appropriate value for α is key to obtaining a good forecast.
- The objective is always to generate an accurate forecast.
- The general approach is to develop trial forecasts with different values of α and select the α that results in the lowest MAD.

Exponential Smoothing

Port of Baltimore Exponential Smoothing Forecast for α=*0.1 and* α=*0.5.*

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST USING α =0.10	FORECAST USING α =0.50
1	180	175	175
2	168	175.5 = 175.00 + 0.10(180 – 175)	177.5
3	159	174.75 = 175.50 + 0.10(168 – 175.50)	172.75
4	175	173.18 = 174.75 + 0.10(159 – 174.75)	165.88
5	190	173.36 = 173.18 + 0.10(175 – 173.18)	170.44
6	205	175.02 = 173.36 + 0.10(190 – 173.36)	180.22
7	180	178.02 = 175.02 + 0.10(205 – 175.02)	192.61
8	182	178.22 = 178.02 + 0.10(180 – 178.02)	186.30
9	?	178.60 = 178.22 + 0.10(182 – 178.22)	184.15

Table 5.5

Exponential Smoothing

Absolute Deviations and MADs for the Port of Baltimore Example

QUARTER	ACTUAL TONNAGE UNLOADED	FORECAST WITH $\alpha = 0.10$	ABSOLUTE DEVIATIONS FOR $\alpha = 0.10$	FORECAST WITH $\alpha = 0.50$	ABSOLUTE DEVIATIONS FOR $\alpha = 0.50$
1	180	175	5	175	5
2	168	175.5	7.5	177.5	9.5
3	159	174.75	15.75	172.75	13.75
4	175	173.18	1.82	165.88	9.12
5	190	173.36	16.64	170.44	19.56
6	205	175.02	29.98	180.22	24.78
7	180	178.02	1.98	192.61	12.61
8	182	178.22	3.78	186.30	4.3
Sum of absolute deviations			82.45		98.63
		$MAD = \dfrac{\Sigma\lvert\text{deviations}\rvert}{n} =$	$\boxed{10.31}$	$MAD =$	12.33

Table 5.6

Best choice

Port of Baltimore Exponential Smoothing Example in Excel QM

If initial forecast is given, enter it here. If you do not want to include the error for this initial forecast, cells E10:H10.

Enter alpha (between 0 and 1), enter the past demands in the shaded column then enter a starting forecast. If the starting forecast is not in the first period then delete the error analysis for all rows above the starting forecast.

Enter the data and alpha.

The forecast for quarter 9 is here.

	A	B	C	D	E	F	G	H
1	**Port of Baltimore**							
2								
3	**Forecasting**		**Exponential smoothing**					
4								
5								
6								
7	Alpha							
8	**Data**			**Forecasts and Error Analysis**				
9	Period	Demand		Forecast	Error	Absolut	Squared	Abs Pct E
10	Quarter 1	180		175	5	5	25	02.78%
11	Quarter 2	168		175.5	-7.5	7.5	56.25	04.46%
12	Quarter 3	159		174.75	-15.75	15.75	248.06	09.91%
13	Quarter 4	175		173.175	1.825	1.825	3.3306	01.04%
14	Quarter 5	190		173.358	16.643	16.643	276.97	08.76%
15	Quarter 6	205		175.022	29.978	29.978	898.7	14.62%
16	Quarter 7	180		178.02	1.9804	1.9804	3.9221	01.10%
17	Quarter 8	182		178.218	3.7824	3.7824	14.306	0.02078
18				**Total**	**35.959**	**82.459**	**1526.5**	**44.75%**
19				**Average**	**4.4948**	**10.307**	**190.82**	**05.59%**
20					**Bias**	**MAD**	**MSE**	**MAPE**
21						**SE**	**15.951**	
22	**Next peri**	**178.596**						

Program 5.2

Exponential Smoothing with Trend Adjustment

- Like all averaging techniques, exponential smoothing does not respond to trends.
- A more complex model can be used that adjusts for trends.
- The basic approach is to develop an exponential smoothing forecast, and then adjust it for the trend.

Forecast including trend (FIT_{t+1}) = Smoothed forecast (F_{t+1})
+ Smoothed Trend (T_{t+1})

Exponential Smoothing with Trend Adjustment

- **The equation for the trend correction uses a new smoothing constant β.**
- **T_t must be given or estimated. T_{t+1} is computed by:**

$$T_{t+1} = (1-\beta)T_t + \beta(F_{t+1} - FIT_t)$$

where

T_t = smoothed trend for time period *t*
F_t = smoothed forecast for time period *t*
FIT_t = forecast including trend for time period *t*
α = smoothing constant for forecasts
β = smoothing constant for trend

Selecting a Smoothing Constant

- As with exponential smoothing, a high value of β makes the forecast more responsive to changes in trend.
- A low value of β gives less weight to the recent trend and tends to smooth out the trend.
- Values are generally selected using a trial-and-error approach based on the value of the MAD for different values of β.

Midwestern Manufacturing

- Midwest Manufacturing has a demand for electrical generators from 2004 – 2010 as given in the table below.
- To forecast demand, Midwest assumes:
  - F_1 is perfect.
  - $T_1 = 0$.
  - $\alpha = 0.3$
  - $\beta = 0.4$.

YEAR	ELECTRICAL GENERATORS SOLD
2004	74
2005	79
2006	80
2007	90
2008	105
2009	142
2010	122

Table 5.7

Midwestern Manufacturing

- According to the assumptions,

  $$FIT_1 = F_1 + T_1 = 74 + 0 = 74.$$

- Step 1: Compute F_{t+1} by:

  $$FIT_{t+1} = F_t + \alpha(Y_t - FIT_t)$$
  $$= 74 + 0.3(74\text{-}74) = 74$$

- Step 2: Update the trend using:

  $$T_{t+1} = T_t + \beta(F_{t+1} - FIT_t)$$
  $$T_2 = T_1 + .4(F_2 - FIT_1)$$
  $$= 0 + .4(74 - 74) = 0$$

Midwestern Manufacturing

- **Step 3: Calculate the trend-adjusted exponential smoothing forecast (F_{t+1}) using the following:**

$$FIT_2 = F_2 + T_2$$
$$= 74 + 0 = 74$$

Midwestern Manufacturing

- For 2006 (period 3) we have:
- Step 1: $F_3 = FIT_2 + 0.3(Y_2 – FIT_2)$
  $= 74 + .3(79 – 74)$
  $= 75.5$
- Step 2: $T_3 = T_2 + 0.4(F_3 – FIT_2)$
  $= 0 + 0.4(75.5 – 74)$
  $= 0.6$
- Step 3: $FIT_3 = F_3 + T_3$
  $= 75.5 + 0.6$
  $= 76.1$

Midwestern Manufacturing Exponential Smoothing with Trend Forecasts

Time (t)	Demand (Y_t)	$FIT_{t+1} = F_t + 0.3(Y_t - FIT_t)$	$T_{t+1} = T_t + 0.4(F_{t+1} - FIT_t)$	$FIT_{t+1} = F_{t+1} + T_{t+1}$
1	74	74	0	74
2	79	74=74+0.3(74-74)	0 = 0+0.4(74-74)	74 = 74+0
3	80	75.5=74+0.3(79-74)	0.6 = 0+0.4(75.5-74)	76.1 = 75.5+0.6
4	90	77.270=76.1+0.3(80-76.1)	1.068 = 0.6+0.4(77.27-76.1)	78.338 = 77.270+1.068
5	105	81.837=78.338+0.3(90-78.338)	2.468 = 1.068+0.4(81.837-78.338)	84.305 = 81.837+2.468
6	142	90.514=84.305+0.3(105-84.305)	4.952 = 2.468+0.4(90.514-84.305)	95.466 = 90.514+4.952
7	122	109.426=95.466+0.3(142-95.466)	10.536 = 4.952+0.4(109.426-95.466)	119.962 = 109.426+10.536
8		120.573=119.962+0.3(122-119.962)	10.780 = 10.536+0.4(120.573-119.962)	131.353 = 120.573+10.780

Table 5.8

Midwestern Manufacturing

Midwestern Manufacturing Trend-Adjusted Exponential Smoothing in Excel QM

Input values for the smoothing constants.

You may enter initial values for F_1 and T_1.

Enter the past observations.

The forecast for the next year is given here.

	A	B	C	D	E	F	G	H	I	J
1	**Midwestern Manufacturing**									
2										
3	**Forecasting**		**Trend adjusted exponential smoothing**							
4	Enter alpha and beta (between 0 and 1), enter the past demands in the shaded column then enter a starting forecast. If the starting forecast is not in the first period then delete the error analysis for all rows above the starting forecast.									
5										
6										
7	Alpha	0.3								
8	Beta	0.4								
9	**Data**			**Forecasts and Error Analysis**						
10	Period	Demand		Smoothed Forecast, F_t	Smoothed Trend, T_t	Forecast Including Trend, FIT_t	Error	Absolute	Squared	Abs Pct Err
11	Period 1	74		74		74	0	0	0	00.00%
12	Period 2	79		74	0	74	5	5	25	06.33%
13	Period 3	80		75.5	0.6	76.1	4.5	4.5	20.25	05.63%
14	Period 4	90		77.27	1.068	78.338	12.73	12.73	162.053	14.14%
15	Period 5	105		81.8366	2.46744	84.30404	23.1634	23.1634	536.543	22.06%
16	Period 6	142		90.512828	4.9509552	95.4637832	51.4872	51.4872	2650.93	36.26%
17	Period 7	122		109.424648	10.5353012	119.9599495	12.5754	12.5754	158.139	0.1030767
18		**Next period**		**120.571965**	**10.7801073**	**131.3520719**				
19				Total			109.456	109.456	3552.91	94.73%
20				**Average**			**15.6366**	**15.6366**	**507.559**	**13.53%**
21							**Bias**	**MAD**	**MSE**	**MAPE**
22								**SE**	**26.6568**	

Program 5.3

Trend Projections

- Trend projection fits a trend line to a series of historical data points.
- The line is projected into the future for medium- to long-range forecasts.
- Several trend equations can be developed based on exponential or quadratic models.
- The simplest is a linear model developed using regression analysis.

Trend Projection

The mathematical form is

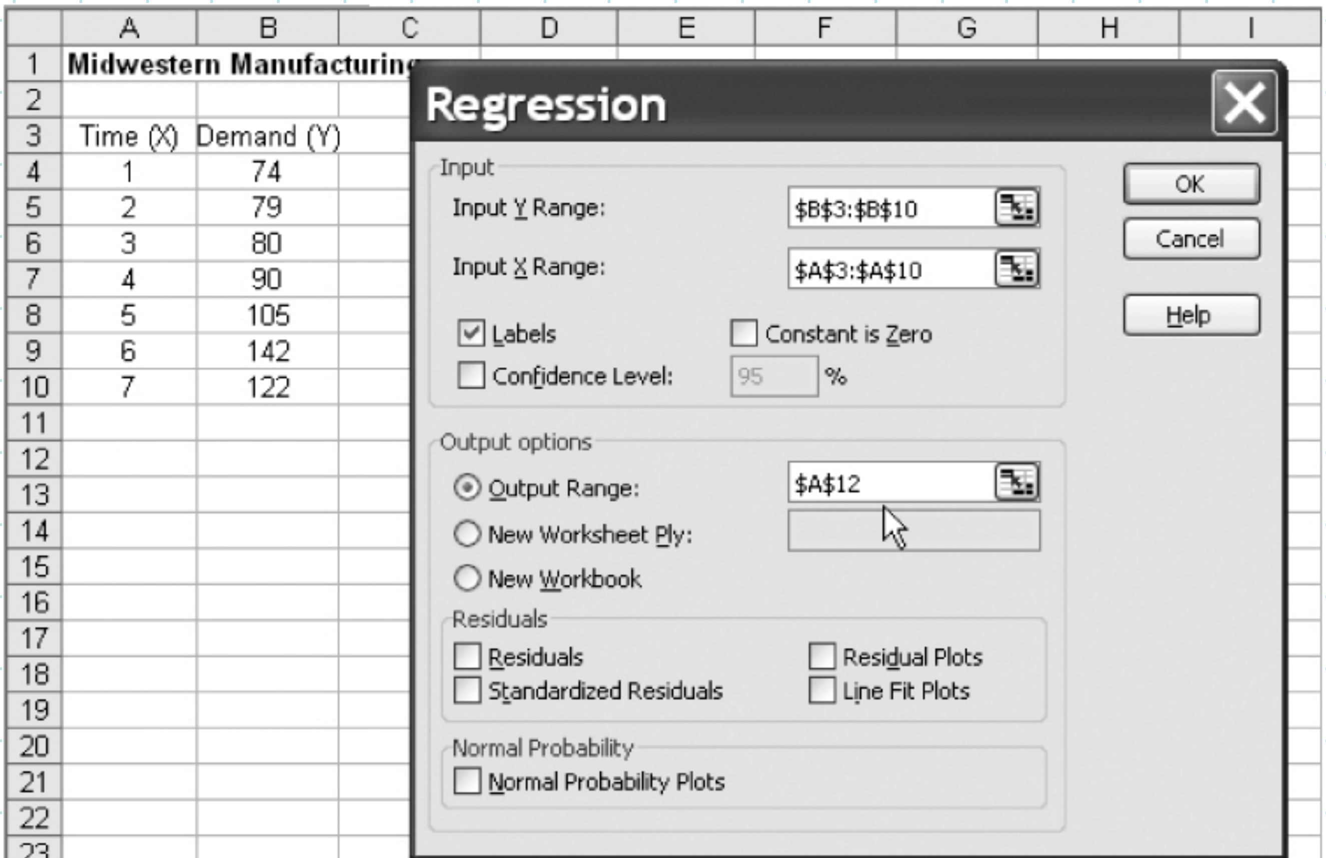
$$\hat{Y} = b_0 + b_1 X$$

Where

$\hat{Y}$ = predicted value
b_0 = intercept
b_1 = slope of the line
X = time period (i.e., X = 1, 2, 3, …, n)

Midwestern Manufacturing

Excel Input Screen for Midwestern Manufacturing Trend Line



Program 5.4A

Midwestern Manufacturing

Excel Output for Midwestern Manufacturing Trend Line

	A	B	C	D	E	F	G	H	I
1	**Midwestern Manufacturing**								
2									
3	Time (X)	Demand (Y)							
4	1	74							
5	2	79							
6	3	80							
7	4	90							
8	5	105							
9	6	142							
10	7	122							
11									
12	SUMMARY OUTPUT								
13									
14	*Regression Statistics*								
15	Multiple R	0.89491							
16	R Square	0.80086							
17	Adjusted F	0.76104							
18	Standard E	12.43239							
19	Observatio	7							
20									
21	ANOVA								
22		*df*	*SS*	*MS*	*F*	*Significance F*			
23	Regressior	1	3108.0357	3108.0357	20.1084	0.0065			
24	Residual	5	772.8214	154.5643					
25	Total	6	3880.8571						
26									
27		*Coefficients*	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*ower 95.0%*	*Upper 95.0%*
28	Intercept	56.71429	10.50729	5.39762	0.00295	29.70449	83.72408	29.70449	83.72408
29	Time (X)	10.53571	2.34950	4.48424	0.00649	4.49614	16.57529	4.49614	16.57529

The next year will be time period 8.

The slope of the trend line is 10.54.

Program 5.4B

Midwestern Manufacturing Company Example

- The forecast equation is

$$\hat{Y} = 56.71 + 10.54X$$

- To project demand for 2011, we use the coding system to define $X = 8$

$$\text{(sales in 2011)} = 56.71 + 10.54(8)$$
$$= 141.03\text{, or 141 generators}$$

- Likewise for $X = 9$

$$\text{(sales in 2012)} = 56.71 + 10.54(9)$$
$$= 151.57\text{, or 152 generators}$$

Midwestern Manufacturing

Electrical Generators and the Computed Trend Line

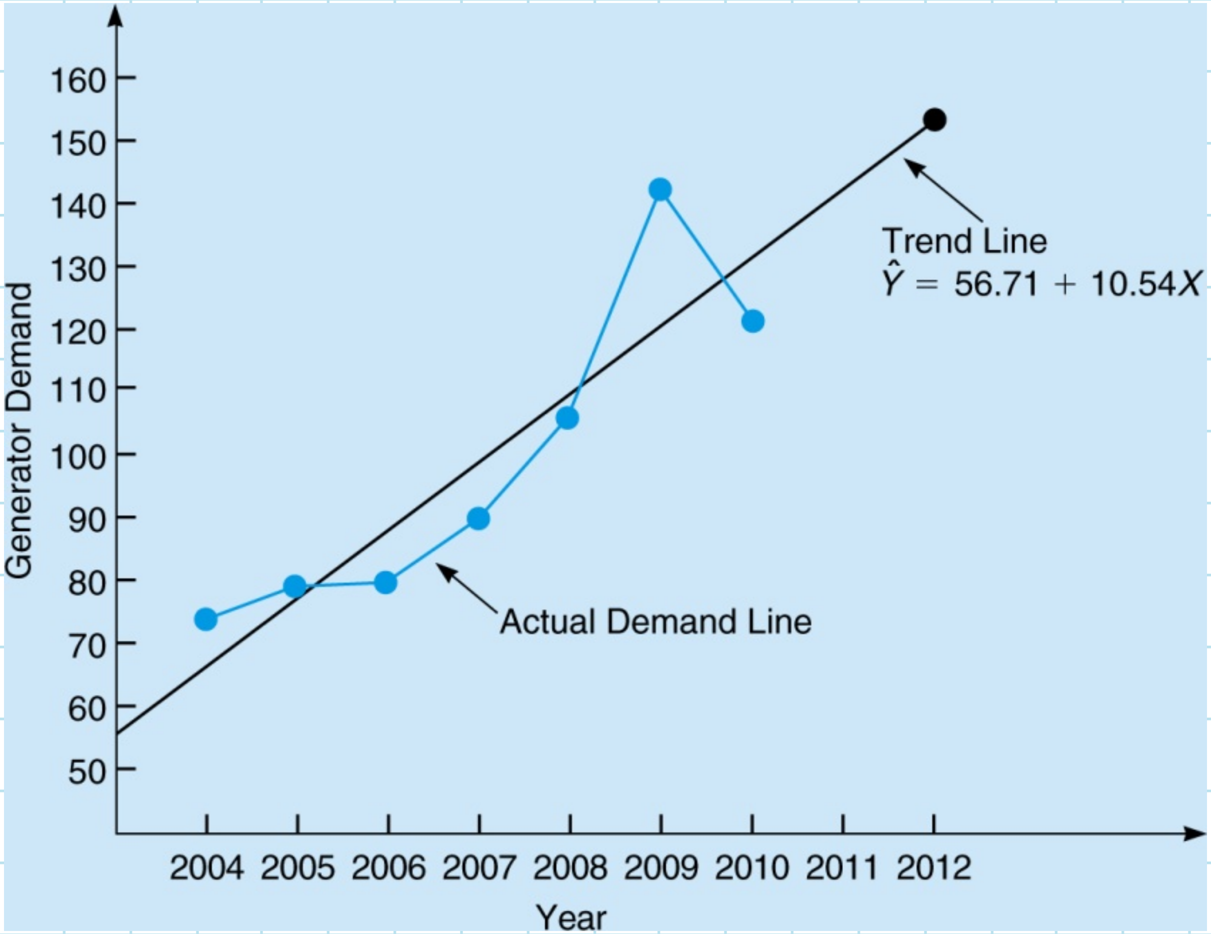


Figure 5.4

Midwestern Manufacturing

Excel QM Trend Projection Model

	A	B	C	D	E	F	G	H	I
1	**Midwestern Manufacturing**								
2									
3	**Forecasting**		**Regression/Trend analysis**						
4					…emands in the demand column. If this				
5					…st and enter a new value of x at the				
6									
7									
8	**Data**				**Forecasts and Error Analysis**				
9	Period	Demand (y)	Period(x)		Forecast	Error	Absolute	Squared	Abs Pct Err
10	Year 1	74	1		67.25	6.75	6.75	45.5625	09.12%
11	Year 2	79	2		77.7857	1.2143	1.2143	1.4745	01.54%
12	Year 3	80	3		88.3214	-8.3214	8.3214	69.2462	10.40%
13	Year 4	90	4		98.8571	-8.8571	8.8571	78.4490	09.84%
14	Year 5	105	5		109.3929	-4.3929	4.3929	19.2972	04.18%
15	Year 6	142	6		119.9286	22.0714	22.0714	487.1480	15.54%
16	Year 7					-8.4643	8.4643	71.6441	06.94%
17						…5326E-14	60.0714	772.8214	57.57%
18	**Intercept**	**56.71429**			**Average**	**-6.09037E-15**	**8.5816**	**110.4031**	**08.22%**
19	**Slope**	**10.53571**							
20									
21	**Next perioc**	**141**	8						
22							**Correlatic**	**0.89491**	

Past forecasts and errors are shown here.

Input the past data and the time periods.

The intercept (b_0) and slope (b_1) are found here.

To obtain a forecast for a future period, enter the time period here.

Program 5.5

Seasonal Variations

- Recurring variations over time may indicate the need for seasonal adjustments in the trend line.
- A seasonal index indicates how a particular season compares with an average season.
- When no trend is present, the seasonal index can be found by dividing the average value for a particular season by the average of all the data.

Eichler Supplies

- Eichler Supplies sells telephone answering machines.
- Sales data for the past two years has been collected for one particular model.
- The firm wants to create a forecast that includes seasonality.

Eichler Supplies Answering Machine Sales and Seasonal Indices

MONTH	SALES DEMAND YEAR 1	SALES DEMAND YEAR 2	AVERAGE TWO-YEAR DEMAND	MONTHLY DEMAND	AVERAGE SEASONAL INDEX
January	80	100	90	94	0.957
February	85	75	80	94	0.851
March	80	90	85	94	0.904
April	110	90	100	94	1.064
May	115	131	123	94	1.309
June	120	110	115	94	1.223
July	100	110	105	94	1.117
August	110	90	100	94	1.064
September	85	95	90	94	0.957
October	75	85	80	94	0.851
November	85	75	80	94	0.851
December	80	80	80	94	0.851
			Total average demand = 1,128		

$$\text{Average monthly demand} = \frac{1{,}128}{12 \text{ months}} = 94$$

$$\text{Seasonal index} = \frac{\text{Average two-year demand}}{\text{Average monthly demand}}$$

Table 5.9

Seasonal Variations

- The calculations for the seasonal indices are

Jan.	$\dfrac{1{,}200}{12} \times 0.957 = 96$	July	$\dfrac{1{,}200}{12} \times 1.117 = 112$
Feb.	$\dfrac{1{,}200}{12} \times 0.851 = 85$	Aug.	$\dfrac{1{,}200}{12} \times 1.064 = 106$
Mar.	$\dfrac{1{,}200}{12} \times 0.904 = 90$	Sept.	$\dfrac{1{,}200}{12} \times 0.957 = 96$
Apr.	$\dfrac{1{,}200}{12} \times 1.064 = 106$	Oct.	$\dfrac{1{,}200}{12} \times 0.851 = 85$
May	$\dfrac{1{,}200}{12} \times 1.309 = 131$	Nov.	$\dfrac{1{,}200}{12} \times 0.851 = 85$
June	$\dfrac{1{,}200}{12} \times 1.223 = 122$	Dec.	$\dfrac{1{,}200}{12} \times 0.851 = 85$

Seasonal Variations with Trend

- When both trend and seasonal components are present, the forecasting task is more complex.
- Seasonal indices should be computed using a ***centered moving average*** (*CMA*) approach.
- There are four steps in computing CMAs:
  1. Compute the CMA for each observation (where possible).
  2. Compute the seasonal ratio = Observation/CMA for that observation.
  3. Average seasonal ratios to get seasonal indices.
  4. If seasonal indices do not add to the number of seasons, multiply each index by (Number of seasons)/(Sum of indices).

Turner Industries

- The following table shows Turner Industries' quarterly sales figures for the past three years, in millions of dollars:

QUARTER	YEAR 1	YEAR 2	YEAR 3	AVERAGE
1	108	116	123	115.67
2	125	134	142	133.67
3	150	159	168	159.00
4	141	152	165	152.67
Average	131.00	140.25	149.50	140.25

Table 5.10

Definite trend

Seasonal pattern

Turner Industries

- To calculate the CMA for quarter 3 of year 1 we compare the actual sales with an average quarter centered on that time period.
- We will use 1.5 quarters before quarter 3 and 1.5 quarters after quarter 3 – that is we take quarters 2, 3, and 4 and one half of quarters 1, year 1 and quarter 1, year 2.

$$\text{CMA}(q3, y1) = \frac{0.5(108) + 125 + 150 + 141 + 0.5(116)}{4} = 132.00$$

Turner Industries

Compare the actual sales in quarter 3 to the CMA to find the seasonal ratio:

$$\text{Seasonal ratio} = \frac{\text{Sales in quarter 3}}{\text{CMA}} = \frac{150}{132} = 1.136$$

Turner Industries

YEAR	QUARTER	SALES	*CMA*	SEASONAL RATIO
1	1	108		
	2	125		
	3	150	132.000	1.136
	4	141	134.125	1.051
2	1	116	136.375	0.851
	2	134	138.875	0.965
	3	159	141.125	1.127
	4	152	143.000	1.063
3	1	123	145.125	0.848
	2	142	147.875	0.960
	3	168		
	4	165		

Table 5.11

Turner Industries

There are two seasonal ratios for each quarter so these are averaged to get the seasonal index:

Index for quarter 1 = I_1 = (0.851 + 0.848)/2 = 0.85

Index for quarter 2 = I_2 = (0.965 + 0.960)/2 = 0.96

Index for quarter 3 = I_3 = (1.136 + 1.127)/2 = 1.13

Index for quarter 4 = I_4 = (1.051 + 1.063)/2 = 1.06

Turner Industries

Scatterplot of Turner Industries Sales Data and Centered Moving Average

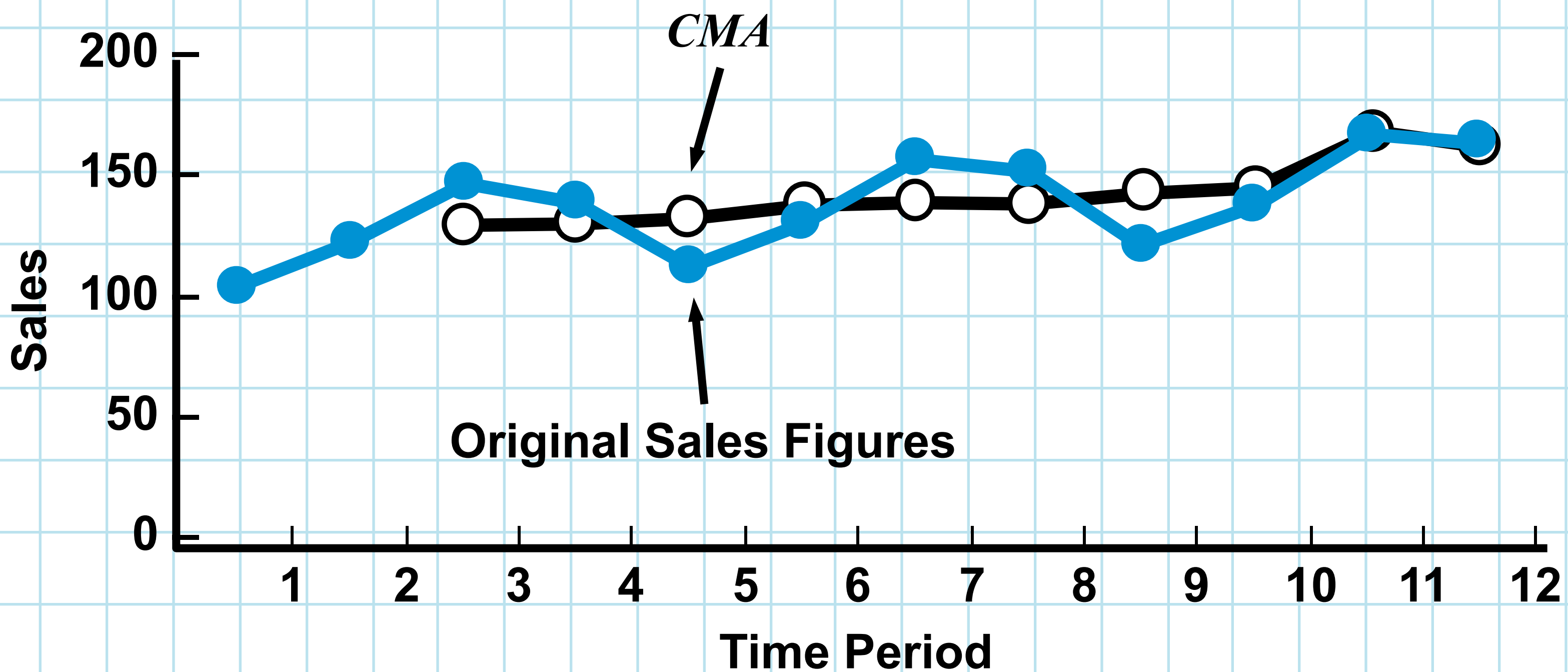


Figure 5.5

The Decomposition Method of Forecasting with Trend and Seasonal Components

- ***Decomposition*** is the process of isolating linear trend and seasonal factors to develop more accurate forecasts.
- There are five steps to decomposition:
  1. **Compute** seasonal indices using CMAs.
  2. **Deseasonalize** the data by dividing each number by its seasonal index.
  3. **Find** the equation of a trend line using the deseasonalized data.
  4. **Forecast** for future periods using the trend line.
  5. **Multiply** the trend line forecast by the appropriate seasonal index.

Deseasonalized Data for Turner Industries

- Find a trend line using the deseasonalized data:

$$b_1 = 2.34 \qquad b_0 = 124.78$$

- Develop a forecast using this trend and multiply the forecast by the appropriate seasonal index.

$$\hat{Y} = 124.78 + 2.34X$$
$$= 124.78 + 2.34(13)$$
$$= 155.2 \text{ (forecast before adjustment for seasonality)}$$

$$\hat{Y} \text{ x } I_1 = 155.2 \text{ x } 0.85 = 131.92$$

Deseasonalized Data for Turner Industries

SALES ($1,000,000s)	SEASONAL INDEX	DESEASONALIZED SALES ($1,000,000s)
108	0.85	127.059
125	0.96	130.208
150	1.13	132.743
141	1.06	133.019
116	0.85	136.471
134	0.96	139.583
159	1.13	140.708
152	1.06	143.396
123	0.85	144.706
142	0.96	147.917
168	1.13	148.673
165	1.06	155.660

Table 5.12

San Diego Hospital

A San Diego hospital used 66 months of adult inpatient days to develop the following seasonal indices.

MONTH	SEASONALITY INDEX	MONTH	SEASONALITY INDEX
January	1.0436	July	1.0302
February	0.9669	August	1.0405
March	1.0203	September	0.9653
April	1.0087	October	1.0048
May	0.9935	November	0.9598
June	0.9906	December	0.9805

Table 5.13

San Diego Hospital

Using this data they developed the following equation:

$$\hat{Y} = 8{,}091 + 21.5X$$

where

$\hat{Y}$ = forecast patient days

X = time in months

Based on this model, the forecast for patient days for the next period (67) is:

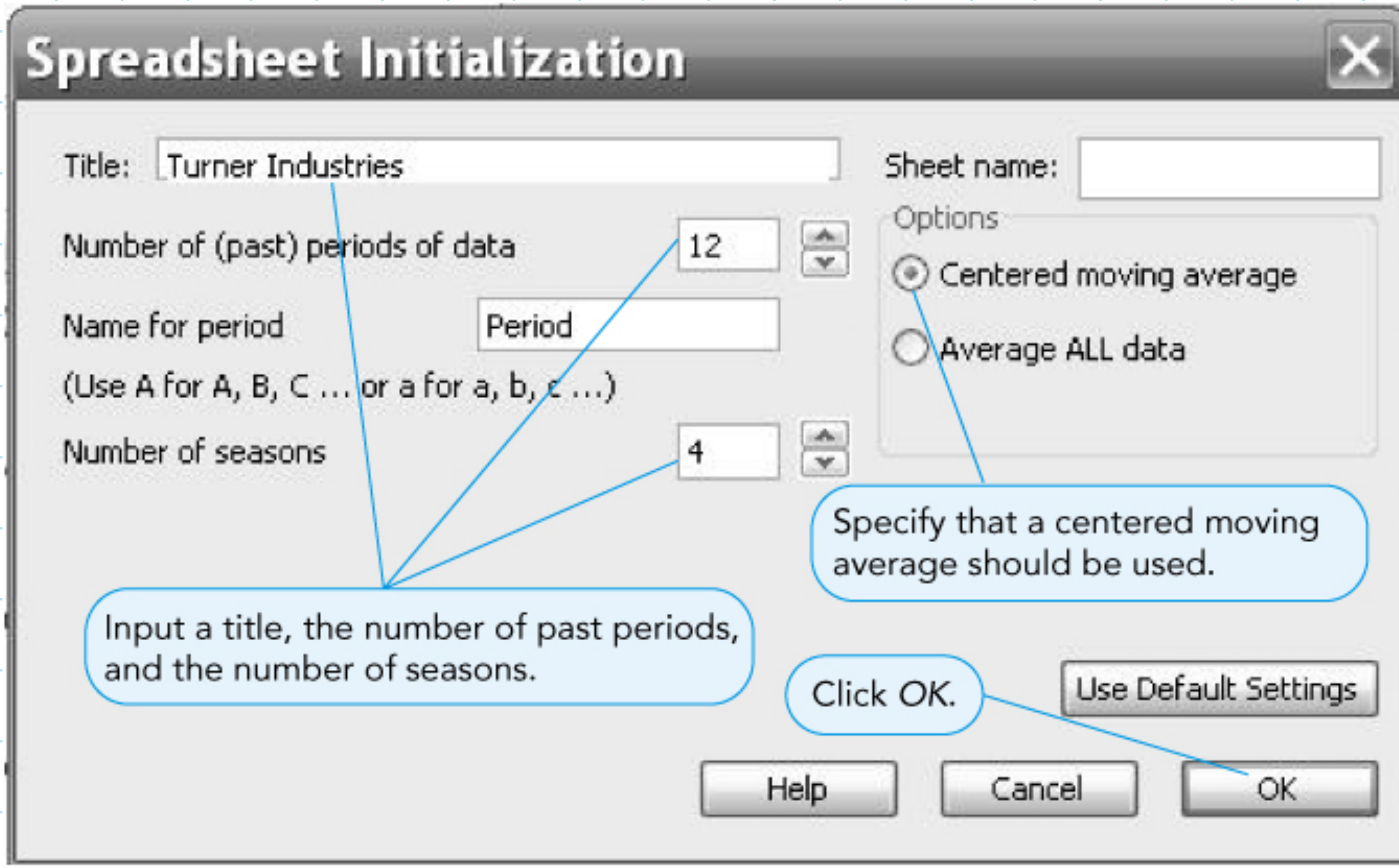
Patient days = 8,091 + (21.5)(67) = 9,532 (trend only)

Patient days = (9,532)(1.0436)

= 9,948 (trend and seasonal)

San Diego Hospital

Initialization Screen for the Decomposition method in Excel QM



Program 5.6A

San Diego Hospital

Turner Industries Forecast Using the Decomposition Method in Excel QM

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	**Turner Industries**													
2														
3	**Forecasting**		**Multiplicative decomposition**											
4														
5	4 seasons													
6														
7														
8	**Data**										**Forecasts and Error Analysis**			
9	Period	Demand (y)	Time (x)		Average	Ratio	Seasonal	Smoothed	Unadjusted	Adjusted	Error	|Error|	Error^2	Abs Pct Err
10	Period 1	108	1				0.8491	127.1979	127.1187	107.9327	0.0673	0.0673	0.0045	00.06%
11	Period 2	125	2				0.9626	129.8589	129.4621	124.6181	0.3819	0.3819	0.1458	00.31%
12	Period 3	150	3	131	132.000	1.136	1.1315	132.5660	131.8056	149.1396	0.8604	0.8604	0.7403	00.57%
13	Period 4	141	4	133	134.125	1.051	1.0571	133.3841	134.1490	141.8086	-0.8086	0.8086	0.6538	00.57%
14	Period 5	116	5	135.25	136.375	0.851	0.8491	136.6200	136.4924	115.8917	0.1083	0.1083	0.0117	00.09%
15	Period 6	134	6	137.5	138.875	0.965	0.9626	139.2087	138.8359	133.6411	0.3589	0.3589	0.1288	00.27%
16	Period 7	159	7	140.25	141.125	1.127	1.1315	140.5199	141.1793	159.7461	-0.7461	0.7461	0.5567	00.47%
17	Period 8	152	8	142	143.000	1.063	1.0571	143.7899	143.5227	151.7175	0.2825	0.2825	0.0798	00.19%
18	Period 9	123	9	144	145.125	0.848	0.8491	144.8643	145.8662	123.8507	-0.8507	0.8507	0.7236	00.69%
19	Period 10	142	10	146.25	147.875	0.960	0.9626	147.5197	148.2096	142.6641	-0.6641	0.6641	0.4410	00.47%
20	Period 11	168	11	149.5			1.1315	148.4739	150.5530	170.3526	-2.3526	2.3526	5.5346	01.40%
21	Period 12	165	12				1.0571	156.0878	152.8965	161.6265	3.3735	3.3735	11.3807	02.04%
22										Total	0.0107	10.8547	20.4014	07.14%
23					Average		Intercept	124.7753			0.0009	0.9046	1.7001	00.59%
24							Slope	2.3434			Bias	MAD	MSE	MAPE
25												SE	1.8439709	
26	**Ratios**													
27		Season 1	Season 2	Season 3	Season 4									
28				1.1364	1.0513									
29		0.8506	0.9649	1.1267	1.0629									
30		0.8475	0.9603											
31	Average	0.8491	0.9626	1.1315	1.0571									
32														
33	**Forecasts**													
34	Period	Unadjusted	Seasonal	Adjusted										
35	13	155.240	0.849	131.810										
36	14	157.583	0.963	151.687										
37	15	159.927	1.132	180.959										
38	16	162.270	1.057	171.535										

Seasonal indices are based on CMAs.

Input the past demand.

The CMAs are here.

The intercept and slope are here.

Program 5.6B

Using Regression with Trend and Seasonal Components

- *Multiple regression* can be used to forecast both trend and seasonal components in a time series.
  - One independent variable is time.
  - Dummy independent variables are used to represent the seasons.
- The model is an additive decomposition model:

$$\hat{Y} = a + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4$$

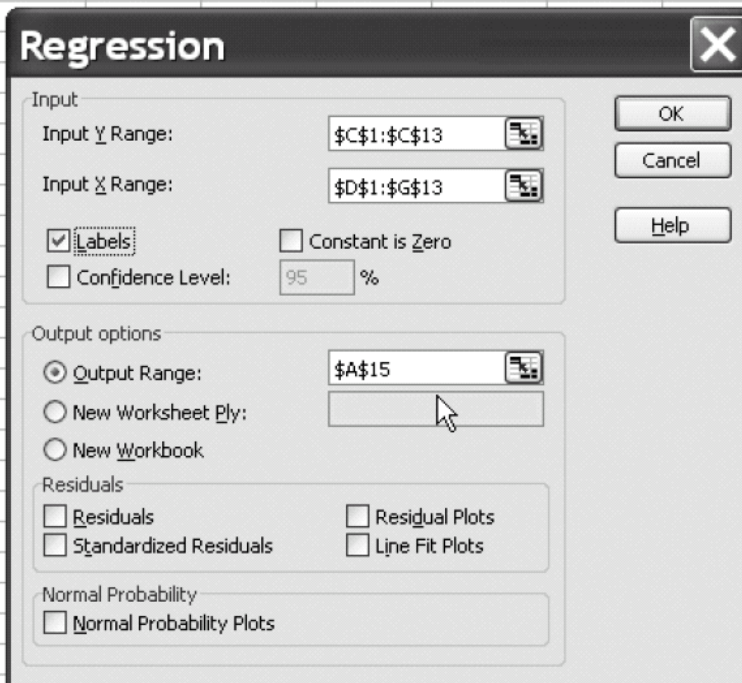
where

X_1 = time period
X_2 = 1 if quarter 2, 0 otherwise
X_3 = 1 if quarter 3, 0 otherwise
X_4 = 1 if quarter 4, 0 otherwise

Regression with Trend and Seasonal Components

Excel Input for the Turner Industries Example Using Multiple Regression

	A	B	C	D	E	F	G
1	Year	Quarter	Sales	Time Peri	X2 Qtr 2	X3 Qtr 3	X4 Qtr 4
2	1	1	108	1	0	0	0
3		2	125	2	1	0	0
4		3	150	3	0	1	0
5		4	141	4	0	0	1
6	2	1	116	5	0	0	0
7		2	134	6	1	0	0
8		3	159	7	0	1	0
9		4	152	8	0	0	1
10	3	1	123	9	0	0	0
11		2	142	10	1	0	0
12		3	168	11	0	1	0
13		4	165	12	0	0	1
14							
15							
16							
17							
18							
19							
20							
21							
22							
23							
24							



Program 5.7A

Using Regression with Trend and Seasonal Components

Excel Output for the Turner Industries Example Using Multiple Regression

	A	B	C	D	E	F	G	H	I
1	Year	Quarter	Sales	X1 Time Period	X2 Qtr 2	X3 Qtr 3	X4 Qtr4		
2	1	1	108	1	0	0	0		
3		2	125	2	1	0	0		
4		3	150	3	0	1	0		
5		4	141	4	0	0	1		
6	2	1	116	5	0	0	0		
7		2	134	6	1	0	0		
8		3	159	7	0	1	0		
9		4	152	8	0	0	1		
10	3	1	123	9	0	0	0		
11		2	142	10	1	0	0		
12		3	168	11	0	1	0		
13		4	165	12	0	0	1		
14									
15	SUMMARY OUTPUT								
16									
17	*Regression Statistics*								
18	Multiple R	0.99718							
19	R Square	0.99436							
20	Adjusted R	0.99114							
21	Standard E	1.83225							
22	Observatio	12							
23									
24	ANOVA								
25		*df*	*SS*	*MS*	*F*	*Significance F*			
26	Regressior	4	4144.75	1.0362E+03	3.0865E+02	6.0284E-08			
27	Residual	7	23.5	3.3571E+00					
28	Total	11	4168.25						
29									
30		*Coefficients*	*Standard Erro*	*t Stat*	*p-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
31	Intercept	104.1042	1.3322	78.1449	0.0000	100.9540	107.2543	100.9540	107.2543
32	X1 Time Pe	2.3125	0.1619	14.2791	0.0000	1.9296	2.6954	1.9296	2.6954
33	X2 Qtr 2	15.6875	1.5048	10.4252	0.0000	12.1293	19.2457	12.1293	19.2457
34	X3 Qtr 3	38.7083	1.5307	25.2882	0.0000	35.0888	42.3278	35.0888	42.3278
35	X4 Qtr4	30.0625	1.5729	19.1123	0.0000	26.3431	33.7819	26.3431	33.7819

Quarter 1 is indicated by letting $X_2 = X_3 = X_4 = 0$.

Program 5.7B

Using Regression with Trend and Seasonal Components

- The resulting regression equation is:

$$\hat{Y} = 104.1 + 2.3X_1 + 15.7X_2 + 38.7X_3 + 30.1X_4$$

- Using the model to forecast sales for the first two quarters of next year:

$$\hat{Y} = 104.1 + 2.3(13) + 15.7(0) + 38.7(0) + 30.1(0) = 134$$

$$\hat{Y} = 104.1 + 2.3(14) + 15.7(1) + 38.7(0) + 30.1(0) = 152$$

- These are different from the results obtained using the multiplicative decomposition method.
- Use MAD or MSE to determine the best model.

Monitoring and Controlling Forecasts

- ***Tracking signals*** can be used to monitor the performance of a forecast.
- A tracking signal is computed as the **running sum of the forecast errors (RSFE)**, and is computed using the following equation:

$$\text{Tracking signal} = \frac{\text{RSFE}}{\text{MAD}}$$

where

$$\text{MAD} = \frac{\sum |\text{forecast error}|}{n}$$

Monitoring and Controlling Forecasts

Plot of Tracking Signals

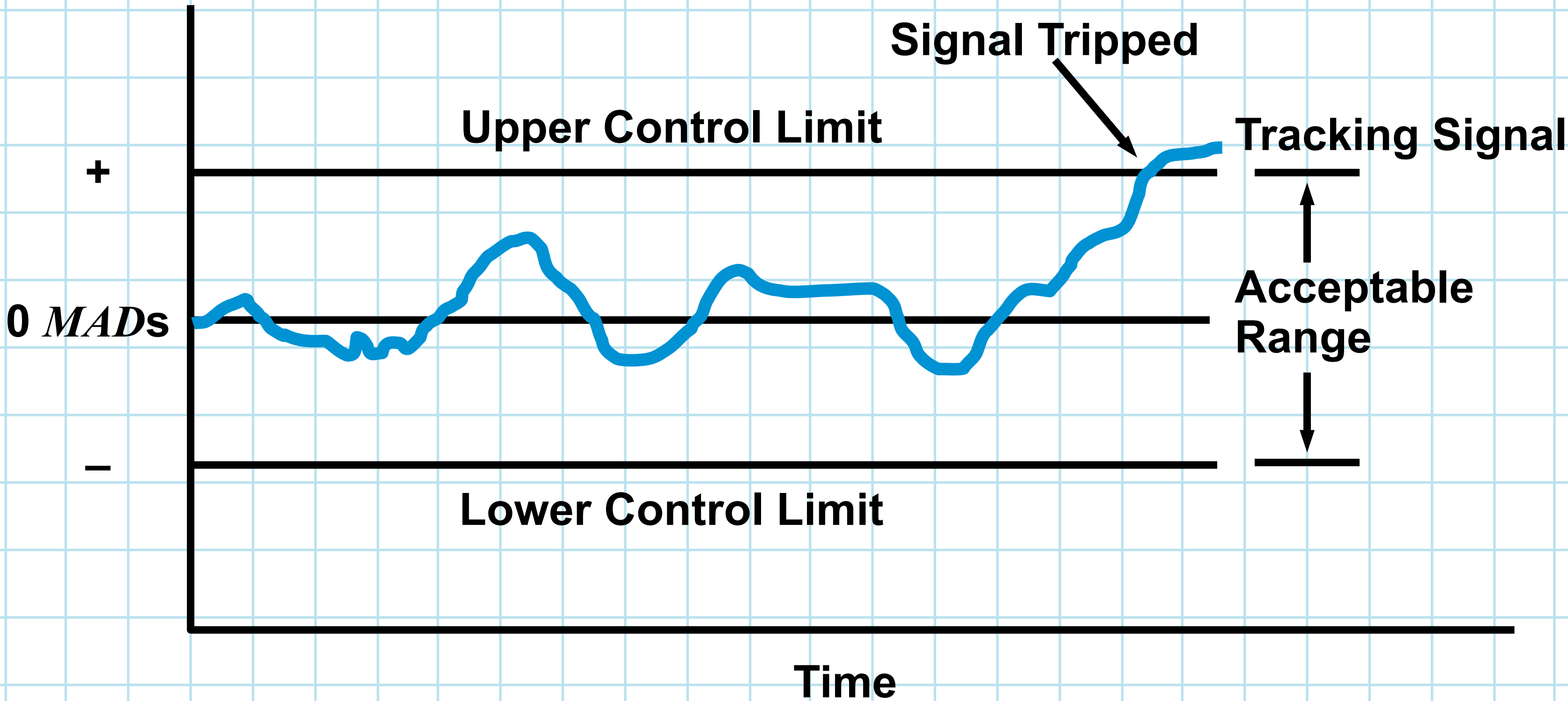


Figure 5.6

Monitoring and Controlling Forecasts

- **Positive tracking signals indicate demand is greater than forecast.**
- **Negative tracking signals indicate demand is less than forecast.**
- **Some variation is expected, but a good forecast will have about as much positive error as negative error.**
- **Problems are indicated when the signal trips either the upper or lower predetermined limits.**
- **This indicates there has been an unacceptable amount of variation.**
- **Limits should be reasonable and may vary from item to item.**

Kimball's Bakery

Quarterly sales of croissants (in thousands):

TIME PERIOD	FORECAST DEMAND	ACTUAL DEMAND	ERROR	RSFE	|FORECAST ERROR|	CUMULATIVE ERROR	MAD	TRACKING SIGNAL
1	100	90	–10	–10	10	10	10.0	–1
2	100	95	–5	–15	5	15	7.5	–2
3	100	115	+15	0	15	30	10.0	0
4	110	100	–10	–10	10	40	10.0	–1
5	110	125	+15	+5	15	55	11.0	+0.5
6	110	140	+30	+35	35	85	14.2	+2.5

$$\text{MAD} = \frac{\sum |\text{forecast error}|}{n} = \frac{85}{6} = 14.2$$

$$\text{Tracking signal} = \frac{\text{RSFE}}{\text{MAD}} = \frac{35}{14.2} = 2.5 MADs$$

Adaptive Smoothing

- ***Adaptive smoothing*** **is the computer monitoring of tracking signals and self-adjustment if a limit is tripped.**
- **In exponential smoothing, the values of α and β are adjusted when the computer detects an excessive amount of variation.**

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