<u>2.6</u> Higher-Order Derivatives

Exercise :

Find fourth derivatives ($f^{(4)}$) of $f(x) = 2x^3 - 4x^2 + 7x - 8$.

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Solution

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$$f'(x) = 6x^2 - 8x + 7$$
$$f''(x) = 12x - 8$$
$$f'''(x) = 12$$
$$f^4(x) = \mathbf{0}$$

<u>Example 1 :</u> If $y = \sin 2x$, find $\frac{d^3y}{dx^3}, \frac{d^4y}{dx^4}$ and $\frac{d^{12}y}{dx^{12}}$

<div align="center">

Solution

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$$\frac{dy}{dx} = 2\cos 2x$$

$$\frac{d^2y}{dx^2} = -4\sin 2x = -2^2 \sin 2x$$

$$\frac{d^3y}{dx^3} = \mathbf{-2^3 \cos 2x}$$

$$\frac{d^4y}{dx^4} = \mathbf{2^4 \sin 2x}$$

$$\frac{d^5y}{dx^5} = 2^5 \cos 2x$$

$$\vdots$$

$$\frac{d^{12}y}{dx^{12}} = \mathbf{2^{12} \sin 2x}$$

❖ Velocity and Acceleration السرعة و التسارع

Example 2 : An object moves along a coordinate line so that its position s satisfies $s = 2t^2 - 12t + 8$, where s is measured in centimeters and t in seconds.

(a) Determine the velocity of the object when $t = 1$ and when $t = 6$.

(b) when is the velocity 0 ?

(c) when is it positive ?

Solution

(a) $v = 4t - 12$,

$$v(1) = 4(1) - 12 = 4 - 12 = -8 \, cm/s$$
$$v(6) = 4(6) - 12 = 24 - 12 = 12 \, cm/s$$

(b) $v = 0$

$$4t - 12 = 0$$
$$4t = 12$$
$$t = 3 \, s$$

(c) $v > 0$

$$4t - 12 > 0$$
$$4t > 12$$
$$t > 3 \, s$$