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أ. محمد رسول الصباغ

Subject رياضة

تمارين تكامل

[١] اوجد التابع الاصلي لكل من التوابع التالية؟

1) $f(x) = \dfrac{1}{\sqrt{x\sqrt{x}}}$

$= \dfrac{1}{\sqrt{x \cdot x^{1/2}}} = \dfrac{1}{\sqrt{x^{3/2}}}$

$= \dfrac{1}{(x^{\frac{3}{2}})^{\frac{1}{2}}} = \dfrac{1}{x^{\frac{3}{4}}} = x^{-\frac{3}{4}}$

$F(x) = \dfrac{x^{-\frac{3}{4}+1}}{-\frac{3}{4}+1} = \dfrac{x^{\frac{1}{4}}}{\frac{1}{4}} = 4\sqrt[4]{x}$

2) $f(x) = \dfrac{1-x^2}{x\sqrt{x}}$

$= \dfrac{1-x^2}{x \cdot x^{\frac{1}{2}}} = \dfrac{1-x^2}{x^{\frac{3}{2}}}$

$= \dfrac{1}{x^{3/2}} - \dfrac{x^2}{x^{3/2}} = x^{-3/2} - x^{\frac{1}{2}}$

$F(x) = \dfrac{x^{-1/2}}{-\frac{1}{2}} - \dfrac{x^{3/2}}{\frac{3}{2}}$

$= -2\dfrac{1}{\sqrt{x}} - \dfrac{2}{3}\sqrt{x^3}$

3) $f(x) = \dfrac{x^3}{\sqrt[3]{x}}$

$= x^3 \cdot x^{-\frac{1}{3}} = x^{\frac{8}{3}}$

$F(x) = \dfrac{x^{\frac{11}{3}}}{\frac{11}{3}} = \dfrac{3}{11}\sqrt[3]{x^{11}}$

4) $f(x) = \sin^3 x \cdot \sin 2x$

$= \sin^3 x \cdot 2\sin x \cdot \cos x$

$= 2\underbrace{\cos x}_{g'} \cdot \underbrace{\sin^4 x}_{g}$

$F(x) = 2\dfrac{\sin^5 x}{5}$

5) $f(x) = 2x + 5 - \csc^2 x$

$\csc x = \dfrac{1}{\sin x} \quad \sec = \dfrac{1}{\cos x}$

$= 2x + 5 - \dfrac{1}{\sin^2 x}$

$F(x) = x^2 + 5x + \text{ct}x$

6) $f(x) = \dfrac{1}{\sqrt{x + x\sqrt{x}}}$

$= \dfrac{1}{\sqrt{x(1+\sqrt{x})}} = \dfrac{1}{\sqrt{x}\sqrt{1+\sqrt{x}}}$

$= \dfrac{1}{\sqrt{x}}(1+\sqrt{x})^{-\frac{1}{2}}$

$= 2\underbrace{\dfrac{1}{2\sqrt{x}}}_{g'}\underbrace{(1+\sqrt{x})}_{g}^{-\frac{1}{2}}$

$F(x) = 2\dfrac{(1+\sqrt{x})^{\frac{1}{2}}}{\frac{1}{2}} = 4\sqrt{1+\sqrt{x}}$

Subject

8) $f(x) = \dfrac{x^4+4}{x^2-4}$

$$\begin{array}{r|l} & x^2-4 \\ \hline x^2-4 & x^4+4 \\ & x^4-4x^2 \\ \hline & -4x^2+4 \\ & -4x^2-16 \\ \hline & 20 \end{array}$$

$f(x) = x^2-4+\dfrac{20}{x^2-4}$

$\dfrac{20}{(x-2)(x+2)} = \dfrac{A}{x-2} + \dfrac{B}{x+2}$

$20 = A(x+2) + B(x-2)$

$x = 2$: A نجد

$20 = 4A \Rightarrow A = 5$

$x = -2$: B نجد

$20 = -4B \Rightarrow B = -5$

ومنه

$f(x) = x^2-4+\dfrac{5}{x-2}-\dfrac{5}{x+2}$

$F(x) = \dfrac{x^3}{3} - 4x + 5\ln|x-2|$
$-5\ln|x+2|$

$= \dfrac{x^3}{3} - 4x + 5\ln\left|\dfrac{x-2}{x+2}\right|$

7) $f(x) = 2x\cos^2 x$

$= \cancel{2}x \cdot \dfrac{1+\cos 2x}{\cancel{2}}$

$= x + x\cos 2x$

$F(x) = \int_0^x f(t)\,dt$

$= \int_0^x (t + \cos 2t)$

$= \int_0^x t\,dt + \underbrace{\int_0^x t\cos 2t}_{G(x)}$ ---*

$G(x) = \int_0^x t\cos 2t\,dt$

$u = t \qquad u' = 1$

$v' = \cos 2t \qquad v = \frac{1}{2}\sin 2t$

$G(x) = \left[\frac{t}{2}\sin 2t\right]_0^x - \int_0^x \frac{1}{2}\sin 2t$

$= \frac{x}{2}\sin 2x + \frac{1}{4}\left[\cos 2t\right]_0^x$

$= \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x - \cancel{\frac{1}{4}}$

$= \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x$

نعوض في *

$F(x) = \left[\frac{t^2}{2}\right]_0^x + \frac{x}{2}\sin 2x +$
$\frac{1}{4}\cdot\cos 2x$

$= \frac{x^2}{2} + \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x$

$\cot x \to \frac{-1}{\sin^2 x}$

$\to -(1+\cot^2 x)$

$(\cot 3x)' = \frac{-3}{\sin^2 3x} = -3(1+\cot^2 3x)$

Subject

$= 1 + \frac{\cos^2 3x}{\sin^2 3x}$

$= 1 + \cot^2 3x$

$F(x) = -\frac{1}{3} \cot 3x$

١١) $f(x) = x\sqrt{x+1}$ [illegible]

$= x(x+1)^{\frac{1}{2}}$

$= (x+1-1)(x+1)^{\frac{1}{2}}$

$= (x+1)(x+1)^{\frac{1}{2}} - (x+1)^{\frac{1}{2}}$

$= (x+1)^{\frac{3}{2}} - (x+1)^{\frac{1}{2}}$

$F(x) = \frac{(x+1)^{\frac{5}{2}}}{\frac{5}{2}} - \frac{(x+1)^{\frac{3}{2}}}{\frac{3}{2}}$

$= \frac{2}{5}\sqrt{(x+1)^5} - \frac{2}{3}\sqrt{(x+1)^3}$

١٢) $f(x) = \sqrt[3]{x^2} + 5\sqrt{x}$

$= (x^2)^{\frac{1}{3}} + 5x^{\frac{1}{2}}$

$= x^{\frac{2}{3}} + 5x^{\frac{1}{2}}$

$F(x) = \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + 5\frac{x^{\frac{3}{2}}}{\frac{3}{2}}$

$= \frac{3}{5}\sqrt[3]{x^5} + \frac{10}{3}\sqrt{x^3}$

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٩) $f(x) = \frac{x^3+2}{x^2-1}$

$$\begin{array}{r|l} & x \\ x^2-1 & x^3+2 \\ & x^3-x \\ \hline & -x+2 \end{array}$$

$f(x) = x + \frac{-x+2}{x^2-1}$

$\frac{-x+2}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$

$-x+2 = A(x+1) + B(x-1)$

$x = 1$; A نجد

$1 = 2A \Rightarrow A = \frac{1}{2}$

$x = -1$ B نجد

$3 = -2B \Rightarrow B = \frac{-3}{2}$

$f(x) = x + \frac{\frac{1}{2}}{x-1} - \frac{\frac{3}{2}}{x+1}$

$= x + \frac{1}{2}\frac{1}{x-1} - \frac{3}{2}\frac{1}{x+1}$

$F(x) = \frac{x^2}{2} + \frac{1}{2}\ln|x-1| -$

$\frac{3}{2}\ln|x+1|$

١٥) $f(x) = \frac{1}{\sin^2 3x}$

$= \frac{\sin^2 3x + \cos^2 3x}{\sin^2 3x}$

$g' \sin(g) \xrightarrow{\text{تكامل}} \sin(g)$

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Subject

13) $f(x) = \frac{(1-x)^2}{x\sqrt{x}}$

$= \frac{1-2x-x^2}{x^{3/2}}$

$= \frac{1}{x^{\frac{3}{2}}} - \frac{2x}{x^{\frac{3}{2}}} - \frac{x^2}{x^{3/2}}$

$= x^{-3/2} - 2x.x^{-3/2} - x^2.x^{-3/2}$

$= x^{-\frac{3}{2}} - 2x^{-\frac{1}{2}} - x^{\frac{1}{2}}$

$F(x) = \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} - 2\frac{x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}}$

$= -2\frac{1}{\sqrt{x}} - 4\sqrt{x} - \frac{2}{3}\sqrt{x^3}$

14) $f(x) = \sin 4x . \cos 3x$

دساتير التحويل

$f(x) = \frac{1}{2}[\sin 7x + \sin x]$

$F(x) = \frac{1}{2}[\frac{-1}{7}\cos 7x - \cos x]$

$= \frac{-1}{14}\cos 7x - \frac{1}{2}\cos x$

15) $f(x) = \frac{(\ln x)^2}{x}$

$= \frac{1}{x}(\ln x)^2$

$F(x) = \frac{(\ln x)^3}{3}$

16) $f(x) = \frac{1}{x^2}\sin(\frac{1}{x})$

$= - \frac{-1}{x^2}\sin(\frac{1}{x})$

$F(x) = \cos(\frac{1}{x})$

17) $f(x) = \frac{x}{\sqrt{2x+1}}$

$= x(2x+1)^{-\frac{1}{2}}$

$F(x) = \int_0^x f(t)\,dt$

$= \int_0^x t(2t+1)^{-\frac{1}{2}}$

$u = t \qquad u' = 1$

$v' = (2t+1)^{-\frac{1}{2}} = \frac{1}{2}\, 2(2t+1)^{-\frac{1}{2}}$

$v = \frac{1}{2}\frac{(2t+1)^{\frac{1}{2}}}{\frac{1}{2}}$

$= (2t+1)^{\frac{1}{2}}$

$F(x) = [t\sqrt{2t+1}]_0^x - \int_0^x (2t+1)^{\frac{1}{2}}$

$= x\sqrt{2x+1} - \frac{1}{2}\int_0^x 2(2t+1)^{\frac{1}{2}}$

$= x\sqrt{2x+1} - \frac{1}{2}\left[\frac{(2t+1)^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^x$

$= x\sqrt{2x+1} - \frac{1}{3}\sqrt{(2x+1)^3}$

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Subject

١٨) $f(x) = \ln x$

$F(x) = \int_1^x \ln t$

$u = \ln t \qquad u' = \frac{1}{t}$

$v' = 1 \qquad v = t$

$F(x) = [t.\ln t]_1^x - \int_1^x 1 \, dt$

$= x.\ln x - [t]_1^x$

$= x.\ln x - x + 1$

$F(x) = x.\ln x - x$

١٩) $f(x) = \frac{\ln x}{x}$

$= \underbrace{\frac{1}{x}}_{g'} \underbrace{\ln x}_{g}$

$F(x) = \frac{\ln^2 x}{2}$

٢٠) $f(x) = \frac{1}{x.\ln x}$

$f(x) = \frac{\frac{1}{x} \ ) \ g'}{\ln x \ ) \ g}$

$F(x) = \ln|\ln x|$

٢) احسب كل من التكاملات التالية ؟

$i) I = \int_0^1 e^{\sqrt{x}} \, dx$

سنحل هذا التكامل بطريقة تغيير المتحول

بفرض $\sqrt{x} = t$

نشتق $\frac{1}{2\sqrt{x}} = \frac{dt}{dx} \Rightarrow$

$dx = 2\sqrt{x} \, dt = 2t \, dt$

نعوض في الفرضية

$x = 0 \Rightarrow t = 0$

$x = 1 \Rightarrow t = 1$

نعوض في التكامل

$\int_0^1 e^t . 2t . dt \Rightarrow$

$\int_0^1 2t e^t \, dt$

$u = 2t \qquad u' = 2$

$v' = e^t \qquad v = e^t$

$I = [2te^t]_0^1 - \int_0^1 2e^t$

$= 2e - 0 - 2[e^t]_0^1$

$= 2e - 2[e-1]$

$= 2e - 2e + 2$

$= 2$

$1 - \cos 2x = 2\sin^2 x$

$1 - \cos^2 x = \sin^2 x$

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Subject

4) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\cos x - \cos^3 x}\ dx$

$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\cos x(1-\cos^2 x)}$

$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{\cos x . \sin^2 x}$

$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\sin x| . \sqrt{\cos x}$

$= \int_{-\frac{\pi}{2}}^{0} |\sin x| . \cos x^{\frac{1}{2}} + \int_{0}^{\frac{\pi}{2}} |\sin x| \cos x^{\frac{1}{2}}$

$= \int_{-\frac{\pi}{2}}^{0} -\sin x \cos x^{\frac{1}{2}} + \int_{0}^{\frac{\pi}{2}} \sin x . \cos x^{\frac{1}{2}}$

$= \int_{-\frac{\pi}{2}}^{0} -\sin x \cos x - \int_{0}^{\frac{\pi}{2}} -\sin x . \cos x^{\frac{1}{2}}$

$= \left[\frac{\cos x^{\frac{3}{2}}}{\frac{3}{2}} \right]_{-\frac{\pi}{2}}^{0} - \left[\frac{\cos x^{\frac{3}{2}}}{\frac{3}{2}} \right]_{0}^{\frac{\pi}{2}}$

$= \frac{2}{3} \left[\sqrt{\cos^3 x} \right]_{-\frac{\pi}{2}}^{0} - \frac{2}{3} \left[\sqrt{\cos^3 x} \right]_{0}^{\frac{\pi}{2}}$

$= \frac{2}{3}(1-0) - \frac{2}{3}(0-1)$

$= \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$

2) $I = \int_{1}^{2} \sqrt[3]{1-x}\ dx$

$\int_{1}^{2} \sqrt[3]{1-x} = \int_{1}^{2} (1-x)^{\frac{1}{3}} dx$

$= -\int_{1}^{2} -(1-x)^{\frac{1}{3}} = -\left[\frac{(1-x)^{\frac{4}{3}}}{\frac{4}{3}} \right]_{1}^{2}$

$= -\frac{3}{4} \left[\sqrt[3]{(1-x)^4} \right]_{1}^{2}$

$= \frac{-3}{4}[1-0] = -\frac{3}{4}$

3) $I = \int_{1}^{4} \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}$

$= \int_{1}^{4} \frac{1}{\sqrt{x}} (1+\sqrt{x})^{\frac{1}{2}}$

$= 2\int_{1}^{4} \underbrace{\frac{1}{2\sqrt{x}}}_{g'} (\underbrace{1+\sqrt{x}}_{g})^{\frac{1}{2}} dx$

$= 2\left[\frac{(1+\sqrt{x})^{\frac{3}{2}}}{\frac{3}{2}} \right]_{1}^{4}$

$= \frac{4}{3} \left[\sqrt{(1+\sqrt{x})^3} \right]_{1}^{4}$

$= \frac{4}{3} [\sqrt{27} - \sqrt{8}]$

$= \frac{4}{3} [3\sqrt{3} - 2\sqrt{2}]$

KORAN

Subject

$$\int_0^{\frac{\pi}{4}} \underbrace{\frac{1}{\cos^2 x}}_{g'} \cdot \underbrace{\tan^{3\,)\,n} x}_{g}$$

$$\left[\frac{\tan^4 x}{4}\right]_0^{\frac{\pi}{4}}$$

$$= \frac{1}{4} - 0 = \frac{1}{4}$$

$$8) \int_0^{\ln 2} \frac{1}{1+e^x}\, dx$$

(نضرب بسط ومقام) ب e^{-x}

$$\int_0^{\ln 2} \frac{e^{-x}}{e^{-x}+1} = -\int_0^{\ln 2} \frac{-e^{-x}}{e^{-x}+1}$$

$$= -\left[\ln(e^{-x}+1)\right]_0^{\ln 2}$$

$$= -\left[\ln(e^{-\ln 2}+1) - \ln(2)\right]$$

$$= -\left[\ln\left(\frac{1}{2}+1\right) - \ln 2\right]$$

$$= -\left[\ln\frac{3}{2} - \ln 2\right]$$

$$= -\ln\frac{3}{4}$$

$$\boxed{e^{-\ln a} = \frac{1}{a}}$$

$$5) \int_1^{e^3} \frac{1}{x\sqrt{1+\ln x}}\, dx$$

$$= \int_1^{e^3} \frac{1}{x}(1+\ln x)^{\frac{1}{2}}\, dx$$

$$= \left[\frac{(1+\ln x)^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^{e^3}$$

$$= \frac{2}{3}\left[\sqrt{(1+\ln x)^3}\right]_1^{e^3}$$

$$= \frac{2}{3}\left[\sqrt{64} - 1\right] = \frac{14}{3}$$

$$6) \int_0^1 \frac{e^x}{1+e^x}\, dx$$

$$= \left[\ln(1+e^x)\right]_0^1$$

$$= \ln(1+e) - \ln 2$$

$$= \ln\left(\frac{1+e}{2}\right)$$

$$7) \int_0^{\frac{\pi}{4}} \frac{\sin^3 x}{\cos^5 x}$$

$$\int_0^{\frac{\pi}{4}} \frac{\sin^3 x}{\cos^2 x \cdot \cos^3 x}\, dx$$

$$= \int_0^{\frac{\pi}{4}} \frac{1}{\cos^2 x} \cdot \frac{\sin^3 x}{\cos^3 x}\, dx$$

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Subject

12) $\int_0^{\frac{\pi}{2}} \sin^2 x \, dx$

$= \int_0^{\frac{\pi}{2}} (\frac{1}{2} - \frac{1}{2}\cos 2x)$

$= [\frac{1}{2}x - \frac{1}{4}\sin 2x]_0^{\frac{\pi}{2}}$

$= \frac{1}{2}\frac{\pi}{2} - \frac{1}{4}\sin\pi - 0 = \frac{\pi}{4}$

13) $\int_0^9 \frac{x-1}{1+\sqrt{x}} \, dx$

$= \int_0^9 \frac{\cancel{(\sqrt{x}+1)}(\sqrt{x}-1)}{\cancel{1+\sqrt{x}}}$

$= \int_0^9 (\sqrt{x}-1) \, dx$

$= \int_0^9 (x)^{\frac{1}{2}} - 1$

$= [\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - x]_0^9$

$= [\frac{2}{3}\sqrt{x^3} - x]_0^9 = [\frac{2}{3}\sqrt{729} - 9] - 0$

$= \frac{2}{3} \times 27 - 9 = 18 - 9$

$= 9$

$\int g' \cdot \cos g = \sin g$

$\int g' \sin g = -\cos g$

9) $\int_1^2 x \cdot \cos(x^2) \, dx$

$= \int_1^2 \frac{1}{2} 2x \cos(x^2) \, dx$

$= \frac{1}{2}[\sin(x^2)]_1^2$

$= \frac{1}{2}[\sin(4) - \sin(1)]$

10) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin x \cdot \cos^2 x$

$= -\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \underbrace{-\sin x}_{g'} \cdot \underbrace{\cos^{2=n} x}_{g}$

$= -[\frac{\cos^3 x}{3}]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$

$= -[0 - \frac{\frac{3\sqrt{3}}{8}}{3}] = \frac{\sqrt{3}}{8}$

11) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\cos x}{\sqrt{\sin x}} \, dx$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos x \cdot \sin x^{-\frac{1}{2}}$

$= [\cos x]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \frac{1}{2} - \frac{\sqrt{3}}{2}$

$= \frac{1-\sqrt{3}}{2}$

Subject

أوجد التابع الأصلي لكل من التوابع الآتية:

1) $f(x)=\frac{\sqrt[3]{\ln x-5}}{x}$

$$=\underbrace{\frac{1}{x}}_{g'}\underbrace{(\ln x-5)}_{g}^{\frac{1}{3}}$$

$$F(x)=\frac{(\ln x-5)^{\frac{4}{3}}}{\frac{4}{3}}$$

$$F(x)=\frac{3}{4}\sqrt[3]{(\ln x-5)^4}$$

2) $f(x)=\frac{x^2-4}{x+2}$

$$=\frac{(x-2)(x+2)}{x+2}=x-2$$

$$F(x)=\frac{x^2}{2}-2x$$

3) $f(x)=\frac{\cos^4 x-\sin^4 x}{\sqrt{1+\cos 4x}}$

$$=\frac{\cos^4 x-\sin^4 x}{\sqrt{2\cos^2 2x}}$$

$$=\frac{(\cos^2 x+\cos^2 x)(\cos^2 x-\sin^2 x)}{\sqrt{2}\cos 2x}$$

$$=\frac{1(\cos 2x)}{\sqrt{2}\cos 2x}=\frac{1}{\sqrt{2}}$$

$$F(x)=\frac{1}{\sqrt{2}}x$$

4) $f(x)=\frac{\tan^2\sqrt{x}}{\sqrt{x}}$

$$=\frac{-1+1+\tan^2\sqrt{x}}{\sqrt{x}}$$

$$=\frac{1}{\sqrt{x}}(-1+1+\tan^2\sqrt{x})$$

$$=\frac{-1}{\sqrt{x}}+\frac{1}{\sqrt{x}}(1+\tan^2\sqrt{x})$$

$$=-x^{-\frac{1}{2}}+2\frac{1}{2\sqrt{x}}(1+\tan^2\sqrt{x})$$

$$F(x)=\frac{-x^{\frac{1}{2}}}{\frac{1}{2}}+2\tan\sqrt{x}$$

$$=-2\sqrt{x}+2\tan\sqrt{x}$$

$$\tan(g(x))\underset{\xleftarrow[\text{تكامل}]{}}{\xrightarrow{\text{مشتق}}}g'(x)(1+\tan^2 g(x))$$

5) $f(x)=(x+2)^5(x+3)$

$$=(x+2)^5(x+2+1)$$

$$=(x+2)^5(x+2)+(x+2)^5$$

$$=(x+2)^6+(x+2)^5$$

$$F(x)=\frac{(x+2)^7}{7}+\frac{(x+2)^6}{6}$$

Subject

$$f(x) = \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{\sqrt{x}(\sqrt{x}-1)}$$

$$= \frac{\sqrt{x}+1}{\sqrt{x}} = 1 + \frac{1}{\sqrt{x}}$$

$$= 1 + x^{-\frac{1}{2}}$$

$$F(x) = x + \frac{x^{\frac{1}{2}}}{\frac{1}{2}}$$

$$= x + 2\sqrt{x}$$

10) $f(x) = \dfrac{e^{-\tan x}}{\cos^2 x}$

$$= \frac{1}{\cos^2 x} e^{-\tan x}$$

$$\int g' e^{g} = e^{g}$$

$$= -\frac{-1}{\cos^2 x} e^{-\tan x}$$

$$F(x) = -e^{-\tan x}$$

11) $f(x) = \dfrac{\cos 5x}{e^{\sin 5x}}$

$$= \cos 5x \, e^{-\sin 5x}$$

$$= \frac{1}{-5}(-5\cos 5x) e^{-\sin 5x}$$

$$F(x) = -\frac{1}{5} e^{-\sin 5x}$$

6) $f = \dfrac{\tan\sqrt{x}}{\sqrt{x}}$

$$= \frac{1}{\sqrt{x}} \tan\sqrt{x}$$

$$= \frac{1}{\sqrt{x}} \cdot \frac{\sin\sqrt{x}}{\cos\sqrt{x}}$$

$$= \frac{\frac{1}{\sqrt{x}}\sin\sqrt{x}}{\cos\sqrt{x}}$$

$$= -2\,\frac{\frac{1}{-2\sqrt{x}}\sin\sqrt{x}}{\cos\sqrt{x}}$$

$$F(x) = -2\ln|\cos\sqrt{x}|$$

7) $f = x^4 \cdot e^{-\ln x}$

$$= x^4 \cdot \frac{1}{x} = x^3$$

$$F(x) = \frac{x^4}{4}$$

8) $f = \dfrac{1}{x\sqrt{\ln x}}$

$$= \frac{1}{x}(\ln x)^{-\frac{1}{2}}$$

$$F(x) = \frac{(\ln x)^{\frac{1}{2}}}{\frac{1}{2}}$$

$$= 2\sqrt{\ln x}$$

9) $f = \dfrac{x-1}{x-\sqrt{x}}$

Subject

15) $f(x) = \sqrt{e^{2x-4}}$

$= \sqrt{(e^{x-2})^2}$

$= e^{x-2}$

$F(x) = e^{x-2}$

16) $f(x) = \underbrace{(1+e^x)}_{g}\underbrace{e^x}_{g'}$

$F(x) = \frac{(1+e^x)^2}{2}$

17) $f(x) = \frac{e^{3x}}{\ln 3 + e^{3x}}$

$= \frac{1}{3} \frac{3e^{3x}}{\ln 3 + e^{3x}}$

$F(x) = \frac{1}{3} \ln(\ln 3 + e^{3x})$

18) $f(x) = \frac{1}{\frac{1}{\sin 2x} - \cot 2x}$

$= \frac{1}{\frac{1}{\sin 2x} - \frac{\cos 2x}{\sin 2x}}$

$= \frac{1}{\frac{1-\cos 2x}{\sin 2x}} = \frac{\sin 2x}{1-\cos 2x}$

$= \frac{\cancel{2}\cancel{\sin x}\cdot\cos x}{\cancel{2}\sin^{\cancel{2}} x} = \frac{\cos x}{\sin x}$

$F(x) = \ln|\sin x|$

12) $f(x) = \frac{\cos^3 x}{\sin^2 x}$

$= \cos^3 x \cdot \sin^{-2} x$

$= \cos x \cdot \cos^2 x \cdot \sin^{-2} x$

$= \cos x(1-\sin^2 x)\sin^{-2} x$

$= \cos x \cdot \sin^{-2} x - \cos x \cdot \sin^0 x$

$= \cos x \cdot \sin^{-2} x - \cos x$

$F(x) = \frac{\sin^{-1} x}{-1} - \sin x$

$= \frac{-1}{\sin x} - \sin x$

13) $f(x) = \frac{1-\tan x}{1+\tan x}$

$= \frac{1-\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}}$

$= \frac{\frac{\cos x - \sin x}{\cancel{\cos x}}}{\frac{\cos x + \sin x}{\cancel{\cos x}}}$

$= \frac{\cos x - \sin x}{\cos x + \sin x}$

$F(x) = \ln|\cos x + \sin x|$

14) $f(x) = \sqrt{e^{2x} + e^{-2x} + 2}$

$= \sqrt{(e^{x} + e^{-x})^2}$

$= e^{x} + e^{-x}$

$F(x) = e^{x} - e^{-x}$

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أ. محمد رسول الصباغ ٠٩٣٤١٣١١٠٩

Subject

19) $f(x) = \sqrt{\sin x} \cdot \cos^3 x$

$= \sin x^{\frac{1}{2}} \cdot \cos^3 x$

$= \sin^{\frac{1}{2}} x \cdot \cos x \cdot \cos^2 x$

$= \sin^{\frac{1}{2}} x \cos x (1 - \sin^2 x)$

$= \underbrace{\sin^{\frac{1}{2}} x}_{g} \underbrace{\cos x}_{g'} - \underbrace{\cos x}_{g'} \underbrace{\sin^{\frac{5}{2}} x}_{g}$

$F(x) = \frac{\sin^{\frac{3}{2}} x}{\frac{3}{2}} - \frac{\sin^{\frac{7}{2}} x}{\frac{7}{2}}$

$= \frac{2}{3} \sqrt{\sin^3 x} - \frac{2}{7} \sqrt{\sin^7 x}$

20) $f(x) = (\cos x - \sin 2x)^2$

$= \cos^2 x - 2\cos x \sin 2x + \sin^2 2x$

$= \frac{1}{2} + \frac{1}{2}\cos 2x - 2\cos x . 2\sin x \cos x + \frac{1}{2} - \frac{1}{2}\cos 4x$

$= 1 + \frac{1}{2}\cos 2x - \frac{1}{2}\cos 4x - 4\sin x \cos^2 x$

$F(x) = x + \frac{1}{4}\sin 2x - \frac{1}{8}\sin 4x + 4\frac{\cos^3 x}{3}$

21) $f(x) = \frac{\ln^2 x - 1}{x \ln x + x}$

$= \frac{(\ln x - 1)\cancel{(\ln x + 1)}}{x\cancel{(\ln x + 1)}}$

$= \frac{\ln x - 1}{x} = \frac{\ln x}{x} - \frac{1}{x}$

$f(x) = \frac{1}{x} \ln x - \frac{1}{x}$

$F(x) = \frac{\ln^2 x}{2} - \ln|x|$

22) $f(x) = \frac{e^x}{\sqrt{1 - e^x}}$

$= e^x (1 - e^x)^{-\frac{1}{2}}$

$= - \cdot - \underbrace{e^x}_{g'} (\underbrace{1 - e^x}_{g})^{-\frac{1}{2}}$

$F = -\frac{(1 - e^x)^{\frac{1}{2}}}{\frac{1}{2}}$

$= -2\sqrt{1 - e^x}$

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23) $f(x) = (x+1)^2\sqrt{x}$

$= (x^2+2x+1)\sqrt{x}$

$= x^2 \cdot x^{\frac12} + 2x \cdot x^{\frac12} + x^{\frac12}$

$= x^{\frac52} + 2x^{\frac32} + x^{\frac12}$

$F(x) = \dfrac{x^{\frac72}}{\frac72} + 2\dfrac{x^{\frac52}}{\frac52} + \dfrac{x^{\frac32}}{\frac32}$

$= \frac27\sqrt{x^7} + \frac45\sqrt{x^5} + \frac23\sqrt{x^3}$

24) $f(x) = \dfrac{\tan 2x}{\sqrt{\cos 2x}}$

$= \tan 2x\,(\cos 2x)^{-\frac12}$

$= \dfrac{\sin 2x}{\cos 2x}(\cos 2x)^{-\frac12}$

$= \sin 2x\,(\cos 2x)^{-1}(\cos 2x)^{-\frac12}$

$= \sin 2x\,(\cos 2x)^{-\frac32}$

$= \frac{1}{-2}(-2\sin 2x)(\cos 2x)^{-\frac32}$

$F(x) = -\frac12 \dfrac{(\cos 2x)^{-\frac12}}{-\frac12}$

$= \dfrac{1}{\sqrt{\cos 2x}}$

25) $f = \sin^2 x(1+\cot^2 x)$

$= \sin^2 x\left(1 + \dfrac{\cos^2 x}{\sin^2 x}\right)$

$= \sin^2 x + \cos^2 x$

$= 1$

ومنه $F(x) = x$

26) $f(x) = x^r \cdot \ln x$

$x \qquad r \in Q \setminus \{-1\}$

$F(x) = \int_1^x f(t)\,dt$

$= \int_1^x t^r \ln t$

$u = \ln t \qquad u' = \frac{1}{t}$

$v' = t^r \qquad v = \dfrac{t^{r+1}}{r+1}$

$F(x) = \left[\dfrac{t^{r+1}}{r+1}\ln t\right]_1^x - \int_1^x \frac{1}{r+1}t^r$

$= \dfrac{x^{r+1}}{r+1}\ln x - \dfrac{1}{(r+1)^2}\left[t^{r+1}\right]_1^x$

$= \dfrac{x^{r+1}}{r+1}\ln x - \dfrac{1}{(r+1)^2}(x^{r+1}-1)$

$F(x) = \dfrac{x^{r+1}}{r+1}\ln x - \dfrac{x^{r+1}}{(r+1)^2}$