مدونة المناهج السعودية

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الموقع التعليمي لجميع المراحل الدراسية

في المملكة العربية السعودية

Math 100

Mada Altiary

خطة مقرر رياضيات ١

SCAN
ME

Relating Absolute Value and Distance

› DEFINITION 1 Absolute Value

$$|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

$|-3| = -(-3) = 3$

$|4| = 4$

[*Note:* $-x$ is positive if x is negative.]

Example: Write without the absolute value:

(A) $|\pi - 3| = \pi - 3$

(B) $|3 - \pi| = -(3 - \pi) = \pi - 3$

Note:
$\pi = 3.14$ So
$3.14 - 3 = 0.14$
positive

Remark:

$$|b - a| = |a - b|$$

› DEFINITION 2 Distance Between Points *A* and *B*

Let A and B be two points on a real number line with coordinates a and b, respectively. The **distance between A and B** is given by

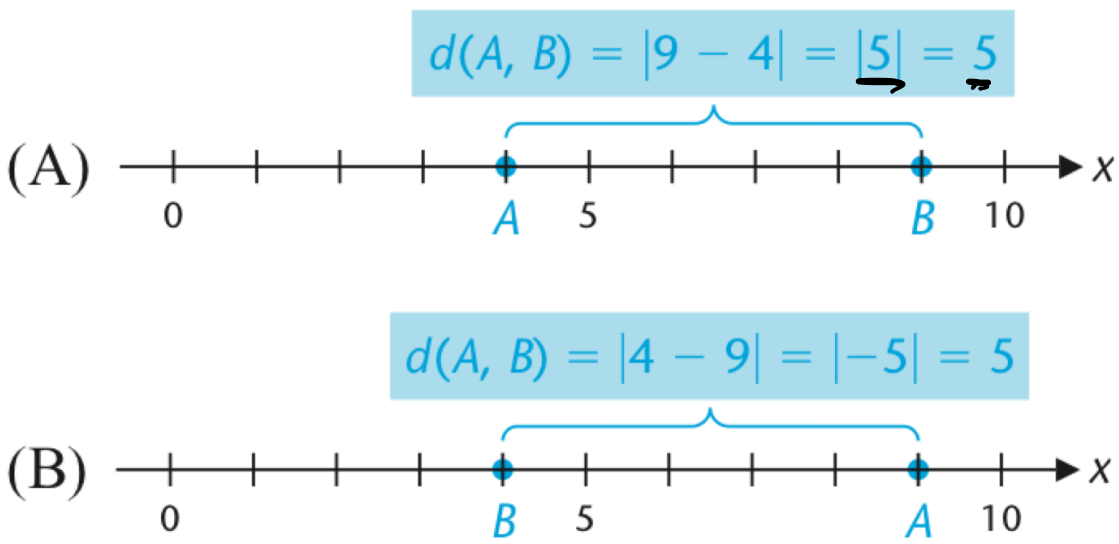
$$d(A, B) = |b - a|$$

This distance is also called the **length of the line segment** joining A and B.

Example: Find the distance between given points

(A) $a = 4, b = 9$ (B) $a = 9, b = 4$ (C) $a = 0, b = 6$

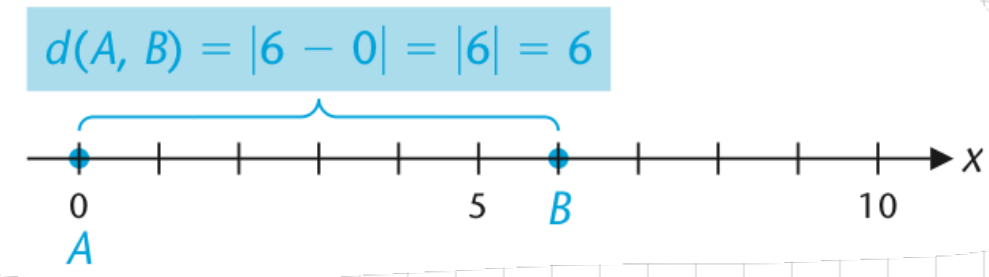
Solution:



Remark:

$d(A, B) = d(B, A)$

(C)



Remark:

$$d(O, B) = |b - 0| = |b|$$

O ← نقطة الأصل

Example: Express each verbal statement as an absolute value equation or inequality.

(A) x is 4 units from 2.

(B) y is less than 3 units form -5.

(C) t is no more than 5 units from 7.

(D) w is no less than 2 units form -1.

SOLUTIONS

(A) $d(x, 2) = |x - 2| = 4$

(B) $d(y, -5) = |y + 5| < 3$

(C) $d(t, 7) = |t - 7| \leq 5$

(D) $d(w, -1) = |w + 1| \geq 2$

Solving Absolute Value Equations and Inequalities

Steps for Solving Absolute Value Equation:

- Isolate the absolute value
- Analyze the equation " Is it possible to solve?"
- Solve the equation
- Check your answer

ملاحظة: إذا كانت المعادلة تساوي عدد سالب فالمعادلة مستحيلة الحل

Example: Solve the following Equations

1) $|x-3|=5$

Step 1 : ✅

Step 2 : ✅

Step 3 :

$x-3=5$ or $-(x-3)=5$

بتطبيق تعريف الداله المطلقه

$x=5+3$ or $-x+3=5$

$x=8$ or $-x=5-3$

$-x=2$

$x=-2$

Step 4:

$x=8$

$|8-3|=5$

$|5|=5$

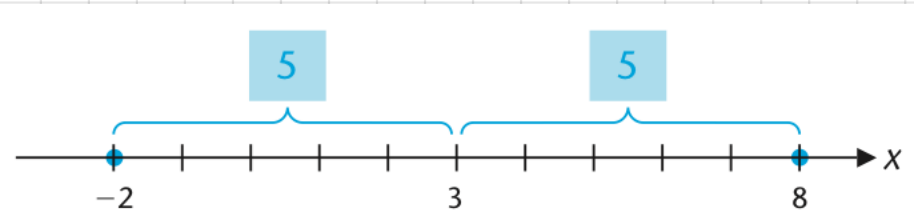
$5=5$

$x=$

$\therefore x=\{-2,8\}$

يسمى هذا النوع من الأقواس رمز المجموعة
Set notation

التمثيل البياني:



2) $|3x-7|+7=2$

Step 1: $|3x-7|=2-7$

$|3x-7|=-5$

Step 2: No Solution or $\emptyset$

H.W: Solve

$|x+1|=0$

3) $|3x-7|+7=9$

Step 1: $|3x-7|=9-7$

$|3x-7|=2$

Step 2: ✅

Step 3: $3x-7=2$ or $-(3x-7)=2$

$3x=2+7$ or $-3x+7=2$

$3x=9$ or $-3x=2-7$

$x=3$ or $-3x=-5$

$x=5/3$

Step 4:

$x=3$

$|3.3-7|=2$

$|9-7|=2$

$|2|=2$

$2=2$

$x=5/3$

$|3.\frac{5}{3}-7|=2$

$|5-7|=2$

$|-2|=2$

$2=2$

$\therefore x=\{3, 5/3\}$

Steps for Solving Absolute Value Inequalities:

- Isolate the absolute value
- Analyze the Inequality " Is it possible to solve?"
- Solve the absolute value inequality
- Check your answer

ملاحظة: اذا كانت المتراجحة اقل من الصفر تكون مستحيلة الحل

Example: Solve the following Inequalities

1) $|x-3| < 5$

Step 1 : ✅

Step 2 : ✅

Step 3 : $x-3 < 5$ and $-(x-3) < 5$

$x < 5+3$ and $-x+3 < 5$

$x < 8$ and $-x < 5-3$

$-x < 2$

$x > -2$

ملاحظة: عند ضرب المتراجحة بعدد سالب نعكس إشارة المتراجحة

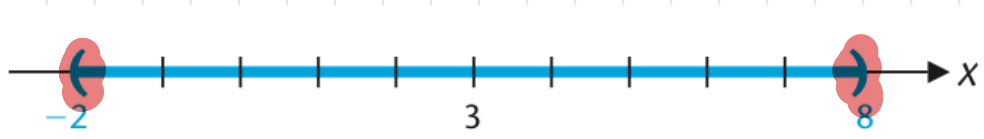
Step 4 :

$x < 8$	$x > -2$
$\|7-3\| < 5$	$\|-1-3\| > -2$
$\|4\| < 5$	$\|-4\| > -2$
$4 < 5$ works!	$4 > -2$ works!

$\therefore x = (-2, 8)$

يسمى هذا النوع من الأقواس رمز الفترة

Interval notation



جميع الأعداد مابين ٨ و ٢- تحقق المتراجحة

2) $0 < |x-3| < 5$

$0 < |x-3|$

or

$|x-3| > 0$

If $x = 3$ then

$|3-3| > 0$

$|0| > 0$

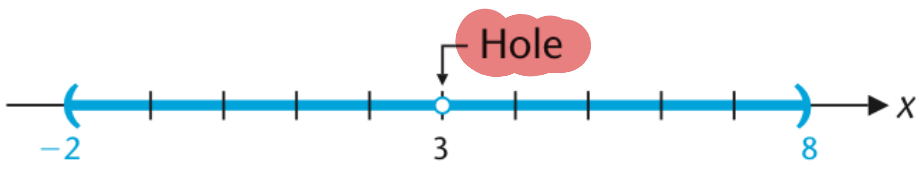
$0 > 0$ does not work

$|x-3| < 5$

تم حلها في المثال السابق
وكانت النتيجة كالتالي

$x = (-2, 8)$

So $x = (-2, 3) \cup (3, 8)$



H.W: Solve

1) $0 < |x+2| < 6$

2) $|x+2| \geq 0$

3) $|x-3| > 5$

Step 1 : ✅

Step 2: ✅

Step 3: $x - 3 > 5$ or $-(x-3) < 5$

$x > 5 + 3$ or $-x + 3 < 5$

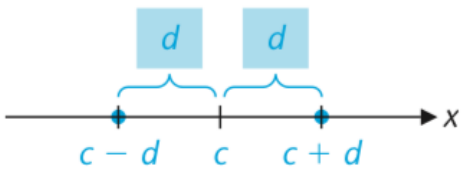
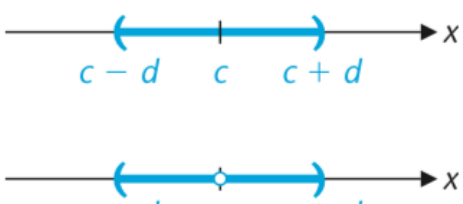
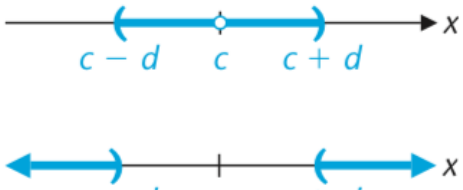
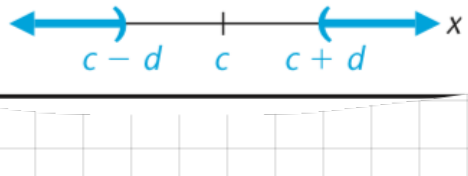
$x > 8$ or $-x < 5 - 3$

$-x < 2$

$x < 2$

Step 4 :

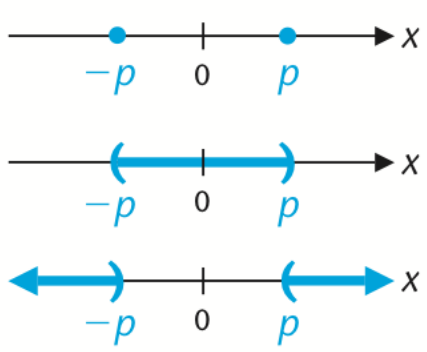
$\therefore x = (-\infty, -2) \cup (8, \infty)$

Form ($d > 0$)	Geometric interpretation	Solution	Graph
$\lvert x - c\rvert = d$	Distance between x and c is equal to d.	$\{c - d, c + d\}$ → Set notation	
$\lvert x - c\rvert < d$	Distance between x and c is less than d.	$(c - d, c + d)$ Interval notation.	
$0 < \lvert x - c\rvert < d$	Distance between x and c is less than d, but $x \neq c$.	$(c - d, c) \cup (c, c + d)$	
$\lvert x - c\rvert > d$	Distance between x and c is greater than d.	$(-\infty, c - d) \cup (c + d, \infty)$	

THEOREM 2 Properties of Equations and Inequalities Involving $|x|$

For $p > 0$:

1. $|x| = p$ is equivalent to $x = p$ or $x = -p$.
2. $|x| < p$ is equivalent to $-p < x < p$.
3. $|x| > p$ is equivalent to $x < -p$ or $x > p$.



THEOREM 3 Properties of Equations and Inequalities Involving $|ax + b|$

For $p > 0$:

1. $|ax + b| = p$ is equivalent to $ax + b = p$ or $ax + b = -p$.
2. $|ax + b| < p$ is equivalent to $-p < ax + b < p$.
3. $|ax + b| > p$ is equivalent to $ax + b < -p$ or $ax + b > p$.

Continuous: Solving Absolute Value Problems

Example: Solve each equation or inequality

A) $|3x+5| = 4$

B) $|x| < 5$

C) $|2x-1| < 3$

D) $|7-3x| \leq 2$

Solution: Step 1 and Step 2 are done.

Step 3:

By applying definition ← **(A). $|3x+5| = 4$** → By applying theorem3

$3x+5 = 4$ or $-(3x+5) = 4$

$3x = 4-5$ or $-3x-5 = 4$

$3x = -1$ or $-3x = 9$

$x = -\frac{1}{3}$ or $x = -3$

$3x+5 = 4$ or $3x+5 = -4$

$3x = 4-5$ or $3x = -9$

or $x = -3$

Step 4: ✅ check!!

$\therefore x = \{-\frac{1}{3}, -3\}$

(B) $|x| < 5$

$x < 5$ and $-x < 5$

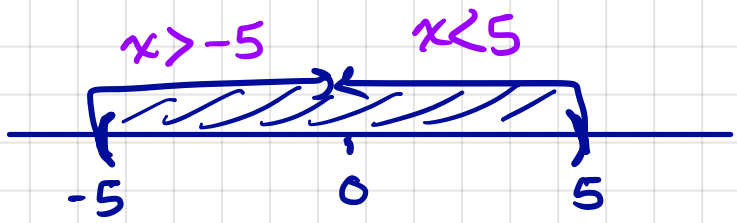
$x > -5$

$-5 < x < 5$

$\therefore x = (-5, 5)$

Step 4: ✅ check!

$\therefore x = (-5, 5)$



(C) $|2x-1| < 3$

$2x-1 < 3$ and $-(2x-1) < 3$

$2x < 4$ and $-2x+1 < 3$

$x < 2$ and $-2x < 2$

$-x < 1$

$x > -1$

Step 4 ✅

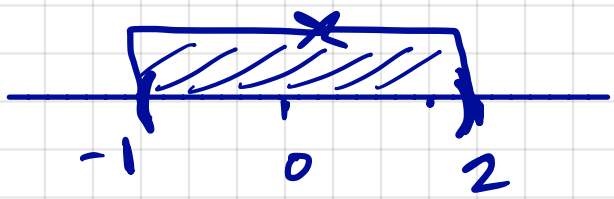
$\therefore x = (-1, 2)$

$-3 < 2x-1 < 3$

$-3+1 < 2x < 3+1$

$-2 < 2x < 4$

$-1 < x < 2$



H.W

D) $|7-3x| \leq 2$

$-7 \quad -7 \quad -7$

$-2 \leq 7-3x \leq 2$

$-9 \leq -3x \leq -5$

$3 \geqslant x \geqslant \frac{5}{3}$

$\frac{5}{3} \leqslant x \leqslant 3$

$\therefore x = [\frac{5}{3}, 3]$

ماذا لاحظتي في B,C,D

Example: Solve the following :

(A) $|x| > 3$

$x > 3$ or $x < -3$

$(-\infty, -3) \cup (3, \infty)$

(B) $|2x - 1| \geq 3$

$2x - 1 \geq 3$ or $2x - 1 \leq -3$

$2x \geq 3 + 1$ or $2x \leq -3 + 1$

$2x \geq 4$ or $2x \leq -2$

$x \geq 2$ or $x \leq -1$

ماذا لاحظتي؟

$\therefore x = (-\infty, -1] \cup [2, \infty)$

C) $|7 - 3x| > 2$

$7 - 3x > 2$ or $7 - 3x < -2$

$-3x > 2 - 7$ or $-3x < -2 - 7$

$-3x > -5$ or $-3x < -9$

$x < \frac{5}{3}$ or $x > 3$

$\therefore x = (-\infty, \frac{5}{3}) \cup (3, \infty)$

Example: Solve $|x+4| = 3x - 8$

$x+4 = 3x-8$ or $-(x+4) = 3x-8$

$4+8 = 3x-x$ or $-x-4 = 3x-8$

$12 = 2x$ or $-4+8 = 3x+x$

$6 = x$ or $4 = 4x$

$1 = x$

ملاحظه : في هذة المسألة لايمكن تطبيق نظرية خصائص القيمة المطلقه وذلك لوجود x في الطرف الاخر وهذا يعني لا نعلم ما اذا كانت قيمة x موجبة او سالبة.

Check:

$x = 6$

$|6+4| = 3(6)-8$

$|10| = 18-8$

$10 = 10$ ✓

$x = 1$

$|1+4| = 3(1)-8$

$|5| = -5$

$5 \neq -5$

H.W: Solve $|3x-4| = x+5$

$\therefore x = \{6\}$

Absolute Value and Radical Inequalities

Definition: For any real number

$$\sqrt{x^2} = \begin{cases} -x & \text{if } x<0 \\ x & \text{if } x \geq 0 \end{cases}$$

For example :

$$\sqrt{(2)^2} = \sqrt{(-2)^2} = \sqrt{4} = 2$$

Remark: $\sqrt{x^2} = |x|$

Example: Solve $\sqrt{(x-2)^2} \leq 5$

Solution:

$$|x-2| \leq 5$$

$$-5 \leq x-2 \leq 5$$

$$-5+2 \leq x \leq 5+2$$

$$-3 \leq x \leq 7$$

$$\therefore x = [-3, 7]$$

H.W: Solve $\sqrt{(x+2)^2} < 3$

ملاحظة: الأسئلة (هاي لايت اخضر) متعلقة بدرس الأعداد المركبه