Student ID #: [redacted]

Question 3 (30 Points):

Q.3 30

Given the vector current density $\mathbf{J} = 10\rho^2 z\,\mathbf{a}_\rho - 4\rho\cos^2\phi\,\mathbf{a}_\phi$ mA/m^2: (*a*) find the current density at $P(\rho = 3, \phi = 30^\circ, z = 2)$; (*b*) determine the total current flowing outward through the circular band $\rho = 3$, $0 < \phi < 2\pi$, $2 < z < 2.8$.

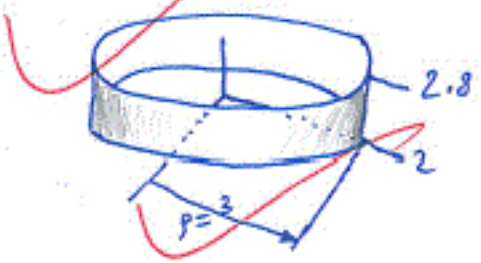
a) $\bar{J} = 10\rho^2 z\,\bar{a}_\rho - 4\rho\cos^2\phi\,\bar{a}_\phi$ mA/m2

$J = 10(3)^2(2)\,\bar{a}_\rho - 4(3)\cos^2(30)\,\bar{a}_\phi$ mA/m^2

$= 180\,\bar{a}_\rho - 9\,\bar{a}_\phi$ mA/m^2

b) $I = \int_S \bar{J}\cdot d\bar{S}$ $\quad d\bar{S} = \rho\,d\phi\,dz\,\bar{a}_\rho$

$\bar{J} = 10\rho^2 z\,\bar{a}_\rho - 4\rho\cos^2\phi\,\bar{a}_\phi$



$$I = 3\int_2^{2.8}\int_0^{2\pi} 10\times10^{-3}\rho^2 z\,d\phi\,dz$$

$$= 3^3\times10^{-2}\left[\phi\right]_0^{2\pi}\left[\frac{z^2}{2}\right]_2^{2.8} = 3.26\ \text{A}$$

$$(z-z_B)=\frac{z_A-z_B}{y_A-y_B}(y-y_B)$$

A (2, 18, 6)
B (1, 8, 5)

$$(z-5)=\frac{6-5}{18-8}(y-8)$$

$$\boxed{y=10x-2}$$

$$z-5=\frac{1}{10}(y-8)$$

$$z=\frac{y}{10}-\frac{8}{10}-5$$

$$W=-Q\int \bar{E}\cdot d\bar{L}$$

$$z=$$

$$\boxed{10z = y-58}$$

$$W=-6\left[\int_1^2 -8xy\,dx+\int_8^{18}-4x^2\,dy+\int_5^6 dz\right]$$

$$=-6\left[\int_1^2 -80x^2+16x\,dx-40x^2\,dx+\int_5^6 dz\right]$$

$$=-6\left[\left[\frac{-80x^2}{2}+\frac{16x^2}{2}-\frac{40x^3}{3}\right]_1^2+\left[z\right]_5^6\right]$$

$$=1530\ \text{J}$$

Student ID #: [redacted]

Question 2 (40 Points):

40

Apply the **concept** of **electric potential** and **find** the **work done** in both the following cases; if an electrostatic field is given by $\bar{E} = -8xy\,\bar{a}_x - 4x^2\,\bar{a}_y + \bar{a}_z$ V/m. The charge of 6C is to be moved from B(1, 8, 5) to A(2, 18, 6). The cases are:

a) **The path selected is $y = 3x^2 + z,\ z = x + 4$**
b) **The straight line from B to A.**

A(2, 18, 6)
B(1, 8, 5)

$dy = (6x+1)\,dx$

$\bar{E} = -8xy\,\bar{a}_x - 4x^2\,\bar{a}_y + \bar{a}_z$

$W = -Q\int \bar{E}\cdot d\bar{L}$

$= -6\left[\int_1^2 -8xy\,dx - \int_8^{18} 4x^2\,dy + \int_5^6 dz\right]$

$= -6\left[\int_1^2 -8x(3x^2+x+4) - 4x^2(6x+1)\,dx + \int_5^6 dz\right]$

$= -6\left[\int_1^2 (-24x^3 - 8x^2 - 32x - 24x^3 + 4x^2)\,dx + \int_5^6 dz\right]$

$= -6\left[\int_1^2 (-48x^3 - 12x^2 - 32x)\,dx + \int_5^6 dz\right]$

$= -6\left[\left[\frac{-48x^4}{4} - \frac{12x^3}{3} - \frac{32x^2}{2}\right]_1^2 + [z]_5^6\right] =$

a) $= -6[-256 + 1] = \boxed{1530}$

$y - y_B = \frac{y_A - y_B}{x_A - x_B}(x - x_B)$

$(y - 8) = \frac{18 - 8}{2 - 1}(x - 1)$

$y - 8 = 10(x - 1)$

$y = 10x - 2$

$(x - x_B) = \frac{x_A - x_B}{z_A - z_B}(z - z_B)$

$(x - 1) = \frac{2 - 1}{6 - 5}(z - 5)$

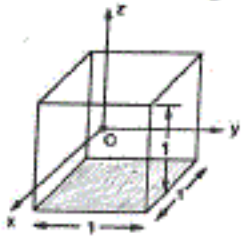
P18. See the solution in Book.

Question 1 (30 Points):

Apply the divergence theorem and find the divergence of the vector function:

$$\bar{A} = x^2\bar{a}_x + (xy)^2\,\bar{a}_y + 24(xyz)^2\,\bar{a}_z$$

Evaluate the volume integral of $\nabla.\bar{A}$ through the volume of a unit cube centered at the origin.



30

$$\nabla.\bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$= 2x + 2(xy)(x) + 48(xyz)(xy)$$

$$= 2x + 2x^2y + 48(x^2y^2z) \Rightarrow \int_v \nabla.\bar{A}\,dx\,dy\,dz$$

$$= \int_0^1 (2x + 2x^2y + 48x^2y^2z)\,dx$$

$$= \left[\frac{2x^2}{2} + \frac{2yx^3}{3} + \frac{48y^2z\,x^3}{3}\right]_0^1$$

for 0 → 1 limit this is correct! but incomplete

$$\int_0^1\int_0^1\int_0^1 2x\,dx\,dy\,dz + \int_0^1\int_0^1\int_0^1 2x^2y\,dx\,dy\,dz + \int_0^1\int_0^1\int_0^1 48x^2y^2z\,dx\,dy\,dz$$

$$= \left[\frac{2x^2}{2}\right]_0^1 \times 1 \times 1 + 2\left[\frac{x^3}{3}\right]_0^1\left[\frac{y^2}{2}\right]_0^1 \times 1 + 48\left[\frac{x^3}{3}\right]_0^1\left[\frac{y^3}{3}\right]_0^1\left[\frac{z^2}{2}\right]_0^1$$

$$= 1 + \frac{1}{3} + \frac{8}{3} = 4$$

Islamic University Faculty of Engineering Department of Electrical Engineering		الجامعة الإسلامية كلية الهندسة قسم الهندسة الكهربائية

EE 282 – ELECTROMAGNETIC FIELD THEORY

Fall Semester 2017 - 2018

Mid Term # 02

Date: December 07th, 2017; Duration: 70 minutes

Student's Full Name: [redacted]

Student ID #: [redacted] **Section #:1052** **Signature:** [redacted]

Instructions:

- **Write your student ID number on the top of each page.**
- **Write the solution in the space provided under each question.**
- **Show all the steps of your calculations.**
- **Bring your own Calculators, use of mobile phone as calculators and sharing of calculators are strictly NOT allowed.**

Qucstion No.	Points Assigned	Points Awarded
1. [CO_3, PI_5_23, SO_5]	30	20
2. [CO_4, PI_5_22, SO_5]	40	40
3. [CO_5, PI_5_24, SO_5]	30	30
Total	100	90/100

Instructor's Full Name	Dr. Khawaja Bilal Mahmood
Signature	