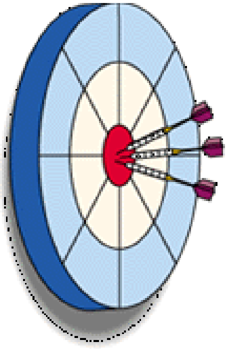
Chapter one

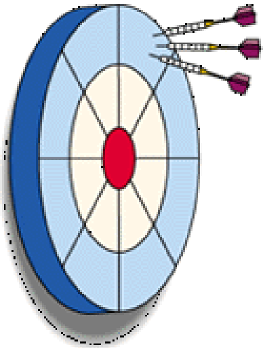
Units and Measurements

Measurement and Uncertainty; Significant Figures

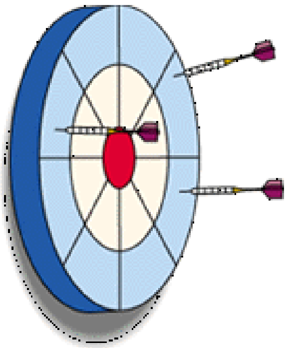
NO measurement is absolutely *precise* OR *accurate*



Good accuracy
Good precision



Poor accuracy
Good precision



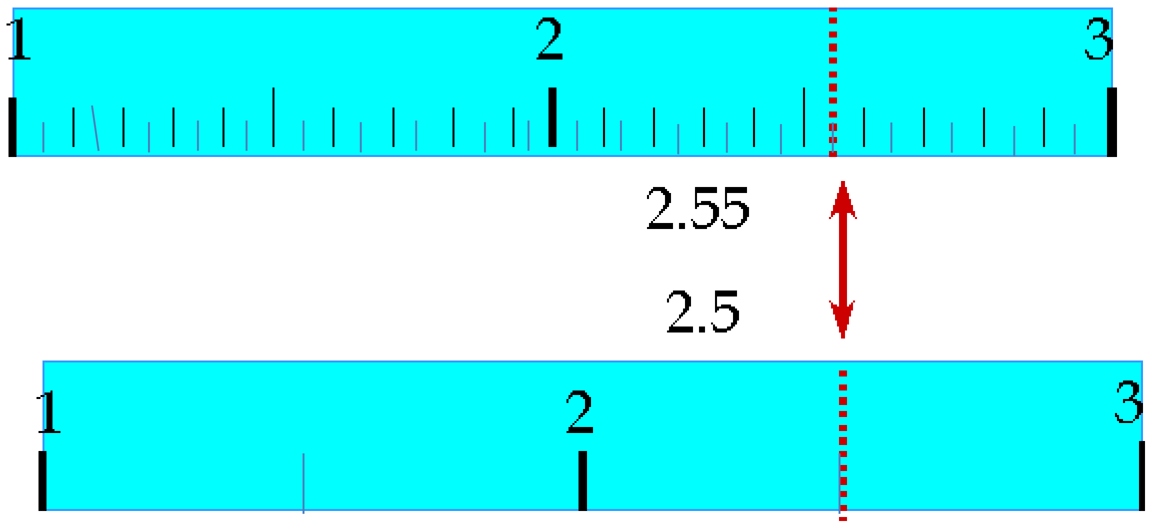
Poor accuracy
Poor precision

Precision refers to the repeatability of a measurement using a given instrument. Accuracy refers to how close a measurement is to the true value

Measurement and Uncertainty; Significant Figures

Main sources of uncertainty (errors):

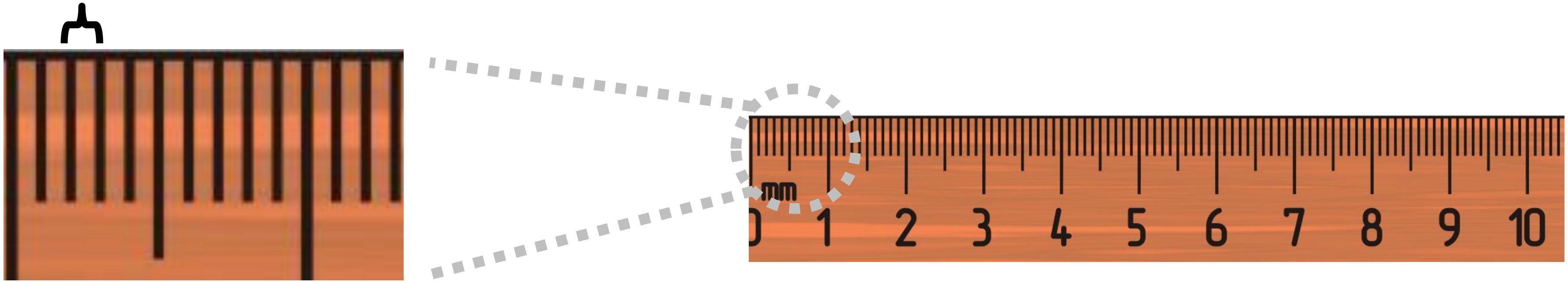
- Human errors:
- Limited Instrument accuracy (*systematic error*)



Relatively accurate ruler with least reading ~ 0.05 unit

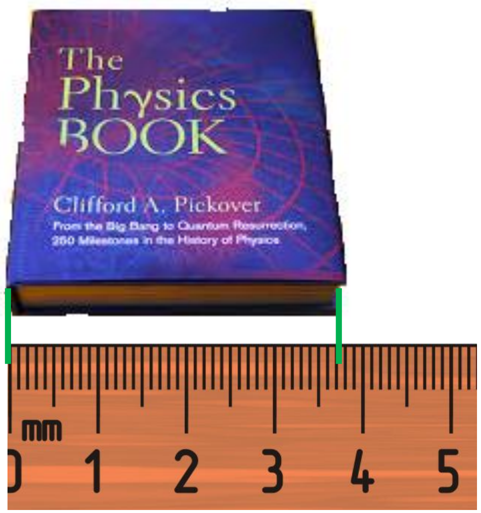
Less accurate ruler. Its least reading ~ 0.5 unit

Smallest division = 1 mm = 0.1 cm



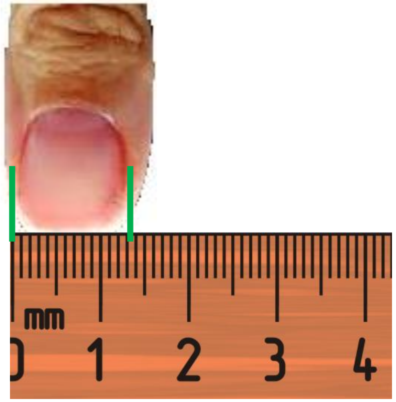
The ruler is ***precise*** to within 0.1 cm,

⇒ **estimated uncertainty (error) = ± 0.1 cm**



The tiny book is **3.7 ± 0.1 cm** wide

⇒ its <u>*true*</u> width likely lies between **3.8 and 3.6 cm**



Width of thumbnail = **1.3 ± 0.1 cm**

⇒ it lies between **1.4 and 1.2 cm**

Therefore measurement result is expressed as:

(Result ± Error) unit

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(Result ± Error) unit

$$\text{The percent uncertainty} = \frac{\text{Error}}{\text{Result}} \times 100\ \%$$

Example 1:

Calculate the percent uncertainty in the measurement:
L = 20.2 ± 0.4 cm

$$\text{The Percent Uncertainty (P.U.)} = \frac{\text{Error}}{\text{Result}} \times 100\ \%$$

$$Error = 0.4cm \ ; \ result = 20.2\ cm$$

$$\therefore \text{P.U.} = \frac{0.4cm}{20.2\ cm} \times 100\ \% = 1.9801\% \approx 2\%$$

Example 2:

What is the percent uncertainty in the measurement of an area A:
$A = 2.03\ \text{m}^2$

Result = $2.03\ \text{m}^2$ (with ***two decimal places***)

⇒In this case the ***error*** is taken as the smallest number with ***two decimal places.***

⇒ Error = $0.01\ \text{m}^2$

$$\text{The Percent Uncertainty (P.U.)} = \frac{\text{Error}}{\text{Result}} \times 100\ \%$$

$$\therefore\ \text{P.U.} = \frac{0.01\text{m}^2}{2.03\text{m}^2} \times 100\ \% \approx 0.5\%$$

Significant figure

The number of reliably known digits in a number

For example: 2 cm, 2.0 cm, & 2.00 cm are mathematically the same but experimentally different: *They have different significant figures*.

Counting rules

Rule 1: **All nonzero digits (1, 2, 3, …, 7, 8, 9) are significant figures**

Q. determine the number of significant figures in:
11 and **235.68**

Ans. $\underset{\uparrow}{1}\underset{\uparrow}{1}$ Has *two* significant figures

$\underset{\uparrow}{2}\underset{\uparrow}{3}\underset{\uparrow}{5}.\underset{\uparrow}{6}\underset{\uparrow}{8}$ Has *five* significant figures

Rule 2:

Zeros

Trailing zeros (*to the right of a number*)

Number *with* decimal point

⇒ **Count**

$\underset{\uparrow}{1}\underset{\uparrow}{0}.\underset{\uparrow}{0}\underset{\uparrow}{0}$

$\underset{\uparrow}{6}\underset{\uparrow}{3}\underset{\uparrow}{0}.$

$\underset{\uparrow}{1}\underset{\uparrow}{7}.\underset{\uparrow}{3}\underset{\uparrow}{0}\underset{\uparrow}{0}$

Number *without* decimal point

⇒ **Don’t count**

$\underset{\uparrow}{1}\underset{\times}{0}$

$\underset{\uparrow}{6}\underset{\uparrow}{3}\underset{\times}{0}$

$\underset{\uparrow}{5}\underset{\uparrow}{4}\underset{\times}{0}\underset{\times}{0}\underset{\times}{0}\underset{\times}{0}$

Zeros in-between digits

⇒ **Count**

$\underset{\uparrow}{2}\underset{\uparrow}{0}\underset{\uparrow}{3}\underset{\uparrow}{3}$

$\underset{\uparrow}{1}\underset{\uparrow}{0}\underset{\uparrow}{0}\underset{\uparrow}{1}$

$\underset{\times}{0}\underset{\uparrow}{1}\underset{\uparrow}{0}\underset{\uparrow}{6}.\underset{\uparrow}{2}\underset{\uparrow}{2}$

Leading zeros (*to the left of a number*)

⇒ **Don’t count**

$\underset{\times}{0}\underset{\times}{0}\underset{\uparrow}{9}\underset{\uparrow}{1}$

$\underset{\times}{0}.\underset{\uparrow}{1}$

$\underset{\times}{0}.\underset{\times}{0}\underset{\times}{0}\underset{\uparrow}{5}\underset{\uparrow}{1}$

Trailing zeros: *special case*

- A whole number (*without decimal point*) preceded by the word ***about*** ***(approximately, etc …)*** ⇒ **don't count zeros**

About $\underset{\uparrow}{8}\underset{\times}{0}$ cm

Implying ±10 cm uncertainty

About $\underset{\uparrow}{1}\underset{\uparrow}{3}\underset{\times}{0}\underset{\times}{0}$ km

Implying ±100 km uncertainty

- A whole number (*without decimal point*) preceded by the word ***precisely*** ***(accurately, etc …)*** ⇒ **Count zeros**

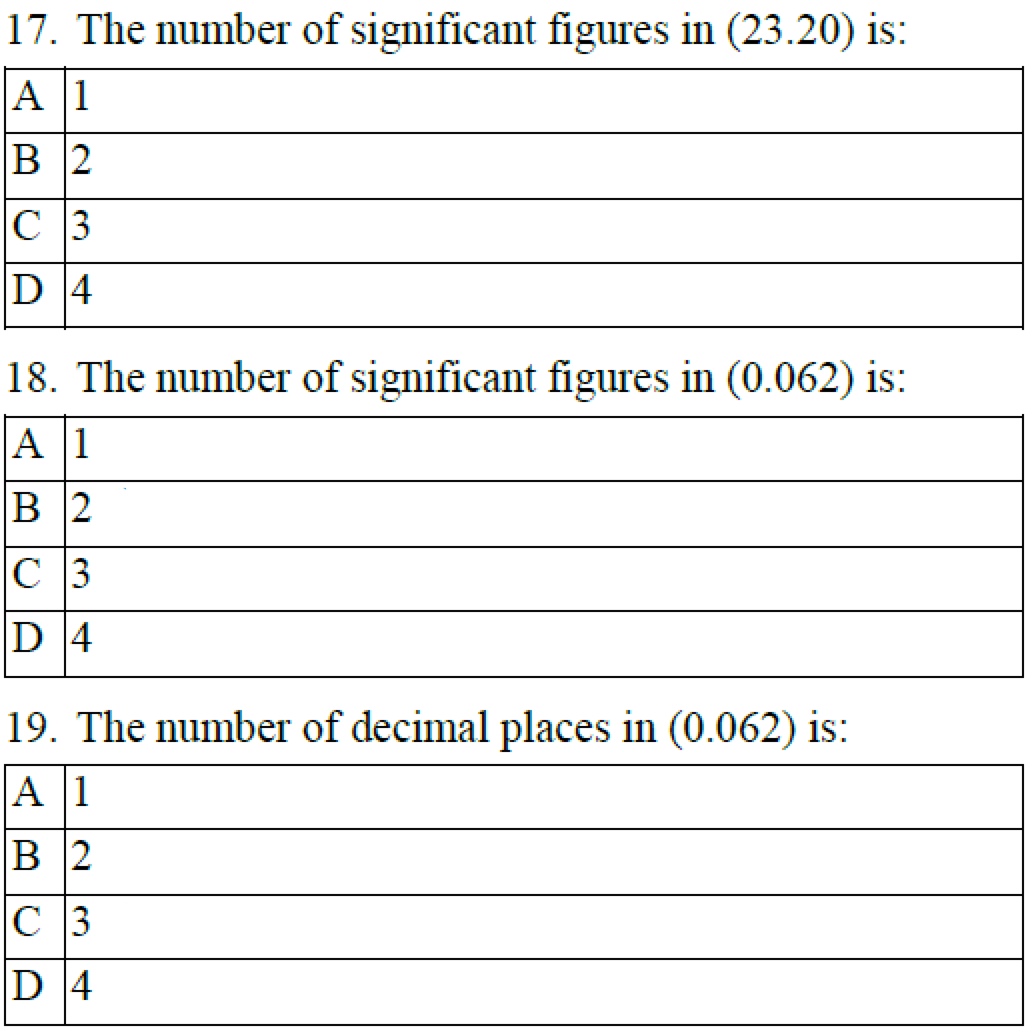
precisely $\underset{\uparrow}{8}\underset{\uparrow}{0}$ cm

Implying ±1 cm uncertainty

precisely $\underset{\uparrow}{1}\underset{\uparrow}{3}\underset{\uparrow}{0}\underset{\uparrow}{0}$ km

Implying ±1 km uncertainty

Check your understanding



17. The number of significant figures in (23.20) is:

A	1
B	2
C	3
D	4

18. The number of significant figures in (0.062) is:

A	1
B	2
C	3
D	4

19. The number of decimal places in (0.062) is:

A	1
B	2
C	3
D	4

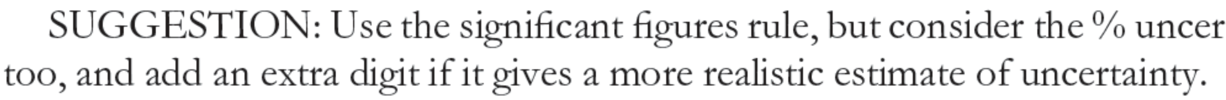
Significant figure: *mathematical operations*

Carry out intermediate calculations without rounding. Only round the final answer (outcome) according to the following rules:

1. Multiplications and divisions

Final result presented with ***significant figures*** similar to that for the number with ***least significant figure*** used in the operation.

$$11.3\,\text{cm}\times\underset{\uparrow}{2}.\underset{\uparrow}{0}\,\text{cm}=22.6\,\text{cm}^2\xrightarrow{\text{Round}}\underset{\uparrow}{2}\underset{\uparrow}{3}\,\text{cm}^2$$



SUGGESTION: Use the significant figures rule, but consider the % uncertainty too, and add an extra digit if it gives a more realistic estimate of uncertainty.

See page 6 in your book for more details

2. Addition and subtraction

Final result presented with ***decimal places*** similar to that for the number with ***least decimal places*** used in the operation.

$$9.300\,\text{cm}+0.\underset{\uparrow}{0}\underset{\uparrow}{1}\,\text{cm}=9.310\,\text{cm}\xrightarrow{\text{Round}}9.\underset{\uparrow}{3}\underset{\uparrow}{1}\,\text{cm}$$

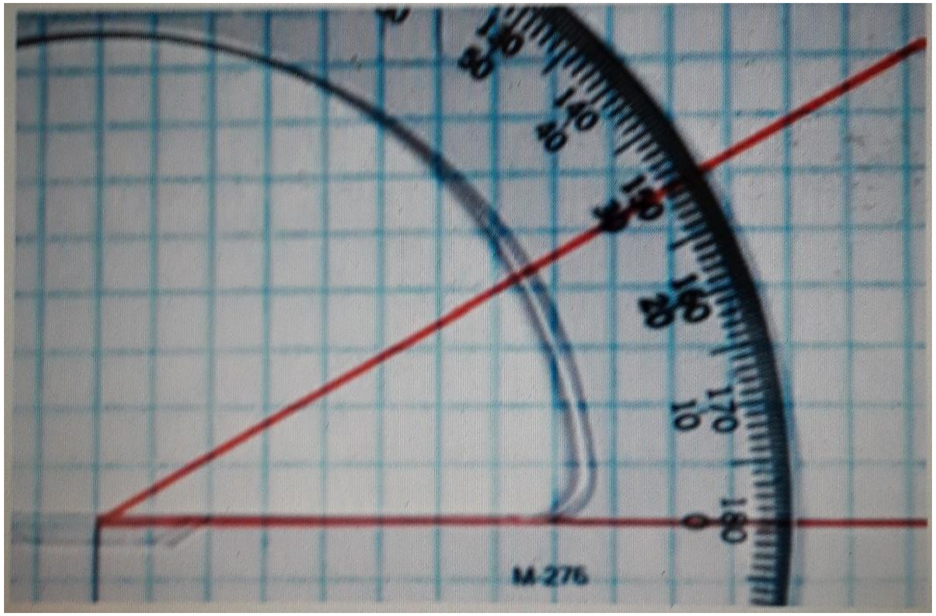
Conceptual Example 1.2

Using protractor, you measure an angle to be 30°.

(a) How many significant figures should you quote in this measurement.

(1° ⇒ we write 30° not 30.0°)

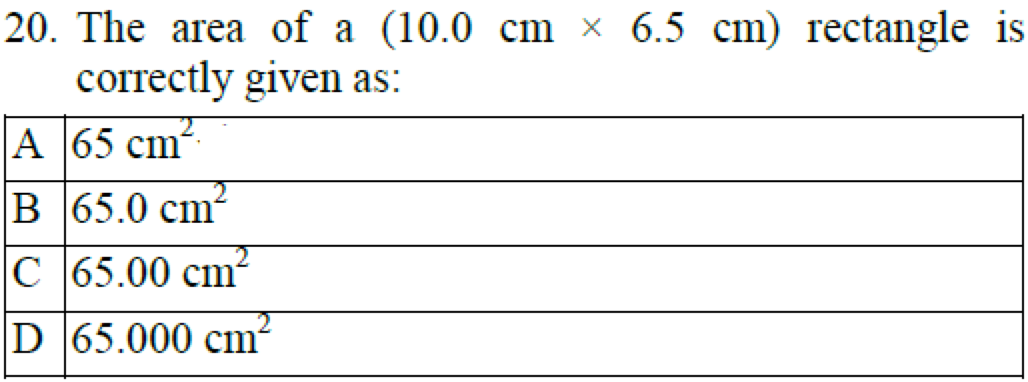
(b) Use a calculator to find the cosine of the angle you measure.



Answer:

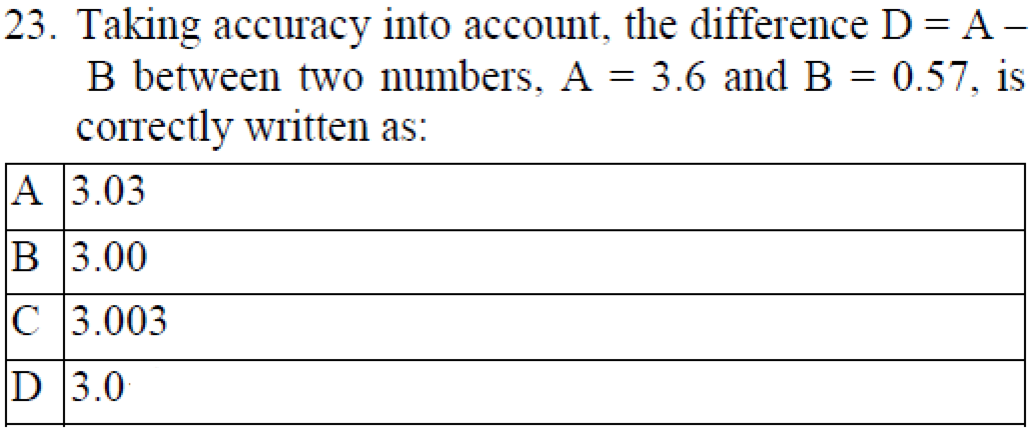
A. 0.86602540378443864676372317075294
B. 0.8660
C. 0.866
D. 0.87
E. 0.9
F. 1

Check your understanding



20. The area of a (10.0 cm × 6.5 cm) rectangle is correctly given as:

A	65 cm^2
B	65.0 cm^2
C	65.00 cm^2
D	65.000 cm^2



23. Taking accuracy into account, the difference D = A – B between two numbers, A = 3.6 and B = 0.57, is correctly written as:

A	3.03
B	3.00
C	3.003
D	3.0

Powers of 10

(Scientific Notation)

- Common to express very large or very small numbers using powers of 10 notation.
- Examples: **39,600 = 3.96×10^4**

  (moved decimal 4 places to left)

  0.0021 = 2.1×10^{-3}

  (moved decimal 3 places to right)
- Useful for controlling significant figures:

$$39600 \equiv \underset{\uparrow}{3}.\underset{\uparrow}{9}\underset{\uparrow}{6}\underset{\uparrow}{0} \times 10^4 = \underset{\uparrow}{3}.\underset{\uparrow}{9}\underset{\uparrow}{6}\underset{\uparrow}{0}\underset{\uparrow}{0}\underset{\uparrow}{0}\underset{\uparrow}{0} \times 10^4 \approx \underset{\uparrow}{4} \times 10^4$$

Units, Standards, SI System

- All measured physical quantities have units.
- Units are ***VITAL*** in physics!!
- The **SI** **system of units**:

  SI = **“Systéme International”** (French)

  More commonly called the “**MKS system**” (meter-kilogram-second) or more simply, “**the metric system**”

SI or MKS System

- Defined in terms of **standards (a standard ≡ *one unit of a physical quantity*)** for length, mass, time, … .
- **Length unit: Meter (m)** (kilometer = km = 1000 m)
  - **Standard meter**. Newest definition in terms of speed of light ≡ Length of path traveled by light in vacuum in (1/299,792,458) of a second!
- **Time unit: Second (s)**
  - **Standard second**. Newest definition ≡ time required for 9,192,631,770 oscillations of radiation emitted by cesium atoms!
  - An earlier definition is terms of the **solar day: (1 sec = 1/86400 of the solar day)**
- **Mass unit: Kilogram (kg)** (kilogram = kg = 1000 g)
  - **Standard kg**. A particular platinum-iridium cylinder whose mass is defined as exactly 1 kg

SI Base Quantities and Units

	Quantity	Unit	Unit Abbreviation
1.	Length	meter	m
2.	Time	second	s
3.	Mass	kilogram	kg
4.	Electric current	ampere	A
5.	Temperature	kelvin	K
6.	Amount of substance	mole	mol
7.	Luminous intensity	candela	cd

SI Derived Quantities and Units

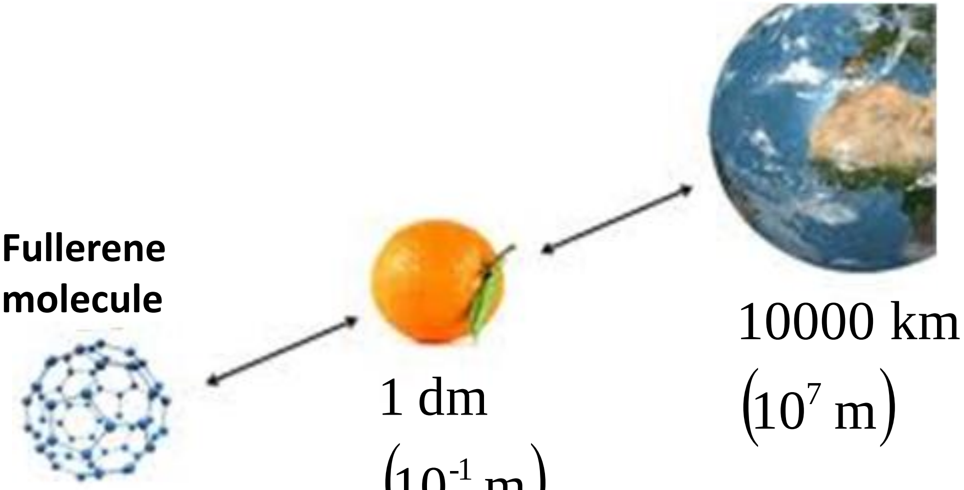
All physical quantities are *defined* in terms of the *base quantities*

Example: Derived units for *speed, acceleration and force:*

$$\text{Speed (m/s)} = \frac{\text{Distance (m)}}{\text{Time (s)}} \qquad \text{Acceleration (m/s}^2\text{)} = \frac{\Delta\ \text{velocity (m/s)}}{\text{Time (s)}}$$

$$\text{Force (Newton, N)} = \text{Mass (kg)} \times \text{Acceleration (m/s}^2\text{)}$$

Larger & smaller units defined from SI standards by powers of 10 & Greek prefixes

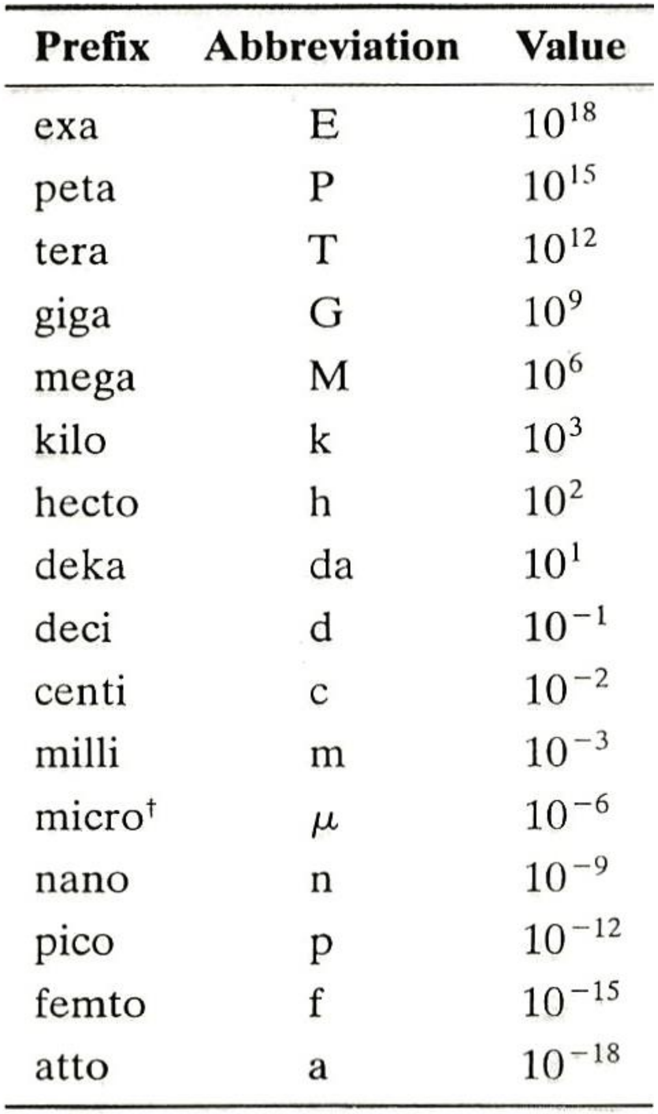


Fullerene molecule

1 nm $(10^{-9}\ \mathrm{m})$

1 dm $(10^{-1}\ \mathrm{m})$

10000 km $(10^{7}\ \mathrm{m})$



Prefix	Abbreviation	Value
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
hecto	h	10^{2}
deka	da	10^{1}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro[†]	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}

Other Systems of Units

- **CGS** (centimeter-gram-second) system
  - **Centimeter** = 0.01 meter
  - **Gram** = 0.001 kilogram
- **British** (foot-pound-second) system
  - Our "everyday life" system of units
  - Still used in some countries like USA

Converting Units

Suppose you are to convert 15 US Dollar ($) into Saudi Ryal (SR) :

1st Find conversion factor: 1 $ = 3.75 SR

2nd
$$\frac{1\cancel{\$}}{1\cancel{\$}} = \frac{3.75\text{SR}}{1\$} \Rightarrow 1 = 3.75\,\frac{\text{SR}}{\$}$$

3rd
$$15\$ = 15\$\times 1 = 15\cancel{\$}\times 3.75\,\frac{\text{SR}}{\cancel{\$}} = 56.25\text{SR}$$

Likewise convert 21.5 inches (in) into cm :

1st Conversion factor: 1 in = 2.54 cm

2nd
$$\frac{1\cancel{\text{in}}}{1\cancel{\text{in}}} = \frac{2.54\,\text{cm}}{1\,\text{in}} \Rightarrow 1 = 2.54\,\frac{\text{cm}}{\text{in}}$$

3rd
$$21.5\,\text{in} = 21.5\,\text{in}\times 1 = 21.5\cancel{\text{in}}\times 2.54\,\frac{\text{cm}}{\cancel{\text{in}}} = 54.6\,\text{cm}$$

Example 1:

A distance of 10 ft. is equal to:

(Hint: **1 ft = 12 in and 1 in = 2.54 cm**)

= 10 x 12 x 2.54

= 305 cm or ≈ 3 m

Example 2:

The maximum capacity in liters of a 3-m^3 water tank is:

(Hint: **1 m^3 = 1000 L**)

= 3000 L

Order of Magnitude; Rapid Estimating

- Sometimes, we are interested in only an approximate value for a quantity. We are interested in obtaining rough or **order of magnitude estimates**.
- **Order of magnitude estimates:** Made by rounding off all numbers in a calculation to 1 significant figure, along with power of 10.
  - Can be accurate to within a factor of 10 (often better)

56. In the world, the 14 highest peaks are between 8000 m and 9000 m high. The order-of-magnitude of their height (ارتفاع) is:

A	1×10^4 m✓
B	0.1×10^4 m
C	2×10^4 m
D	10×10^4 m

Explanation:

$9000\,\text{m} \sim 10000\,\text{m} = 10^4$ m

$\vdots$

$8500\,\text{m} \sim 9000\,\text{m} \sim 10000 = 10^4$ m

$\vdots$

$8000 \sim 10000 = 10^4$ m

58. The thickness (سماكة) of a 200-page book is 1.0 cm. The thickness of one sheet of this book can be estimated as:

A	0.001 mm
B	0.01 mm
C	0.1 mm✓
D	1 mm

Dimensions and Dimensional Analysis‡

The dimension of a physical quantity is the type of units or ***base quantities*** that make it up.

Base quantity	Dimension abbreviation
Length	**[L]**
Time	**[T]**
Mass	**[M]**
…	**…**

Dimension of the velocity & speed = $[\mathrm{V}] = \dfrac{[\mathrm{L}]}{[\mathrm{T}]}$

Dimension of the acceleration = $\dfrac{[\mathrm{L}]}{[\mathrm{T}^2]}$

Dimensional analysis:

Example:

$$\underbrace{V_{final}}_{\text{Left Hand Side (LHS)}} = \underbrace{V_{intial} + a \cdot t^2}_{\text{Right Hand Side (RHS)}}$$

$$\underbrace{\frac{[L]}{[T]}}_{\text{LHS dimension}} \overset{?}{=} \underbrace{\frac{[L]}{[T]} + \frac{[L]}{[T^2]} \cdot [T^2]}_{\text{RHS dimension}} = \underbrace{\frac{[L]}{[T]} + [L]}_{\text{RHS dimension}}$$

$$\Rightarrow \text{LHS dimension} \neq \text{RHS dimension}$$

⇒ The equation is *incorrect*

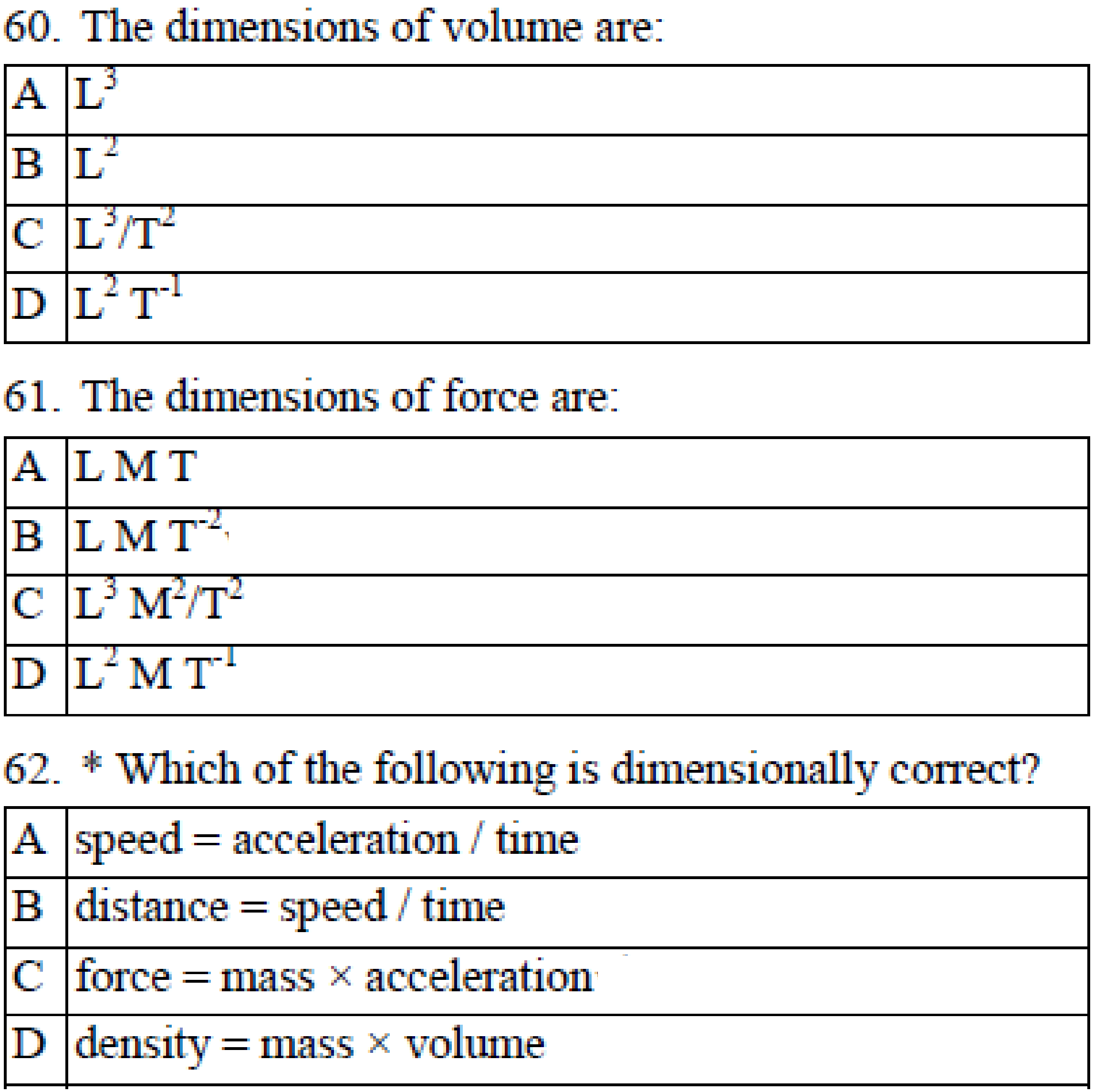
If $\text{LHS dimension} = \text{RHS dimension}$

$$V_{final} = V_{intial} + a \cdot t$$

⇒ The equation is *dimensionally correct*

(But could be physically incorrect)

Examples



60. The dimensions of volume are:

A	L^3
B	L^2
C	L^3/T^2
D	$L^2\ T^{-1}$

61. The dimensions of force are:

A	L M T
B	$L\ M\ T^{-2}$
C	$L^3\ M^2/T^2$
D	$L^2\ M\ T^{-1}$

62. * Which of the following is dimensionally correct?

A	speed = acceleration / time
B	distance = speed / time
C	force = mass × acceleration
D	density = mass × volume