

الفصل الثالث المتطابقات والمعادلات المثلثية

اختبار سريع:

حل كل عبارة فيما يأتي تحليلاً تاماً، وإذا لم يكن ذلك ممكناً فأكتب " أولية "

$$(1) -4a(4a - 1)$$

$$(2) 5(x + 2)(x - 2)$$

(3) أولية

$$(4) (2y + 5)(y - 3)$$

(5) هندسة:

$$\frac{x^2 + 6x + 8}{x + 4} = \frac{(x + 2)(x + 4)}{x + 4} = x + 2$$

حل كلاً من المعادلات الآتية باستعمال التحليل:

$$(6) x^2 + 6x = 0$$

$$x^2 + 6x = 0$$

$$\therefore x(x + 6) = 0$$

$$\therefore x = 0 , x + 6 = 0$$

$$\therefore x = 0 , x = -6$$

$$(7) x^2 + 2x - 35 = 0$$

$$x^2 + 2x - 35 = 0$$

$$\therefore (x + 7)(x - 5) = 0$$

$$\therefore x = -7 , x = 5$$

$$x^2 - 9 = 0 \quad (8)$$

$$x^2 - 9 = 0$$

$$\therefore (x - 3)(x + 3) = 0$$

$$\therefore x = 3 , x = -3$$

$$x^2 - 7x + 12 = 0 \quad (9)$$

$$x^2 - 7x + 12 = 0$$

$$\therefore (x - 3)(x - 4) = 0$$

$$\therefore x = 3 , x = 4$$

(10) حدائق:

$$x(x + 1) = 42$$

$$\therefore x^2 + x = 42$$

$$\therefore x^2 + x - 42 = 0$$

$$\therefore (x + 7)(x - 6) = 0$$

$$\therefore x = -7 , x = 6$$

قيمة x الممكنة هي $x = 6$ ft حيث لا يوجد طول بالسالب.

أوجد القيمة الدقيقة لكل دالة مثلثية فيما يأتي:

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \quad (11)$$

$$\frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2} \quad (12)$$

$$\frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3} \quad (13)$$

$$\frac{\sqrt{3}}{2} \quad (14)$$

قصر المصمك: (15)

$$36 \sin 30 = 36 \times \frac{1}{2} = 18m$$

(3-1) المتطابقات المثلثية

■ **تحقق من فهمك:**

(1)

(1A)

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\therefore \sin^2 \theta + \frac{1}{3} = 1$$

$$\therefore \sin^2 \theta = 1 - \frac{1}{3} = \frac{2}{3}$$

$$\therefore \sin \theta = \pm \frac{\sqrt{6}}{3}$$

وحيث θ تقع في الربع الرابع فإن $\therefore \sin \theta = -\frac{\sqrt{6}}{3}$

(1B)

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\therefore \frac{-2}{7} + \cos^2 \theta = 1$$

$$\therefore \cos^2 \theta = 1 + \frac{2}{7} = \frac{9}{7}$$

$$\therefore \cos \theta = \pm \frac{3}{\sqrt{7}}$$

وحيث θ تقع في الربع الثالث فإن $\therefore \cos \theta = -\frac{3}{\sqrt{7}} = -\frac{3\sqrt{7}}{7}$

■ **تحقق من فهمك:**

(2)
(2A)

$$\frac{\tan^2 \theta \csc^2 \theta - 1}{\sec^2 \theta} = \frac{\frac{\sin^2 \theta}{\cos^2 \theta} \cdot \frac{1}{\sin^2 \theta} - 1}{\frac{1}{\cos^2 \theta}}$$
$$= \frac{1 - \cos^2 \theta}{\cancel{\cos^2 \theta}} \cdot \cancel{\cos^2 \theta} = 1 - \cos^2 \theta = \sin^2 \theta$$

(2B)

$$\frac{\sec \theta}{\sin \theta} (1 - \cos^2 \theta) = \frac{\sec \theta}{\cancel{\sin \theta}} \cdot \sin^2 \theta$$
$$= \sec \theta \cdot \sin \theta = \frac{1}{\cos \theta} \cdot \frac{\sin \theta}{1} = \tan \theta$$

تحقق من فهمك:

(3)

$$T = fr \sin \theta$$

$$\frac{T}{r \sin \theta} = \frac{fr \sin \theta}{r \sin \theta}$$

$$f = \frac{T}{r \sin \theta}$$

تدرب وحل المسائل

أوجد القيمة الدقيقة لكل من النسب المثلثية الآتية:

(1)

$$\tan \theta = \frac{1}{\cot \theta} = \frac{1}{2}$$

(2)

$$\csc \theta = \frac{1}{\sin \theta} = \frac{3\sqrt{5}}{5}$$

(3)

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\therefore \sin^2 \theta = 1 - \frac{25}{169} = \frac{144}{169}$$

$$\therefore \sin \theta = \frac{-12}{13}$$

(4)

$$\sec^2 \theta = \tan^2 \theta + 1$$

$$\therefore \sec^2 \theta = 1 + 1 = 2$$

$$\therefore \sec \theta = \sqrt{2}$$

(5)

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{3}$$

(6)

$$\cot^2 \theta + 1 = \csc^2 \theta$$

$$\therefore \frac{1}{16} + \frac{16}{16} = \csc^2 \theta$$

$$\therefore \csc^2 \theta = \frac{17}{16}$$

$$\therefore \csc \theta = \frac{\sqrt{17}}{4}$$

(7)

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\therefore \cos^2 \theta = 1 - \sin^2 \theta$$

$$\therefore \cos^2 \theta = 1 - \frac{16}{25}$$

$$\therefore \cos \theta = \frac{3}{5}$$

(8)

$$\sin \theta = \frac{1}{\csc \theta}$$

$$\therefore = \frac{2}{9} \sin^2 \theta$$

$$\therefore \cot^2 \theta = \csc^2 \theta - 1$$

$$\therefore \cos \theta = -\frac{\sqrt{27}}{9}$$

بسٹ کل عبارتہ مما یأتی:

(9)

$$\begin{aligned} \tan \theta \cos^2 \theta &= \frac{\sin \theta}{\cancel{\cos \theta}} \square \cos^2 \theta \\ &= \sin \theta \cos \theta \end{aligned}$$

(10)

$$\begin{aligned} \csc^2 \theta - \cot^2 \theta &= \frac{1}{\sin^2 \theta} - \frac{\cos^2 \theta}{\sin^2 \theta} \\ &= \frac{1 - \cos^2 \theta}{\sin^2 \theta} = \frac{\sin^2 \theta}{\sin^2 \theta} = 1 \end{aligned}$$

(11)

$$\frac{\cos \theta \csc \theta}{\tan \theta} = \frac{\cos \theta \times \frac{1}{\sin \theta}}{\frac{\sin \theta}{\cos \theta}}$$

$$= \frac{\cos^2 \theta}{\sin^2 \theta} = \cot^2 \theta$$

(12)

$$\sec \theta \tan^2 \theta + \sec \theta = \frac{1}{\cos \theta} \times \frac{\sin^2 \theta}{\cos^2 \theta} + \frac{1}{\cos \theta}$$

$$= \frac{\sin^2 \theta + \cos^2 \theta}{\cos^3 \theta} = \frac{1}{\cos^3 \theta} = \sec^3 \theta$$

(13)

$$\sin \theta (1 + \cot^2 \theta) = \sin \theta \left(1 + \frac{\cos^2 \theta}{\sin^2 \theta} \right)$$

$$= \sin \theta \left(\frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta} \right) = \cancel{\sin \theta} \left(\frac{1}{\cancel{\sin} \theta} \right)$$

$$= \frac{1}{\sin \theta} = \csc \theta$$

(14)

$$\sin \left(\frac{\pi}{2} - \theta \right) \sec \theta = \cancel{\cos \theta} \times \frac{1}{\cancel{\cos \theta}} = 1$$

(15)

$$\frac{\cos(-\theta)}{\sin(-\theta)} = \frac{\cos \theta}{-\sin \theta} = -\cot \theta$$

(16)

$$(1 + \sin \theta)(1 - \sin \theta) = 1 - \sin^2 \theta = \cos^2 \theta$$

(17)

$$2 - 2\sin^2 \theta$$

$$= 2(1 - \sin^2 \theta) = 2\cos^2 \theta$$

(18)

$$\csc \theta - \cos \theta \cot \theta$$

$$= \frac{1}{\sin \theta} - \cos \theta \left(\frac{\cos \theta}{\sin \theta} \right)$$

$$= \frac{1 - \cos^2 \theta}{\sin \theta} = \sin \theta$$

(19) بصريات:

$$I = I_0 \cos^2 \theta \quad (a)$$

أي أن شدة الضوء تساوي ثلاث أرباع شدة الضوء قبل مرورها بالعدسة الثانية. $I = \frac{3}{4} I_0$ (b)

(20) الشمس:

$$W = \frac{eAS}{\sec \theta} = eAS \cos \theta \quad (a)$$

$$W = eAS \cos \theta = 0.8 \times 0.75 \times 1000 \times \cos 40^\circ$$

$$= 459.63w$$

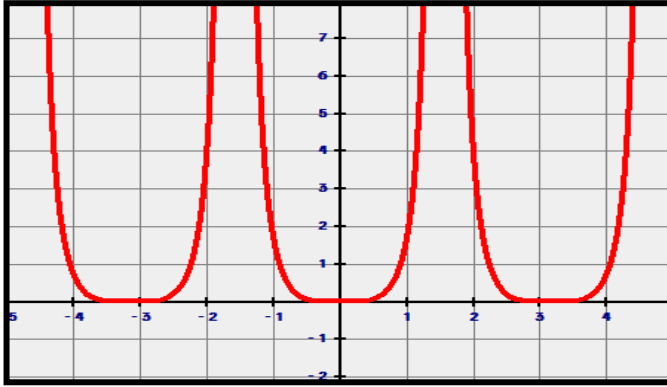
(b)

(21) تمثيلات متعددة:

60°	45°	30°	0°	θ
$\frac{9}{4}$	$\frac{1}{2}$	$\frac{1}{12}$	0	$\tan^2 \theta - \sin^2 \theta$
$\frac{9}{4}$	$\frac{1}{2}$	$\frac{1}{12}$	0	$\tan^2 \theta \sin^2 \theta$

(a) جدولياً:

(b) بيانياً:



متطابقان

(c) تحليلياً:

نعم التمثيلان متطابقان.

(d) تحليلياً:

نعم تمثل متطابقة.

(22) التزلج على الجليد:

$$\mu_k = \frac{mg \sin \theta}{F_n} = \frac{\cancel{mg} \sin \theta}{\cancel{mg} \cos \theta} = \tan \theta$$

بسط كلاً مما يأتي:

(23)

$$\frac{\cos\left(\frac{\pi}{2} - \theta\right) - 1}{1 + \sin(-\theta)} = \frac{\sin \theta - 1}{1 - \sin \theta} = -1$$

(24)

$$\begin{aligned} \frac{\sec \theta \sin \theta + \cos\left(\frac{\pi}{2} - \theta\right)}{1 + \sec \theta} &= \frac{\frac{1}{\cos \theta} \sin \theta + \sin \theta}{1 + \frac{1}{\cos \theta}} \\ &= \frac{\sin \theta + \cos \theta \sin \theta}{\cos \theta + 1} = \frac{\sin \theta (1 + \cos \theta)}{\cos \theta + 1} = \sin \theta \end{aligned}$$

(25) إكتشف الخطأ:

أحمد،

لم يبرهن سعيد صحة المتطابقة عند جميع قيم θ ، وقد يكون هناك قيم أخرى لا تحقق المعادلة.

(26) تحد:

ليست متطابقة عند $x = 45^\circ$

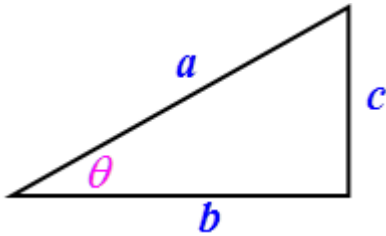
(27) تبرير:

$$\sec \theta = \frac{I}{ER^2}$$

$$\therefore \frac{1}{\cos \theta} = \frac{I}{ER^2}$$

$$\therefore I \cos \theta = ER^2$$

$$\therefore \cos \theta = \frac{ER^2}{I}$$



$$\begin{aligned} \sin^2 \theta + \cos^2 \theta &= \frac{c^2}{a^2} + \frac{b^2}{a^2} = \frac{c^2 + b^2}{a^2} \\ &= \frac{a^2}{a^2} = 1 \end{aligned}$$

(28) أكتب:

(29) برهان:

$$\tan \theta(-a) = \frac{\sin(-a)}{\cos(-a)} = \frac{-\sin a}{\cos a} = -\tan a$$

(30) مسألة مفتوحة:

$$\tan \theta \sin \theta = \frac{\sin^2 \theta}{\cos \theta} = \sin^2 \theta \sec \theta$$

(31) تبرير:

أقسم جميع الحدود على $\sin^2 \theta$

(32) إكتشف الخطأ:

إجابة سامي هي الصحيحة: لأن علاء إستخدم علاقة خاطئة.

مراجعة تراكمية

أوجد قيمة كل مما يأتي، اكتب قياس الزاوية بالراديان، وقرب الناتج إلى أقرب جزء من مئة إذا:

(33) 2.09

(34) 0.60

$$0.50 \quad (35)$$

$$0.80 \quad (36)$$

$$(37)$$

$$K + x^2 = 3x + 2$$

$$\therefore K + 25 = 15 + 2$$

$$\therefore K = 17 - 25 = -8$$

$$(38)$$

$$2^x = 32^{x-2}$$

$$\therefore 2^x = 2^{5x-10}$$

$$\therefore x = 5x - 10$$

$$\therefore 10 = 4x$$

$$\therefore x = 2.5$$

تدریب علی إختبار

$$(39)$$

$$5 \leftarrow A$$

$$(40)$$

$$\frac{m \sqrt{1-m^2}}{1-m^2} \leftarrow B$$

(3-2) إثبات صحة المتطابقات المثلثية.

■ تحقق من فهمك:

(1)

$$\begin{aligned} & \cot^2 \theta - \cos^2 \theta \\ &= \frac{\cos^2 \theta}{\sin^2 \theta} - \cos^2 \theta = \cos^2 \theta \left(\frac{1}{\sin^2 \theta} - 1 \right) \\ &= \cos^2 \theta (\csc^2 \theta - 1) = \cos^2 \theta \cot^2 \theta \end{aligned}$$

تحقق من فهمك:

$$\cos^2 \theta \leftarrow c \quad (2)$$

$$\tan^2 \theta (\cot^2 \theta - \cos^2 \theta)$$

$$= \frac{\sin^2 \theta}{\cos^2 \theta} \left(\frac{\cos^2 \theta}{\sin^2 \theta} - \cos^2 \theta \right) = \frac{\cancel{\sin^2 \theta}}{\cos^2 \theta} \left(\frac{\cos^2 \theta - \cos^2 \theta \cancel{\sin^2 \theta}}{\cancel{\sin^2 \theta}} \right)$$

$$= 1 - \sin^2 \theta = \cos^2 \theta$$

تحقق من فهمك:

(3)

$$\csc^2 \theta - \cot^2 \theta$$

$$= \left(\frac{1}{\sin^2 \theta} - \frac{\cos^2 \theta}{\sin^2 \theta} \right) = \left(\frac{1 - \cos^2 \theta}{\sin^2 \theta} \right) = \frac{\sin^2 \theta}{\sin^2 \theta} = 1$$

$$\cot \theta \tan \theta$$

$$= \frac{\cos \theta}{\sin \theta} \cdot \frac{\sin \theta}{\cos \theta} = 1$$

تدرب وحل المسائل

اثبت أن كل معادلة مما يأتي تمثل متطابقة.

(1)

$$\cos^2 \theta + \tan^2 \theta \cos^2 \theta$$

$$= \cos^2 \theta + \frac{\sin^2 \theta}{\cos^2 \theta} \cdot \cos^2 \theta$$

$$= \cos^2 \theta + \sin^2 \theta = 1$$

(2)

$$\cot \theta (\cot \theta + \tan \theta)$$

$$= \cot^2 \theta + \cot \theta \tan \theta = \cot^2 \theta + 1$$

$$= \frac{\cos^2 \theta}{\sin^2 \theta} + 1 = \frac{\cos^2 \theta + \sin^2 \theta}{\sin^2 \theta} = \frac{\sin^2 \theta}{\sin^2 \theta}$$

$$= \frac{1}{\sin^2 \theta} = \csc^2 \theta$$

(3)

$$1 + \sec^2 \theta$$

$$= 1 + \frac{1}{\cos^2 \theta} \square \sin^2 \theta$$

$$= \frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta} = \sec^2 \theta$$

(4)

$$\sin \theta \square \sec \theta \square \cot \theta$$

$$= \sin \theta \square \frac{1}{\cos \theta} \square \frac{\cos \theta}{\sin \theta} = \frac{\cos \theta \sin \theta}{\cos \theta \sin \theta} = 1$$

(5)

$$\begin{aligned}
& (\sec \theta - \cot \theta)^2 \\
&= \sec^2 \theta - 2 \sec \theta \cot \theta + \cot^2 \theta \\
&= \frac{1}{\sin^2 \theta} - 2 \frac{1}{\sin \theta} \frac{\cos \theta}{\sin \theta} + \frac{\cos^2 \theta}{\sin^2 \theta} = \frac{1 - 2 \cos \theta + \cos^2 \theta}{\sin^2 \theta} \\
&= \frac{(1 - \cos \theta)^2}{\sin^2 \theta} = \frac{(1 - \cos \theta)(1 - \cos \theta)}{1 - \cos^2 \theta} \\
&= \frac{\cancel{(1 - \cos \theta)} (1 - \cos \theta)}{\cancel{(1 - \cos \theta)} (1 + \cos \theta)} = \frac{1 - \cos \theta}{1 + \cos \theta}
\end{aligned}$$

(6)

$$\begin{aligned}
& \frac{1 - 2 \cos^2 \theta}{\sin \theta \cos \theta} \\
&= \frac{(1 - \cos^2 \theta) - \cos^2 \theta}{\sin \theta \cos \theta} \\
&= \frac{\sin^2 \theta - \cos^2 \theta}{\sin \theta \cos \theta} = \frac{\sin \theta}{\cos \theta} - \frac{\cos \theta}{\sin \theta} \\
&= \tan \theta \cot \theta
\end{aligned}$$

(7)

$$\begin{aligned}
& \frac{\sec \theta}{\csc \theta} \\
&= \frac{\frac{1}{\cos \theta}}{\frac{1}{\sin \theta}} = \frac{\sin \theta}{\cos \theta} = \tan \theta
\end{aligned}$$

(8)

$$\begin{aligned}
& \sin \theta \cot \theta \\
&= \cancel{\sin \theta} \frac{\cos \theta}{\cancel{\sin \theta}} = \cos \theta
\end{aligned}$$

(9)

$$\begin{aligned} & (\sin \theta - 1)(\tan \theta + \sec \theta) \\ &= \sin \theta \tan \theta + \sin \theta \sec \theta - \tan \theta - \sec \theta \\ &= \frac{\sin^2 \theta}{\cos \theta} + \frac{\cancel{\sin \theta}}{\cancel{\cos \theta}} - \frac{\cancel{\sin \theta}}{\cancel{\cos \theta}} - \frac{1}{\cos \theta} \\ &= \frac{\sin^2 \theta - 1}{\cos \theta} = \frac{\cancel{\cos^2 \theta}}{\cancel{\cos \theta}} = \cos \theta \end{aligned}$$

(10)

$$\begin{aligned} & \cos \theta \cos(-\theta) - \sin \theta \sin(-\theta) \\ &= \cos^2 \theta + \sin^2 \theta = 1 \end{aligned}$$

(11) إختيار من متعدد:

$$\csc^2 \theta \leftarrow D$$

اثبت أن كل معادلة مما يأتي تمثل متطابقة.

(12)

$$\begin{aligned} & \sec \theta - \tan \theta \\ &= \frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta} = \frac{1 - \sin \theta}{\cos \theta} \end{aligned}$$

(13)

$$\frac{1 + \tan \theta}{\sin \theta + \cos \theta}$$

$$= \frac{1 + \frac{\sin \theta}{\cos \theta}}{\sin \theta + \cos \theta} = \frac{\cancel{\cos \theta + \sin \theta}}{\cos \theta (\cancel{\sin \theta + \cos \theta})}$$

$$\frac{1}{\cos \theta} = \sec \theta$$

(14)

$$\sec \theta \csc \theta$$

$$= \frac{1}{\cos \theta} \cdot \frac{1}{\sin \theta} = \frac{1}{\cos \theta \sin \theta}$$

$$\tan \theta + \cot \theta$$

$$= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta}$$

$$= \frac{1}{\cos \theta \sin \theta}$$

(15)

$$\begin{aligned}
& \frac{2\sin^2 \theta - 1}{\sin \theta - \cos \theta} \\
&= \frac{2\sin^2 \theta - \sin^2 \theta - \cos^2 \theta}{\sin \theta - \cos \theta} \\
&= \frac{\sin^2 \theta - \cos^2 \theta}{\sin \theta - \cos \theta} = \frac{(\cancel{\sin \theta - \cos \theta})(\sin \theta + \cos \theta)}{\cancel{\sin \theta - \cos \theta}} \\
&= \sin \theta + \cos \theta
\end{aligned}$$

(16)

$$\frac{2 + \sec \theta \csc \theta}{\sec \theta \csc \theta}$$

$$\begin{aligned}
&= \frac{2 + \frac{1}{\cos \theta \sin \theta}}{\frac{1}{\cos \theta \sin \theta}} = 2 \cos \theta \sin \theta + 1 \\
&= 2 \cos \theta \sin \theta + \sin^2 \theta + \cos^2 \theta \\
&= (\sin \theta + \cos \theta)^2
\end{aligned}$$

(17)

$$\frac{1 + \sin \theta}{\cos \theta}$$

$$\begin{aligned}
&= \frac{1 + \sin \theta}{\cos \theta} \cdot \frac{1 - \sin \theta}{1 - \sin \theta} = \frac{1 - \sin^2 \theta}{\cos \theta (1 - \sin \theta)} \\
&= \frac{\cancel{\cos \theta} (1 - \sin \theta)}{\cancel{\cos \theta} (1 - \sin \theta)} = \frac{\cos \theta}{1 - \sin \theta}
\end{aligned}$$

(18)

$$\begin{aligned}
& \frac{\cot^2 \theta}{\csc \theta + 1} \\
&= \frac{\csc^2 \theta - 1}{\csc \theta + 1} = \frac{(\csc \theta - 1) \cancel{(\csc \theta + 1)}}{\cancel{\csc \theta + 1}} \\
&= \csc \theta - 1
\end{aligned}$$

(19)

$$\begin{aligned}
& \csc^2 \theta - \cot^2 \theta \\
&= 1 \\
& \sec^2 \theta - \tan^2 \theta \\
&= 1
\end{aligned}$$

(20)

$$\begin{aligned}
& \sin \theta \cos \theta \tan \theta + \cos^2 \theta \\
&= \sin \theta \cos \theta \square \frac{\sin \theta}{\cos \theta} + \cos^2 \theta \\
&= \sin^2 \theta + \cos^2 \theta = 1
\end{aligned}$$

(21)

$$\sec \theta - \cos \theta$$

$$= \frac{1}{\sin \theta} - \cos \theta = \frac{1 - \cos^2 \theta}{\cos \theta}$$

$$= \frac{\sin^2 \theta}{\cos \theta}$$

$$\sec \theta - \cos \theta$$

$$= \frac{\sin \theta}{\cos \theta} \times \sin \theta$$

$$= \frac{\sin^2 \theta}{\cos \theta}$$

(22)

$$\cot^2 \theta + \sin \theta \csc \theta$$

$$= \cot^2 \theta + \sin \theta \times \frac{1}{\sin \theta}$$

$$= \cot^2 \theta + 1 = \csc^2 \theta$$

(23)

$$\frac{\sec \theta - \csc \theta}{\sec \theta \csc \theta}$$

$$\frac{\sec \theta - \csc \theta}{\sec \theta \csc \theta}$$

$$= \frac{1}{\csc \theta} - \frac{1}{\sec \theta} = \sin \theta - \cos \theta$$

(24) ألغاب:

$$L = \frac{g \tan \theta}{\omega^2 \sin \theta} = \frac{g \frac{\sin \theta}{\cos \theta}}{\omega^2 \sin \theta} = \frac{g}{\omega^2 \cos \theta} = \frac{g \sec \theta}{\omega^2}$$

نعم الصيغة $L = \frac{g \tan \theta}{\omega^2 \sin \theta}$ تمثل علاقة بين L ، θ

(25) **جری:**

$$\cos^2 \theta = 1 - \sin^2 \theta = 1 - \frac{1}{16} = \frac{15}{16}$$

$$\therefore \cos \theta = 0.968 \quad \therefore \tan \theta = 0.258$$

$$v^2 = gR \tan \theta = 9.8 \times 16.7 \times 0.258 = 42.22$$

$$\therefore v = 6.5 \text{ m/s}$$

بسط كلاً من العبارات الآتية لتحصل على الناتج 1 أم -1.

1 (26)

-1 (27)

1 (28)

1 (29)

1 (30)

-1 (31)

1 (32)

-1 (33)

بسّط كل مما يأتي إلى قيمة عددية، أو إلى دالة مثلثية أساسية:

(34)

$$\frac{\tan\left(\frac{\pi}{2} - \theta\right) \cancel{\csc \theta}}{\cancel{\csc \theta}} = \frac{\cot \theta}{\csc \theta} = \frac{\frac{\cos \theta}{\sin \theta}}{\frac{1}{\sin \theta}} = \cos \theta$$

(35)

$$\frac{1 + \tan \theta}{1 + \cot \theta} = \frac{1 + \frac{\sin \theta}{\cos \theta}}{1 + \frac{\cos \theta}{\sin \theta}} = \frac{\frac{\cos \theta + \sin \theta}{\cos \theta}}{\frac{\sin \theta + \cos \theta}{\sin \theta}} = \frac{\sin \theta}{\cos \theta} = \tan \theta$$

(36)

$$\frac{\sec^2 \theta - \tan^2 \theta}{\cos^2 \theta + \sin^2 \theta} = \frac{1}{1} = 1$$

(37)

$$\tan \theta \cos \theta$$

$$= \frac{\sin \theta}{\cancel{\cos \theta}} \square \cancel{\cos \theta} = \sin \theta$$

$$\cot \theta \tan \theta = 1$$

(38)

(39)

$$\sec \theta \sin\left(\frac{\pi}{2} - \theta\right) = \frac{1}{\cancel{\cos \theta}} \square \cancel{\cos \theta} = 1$$

(40)

$$\begin{aligned} (\sec^2 \theta + \csc^2 \theta) - (\tan^2 \theta + \cot^2 \theta) &= (\sec^2 \theta - \tan^2 \theta) + (\csc^2 \theta - \cot^2 \theta) \\ &= 1 + 1 = 2 \end{aligned}$$

(41) فيزياء:

$$y = \frac{-gx^2}{2v_0^2} (1 + \tan^2 \theta) + x \tan \theta$$

(42) إلكترونيات:

$$P = I_0^2 R (1 - \cos^2 2\pi ft) \quad (a)$$

$$P = \frac{I_0^2 R}{\csc^2 2\pi ft} \quad (b)$$

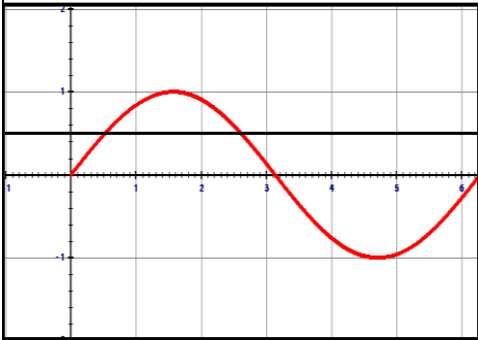
(43) تمثيلات متعددة:

(a) جبريا: $\sin x = 0.5$

(b) بيانيا:

يتقاطع التمثيل البياني للدالتين $y = \sin x$ ، $y = 0.5$

عند النقاط $x = \frac{\pi}{6}$ ، $x = \frac{5\pi}{6}$ على الفترة $[0, 2\pi)$

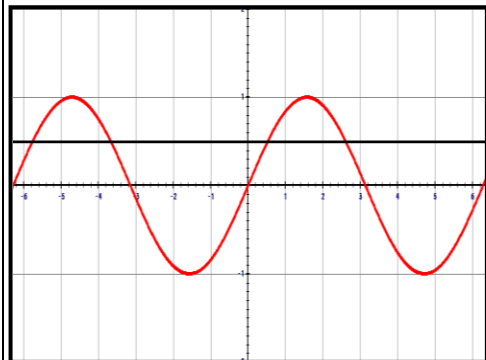


(c) بيانيا:

يتقاطع التمثيل البياني للدالتين $y = \sin x$ ، $y = 0.5$

عند النقاط $\frac{-11\pi}{6}$ ، $\frac{-7\pi}{6}$ ، $\frac{\pi}{6}$ ، $\frac{5\pi}{6}$

على الفترة $[-2\pi, 2\pi)$



(d) بما أن الجيب دالة دورية تكون حلول المعادلة هي $x = \frac{\pi}{6} + 2n\pi$ ، $x = \frac{5\pi}{6} + 2n\pi$

حيث عدد n صحيح.

(44) إكتشف الخطأ:

باقي المعادلات هي متطابقة فيثاغورس ولكن هذه المعادلة ليست منها. $\sin^2 \theta - \cos^2 \theta = 2 \sin^2 \theta$

(45) تبرير:

لأن $\sin^2 \theta + \cos^2 \theta = 1$ هي متطابقة فيثاغورس أما الدالة $\sin \theta = \sqrt{1 - \cos^2 \theta}$ فليست منها

(46) أكتب سؤالاً:

هل استعملت المتطابقة $\sin^2 \theta + \cos^2 \theta = 1$ ؟

(47) أكتب:

لأنهما أكثر دالتين مثلثيتين شيوياً في الإستخدام.

(48) تحذ:

α , β زاويتين متتامتين لذا فإن

$$\begin{aligned} \cos^2 \alpha + \cos^2 \beta &= \cos^2 \alpha + \cos^2 \left(\frac{\pi}{2} - \alpha \right) \\ &= \cos^2 \alpha + \sin^2 \alpha = 1 \end{aligned}$$

(49) تبرير:

$$1 + \tan^2 \theta = 1 + \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta} = \sec^2 \theta$$

$$1 + \cot^2 \theta = 1 + \frac{\cos^2 \theta}{\sin^2 \theta} = \frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta} = \csc^2 \theta$$

مراجعة تراكمية

أوجد القيمة الدقيقة لكل مما يأتي:

$$\frac{\sqrt{5}}{3} \quad (50)$$

$$\frac{5}{4} \quad (51)$$

$$\frac{3}{5} \quad (52)$$

(53) هندسة معمارية:

$$\theta = 30^\circ$$

بسّط العبارتين الآتيتين:

(54)

$$\sin \theta \cos \theta (1 + \cot^2 \theta) = \sin \theta \cos \theta \csc^2 \theta = \cancel{\sin \theta} \cos \theta \frac{1}{\sin^2 \theta} = \cot \theta$$

(55)

$$\frac{\sin^4 \theta - \cos^4 \theta}{\sin^2 \theta - \cos^2 \theta} = \frac{(\cancel{\sin^2 \theta - \cos^2 \theta}) (\sin^2 \theta + \cos^2 \theta)}{\cancel{\sin^2 \theta - \cos^2 \theta}}$$
$$= \sin^2 \theta + \cos^2 \theta = 1$$

تدرب على إختبار

(56)

$$\tan \theta \csc \theta \quad D$$

(57) سؤال ذو إجابة قصيرة:

$$\sin^3 \theta \cos \theta - \cos^3 \theta \sin \theta = \sin \theta \cos \theta (\sin^2 \theta + \cos^2 \theta)$$
$$= \sin \theta \cos \theta \times 1 = \sin \theta \cos \theta$$

(3-3) المتطابقات المثلثية لمجموع زاويتين و الفرق بينهما

■ تحقق من فهمك:

(1)

(1A)

$$\sin(15) = \sin(60 - 45) = \sin 60 \cos 45 - \cos 60 \sin 45$$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

(1B)

$$\cos(-15) = \cos(45 - 60) = \cos 60 \cos 45 + \sin 60 \sin 45$$

$$= \frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

■ تحقق من فهمك:

(2)

(2A)

$$2\sin(245t) = 2\sin(315t - 30t)$$

(2B)

$$\begin{aligned} 2 \sin(245) &= 2 \sin(315 - 30) = 2(\sin 315 \cos 30 - \cos 315 \sin 30) \\ &= \frac{-\sqrt{6} - \sqrt{2}}{2} \end{aligned}$$

■ تحقق من فهمك:

(3)

(3A)

$$\begin{aligned} \sin(90 - \theta) &= \sin 90 \cos \theta - \cos 90 \sin \theta \\ &= 1 \times \cos \theta = \cos \theta \end{aligned}$$

(3B)

$$\begin{aligned} \tan\left(\frac{\pi}{4} + \theta\right) &= \frac{\tan \frac{\pi}{4} + \tan \theta}{1 - \tan \frac{\pi}{4} \tan \theta} \\ &= \frac{1 + \tan \theta}{1 - \tan \theta} \end{aligned}$$

تدرب وحل المسائل

دون استعمال الآلة الحاسبة، أوجد القيمة الدقيقة لكل مما يأتي:

$$\begin{aligned}\cos 165 &= \cos(120 + 45) = \cos 120 \cos 45 - \sin 120 \sin 45 \\ &= \frac{-\sqrt{2} + \sqrt{6}}{4}\end{aligned}\quad (1)$$

$$\begin{aligned}\cos 105 &= \cos(60 + 45) = \cos 60 \cos 45 - \sin 60 \sin 45 \\ &= \frac{\sqrt{2} - \sqrt{6}}{4}\end{aligned}\quad (2)$$

$$\begin{aligned}\cos 75 &= \cos(30 + 45) = \cos 30 \cos 45 - \sin 30 \sin 45 \\ &= \frac{\sqrt{6} - \sqrt{2}}{4}\end{aligned}\quad (3)$$

(4)

$$\begin{aligned}\cos\left(\frac{\pi}{12}\right) &= \cos(45 - 30) = \cos 45 \cos 30 + \sin 45 \sin 30 \\ &= \frac{1 + \sqrt{3}}{2\sqrt{2}}\end{aligned}$$

(5)

$$\begin{aligned}\sin(-30) &= \sin(60 - 90) = \sin 60 \cos 90 - \cos 60 \sin 90 \\ &= -1 \times \frac{1}{2} = -\frac{1}{2}\end{aligned}$$

(6)

$$\begin{aligned}\sin(-210) &= \sin(60 - 270) = \sin 60 \cos 270 - \cos 60 \sin 270 \\ &= 1 \times \frac{1}{2} = \frac{1}{2}\end{aligned}$$

(7)

$$\cos(135) = \cos(180 - 45) = \cos 180 \cos 45 + \sin 180 \sin 45 = -\frac{\sqrt{2}}{2}$$

(8)

$$\begin{aligned}\tan 195 &= \tan(90 + 105) = \frac{\tan 90 + \tan 105}{1 - \tan 90 (\tan 105)} \\ &= \frac{\tan 90 + \left[\frac{\tan 60 + \tan 45}{1 - \tan 60 \tan 45} \right]}{1 - \tan 90 \left[\frac{\tan 60 + \tan 45}{1 - \tan 60 \tan 45} \right]} = 2 - \sqrt{3}\end{aligned}$$

(9) كهرياء:

$$C = 2 \sin[90t + 30t]$$

(a)

(b)

$$C = 2 \sin(90 + 30) = 2(\sin 90 \cos 30 + \cos 90 \sin 30)$$

$$= 2 \times 1 \times \frac{\sqrt{3}}{2} = \sqrt{3}$$

أمبير

أثبت أن كل معادلة مما يأتي تمثل متطابقة:

(10)

$$\begin{aligned} \sin(90 + \theta) &= \sin 90 \cos \theta + \cos 90 \sin \theta \\ &= 1 \times \cos \theta = \cos \theta \end{aligned}$$

(11)

$$\begin{aligned} \cos\left(\frac{3\pi}{2} - \theta\right) &= \cos \frac{3\pi}{2} \cos \theta + \sin \frac{3\pi}{2} \sin \theta \\ &= \sin \frac{3\pi}{2} \times \sin \theta = \sin \theta \end{aligned}$$

(12)

$$\begin{aligned} \tan\left(\theta + \frac{\pi}{2}\right) &= \frac{\sin\left(\theta + \frac{\pi}{2}\right)}{\cos\left(\theta + \frac{\pi}{2}\right)} = \frac{\sin \theta \cos \frac{\pi}{2} + \cos \theta \sin \frac{\pi}{2}}{\cos \theta \cos \frac{\pi}{2} - \sin \theta \sin \frac{\pi}{2}} \\ &= \frac{\cos \theta \times 1}{-\sin \theta \times 1} = -\cot \theta \end{aligned}$$

(13)

$$\begin{aligned}\sin(\theta + \pi) &= \sin \theta \cos \pi + \cos \theta \sin \pi \\ &= \sin \theta \times -1 = -\sin \theta\end{aligned}$$

(14)

$$\begin{aligned}\cos\left(\frac{\pi}{2} + \theta\right) &= \cos \frac{\pi}{2} \cos \theta - \sin \frac{\pi}{2} \sin \theta \\ &= -1 \times \sin \theta = -\sin \theta\end{aligned}$$

(15)

$$\begin{aligned}\tan(\theta + 45) &= \frac{\tan \theta + \tan 45}{1 - \tan \theta \tan 45} \\ &= \frac{1 + \tan \theta}{1 - \tan \theta}\end{aligned}$$

(16) إلكترونيات:

$$\begin{aligned}y_1 + y_2 &= 10 \sin [2t + 210 + 2t + 30] \\ &= 10 \sin [4t + 240] = 0\end{aligned}$$

تداخل هدام أي أن كلاً من الموجتين تلاشي الأخرى.

دون استعمال الآلة الحاسبة، أوجد القيمة الدقيقة لكل مما يأتي:

(17)

$$\begin{aligned}\tan 165 &= \tan(120 + 45) = \frac{\tan 120 + \tan 45}{1 - \tan 120 \tan 45} \\ &= -2 + \sqrt{3}\end{aligned}$$

(18)

$$\begin{aligned}\sec 1275 &= \frac{1}{\cos 1275} = \frac{1}{\cos 195} = \frac{1}{\cos(135 + 60)} \\ &= \frac{1}{\cos 135 \cos 60 - \sin 135 \sin 60} = \sqrt{2} - \sqrt{6}\end{aligned}$$

(19)

$$\begin{aligned}\sin 735 &= \sin(360 + 375) = \sin 360 \cos 375 + \cos 360 \sin 375 \\ &= \frac{\sqrt{6} - \sqrt{2}}{4}\end{aligned}$$

(20)

$$\tan\left(\frac{23\pi}{12}\right) = -2 + \sqrt{3}$$

(21)

$$\csc\left(\frac{5\pi}{12}\right) = \frac{1}{\sin\left(\frac{5\pi}{12}\right)} = \sqrt{6} - \sqrt{2}$$

(22)

$$\cot\left(\frac{113\pi}{12}\right) = \frac{\cos\left(\frac{113\pi}{12}\right)}{\sin\left(\frac{113\pi}{12}\right)} = 2 - \sqrt{3}$$

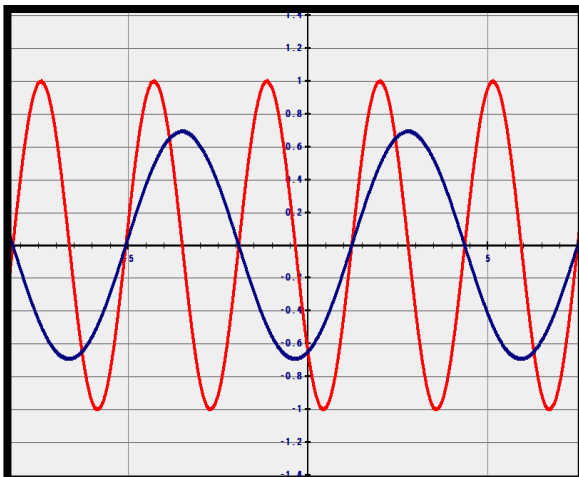
(23)

$$\begin{aligned} & \frac{\sin A + \tan \theta \cos A}{\cos A - \tan \theta \sin A} \\ &= \frac{\left(\frac{\sin A}{\cos A} + \tan \theta \right)}{1 - \tan \theta \frac{\sin A}{\cos A}} \\ &= \frac{(\tan A + \tan \theta)}{1 - \tan \theta \tan A} \\ &= \tan(A + \theta) \end{aligned}$$

(24) تمثيلات متعددة:

(a) جدولياً:

A	B	$\sin A$	$\sin B$	$\sin(A + B)$	$\sin A + \sin B$
30	90	$\frac{1}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{3}{2}$
45	60	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2} + \sqrt{6}}{4}$	$\frac{\sqrt{2} + \sqrt{3}}{2}$
90	30	1	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{3}{2}$



(b) بيانياً:

(c) تحليلاً:

$$\sin(30 + 45) = \sin(30) + \sin(45)$$

الطرف الأيمن $= \frac{\sqrt{3}}{2} + \frac{1}{2}$ أي 1.21 تقريباً، وبما أن قيمة جيب أي زاوية لا يمكن أن يكون أكبر من 1 فإن هذه المعادلة خطأ.

أثبت صحة كل من المتطابقات الآتية:

(25)

$$\frac{\tan A + \tan B}{\sec A \sec B} = \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{\frac{1}{\cos A} \cdot \frac{1}{\cos B}}$$
$$= \sin A \cos B + \sin B \cos A = \sin(A + B)$$

(26)

$$\frac{1 - \tan A \tan B}{\sec A \sec B} = \frac{1 - \frac{\sin A}{\cos A} \cdot \frac{\sin B}{\cos B}}{\frac{1}{\cos A} \cdot \frac{1}{\cos B}}$$
$$= \cos A \cos B - \sin A \sin B = \cos(A + B)$$

(27)

$$\begin{aligned}
\frac{\sec A \sec B}{1 + \tan A \tan B} &= \frac{\frac{1}{\cos A} \cdot \frac{1}{\cos B}}{1 + \frac{\sin A}{\cos A} \cdot \frac{\sin B}{\cos B}} \\
&= \frac{1}{\cos A \cos B + \sin A \sin B} \\
&= \frac{1}{\cos(A - B)} = \sec(A - B)
\end{aligned}$$

(28)

$$\begin{aligned}
&\sin(A + B)\sin(A - B) \\
&= (\sin A \cos B + \cos A \sin B)(\sin A \cos B - \cos A \sin B) \\
&= (\sin A \cos B)^2 - (\cos A \sin B)^2 \\
&= \sin^2 A \cos^2 B - \sin^2 B \cos^2 A \\
&= \sin^2 A \cos^2 B + \sin^2 A \sin^2 B - \sin^2 A \sin^2 B - \sin^2 B \cos^2 A \\
&= \sin^2 A (\cos^2 B + \sin^2 B) - \sin^2 B (\sin^2 A + \cos^2 A) \\
&= \sin^2 A \times 1 - 1 \times \sin^2 B = \sin^2 A - \sin^2 B
\end{aligned}$$

مسائل مهارات التفكير العليا:

(29) تبریر:

$$\begin{aligned} & \sin\left(\frac{\pi}{3} - \theta\right) \cos\left(\frac{\pi}{3} + \theta\right) - \cos\left(\frac{\pi}{3} - \theta\right) \sin\left(\frac{\pi}{3} + \theta\right) \\ &= \sin\left(\frac{\pi}{3} - \theta - \frac{\pi}{3} - \theta\right) = \sin(-2\theta) \end{aligned}$$

(30) تحد:

$$\begin{aligned} \cot(A + B) &= \frac{1}{\tan(A + B)} = \frac{1}{\frac{\tan A + \tan B}{1 - \tan A \tan B}} \\ &= \frac{1 - \tan A \tan B}{\tan A + \tan B} = \frac{1 - \frac{1}{\cot A} - \frac{1}{\cot B}}{\frac{1}{\cot A} + \frac{1}{\cot B}} \\ &= \frac{\cot A \cot B - 1}{\cot A + \cot B} \end{aligned}$$

(31) برهان:

$$\begin{aligned} d &= \sqrt{(\cos A - \cos B)^2 + (\sin A - \sin B)^2} \\ \therefore d^2 &= \cos^2 A - 2\cos A \cos B + \cos^2 B + \sin^2 A - 2\sin A \sin B + \sin^2 B \\ \therefore d^2 &= 1 + 1 - 2\cos A \cos B - 2\sin A \sin B \\ \therefore d^2 &= 2 - 2[\cos A \cos B + \sin A \sin B] \\ \therefore d^2 &= 2 - 2\cos(A + B) \end{aligned}$$

(32) أكتب:

قد تختلف الإجابات من فرد لآخر تبعاً لوجهة نظره.

(33) مسألة مفتوحة:

$$A = 35 , B = 60 , C = 85$$

$$0.7002 + 1.7321 + 11.4301 = 13.86$$

مراجعة تراكمية

بسّط كل من العبارتين الآتيتين:

(34)

$$\begin{aligned} & \sin \theta \csc \theta - \cos^2 \theta \\ &= \sin \theta \frac{1}{\sin \theta} - \cos^2 \theta \\ &= 1 - \cos^2 \theta = \sin^2 \theta \end{aligned}$$

(35)

$$\begin{aligned} & \cos^2 \theta \sec \theta \csc \theta \\ &= \frac{\cos^2 \theta}{\sin \theta \cancel{\cos \theta}} = \cot \theta \end{aligned}$$

أوجد القيمة الدقيقة لكل مما يأتي:

$$\frac{\sqrt{5}}{2} \quad (36)$$

$$-\frac{\sqrt{5}}{2} \quad (37)$$

$$\frac{\sqrt{193}}{12} \quad (38)$$

$$-\frac{\sqrt{7}}{4} \quad (39)$$

$$\frac{\sqrt{39}}{4} \quad (40)$$

أثبت صحة كل من المتطابقتين الآتيتين:

(41)

$$\begin{aligned} \frac{\sin \theta}{\tan \theta} + \frac{\cos \theta}{\cot \theta} &= \frac{\sin \theta}{\frac{\sin \theta}{\cos \theta}} + \frac{\cos \theta}{\frac{\cos \theta}{\sin \theta}} \\ &= \frac{\cos \theta \cancel{\sin \theta}}{\cancel{\sin \theta}} + \frac{\sin \theta \cancel{\cos \theta}}{\cancel{\cos \theta}} \\ &= \cos \theta + \sin \theta \end{aligned}$$

(42)

$$\begin{aligned} &\sec \theta (\sec \theta - \cos \theta) \\ &= \frac{1}{\cos \theta} \left(\frac{1}{\cos \theta} - \cos \theta \right) = \frac{1}{\cos^2 \theta} - 1 \\ &= \sec^2 \theta = \tan^2 \theta \end{aligned}$$

تدرب على إختبار

(43)

$$\frac{\sqrt{3}}{2} \leftarrow \leftarrow \leftarrow \leftarrow B$$

(44)

$$\cos \theta = -0.3 \quad \therefore \sin \theta = \frac{\sqrt{91}}{10}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{3\sqrt{91}}{91}$$

←←←←

إختبار منتصف الفصل

بسّط كل عبارة مما يأتي:

$$\cot \theta \sec \theta$$

$$= \frac{\cancel{\cos \theta}}{\sin \theta} \square \frac{1}{\cancel{\cos \theta}} = \frac{1}{\sin \theta} = \csc \theta$$

(1)

(2)

$$\frac{1 - \cos^2 \theta}{\sin^2 \theta} = \frac{\sin^2 \theta}{\sin^2 \theta} = 1$$

(3)

$$\frac{1}{\cos \theta} - \frac{\sin^2 \theta}{\cos \theta} = \frac{1 - \sin^2 \theta}{\cos \theta} = \frac{\cos^2 \theta}{\cos \theta} = \cos \theta$$

(4)

$$\cos\left(\frac{\pi}{2} - \theta\right) \csc \theta = \cancel{\sin \theta} \square \frac{1}{\cancel{\sin \theta}} = 1$$

أوجد القيمة الدقيقة لكل مما يأتي:

$$\frac{4}{5} \quad (5)$$

$$-\frac{\sqrt{5}}{2} \quad (6)$$

$$\frac{\sqrt{7}}{3} \quad (7)$$

(8) إختيار من متعدد:

$$\sec \theta \leftarrow \leftarrow \leftarrow \leftarrow D$$

(9) مدينة ألعاب:

$$\theta = 11.5^\circ \quad (a) \text{ تقريباً}$$

$$v = 4 \text{ m/sec} \quad (b)$$

أثبت صحة كل من المتطابقات الآتية:

(10)

$$\frac{\cot \theta}{\cos \theta \sin \theta} = \frac{\cancel{\cos \theta}}{\sin \theta \cancel{\cos \theta} \sin \theta} = \frac{1}{\sin^2 \theta} = \csc^2 \theta$$

(11)

$$\frac{\cos \theta \csc \theta}{\cot \theta} = \frac{\cos \theta \cdot \frac{1}{\sin \theta}}{\frac{\cos \theta}{\sin \theta}} = \frac{\cancel{\cos \theta}}{\cancel{\sin \theta}} = 1$$

(12)

$$\begin{aligned}
\frac{\sin \theta \tan \theta}{1 - \cos \theta} &= \frac{\frac{\sin^2 \theta}{\cos \theta}}{1 - \cos \theta} = \frac{\sin^2 \theta}{\cos \theta - \cos^2 \theta} \\
&= \frac{1 - \cos^2 \theta}{\cos \theta (1 - \cos \theta)} = \frac{\cancel{(1 - \cos \theta)} (1 + \cos \theta)}{\cos \theta \cancel{(1 - \cos \theta)}} \\
&= \frac{(1 + \cos \theta)}{\cos \theta} = (1 + \cos \theta) \square \frac{1}{\cos \theta} = (1 + \cos \theta) \square \sec \theta
\end{aligned}$$

(13)

$$\begin{aligned}
\frac{\cos \theta \sin \theta}{1 + \sin \theta} &= \frac{\cos \theta \sin \theta (1 - \sin \theta)}{(1 + \sin \theta)(1 - \sin \theta)} = \frac{\cos \theta \sin \theta (1 - \sin \theta)}{1 - \sin^2 \theta} \\
&= \frac{\cancel{\cos \theta} \sin \theta (1 - \sin \theta)}{\cos^2 \theta} \\
&= \frac{\sin \theta}{\cos \theta} (1 - \sin \theta) = \tan \theta (1 - \sin \theta)
\end{aligned}$$

(14) حاسوب:

(a)

$$h = \sqrt{225 - 144} = \sqrt{81} = 9 \text{ in}$$

(b)

$$\cot \theta = \frac{9}{12}$$

$$\frac{\cos \theta}{\sin \theta} = \frac{12}{15} \div \frac{9}{15} = \frac{12}{9}$$

$$\therefore \cot \theta = \frac{\cos \theta}{\sin \theta}$$

أثبت صحة كل من المتطابقات الآتية:

(15)

$$\frac{\sin \theta \sec \theta (\sec \theta + 1)}{(\sec \theta - 1)(\sec \theta + 1)} = \frac{\sin \theta \cdot \frac{1}{\cos \theta} (\sec \theta + 1)}{\sec^2 \theta - 1}$$

$$= \frac{\cancel{\sin \theta} (\sec \theta + 1)}{\tan^2 \theta} = \frac{\sec \theta + 1}{\tan \theta} = (\sec \theta + 1) \cot \theta$$

(16)

$$\tan^2 \theta - \sin^2 \theta = \frac{\sin^2 \theta}{\cos^2 \theta} - \sin^2 \theta = \frac{\sin^2 \theta - \sin^2 \theta \cos^2 \theta}{\cos^2 \theta}$$

$$= \frac{\sin^2 \theta (1 - \cos^2 \theta)}{\cos^2 \theta} = \frac{\sin^2 \theta}{\cos^2 \theta} \sin^2 \theta = \tan^2 \theta \sin^2 \theta$$

(17)

$$\frac{\cos \theta \sin \theta}{1 + \cos \theta} = \frac{\cos \theta \sin \theta (1 - \cos \theta)}{(1 + \cos \theta)(1 - \cos \theta)} = \frac{\cos \theta \sin \theta (1 - \cos \theta)}{1 - \cos^2 \theta}$$

$$= \frac{\cos \theta \cancel{\sin \theta} (1 - \cos \theta)}{\sin^2 \theta} = \frac{\cos \theta}{\sin \theta} (1 - \cos \theta)$$

$$= \cot \theta (1 - \cos \theta)$$

دون استخدام الآلة الحاسبة أوجد القيمة الدقيقة لكل مما يأتي:

$$\frac{\sqrt{2} - \sqrt{6}}{4} \quad (18)$$

$$-\frac{\sqrt{2}}{2} \quad (19)$$

$$2 - \sqrt{3} \quad (20)$$

$$2 - \sqrt{3} \quad (21)$$

إختيار من متعدد: (22)

$$\frac{\sqrt{6} - \sqrt{2}}{4} \leftarrow \leftarrow \leftarrow \leftarrow (C)$$

(23)

$$\cos 30 \cos \theta + \sin 30 \sin \theta = \frac{\sqrt{3}}{2} \cos \theta + \frac{1}{2} \sin \theta$$

$$\sin 60 \cos \theta + \cos 60 \sin \theta = \frac{\sqrt{3}}{2} \cos \theta + \frac{1}{2} \sin \theta$$

(3-4) المتطابقات المثلثية لضرب الزاوية وخصفها

■ تحقق من فهمك:

(1)

$$\begin{aligned}\sin 2\theta &= 2 \sin \theta \cos \theta \\ &= 2 \left[\frac{2\sqrt{2}}{3} \right] \left[\frac{-1}{3} \right] = -\frac{4\sqrt{2}}{9}\end{aligned}$$

■ تحقق من فهمك:

(2)

(2A)

$$\begin{aligned}\cos 2\theta &= 1 - 2 \sin^2 \theta \\ &= 1 - 2 \left[\frac{8}{9} \right] = -\frac{7}{9}\end{aligned}$$

(2B)

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{4\sqrt{2}}{7}$$

■ تحقق من فهمك:

(3)

$$\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}} = \frac{\sqrt{18 + 6\sqrt{5}}}{6}$$

■ تحقق من فهمك:

(4)

(4A

$$\begin{aligned} g &= 978 + 5.17 \sin^2 L - 0.014 \sin L \cos L \\ &= 978 + 5.17(1 - \cos 2L) - 0.014 \left(\frac{\sin 2L}{2} \right) \\ &= 978 + 5.17 - 5.17 \cos 2L - 0.028 \sin 2L \\ &= g = 983.17 - 5.17 \cos 2L - 0.028 \sin 2L \end{aligned}$$

$$g = 983.17 - 5.17 \cos 90 - 0.028 \sin 90$$

$$g = 983.17 - 0.028 \sin 90$$

$$= 983.142$$

(4B

■ تحقق من فهمك:

(5)

$$4 \cos^2 x - \sin^2 2x$$

$$= 4 \cos^2 x - 4 \sin^2 x \cos^2 x = 4 \cos^2 x (1 - \sin^2 x)$$

$$= 4 \cos^2 x \cos^2 x = 4 \cos^4 x$$

تدرب وحل المسائل

دون استعمال الآلة الحاسبة، أوجد القيمة الدقيقة لكل من $\sin 2\theta$ ، $\cos 2\theta$ ، $\sin \frac{\theta}{2}$ ، $\cos \frac{\theta}{2}$ إذا

كان:

(1)

$$\sin \theta = \frac{1}{4}, \quad \therefore \sin^2 \theta = \frac{1}{16}, \quad \therefore \cos^2 \theta = 1 - \frac{1}{16} = \frac{15}{16}, \quad \therefore \cos \theta = \frac{\sqrt{15}}{4}$$

$$\cos 2\theta = 1 - 2\sin^2 \theta = 1 - 2 \cdot \frac{1}{16} = \frac{7}{8}$$

$$\sin 2\theta = 2\sin \theta \cos \theta = 2 \cdot \frac{1}{4} \cdot \frac{\sqrt{15}}{4} = \frac{\sqrt{15}}{8}$$

$$\cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}} = \pm \sqrt{\frac{1 + \frac{\sqrt{15}}{4}}{2}} = \pm \frac{\sqrt{8 + 2\sqrt{15}}}{4}$$

$$\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}} = \pm \sqrt{\frac{1 - \frac{\sqrt{15}}{4}}{2}} = \pm \frac{\sqrt{8 - 2\sqrt{15}}}{4}$$

$$\sin \theta = \frac{4}{5}, \quad \therefore \sin^2 \theta = \frac{16}{25}, \quad \therefore \cos^2 \theta = 1 - \frac{16}{25} = \frac{9}{25}, \quad \therefore \cos \theta = -\frac{3}{5} \quad (2)$$

$$\cos 2\theta = 1 - 2\sin^2 \theta = 1 - 2\left(\frac{16}{25}\right) = -\frac{7}{25}$$

$$\sin 2\theta = 2\sin \theta \cos \theta = 2\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right) = -\frac{24}{25}$$

$$\cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}} = \pm \sqrt{\frac{1 + \frac{-3}{5}}{2}} = \pm \sqrt{\frac{1}{5}}$$

$$\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}} = \pm \sqrt{\frac{1 - \frac{-3}{5}}{2}} = \pm \sqrt{\frac{4}{5}}$$

$$\cos \theta = \frac{3}{5}, \quad \therefore \cos^2 \theta = \frac{9}{25}, \quad \therefore \sin^2 \theta = 1 - \frac{9}{25} = \frac{16}{25}, \quad \therefore \sin \theta = -\frac{4}{5} \quad (3)$$

$$\cos 2\theta = 1 - 2\sin^2 \theta = 1 - 2\left(\frac{16}{25}\right) = -\frac{7}{25}$$

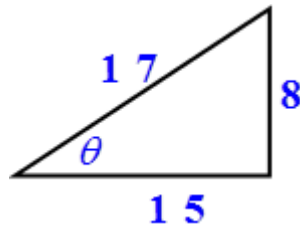
$$\sin 2\theta = 2\sin \theta \cos \theta = 2\left(-\frac{4}{5}\right)\left(\frac{3}{5}\right) = -\frac{24}{25}$$

$$\cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}} = \pm \sqrt{\frac{1 + \frac{3}{5}}{2}} = \pm \sqrt{\frac{4}{5}}$$

$$\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}} = \pm \sqrt{\frac{1 - \frac{3}{5}}{2}} = \pm \sqrt{\frac{2}{5}} \quad (4)$$

$$\tan \theta = \frac{-8}{15} \quad \therefore \cos \theta = -\frac{15}{17}, \quad \therefore \sin \theta = \frac{8}{17}, \quad \therefore \sin^2 \theta = \frac{64}{289}$$

$$\cos 2\theta = 1 - 2\sin^2 \theta = 1 - 2 \cdot \frac{64}{289} = \frac{161}{289}$$



$$\sin 2\theta = 2\sin \theta \cos \theta = 2 \cdot \frac{8}{17} \cdot \frac{-15}{17} = -\frac{240}{289}$$

$$\cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}} = \pm \sqrt{\frac{1 + \frac{-15}{17}}{2}} = \pm \sqrt{\frac{1}{17}}$$

$$\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}} = \pm \sqrt{\frac{1 - \frac{-15}{17}}{2}} = \pm \sqrt{\frac{16}{17}} = \pm \frac{4}{\sqrt{17}} \quad (5)$$

$$\sin \theta = \frac{2}{3}, \quad \therefore \sin^2 \theta = \frac{4}{9}, \quad \therefore \cos^2 \theta = 1 - \frac{4}{9} = \frac{5}{9}, \quad \therefore \cos \theta = -\frac{\sqrt{5}}{3}$$

$$\cos 2\theta = 1 - 2\sin^2 \theta = 1 - 2 \cdot \frac{4}{9} = \frac{1}{9}$$

$$\sin 2\theta = 2\sin \theta \cos \theta = 2 \cdot \frac{2}{3} \cdot \frac{-\sqrt{5}}{3} = -\frac{4\sqrt{5}}{9}$$

$$\cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}} = \pm \sqrt{\frac{1 + \frac{-\sqrt{5}}{3}}{2}} = \pm \sqrt{\frac{3 - \sqrt{5}}{6}}$$

$$\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}} = \pm \sqrt{\frac{1 - \frac{-\sqrt{5}}{3}}{2}} = \pm \sqrt{\frac{3 + \sqrt{5}}{6}}$$

(6)

$$\sin \theta = -\frac{15}{17}, \quad \therefore \sin^2 \theta = \frac{225}{289}, \quad \therefore \cos^2 \theta = 1 - \frac{225}{289} = \frac{64}{289}, \quad \therefore \cos \theta = -\frac{8}{17}$$

$$\cos 2\theta = 1 - 2\sin^2 \theta = 1 - 2 \cdot \frac{225}{289} = -\frac{161}{289}$$

$$\sin 2\theta = 2\sin \theta \cos \theta = 2 \cdot \frac{-15}{17} \cdot \frac{-8}{17} = \frac{240}{289}$$

$$\cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}} = \pm \sqrt{\frac{1 + \frac{-8}{17}}{2}} = \pm \sqrt{\frac{9}{14}}$$

$$\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}} = \pm \sqrt{\frac{1 - \frac{-8}{17}}{2}} = \pm \sqrt{\frac{25}{14}}$$

(7)

$$\tan \theta = -2 \quad \therefore \tan^2 \theta = 4 \quad \therefore \sec^2 \theta = 5 \quad \therefore \sec \theta = -\sqrt{5}$$

$$\therefore \cos \theta = -\frac{1}{\sqrt{5}}, \quad \therefore \sin \theta = \frac{2}{\sqrt{5}}, \quad \therefore \sin^2 \theta = \frac{4}{5}$$

$$\cos 2\theta = 1 - 2\sin^2 \theta = 1 - 2 \cdot \frac{4}{5} = -\frac{3}{5}$$

$$\sin 2\theta = 2 \sin \theta \cos \theta = 2 \left[\frac{2}{\sqrt{5}} \right] \left[\frac{-1}{\sqrt{5}} \right] = -\frac{4}{5}$$

$$\cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}} = \pm \sqrt{\frac{1 + \frac{-1}{\sqrt{5}}}{2}} = \pm \sqrt{\frac{5 - \sqrt{5}}{10}}$$

$$\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}} = \pm \sqrt{\frac{1 - \frac{-1}{\sqrt{5}}}{2}} = \pm \sqrt{\frac{5 + \sqrt{5}}{10}}$$

أوجد القيمة الدقيقة لكل مما يأتي:

$$\sin \frac{\pi}{8} = \sqrt{\frac{1 - \cos \frac{\pi}{4}}{2}} = \frac{\sqrt{2 - \sqrt{2}}}{2} \quad (8)$$

$$\cos 15 = \sqrt{\frac{1 + \cos 30}{2}} = \frac{\sqrt{2 + \sqrt{3}}}{2} \quad (9)$$

$$\sin 75 = \frac{\sqrt{2 + \sqrt{3}}}{2} \quad (10)$$

$$\sin 165 = \sqrt{3} - 2 \quad (11)$$

(12)

$$\sin 165 = 2 + \sqrt{3}$$

(13) كرة قدم:

$$d = \frac{v^2 \sin 2\theta}{g} \quad (a)$$

$$d \approx 81 \text{ ft} \quad (b)$$

أثبت صحة كل من المتطابقات الآتية:

$$\begin{aligned} \frac{1 - \cos 2\theta}{\sin 2\theta} &= \frac{1 - (1 - 2\sin^2 \theta)}{2\sin \theta \cos \theta} = \frac{\cancel{2} \sin^2 \theta}{\cancel{2} \sin \theta \cos \theta} \\ &= \frac{\sin \theta}{\cos \theta} = \tan \theta \end{aligned} \quad (14)$$

$$\begin{aligned} \tan \frac{\theta}{2} &= \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \frac{1 - \cos \theta}{1 + \cos \theta} \\ &= \frac{\sin \theta}{1 + \cos \theta} \end{aligned} \quad (15)$$

(16)

$$\frac{2}{\cot \theta - \tan \theta} = \frac{2}{\cot \theta - \tan \theta} \cdot \frac{\tan \theta}{\tan \theta} = \frac{2 \tan \theta}{\cot \theta \tan \theta - \tan^2 \theta}$$

$$= \frac{2 \tan \theta}{1 - \tan^2 \theta} = \tan 2\theta$$

(17)

$$\sin \frac{\theta}{2} \cos \frac{\theta}{2} = \frac{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2}$$

$$= \frac{\sin 2\left(\frac{\theta}{2}\right)}{2} = \frac{\sin \theta}{2}$$

(18) العدد ماخ:

(a)

$$\frac{1}{M} = \sqrt{\frac{1 - \cos \theta}{2}}$$

$$M = 6$$

(b)

(19) إلكترونيات:

$$P = I_o^2 R \sin^2 t \theta = \frac{1}{2} I_o^2 R - \frac{1}{2} I_o^2 R \cos 2t \theta$$

(20) كرة قدم:

إذا كانت $\theta = 45 + \alpha$

$$\begin{aligned}
 d &= \frac{v^2 \sin 2(45 + \alpha)}{g} = \frac{v^2 \sin(90 + 2\alpha)}{g} \\
 &= \frac{v^2 (\sin 90 \cos \alpha + \cos 90 \sin \alpha)}{g} \\
 &= \frac{v^2 \cos 2\alpha}{g}
 \end{aligned}$$

إذا كانت $\theta = 45 - \alpha$

$$\begin{aligned}
 d &= \frac{v^2 \sin 2(45 - \alpha)}{g} = \frac{v^2 \sin(90 - 2\alpha)}{g} \\
 &= \frac{v^2 (\sin 90 \cos \alpha - \cos 90 \sin \alpha)}{g} \\
 &= \frac{v^2 \cos 2\alpha}{g}
 \end{aligned}$$

أوجد القيمة الدقيقة لكل من $\sin 2\theta$ ، $\cos 2\theta$ ، $\tan 2\theta$ إذا كان:

$$\begin{aligned}
 \cos \theta &= \frac{4}{5} , \therefore \cos^2 \theta = \frac{16}{25} , \therefore \sin^2 \theta = \frac{9}{25} , \therefore \sin \theta = \frac{3}{5} , \therefore \tan \theta = \frac{3}{4} \\
 \cos 2\theta &= 1 - 2\sin^2 \theta = 1 - 2 \cdot \frac{9}{25} = \frac{7}{25}
 \end{aligned}
 \tag{21}$$

$$\sin 2\theta = 2 \sin \theta \cos \theta = 2 \cdot \frac{3}{5} \cdot \frac{4}{5} = \frac{24}{25}$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2 \cdot \frac{3}{4}}{1 - \frac{9}{16}} = \frac{24}{7}$$

(22)

$$\sin \theta = \frac{1}{3}, \quad \therefore \sin^2 \theta = \frac{1}{9}, \quad \therefore \cos^2 \theta = \frac{8}{9}, \quad \therefore \cos \theta = \frac{2\sqrt{2}}{3}, \quad \therefore \tan \theta = \frac{\sqrt{2}}{4}$$

$$\cos 2\theta = 1 - 2 \sin^2 \theta = 1 - 2 \cdot \frac{1}{9} = \frac{7}{9}$$

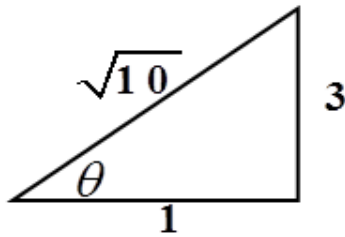
$$\sin 2\theta = 2 \sin \theta \cos \theta = 2 \cdot \frac{1}{3} \cdot \frac{2\sqrt{2}}{3} = \frac{4\sqrt{2}}{9}$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2 \cdot \frac{\sqrt{2}}{4}}{1 - \frac{2}{16}} = \frac{4\sqrt{2}}{7}$$

(23)

$$\sin \theta = \frac{3}{\sqrt{10}}, \quad \therefore \sin^2 \theta = \frac{9}{10}, \quad \therefore \cos \theta = -\frac{1}{\sqrt{10}}, \quad \therefore \tan \theta = -3$$

$$\cos 2\theta = 1 - 2\sin^2 \theta = 1 - 2\left(\frac{9}{10}\right) = -\frac{4}{5}$$



$$\sin 2\theta = 2\sin \theta \cos \theta = 2\left(\frac{3}{\sqrt{10}}\right)\left(\frac{-1}{\sqrt{10}}\right) = -\frac{6}{10} = -\frac{3}{5}$$

$$\tan 2\theta = \frac{2\tan \theta}{1 - \tan^2 \theta} = \frac{2(-3)}{1 - 9} = \frac{6}{8} = \frac{3}{4}$$

(24)

$$\cos \theta = -\frac{3}{4}, \quad \therefore \cos^2 \theta = \frac{9}{16}, \quad \therefore \sin^2 \theta = \frac{7}{16}, \quad \therefore \sin \theta = \frac{\sqrt{7}}{4}, \quad \therefore \tan \theta = -\frac{\sqrt{7}}{3}$$

$$\cos 2\theta = 1 - 2\sin^2 \theta = 1 - 2\left(\frac{7}{16}\right) = \frac{1}{8}$$

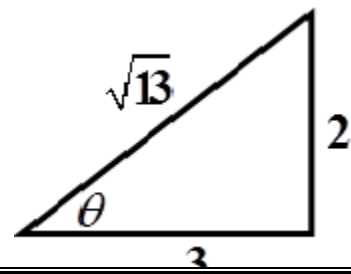
$$\sin 2\theta = 2\sin \theta \cos \theta = 2\left(\frac{\sqrt{7}}{4}\right)\left(\frac{-3}{4}\right) = \frac{-3\sqrt{7}}{8}$$

$$\tan 2\theta = \frac{2\tan \theta}{1 - \tan^2 \theta} = \frac{2\left(\frac{-\sqrt{7}}{3}\right)}{1 - \frac{7}{9}} = -3\sqrt{7}$$

(25)

$$\sin \theta = -\frac{2}{\sqrt{13}}, \quad \therefore \sin^2 \theta = \frac{4}{13}, \quad \therefore \cos \theta = -\frac{3}{\sqrt{13}}, \quad \therefore \tan \theta = \frac{2}{3}$$

$$\cos 2\theta = 1 - 2\sin^2 \theta = 1 - 2\left(\frac{4}{13}\right) = \frac{5}{13}$$



$$\sin 2\theta = 2 \sin \theta \cos \theta = 2 \left[\frac{-2}{\sqrt{13}} \right] \left[\frac{-3}{\sqrt{13}} \right] = \frac{12}{13}$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2 \left[\frac{2}{3} \right]}{1 - \frac{4}{9}} = \frac{12}{5}$$

(26) تمثيلات متعددة:

تختلف الإجابات من شخص لآخر حسب وجهة نظره.

مسائل مهارات التفكير العليا:

(27) إكتشف الخطأ:

كلاهما خطأ، حيث طرح سعيد الجذور التربيعية بطريقة غير صحيحة، كما استعمل سلمان متطابقة نصف الزاوية، ولاكنة خطأ في إيجاد قيمة $\cos 30$ في المتطابقة كلها فكتبها $\frac{1}{2}$

بدلاً من $\frac{\sqrt{3}}{2}$.

(28) تحذ:

الزاوية $\angle PBD$ هي زاوية محيطية تقابل القوس نفسة الذي تقابله الزاوية المركزية $\angle POD$ لذا فإن $m \angle (PBD) = \frac{1}{2} m \angle (PoD)$

وباستعمال المثلث القائم نجد أن

$$\tan \frac{\theta}{2} = \tan (PBA) = \frac{PA}{BA} = \frac{PA}{1+OA}$$

$$\frac{\sin \theta}{1 + \cos \theta} = \frac{\frac{AP}{OP}}{1 + \frac{OA}{OP}} = \frac{AP}{1 + OA}$$

(29) أكتب:

إذا أعطيت فقط قيمة $\cos \theta$ فإن $\cos 2\theta = 2\cos^2 \theta - 1$ هي أفضل متطابقة يمكن استعمالها.
وإذا أعطيت فقط قيمة $\sin \theta$ فإن $\cos 2\theta = 1 - 2\sin^2 \theta$ هي أفضل متطابقة يمكن استعمالها.
وإذا أعطيت كلا من $\sin \theta$ ، $\cos \theta$ فإن $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ هي أفضل متطابقة يمكن استعمالها.

(30) برهان:

$$\begin{aligned}\sin 2\theta &= \sin(\theta + \theta) \\ &= \sin \theta \cos \theta + \sin \theta \cos \theta = 2\sin \theta \cos \theta \\ \cos 2\theta &= \cos(\theta + \theta) \\ &= \cos \theta \cos \theta - \sin \theta \sin \theta = \cos^2 \theta - \sin^2 \theta\end{aligned}$$

(31) تبرير:

$$2\cos^2 \theta - 1 = \cos 2\theta$$

$$\theta = \frac{A}{2} \quad \therefore 2\theta = A$$

$$\therefore 2\cos^2 \frac{A}{2} - 1 = \cos A$$

$$\therefore \cos^2 \frac{A}{2} = \frac{1 + \cos A}{2}$$

$$\therefore \cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$1 - 2\sin^2 \theta = \cos 2\theta$$

$$\theta = \frac{A}{2} \quad \therefore 2\theta = A$$

$$\therefore \sin^2 \frac{A}{2} = \frac{1 - \cos A}{2}$$

$$\therefore \sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$$

(32)

$$d = \frac{2v^2 \sin \theta \cos \theta}{g} = \frac{v^2 \sin 2\theta}{g}$$

تكون أكثر قيمة لـ d عند $\sin 2\theta = 1$ ويكون هذا عند $2\theta = 90$ أو عند $\theta = 45$

مراجعة تراكمية

أثبت صحة كل ن المتطابقات الآتية:

(33)

$$\begin{aligned} & \frac{\cos^2 \theta + \sin \theta}{\sin \theta \cos \theta} \\ &= \frac{\cos^2 \theta}{\sin \theta \cancel{\cos \theta}} + \frac{\cancel{\sin \theta}}{\cancel{\sin \theta} \cos \theta} \\ &= \frac{\cos \theta}{\sin \theta} + \frac{1}{\cos \theta} = \cot \theta + \sec \theta \end{aligned}$$

(34)

$$(1 - \cos^2 \theta) + \frac{\sec^2 \theta}{\csc^2 \theta} =$$

$$\sin^2 \theta + \frac{\frac{1}{\cos^2 \theta}}{\frac{1}{\sin^2 \theta}} = \sin^2 \theta + \frac{\sin^2 \theta}{\cos^2 \theta}$$
$$= \sin^2 \theta + \tan^2 \theta$$

(35)

$$(\sin \theta - \cos \theta)^2 =$$

$$\sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta$$
$$= 1 - 2 \sin \theta \cos \theta$$

أوجد القيمة الدقيقة لكل مما يأتي:

$$\sin 135 = \frac{\sqrt{2}}{2} \quad (36)$$

$$\cos 105 = \frac{\sqrt{2} - \sqrt{6}}{4} \quad (37)$$

$$\sin 285 = \frac{-\sqrt{6} - \sqrt{2}}{4} \quad (38)$$

$$\cos(210) = -\frac{\sqrt{3} + 2}{4} \quad (39)$$

$$\sin(-240) = \frac{\sqrt{3}}{2} \quad (40)$$

$$\cos(-120) = -\frac{1}{2} \quad (41)$$

$$\cos 78 \cos 18 + \sin 78 \sin 18 = \cos(78 - 18) = \cos 60 = \frac{1}{2} \quad (42)$$

تدرب على إختبار

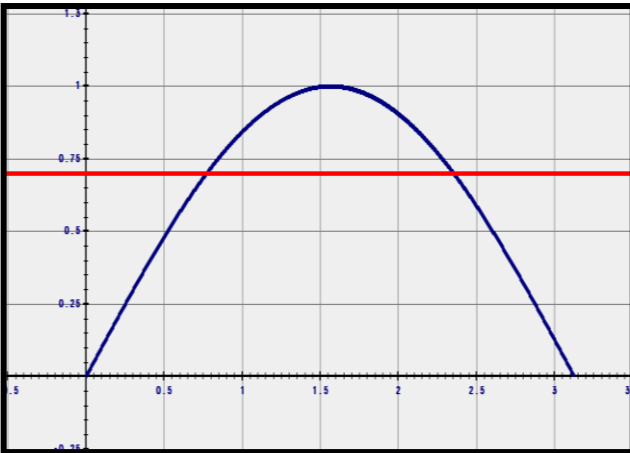
$$\sqrt{3} - 2 \leftarrow \leftarrow \leftarrow \leftarrow B \quad (43)$$

$$y = \frac{1}{3} \cos 2B \quad \leftarrow \leftarrow \leftarrow \leftarrow B \quad (44)$$

معمل الحاسبة البيانية ستكشاف (3-5)

حل المعادلات المثلثية

استعمل الحاسبة البيانية لحل المعادلات الآتية لقيم x جميعها الموضحة بجانب كل منها:

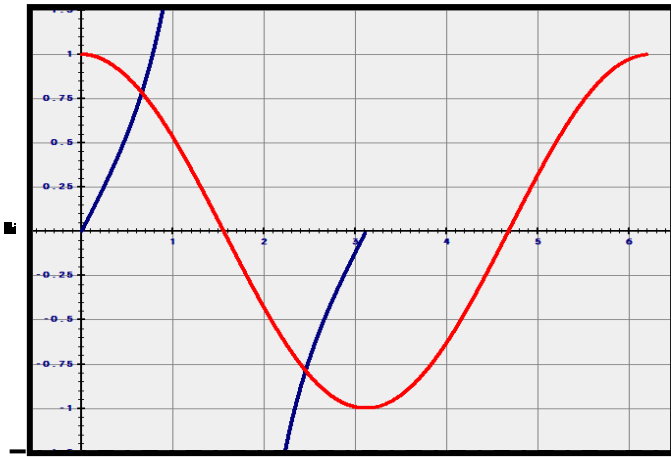


(1)

$$x = 44.4 \quad , \quad x = 135.6$$

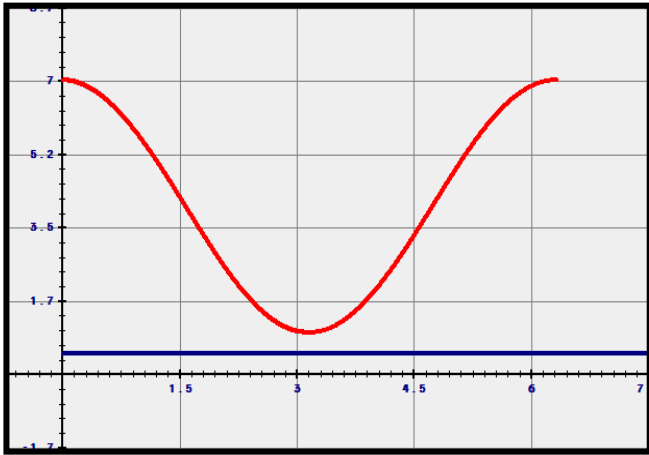
(2)

$$x = \{53.43, 137.81\}$$



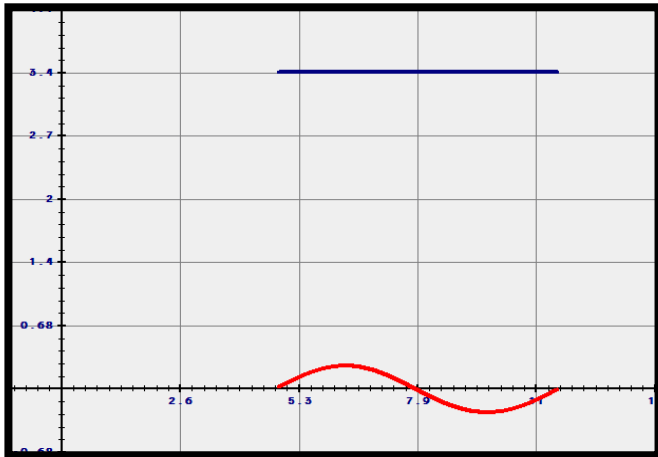
(3)

لا يوجد حل حقيقي



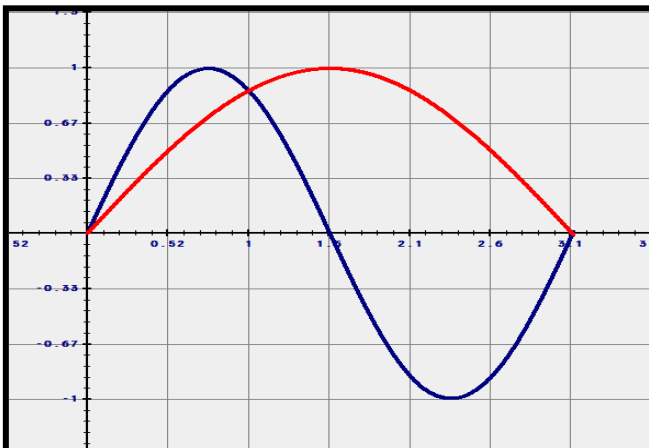
(4)

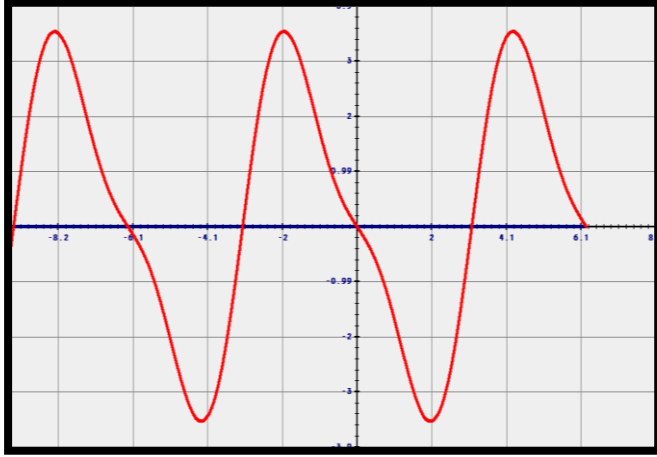
لا يوجد حل حقيقي



(5)

$$x = \{0, 115.38\}$$





(6)

$$x = \{-360, -180, 0, 180\}$$

(3-5) حل المعادلات المثلثية

تحقق من فهمك:

(1)

(1A)

$$\cos x \sin x = 3 \cos x$$

$$\therefore \cos x \sin x - 3 \cos x = 0$$

$$\therefore \cos x (\sin x - 3) = 0$$

$$\therefore \cos \theta = 0 \quad , \quad (\sin \theta - 3) = 0 \rightarrow \text{لها حل}$$

$$\therefore \theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\therefore \theta = \left\{ \frac{\pi}{2}, \frac{3\pi}{2} \right\}$$

(1B)

$$\sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta = 0$$

$$\sin^2 \theta + (1 - \sin^2 \theta) + 2 \sin \theta \cos \theta = 0$$

$$2 \sin \theta \cos \theta = 1$$

$$\sin \theta = \frac{1}{2} \quad , \quad \cos \theta = \frac{1}{2}$$

$$\theta = 30^\circ \quad , \quad \theta = 60^\circ$$

■ تحقق من فهمك:

(2A)

$$4 \sin x = 2 \sin x + \sqrt{2}$$

$$\therefore 4 \sin x - 2 \sin x = \sqrt{2}$$

$$\therefore 2 \sin x = \sqrt{2}$$

$$\therefore \sin x = \frac{\sqrt{2}}{2}$$

$$x = 45^\circ$$

(2B)

$$2 \sin \theta = -1$$

$$\therefore \sin \theta = -\frac{1}{2}$$

$$\therefore \theta = \left\{ \frac{7\pi}{6} + 2k\pi, \frac{11\pi}{6} + 2k\pi \right\}$$

■ تحقق من فهمك:

(3)

$$h = 21 - 20 \cos 3\pi t$$

$$\therefore 41 = 21 - 20 \cos 3\pi t$$

$$\therefore 20 \cos 3\pi t = -20$$

$$\therefore \cos 3\pi t = -1$$

$$\therefore 3\pi t = (2k + 1)\pi$$

$$\therefore t = \frac{2}{3}k + \frac{1}{3}$$

$$\therefore t = 20 \text{ s}$$

■ تحقق من فهمك:

(4) متطابقة، ولها عدد لانتهائي من الحلول

$$\cos^2 \theta + 3 = 4 - \sin^2 \theta$$

$$\cos^2 \theta + 3 = (1 - \sin^2 \theta) + 3 = 4 - \sin^2 \theta$$

■ **تحقق من فهمك:**

(5)

(5A) لا يوجد لها حل حيث:

$$\sin \theta \cot \theta - \cos^2 \theta = 0$$

$$\therefore \cancel{\sin \theta} \frac{\cos \theta}{\cancel{\sin \theta}} - \cos^2 \theta = 0$$

$$\therefore \cos \theta - \cos^2 \theta = 0$$

$$\therefore \cos \theta (1 - \cos \theta) = 0$$

$$\therefore \cos \theta = 0 \text{ or } \cos \theta = 1$$

$$\therefore \theta = \frac{\pi}{2} + k\pi$$

(5B)

$$\frac{\cos \theta}{\cot \theta} + 2 \sin^2 \theta = 0$$

$$\therefore \frac{\cos \theta}{\frac{\cos \theta}{\sin \theta}} + \sin^2 \theta = 0$$

$$\therefore \sin \theta + \sin^2 \theta = 0$$

$$\therefore \sin \theta (1 + \sin \theta) = 0$$

$$\therefore \sin \theta = 0 \text{ or } \sin \theta = -1$$

$$\therefore \theta = \frac{7\pi}{6} + 2k\pi \text{ or } \theta = \frac{11\pi}{6} + 2k\pi$$

تدرب وحل المسائل

حل كل معادلة مما يأتي لقيم θ الموضحة بجانب كل منها.

(1)

$$\cos^2 \theta + 2 \cos \theta + 1 = 0$$

$$\therefore (\cos \theta + 1)^2 = 0$$

$$\therefore \cos \theta = -1$$

$$\therefore \theta = 80^\circ$$

(2)

$$2\cos^2 \theta + \cos \theta = 1$$

$$\therefore (\cos \theta + 1)(2\cos \theta - 1) = 0$$

$$\therefore \cos \theta = -1 \quad \cos \theta = \frac{1}{2}$$

$$\therefore \theta = 180^\circ, 60^\circ, 300^\circ$$

(3)

$$-2\sin^2 \theta = 7 - 15\sin \theta$$

$$\therefore 2\sin^2 \theta - 15\sin \theta + 7 = 0$$

$$\therefore (2\sin \theta - 1)(\sin \theta - 7) = 0$$

$$\therefore \sin \theta = \frac{1}{2} \quad \sin \theta = 7$$

$$\therefore \theta = 150^\circ, 30^\circ$$

(4)

$$\cos \theta + \frac{\sqrt{3}}{2} = 0$$

$$\therefore \cos \theta = -\frac{\sqrt{3}}{2}$$

$$\therefore \theta = 150^\circ, 30^\circ$$

حل كل معادلة مما يأتي لقيم θ جميعها إذا كان قياس θ بالراديان.

(5)

$$4\sin^2 \theta - 1 = 0$$

$$\therefore \sin^2 \theta = \frac{1}{4}$$

$$\therefore \sin \theta = \frac{1}{2}$$

$$\therefore \theta = \left\{ \pm \frac{\pi}{6} + 2k\pi, \pm \frac{5\pi}{6} + 2k\pi \right\}$$

(6)

$$2\cos^2 \theta = 1$$

$$\therefore \cos^2 \theta = \frac{1}{2}$$

$$\therefore \cos \theta = \frac{1}{\sqrt{2}}$$

$$\therefore \theta = \left\{ \frac{\pi}{4} + k \frac{\pi}{2} \right\}$$

(7)

$$\sin \frac{\theta}{2} - 2\sin^2 \frac{\theta}{2} = 0$$

$$\sin \frac{\theta}{2} (1 - 2\sin \frac{\theta}{2}) = 0$$

$$\sin \frac{\theta}{2} = 0 \quad 1 - 2\sin \frac{\theta}{2} = 0$$

$$\frac{\theta}{2} = 0, 180 \quad \sin \frac{\theta}{2} = -\frac{1}{2}$$

$$\theta = 0^\circ, 360^\circ \quad \frac{\theta}{2} = 30$$

$$\theta = 60^\circ$$

(8)

$$\therefore 2\cos^2 \theta + 4\cos \theta = -2$$

$$\therefore \cos^2 \theta + 2\cos \theta + 1 = 0$$

$$\therefore (\cos \theta + 1)^2 = 0$$

$$\therefore \cos \theta = -1$$

$$\therefore \theta = \{ \pi + 2k\pi \}$$

حل كل معادلة مما يأتي لقيم θ جميعها إذا كان قياس θ بالدرجات.

(9)

$$\cos 2\theta - \sin^2 \theta + 2 = 0$$

$$\therefore 1 - 2\sin^2 \theta - \sin^2 \theta + 2 = 0$$

$$\therefore 3 - 3\sin^2 \theta = 0$$

$$\therefore \sin^2 \theta = 1$$

$$\therefore \sin \theta = 1$$

$$\therefore \theta = \{ 90^\circ + k 180^\circ \}$$

(10)

$$\sin^2 \theta - \sin \theta = 0$$

$$\therefore \sin \theta (1 - \sin \theta) = 0$$

$$\therefore \sin \theta = 0, \quad \sin \theta = 1$$

$$\therefore \theta = \{ k 180^\circ, 90^\circ + k 360^\circ \}$$

(11)

$$2\sin^2 \theta - 1 = 0$$

$$\therefore \sin^2 \theta = \frac{1}{2}$$

$$\therefore \sin \theta = \frac{1}{\sqrt{2}}$$

$$\therefore \theta = \{ 45^\circ + k 90^\circ \}$$

(12)

$$\theta = \{ 30^\circ + k 360^\circ , 150^\circ + k 360^\circ , 90^\circ + k 180^\circ \}$$

(13) الليل والنهار:

(a) عدد ساعات النهار 10.5 ساعة، ويكون ذلك بعد 213 أو 335 يوماً بعد يوم 21 مارس. وهذا يعني أن ساعات النهار ستكون في 20 أكتوبر أو 19 أكتوبر. ستكون عدد ساعات النهار 10.5 ساعة.

(b) كل يوم منذ 19 فبراير إلى 20 أكتوبر. بما أن أطول نهار في السنة يحدث تقريباً يوم 22 يونيو، لذا فإن الأيام بين 19 فبراير إلى 20 أكتوبر تتزايد في الطول حتى يوم 22 يونيو، ثم يبدأ النهار بالنقصان إلى يوم 20 أكتوبر.

حل كل معادلة مما يأتي:

(14)

$$\sin^2 2\theta + \sin^2 \theta = 0$$

$$\therefore 4\sin^2 \theta \cos^2 \theta + \sin^2 \theta = 0$$

$$\therefore \sin^2 \theta (4\cos^2 \theta + 1) = 0$$

$$\therefore \sin \theta = 0 \quad , \quad \cos^2 \theta = -\frac{1}{4}$$

$$\therefore \theta = \{ 90^\circ + k(180^\circ) \}$$

(15)

$$\sin 2\theta - \cos \theta = 0$$

$$\therefore 2\sin \theta \cos \theta - \cos \theta = 0$$

$$\therefore \cos \theta (2\sin \theta - 1) = 0$$

$$\therefore \cos \theta = 0 \quad , \quad \sin \theta = \frac{1}{2}$$

$$\therefore \theta = \{ k(180^\circ), 30^\circ + k(360^\circ), 150^\circ + k(360^\circ) \}$$

(16)

$$\tan \theta = 1$$

$$\therefore \theta = \{ 45^\circ + k180^\circ, 45^\circ + k(180^\circ) \}$$

(17)

$$\cos^2 \theta = \frac{1}{4}$$

$$\therefore \cos \theta = \frac{1}{2}$$

$$\therefore \theta = \{ 60^\circ, 120^\circ, 240^\circ, 300^\circ, 135^\circ, 225^\circ \}$$

(18)

$$2 \sin^2 \theta = 1$$

$$\therefore \sin^2 \theta = \frac{1}{2}$$

$$\therefore \sin \theta = \frac{1}{\sqrt{2}}$$

$$\therefore \theta = \{ 135^\circ, 225^\circ \}$$

(19)

$$\sin 2\theta - \cos \theta = 0$$

$$\therefore 2 \sin \theta \cos \theta - \cos \theta = 0$$

$$\therefore \cos \theta (2 \sin \theta - 1) = 0$$

$$\therefore \cos \theta = 0, \quad \sin \theta = \frac{1}{2}$$

$$\therefore \theta = \left\{ \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2} \right\}$$

(20)

$$4\sin^2 \theta - 1 = 0$$

$$\therefore \sin^2 \theta = \frac{1}{4}$$

$$\therefore \sin \theta = \frac{1}{2}$$

$$\therefore \theta = \{210^\circ, 330^\circ\}$$

(21)

$$\tan \theta - \sin \theta = 0$$

$$\therefore \frac{\sin \theta}{\cos \theta} - \sin \theta = 0$$

$$\therefore \frac{\sin \theta - \cos \theta \sin \theta}{\cos \theta} = 0$$

$$\therefore \sin \theta - \cos \theta \sin \theta$$

$$\therefore \sin \theta (1 - \cos \theta) = 0$$

$$\therefore \sin \theta = 0, \cos \theta = 1$$

$$\therefore \theta = \{0^\circ + k 180^\circ\}$$

(22)

$$\theta = \{30^\circ + k 360^\circ, 150^\circ + k 360^\circ\}$$

ناطحات سحاب: (23)

$$\tan \theta = \frac{876}{685} = 1.28$$

$$\therefore \theta = 21^\circ$$

أنهار: (24)

$$y = 11 m$$

(a)

(b) في الساعة 7 صباحاً، 7 مساءً.

حل كل معادلة مما يأتي لقيم θ جميعها إذا كان قياس θ بالراديان.

(25)

$$\cos \theta \sin 2\theta - 2 \sin \theta + 2 = 0$$

$$\therefore \cos \theta (2 \sin \theta \cos \theta) - 2 \sin \theta + 2 = 0$$

$$\therefore 2 \sin \theta \cos^2 \theta - 2 \sin \theta + 2 = 0$$

$$\therefore \sin \theta \cos^2 \theta - \sin \theta + 1 = 0$$

$$\therefore \sin \theta (\cos^2 \theta - 1) + 1 = 0$$

$$\therefore \sin \theta (-\sin^2 \theta) + 1 = 0$$

$$\therefore -\sin^3 \theta = -1$$

$$\therefore \sin \theta = 1$$

$$\therefore \theta = \left\{ \frac{\pi}{2} + 2k\pi \right\}$$

(26)

$$\theta = \left\{ \frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi \right\}$$

(27)

$$2 \sin \theta = \sin 2\theta$$

$$\therefore 2 \sin \theta = 2 \sin \theta \cos \theta$$

$$\therefore 2 \sin \theta - 2 \sin \theta \cos^2 \theta = 0$$

$$\therefore 2 \sin \theta (1 - \cos^2 \theta) = 0$$

$$\therefore \sin \theta = 0 \quad , \quad \therefore \sin \theta = 1$$

$$\therefore \theta = \left\{ \frac{5\pi}{6} + 2k\pi , \frac{7\pi}{4} + 2k\pi \right\}$$

حل كل معادلة مما يأتي لقيم θ جميعها إذا كان قياس θ بالدرجات.

(28)

$$\theta = \{ 30^\circ + 360^\circ k , 150^\circ + 360^\circ k , 330^\circ + 360^\circ k \}$$

(29)

$$\theta = \{ 120^\circ + 360^\circ k , 240^\circ + 360^\circ k \}$$

(30) الماس:

(a) حوالي 13.71°

(b) بقياس زاوية السقوط للضوء وانعكاساتها لتحديد معامل إنكسار الضوء، فإذا كان معامل الإنكسار يساوي 2.42 يكون ماساً نقياً.

مسائل مهارات التفكير العليا:

(31) اكتشف الخطأ:

قسمت **حلاً** كلاً من الطرفين على $\sin \theta$ وهذا خطأ، بينما طرحت **شهد** $\sin \theta$ من الطرفين بشكل خطأ أيضاً.

(32) تحديد:

$$\sin 2x < \sin x$$

$$\therefore \sin 2x - \sin x < 0$$

$$\therefore 2\sin x \cos x - \sin x < 0$$

$$\therefore \sin x (2\cos x - 1) < 0$$

$$\therefore \sin x < 0 \quad , \quad \cos x > \frac{1}{2}$$

$$\text{or } \sin x > 0 \quad , \quad \cos x < \frac{1}{2}$$

$$\therefore \frac{5\pi}{3} < x < 2\pi \quad \text{or} \quad \frac{\pi}{3} < x < \pi$$

(33) اكتب:

كل نوع من المعادلات يحتاج إما إلى جمع أو طرح أو ضرب أو قسمة كل طرف على العدد نفسه
تحل المعادلات التربيعية والمثلثية على الأغلب بإستعمال التحليل ولا تحتاج المعادلات الخطية
والتربيعية إلى متطابقات لحلها، ويمكن حلها جبرياً في حين يمكن تمثيل بعض المعادلات المثلثية
بيانياً بسهولة بإستعمال الحاسبة البيانية، أما المعادلات الخطية فلها على الأكثر حل وحيد.
والمعادلة التربيعية لها على الأكثر حلان. أما المعادلات المثلثية فلها عادة عدد لانهاى من الحلول
إلا إذا كانت قيم المتغير مقيدة أو مشروطة.

(34) تبرير:

لأن الدوال المثلثية دورية، فإضافة دورة كاملة لأي حل للمعادلة ينتج حلاً لها.

(35) مسألة مفتوحة:

المعادلة هي: $2\cos \theta = 0$ والحلان هما 270° , $90^\circ = \theta$

(36) تحديد:

نعم لأن

$$\cot x + 1 = 2$$

$$\therefore \sec^2 x = 2 \quad \therefore \sec x = \sqrt{2}$$

$$\therefore \cos x = \frac{1}{\sec x} = \frac{1}{\sqrt{2}}$$

$$\therefore x = \frac{\pi}{4}$$

$$\csc x = \sqrt{2}$$

$$\therefore \sin x = \frac{1}{\csc x} = \frac{1}{\sqrt{2}}$$

$$\therefore x = \frac{\pi}{4}$$

مراجعة تراكمية

أوجد القيمة الدقيقة لكل مما يأتي:

$$\frac{-\sqrt{6}-\sqrt{2}}{4} \quad (37)$$

$$\frac{\sqrt{2}-\sqrt{2}}{2} \quad (38)$$

$$\frac{\sqrt{2}-\sqrt{2}}{2} \quad (39)$$

$$\frac{-\sqrt{6}-\sqrt{2}}{4} \quad (40)$$

أثبت أن كل معادلة مما يأتي متطابقة:

(41)

$$\begin{aligned} & \sin(270 - \theta) \\ &= \sin 270 \cos \theta - \cos 270 \sin \theta \\ &= -1 \times \cos \theta - 0 = -\cos \theta \end{aligned}$$

(42)

$$\begin{aligned} & \cos(90 + \theta) \\ &= \cos 90 \cos \theta - \sin 90 \sin \theta \\ &= 0 - 1 \times \sin \theta = -\sin \theta \end{aligned}$$

(43)

$$\begin{aligned} & \cos(90 - \theta) \\ &= \cos 90 \cos \theta + \sin 90 \sin \theta \\ &= 0 + 1 \times \sin \theta = \sin \theta \end{aligned}$$

(44)

$$\begin{aligned} & \sin(90 - \theta) \\ &= \sin 90 \cos \theta - \cos 90 \sin \theta \\ &= 1 \times \cos \theta - 0 = \cos \theta \end{aligned}$$

(45)

$$\frac{v^2 \tan^2 \theta}{2g \sec^2 \theta} = \frac{v^2 \frac{\sin^2 \theta}{\cos^2 \theta}}{2g \frac{1}{\cos^2 \theta}} = \frac{v^2 \sin^2 \theta}{2g} \quad (a)$$

$$h = \frac{(110)^2 \sin^2 80}{2 \times 9.8} = 598.73 \text{ m} \quad (b)$$

(46)

المجال: $[-5, \infty)$
المدى: $[-2, \infty)$

تدرب على إختبار

(47)

$$\frac{5\pi}{2} \leftarrow \leftarrow \leftarrow \leftarrow A$$

(48)

$$240^\circ \text{ أو } 300^\circ \leftarrow \leftarrow \leftarrow \leftarrow D$$

دليل الدراسة والمراجعة

اختبر مفرداتك:

اكتب المفردة المناسبة لكل عبارة مما يأتي:

- (1) متطابقة الزاويتين المتتامتين
- (2) المتطابقات النسبية
- (3) المتطابقة المثلثية
- (4) متطابقات ضعف الزاوية
- (5) المتطابقات
- (6) المتطابقات المثلثية لجمع زاويتين أو الفرق بينهما
- (7) متطابقات المقلوب
- (8) متطابقات مجموع زاويتين
- (9) متطابقات فيثاغورث

أوجد القيمة الدقيقة لكل من النسب المثلثية الآتية:

(10)

$$\begin{aligned}\cos^2 \theta + \sin^2 \theta &= 1 \\ &= \left(\frac{\sqrt{2}}{2}\right)^2 + \sin^2 \theta = 1 \\ \frac{2}{4} + \sin^2 \theta &= 1 \\ \sin \theta &= \frac{\sqrt{2}}{2}\end{aligned}$$

(11)

$$\sec \theta = \sqrt{\frac{6}{2}}$$

(12)

$$\tan \theta = \frac{1}{\cot \theta}$$

$$\tan \theta = \frac{1}{2}$$

(13)

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\cos^2 \theta + \frac{9}{25} = 1$$

$$\cos^2 \theta = \frac{16}{25}$$

$$\cos \theta = \frac{4}{5}$$

(14)

$$\cot^2 \theta + 1 = \csc^2 \theta$$

$$\frac{16}{25} + 1 = \csc^2 \theta$$

$$\csc^2 \theta = \frac{41}{25}$$

$$\csc \theta = \frac{\sqrt{41}}{5}$$

(15) كرة قدم:

$$\sin \theta = \frac{75}{133}$$

بسّط كل عبارة مما يأتي:

(16)

$$\begin{aligned} & 1 - \tan \theta \sin \theta \cos \theta \\ & 1 - \frac{\sin \theta}{\cos \theta} \sin \theta \cos \theta \\ & 1 - \sin^2 \theta \\ & = \cos^2 \theta \end{aligned}$$

(17)

$$\begin{aligned} & \tan \theta \csc \theta \\ & \frac{\sin \theta}{\cos \theta} \times \frac{1}{\sin \theta} \\ & = \frac{1}{\cos \theta} \\ & = \sec \theta \end{aligned}$$

(18)

$$\begin{aligned} & \sin \theta + \cos \theta \cot \theta \\ & = \sin \theta + \cos \theta \frac{1}{\sin \theta} \\ & = \frac{1}{\cos \theta} \\ & = \sec \theta \end{aligned}$$

(19)

$$\begin{aligned} & \cos \theta (1 + \tan^2 \theta) \\ & = \cos \theta (\sec^2 \theta) \\ & = \cos \theta \times \frac{1}{\cos^2 \theta} \\ & = \sec \theta \end{aligned}$$

أثبت أن كل معادلة مما يأتي تمثل متطابقة:

(20)

$$\begin{aligned} & \tan \theta \cos \theta + \cot \theta \\ &= \frac{\sin \theta}{\cos \theta} \times \cos \theta + \frac{\cos \theta}{\sin \theta} \times \sin \theta \\ &= \sin \theta + \cos \theta \end{aligned}$$

(21)

$$\begin{aligned} & \frac{\cos \theta}{\cot \theta} + \frac{\sin \theta}{\tan \theta} \\ &= \cos \theta \times \frac{\sin \theta}{\cos \theta} + \sin \theta \times \frac{\cos \theta}{\sin \theta} \\ &= \sin \theta + \cos \theta \end{aligned}$$

(22)

$$\begin{aligned} & \sec^2 \theta - 1 \\ &= \tan^2 \theta \\ &= \frac{\sin^2 \theta}{\cos^2 \theta} \\ &= \frac{\sin^2 \theta}{1 - \sin^2 \theta} \end{aligned}$$

(23) هندسة:

$$\cos \theta = \frac{3}{4}, \sin \theta = \frac{\sqrt{7}}{4}$$

$$\tan \theta = \frac{\sqrt{7}}{4} \times \frac{4}{3}$$

$$\tan \theta = \frac{7}{9}$$

$$\tan \theta + 1 = \frac{16}{9}$$

$$\sec^2 \theta = \frac{1}{\cos^2 \theta}$$

$$= \frac{16}{9}$$

دون التعمال الآلة الحاسبة، أوجد القيمة الدقيقة لكل مما يأتي:

(24)

$$\cos(-135) = \cos(45 - 180)$$

$$= \cos 45 \cos 180 + \sin 45 \sin 180$$

$$= \frac{\sqrt{2}}{2} \times 1 + \frac{\sqrt{2}}{2} \times 0$$

$$= \frac{\sqrt{2}}{2}$$

(25)

$$\begin{aligned}\cos(15) &= \cos(45 - 30) \\ &= \cos 45 \cos 30 + \sin 45 \sin 30 \\ &= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2} \\ &= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} \\ &= \frac{\sqrt{6} + \sqrt{2}}{4}\end{aligned}$$

(26)

$$\begin{aligned}\sin(210) &= \sin(180 + 30) \\ &= \sin 180 \cos 30 + \cos 180 \sin 30 \\ &= 0 \times \frac{\sqrt{3}}{2} + 1 \times \frac{1}{2} \\ &= \frac{1}{2}\end{aligned}$$

(27)

$$\begin{aligned}\sin(105) &= \sin(60 + 45) \\ &= \sin 60 \cos 45 + \cos 60 \sin 45 \\ &= \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} + \frac{1}{2} \times \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} \\ &= \frac{\sqrt{6} + \sqrt{2}}{4}\end{aligned}$$

(28)

$$\begin{aligned}\tan(75) &= \tan(45 + 30) \\ &= \frac{\tan 45 + \tan 30}{1 - \tan 45 \tan 30} \\ &= \frac{1 + \sqrt{3}}{1 - \sqrt{3}}\end{aligned}$$

(29)

$$\begin{aligned}\cos(105) &= \cos(60 + 45) \\ &= \cos 60 \cos 45 - \sin 60 \sin 45 \\ &= \frac{1}{2} \times \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} \\ &= \frac{\sqrt{6} + \sqrt{2}}{4}\end{aligned}$$

أثبت صحة كل من المتطابقات الآتية:

(30)

$$\begin{aligned}\sin(\theta + 90) \\ &= \sin \theta \cos 90 + \cos \theta \sin 90 \\ &= \sin \theta \times 0 + \cos \theta \times 1 \\ &= \cos \theta\end{aligned}$$

(31)

$$\begin{aligned} & \sin\left(\frac{3\pi}{2} + \theta\right) \\ &= \sin \frac{3\pi}{2} \cos \theta + \cos \frac{3\pi}{2} \sin \theta \\ &= -1 \times \cos \theta - 0 \times \sin \theta \\ &= -\cos \theta \end{aligned}$$

(32)

$$\begin{aligned} & \tan(\theta - \pi) \\ &= \frac{\tan \theta - \tan \pi}{1 + \tan \theta \tan \pi} \\ &= \frac{\tan \theta - 0}{1 + 0} \\ &= \tan \theta \end{aligned}$$

أوجد القيم الدقيقة لكل من $\sin 2\theta, \cos 2\theta, \sin \frac{\theta}{2}, \cos \frac{\theta}{2}$ إذا علمت أن:

(33)

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$\sin^2 \theta = \frac{9}{25}$$

$$\sin \theta = \frac{12}{13}$$

$$\sin 2\theta = 2\sin \theta \cos \theta$$

$$= 2\left(\frac{4}{5} \times \frac{3}{5}\right)$$

$$\sin 2\theta = \frac{24}{25}$$

$$\cos 2\theta = 2\cos^2 \theta - 1$$

$$\cos 2\theta = 2\left(\frac{16}{25}\right)^2 - 1$$

$$= \frac{7}{25}$$

$$\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\sin \frac{\theta}{2} = \frac{1}{\sqrt{10}}$$

$$\cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\cos \frac{\theta}{2} = \frac{3}{\sqrt{10}}$$

(34)

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$\sin^2 \theta = \frac{9}{25}$$

$$\sin \theta = \frac{\sqrt{15}}{4}$$

$$\sin 2\theta = 2\sin \theta \cos \theta$$

$$= 2\left(-\frac{1}{4} \times \frac{\sqrt{15}}{4}\right)$$

$$\sin 2\theta = -\frac{\sqrt{15}}{8}$$

$$\cos 2\theta = 2\cos^2 \theta - 1$$

$$\cos 2\theta = 2\left(\frac{15}{16}\right) - 1$$

$$= \frac{14}{16}$$

$$\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\sin \frac{\theta}{2} = \frac{1}{2} - \frac{\sqrt{15}}{4}$$

$$\cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\cos \frac{\theta}{2} = \frac{1}{2} + \frac{\sqrt{15}}{4}$$

(35)

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$\sin^2 \theta = \frac{5}{9}$$

$$\sin \theta = \frac{\sqrt{5}}{3}$$

$$\sin 2\theta = 2\sin \theta \cos \theta$$

$$= 2\left(\frac{\sqrt{5}}{3} \times -\frac{2}{3}\right)$$

$$\sin 2\theta = \frac{-4\sqrt{5}}{9}$$

$$\cos 2\theta = 2\cos^2 \theta - 1$$

$$\cos 2\theta = 2\left(-\frac{2}{3}\right)^2 - 1$$

$$= -\frac{1}{9}$$

$$\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\sin \frac{\theta}{2} = \sqrt{\frac{5}{6}}$$

$$\cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\cos \frac{\theta}{2} = \sqrt{\frac{1}{6}}$$

(36) ملاعب:

$$127 \text{ ft} = \text{طول القطر} = (a)$$

$$\cos 45 = \frac{90}{127} \quad (b)$$

$$\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}} \quad (c)$$

$$\sin \frac{\theta}{2} = \sqrt{\frac{74}{127}}$$

$$\frac{\theta}{2} = 22.5$$

حل كل معادلة مما يأتي لقيم θ جميعها الموضحة بجانب كل منها:

(37)

$$2 \cos \theta - 1 = 0$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = 60^\circ, 300^\circ$$

(38)

$$4 \cos^2 \theta - 1 = 0$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

(39)

$$\cos 2\theta + \cos \theta = 0$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = 90^\circ, 210^\circ, 270^\circ, 330^\circ$$

(40)

$$\theta = 270$$

(41)

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

(42)

$$\begin{aligned}\sin \theta &= \frac{\sqrt{145}}{145} \\ \cos \theta &= \frac{12\sqrt{145}}{145}\end{aligned}$$

(43)

$$I = I_0 \left(\frac{1}{1 + \tan^2 \theta} \right)$$

(44)

$$\begin{aligned}r &= \frac{\sin \alpha}{1 - \cos \alpha} \\ &= \frac{\sin \alpha}{1 - \cos \alpha} \cdot \frac{1 + \cos \alpha}{1 + \cos \alpha} \\ &= \frac{\sin \alpha (1 + \cos \alpha)}{\sin \alpha (1 + \cos \alpha)} \\ &= \frac{\sin^2 \alpha}{1 + \cos \alpha} \\ &= \frac{\sin \alpha}{1 + \cos \alpha}\end{aligned}$$

(45) موجات:

$$y_1 + y_2 = 0$$

إذن التداخل هدام

(46) هندسة:

$$\sin 2 = 2 \sin N \cos N$$

$$= 2 \frac{n}{m} \frac{l}{m}$$

$$= \frac{2nl}{m^2}$$

أثبت أن كل من المعادلتين الآتيتين تمثلان متطابقة:

(47)

$$\frac{\sin 2\theta}{2 \sin^2 \theta} = \cot \theta$$

$$\frac{2 \sin \theta \cos \theta}{2 \sin^2 \theta} = \cot \theta$$

$$\frac{\cos \theta}{\sin \theta} = \cot \theta$$

(48)

$$1 + \cos^2 \theta = \frac{2}{1 + \tan^2 \theta}$$

$$1 + 2\cos^2 \theta - 1 = \frac{2}{\sec^2 \theta}$$

$$2\cos^2 \theta = 2\cos^2 \theta$$

(49) مقذوفات:

الزاوية التي قذفت بها الكرة = 60

اختبار الفصل

(1) إختيار من متعدد:

$$\cos \theta \leftarrow \leftarrow \leftarrow \leftarrow D$$

(2)

$$\cos(30 - \theta)$$

$$= \cos 30 \cos \theta + \sin 30 \sin \theta$$

$$= \frac{\sqrt{3}}{2} \cos \theta + \frac{1}{2} \sin \theta$$

$$\sin(60 + \theta)$$

$$= \sin 60 \cos \theta + \cos 60 \sin \theta$$

$$= \frac{\sqrt{3}}{2} \cos \theta + \frac{1}{2} \sin \theta$$

(3)

$$\begin{aligned}
& \cos(\theta - \pi) \\
&= \cos \theta \cos \pi + \sin \theta \sin \pi \\
&= -1 \square \cos \theta + 0 \\
&= -\cos \theta
\end{aligned}$$

(4) إختيار من متعدد:

$$\frac{4}{5} \leftarrow \leftarrow \leftarrow \leftarrow D$$

بدون استعمال الآلة الحاسبة، أوجد القيمة الدقيقة لكل مما يأتي:

$$-\frac{3\sqrt{7}}{7} \quad (5)$$

$$-\sqrt{3} \quad (6)$$

$$\frac{-2\sqrt{3}}{3} \quad (7)$$

$$\frac{2\sqrt{3}}{3} \quad (8)$$

أثبت أن كل معادلة مما يأتي متطابقة:

(9)

$$\begin{aligned}
& \sin \theta (\cot \theta + \tan \theta) \\
&= \sin \theta \left(\frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} \right) \\
&= \cos \theta + \frac{\sin^2 \theta}{\cos \theta} = \frac{\cos^2 \theta + \sin^2 \theta}{\cos \theta} \\
&= \frac{1}{\cos \theta} = \sec \theta
\end{aligned}$$

(10)

$$\begin{aligned} & \frac{\cos \theta}{\sec \theta - \tan \theta} \\ &= \frac{\cos \theta}{\frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta}} = \frac{\cos \theta}{\frac{1 - \sin \theta}{\cos \theta}} \\ &= \frac{\cos^2 \theta}{1 - \sin \theta} \end{aligned}$$

(11)

$$\begin{aligned} & (\tan \theta + \cot \theta)^2 \\ &= \left(\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \right)^2 = \left(\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \sin \theta} \right)^2 \\ &= \left(\frac{1}{\cos \theta \sin \theta} \right)^2 = \frac{1}{\cos^2 \theta \sin^2 \theta} \\ &= \sec^2 \theta \csc^2 \theta \end{aligned}$$

(12)

$$\frac{1 + \sec \theta}{\sec \theta} = \frac{1}{\sec \theta} + \frac{\sec \theta}{\sec \theta} = \cos \theta + 1$$

$$\begin{aligned} & \frac{\sin^2 \theta}{1 - \cos \theta} \\ &= \frac{\sin^2 \theta}{1 - \cos \theta} \cdot \frac{1 + \cos \theta}{1 + \cos \theta} = \frac{\sin^2 \theta (1 + \cos \theta)}{1 - \cos^2 \theta} \\ &= \frac{\cancel{\sin^2 \theta} (1 + \cos \theta)}{\cancel{\sin^2 \theta}} = 1 + \cos \theta = \cos \theta + 1 \end{aligned}$$

(13) إختيار من متعدد:

$$\sqrt{2} - 1 \leftarrow \leftarrow \leftarrow \leftarrow B$$

(14) تاريخ:

(a) بفرض أن إرتفاع المثلث يساوي a

$$a^2 + 9^2 = 18^2$$

$$\therefore a^2 = 18^2 - 9^2$$

$$\therefore a = \sqrt{243} = 9\sqrt{3}$$

(b)

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\therefore \sin 2(30) = 2 \sin 30 \cos 30 = \frac{\sqrt{3}}{2}$$

$$\therefore \sin 60 = \frac{\sqrt{3}}{2}$$

أوجد القيمة الدقيقة لكل مما يأتي:

(15)

$$\cos(-225)$$

$$= \cos(-225 + 360) = \cos 135 = -\frac{\sqrt{2}}{2}$$

(16)

$$\sin 480$$

$$= \sin(480 - 360) = \sin 120 = \frac{\sqrt{3}}{2}$$

(17)

$$\cos 75$$

$$= \cos(120 - 45) = \cos 120 \cos 45 + \sin 120 \sin 45 = \frac{\sqrt{6} - \sqrt{2}}{4}$$

(18)

$$\sin 165$$

$$= \sin(120 + 45) = \sin 120 \cos 45 + \cos 120 \sin 45 = \frac{\sqrt{6} - \sqrt{2}}{4}$$

حل كل من المعادلتين الآتيتين لقيم θ جميعها، إذا كان قياس θ بالراديان:

(19)

$$\frac{\pi}{6} + k\pi$$

(20)

$$\frac{\pi}{18} + k \frac{2\pi}{3}$$

حل كل من المعادلتين الآتيتين حيث $0 \leq \theta \leq 360$:

(21)

$$\theta = \{0^\circ, 360^\circ\}$$

$$\sin \theta \cos \theta - \frac{1}{2} \sin \theta = 0$$

$$\therefore \sin \theta \left(\cos \theta - \frac{1}{2} \right) = 0$$

$$\therefore \sin \theta = 0 \quad , \quad \cos \theta = \frac{1}{2}$$

$$\theta = \{0^\circ, 60^\circ, 180^\circ, 300^\circ, 360^\circ\}$$