15

السعدي ALSAADI

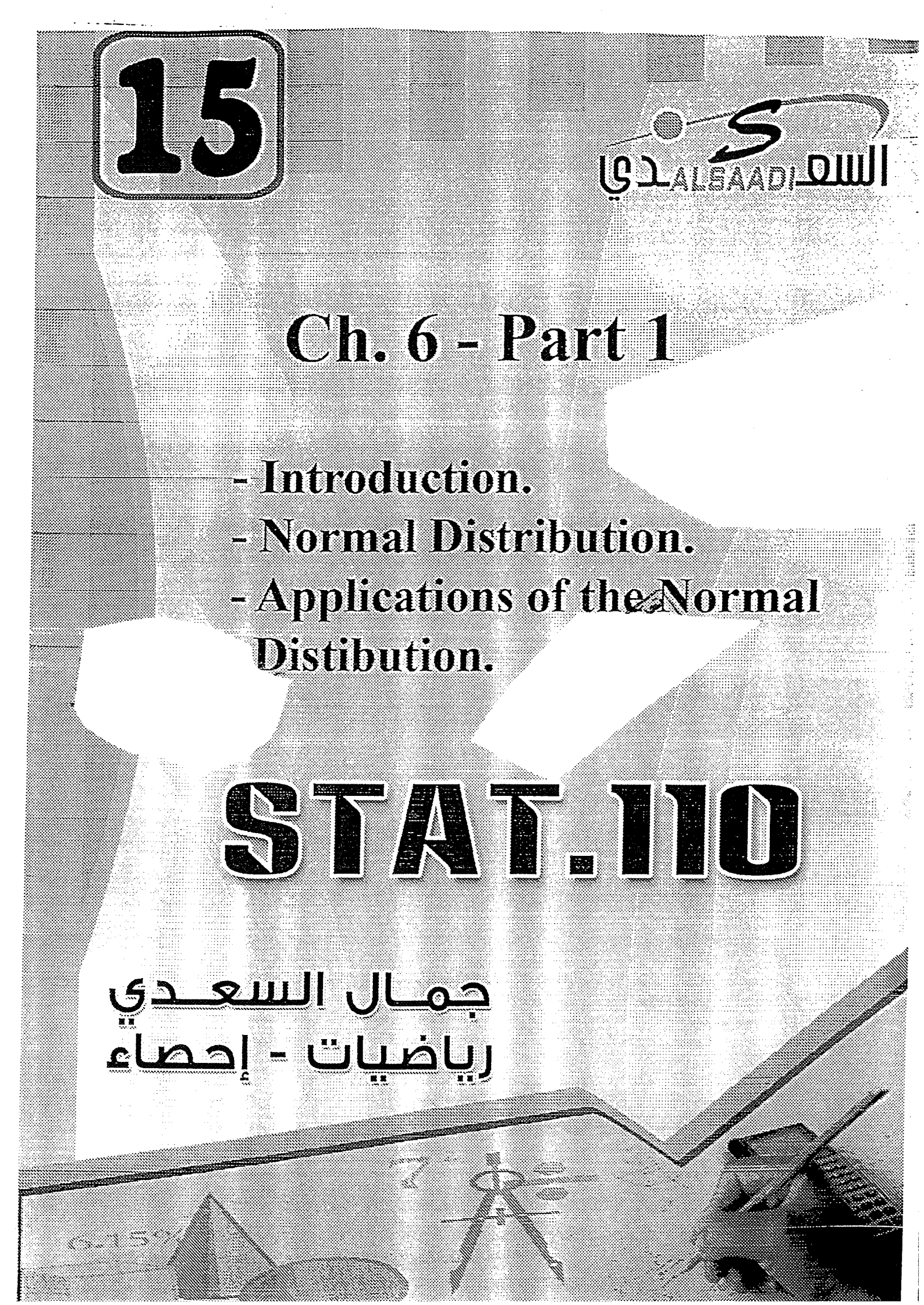
Ch. 6 - Part 1

- Introduction.
- Normal Distribution.
- Applications of the Normal Distibution.

STAT.110

جمال السعدي

رياضيات - إحصاء

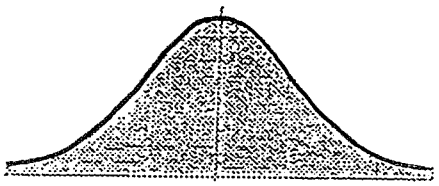


Ch. 6 Part. 1

التوزيع الطبيعي
The Normal Distribution

When the data values are evenly distributed about the mean, a distribution is said to be a symmetric distribution. (A normal distribution is symmetric.).

* تماثل

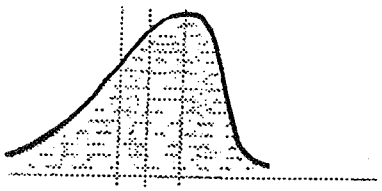


Mean = Median = Mode

Normal

When the majority of the data values fall to the right of the mean, the distribution is said to be a negatively or left-skewed distribution.

* منحرف يسار

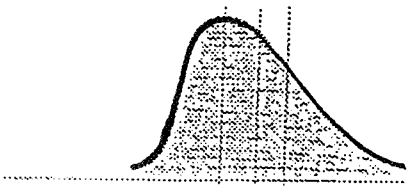


Mean Median Mode
Negatively skewed

Mean < Median < Mode

When the majority of the data values fall to the left of the mean, a distribution is said to be a positively or right-skewed distribution.

* منحرف يمين



Mode Median Mean
Positively skewed

Mode < Median < Mean

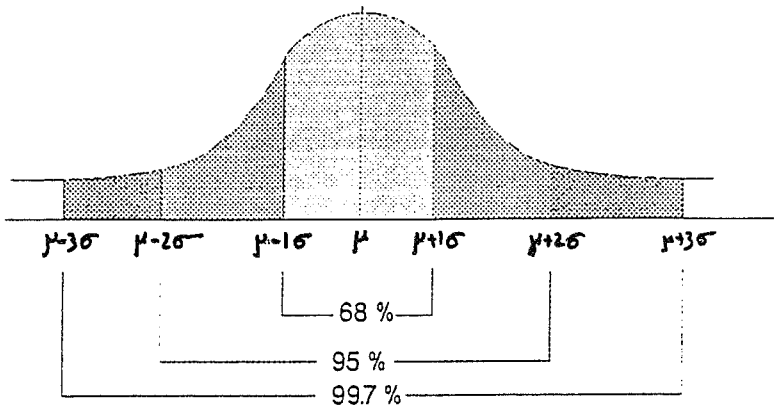
التوزيع الطبيعي

A normal distribution is a continuous, symmetric, bell-shaped distribution of a variable.

ملخص الخواص النظرية للتوزيع الطبيعي

Summary of the Properties of the Theoretical Normal Distribution

1. A normal distribution curve is bell-shaped.
2. The mean, median, and mode are equal and are located at the center of the distribution.
3. A normal distribution curve is unimodal (it has only one mode).
4. The curve is symmetric about the mean
5. The curve is continuous, that is, there are no gaps or holes.
6. The curve never touches the x axis.
7. The total area under a normal distribution curve is equal to 1.00, or 100%.
8. The area under the part of a normal curve that lies within 1 standard deviation of the mean is approximately 0.68, or 68%; within 2 standard deviations, about 0.95, or 95%; and within 3 standard deviations, about 0.997, or 99.7%.



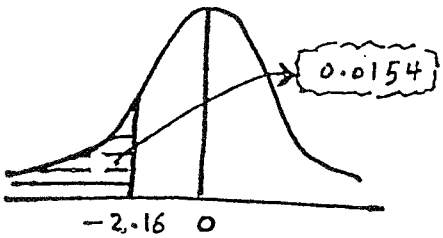
* الجدول الجديد : الكشف عن الموجب والسالب

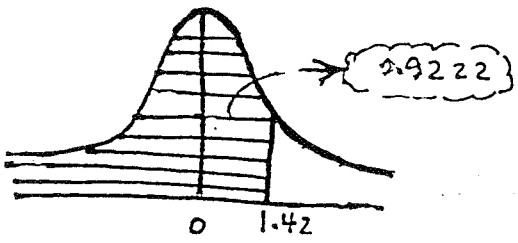
* طريقة استخدام جدول Table I : $\Phi(z)$

الذي يحتوي على z_+ / z_-

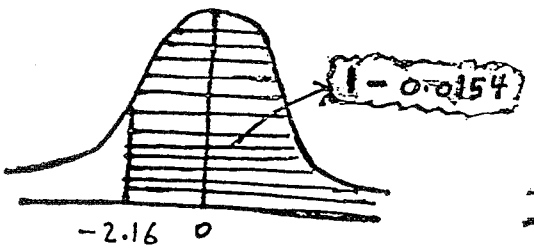
أولاً : اذا علم الـ z والمطلوب حساب الاحتمال P

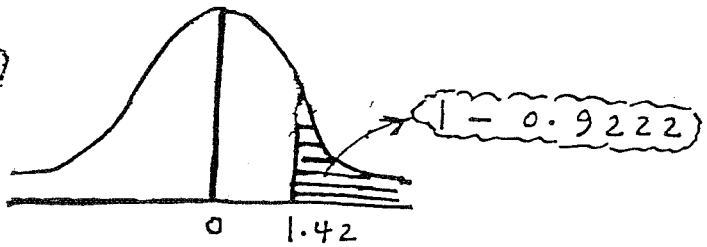
الحالة الأولى : أقل من (يسار) "P = الكشف المباشر"



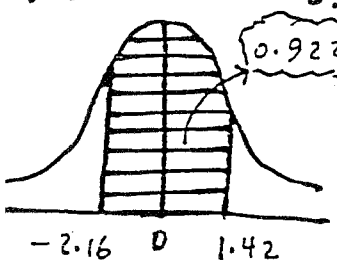


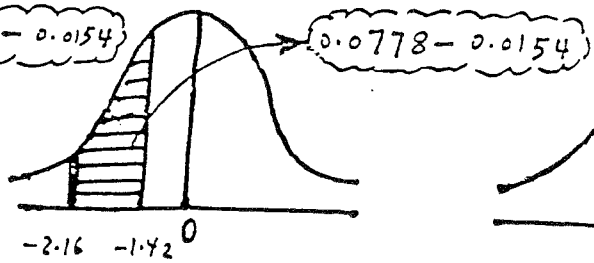
الحالة الثانية : أكبر من (يمين) "P = 1 - ناتج الكشف" ⇐

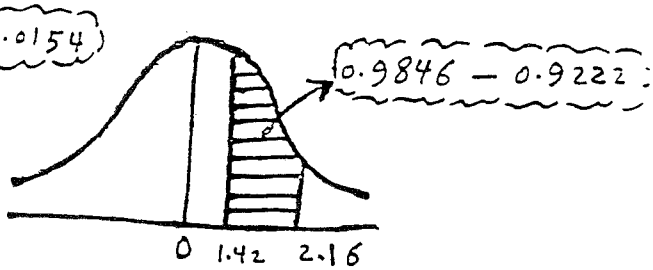




الحالة الثالثة : المساحة المحصورة بين عددين " الكشف عن العددين ثم نطرح ناتج الكشف الأصغر ⇐ ناتج الكشف الأكبر ناقص



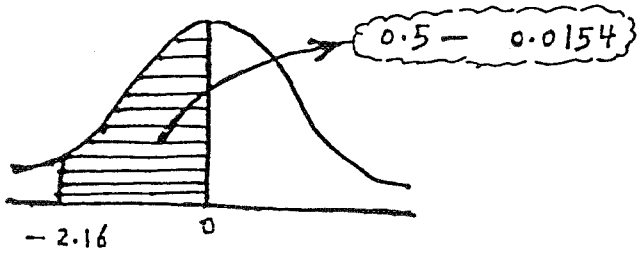




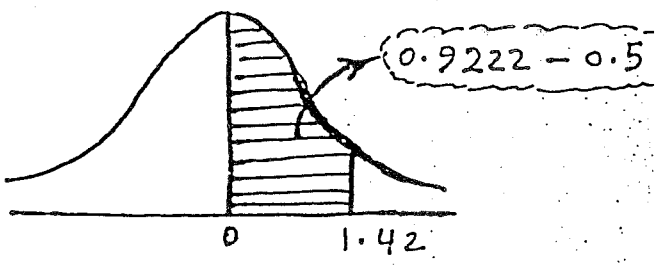
جمال السعدي
استاذ الرياضيات والإحصاء للمرحلة الجامعية
٠٥٦٦٦٦٤٧٩٠

الحاله الرابعه : من z إلى Zero

ناتج الكشف عن z − 0.5

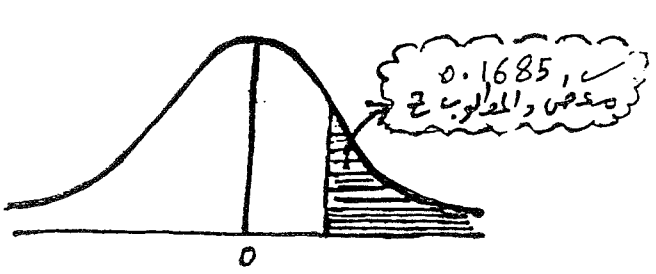


0.5 − ناتج الكشف عن z



ثانياً : اذا علم الاحتمال (المساحه) P والمطلوب ايجاد z

كشف عكسي من الداخل إلى الخارج ←



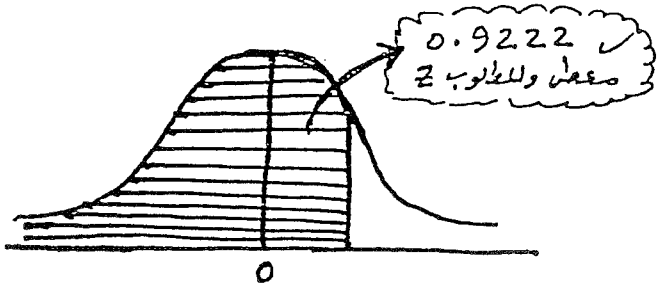
المساحه المطلوبه على اليمين

أولاً : تطرح المساحه المعلومه من 1

ثانياً : نكشف عن الناتج كشف عكسي

ثالثاً : ويكون الناتج + اذا z على اليمين

" " − " " z " اليسار



المساحه المعلومه على اليسار

أولاً : يكون الكشف عكسي مباشر .

ثانياً : الناتج + اذا z على اليمين .

" − " z " اليسار .

اللهم لا سهل إلا ما جعلته سهلاً وأنت تجعل الحزن إن شئت سهلاً

مع تمنياتي لكم بالتوفيق ،،

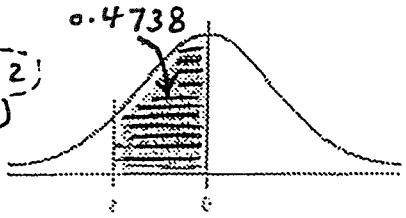
السعدي

جمال السعدي
أستاذ الرياضيات والإحصاء للمرحلة الجامعية
٠٥٦٦٦٦٤٧٩٠

Note: إذا علمت المساحة وطلب قيمة Z نكشف كشف عكسي في الجدول

For Exercises 40 through 45, find the z value that corresponds to the given area

40



تطرح المساحة المعطاة من 0.5

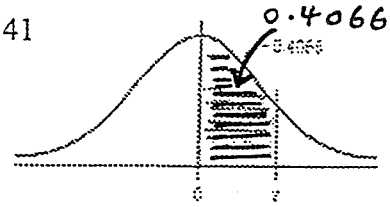
$0.5 - 0.4738 = 0.0262$

نكشف عن هذا الناتج كشف عكسي

$z = -1.94$ فتكون قيمته

$Z = -1.94$

41



يضاف للمساحة 0.4066

0.5

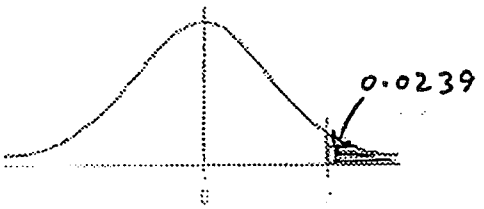
0.9066 فيكون الناتج

نكشف عن هذا الناتج كشف عكسي

$z = 1.32$ فتكون قيمته

$Z = 1.32$

42



∴ التظليل يمين

∴ تطرح المساحة المعطاة من 1

$1 - 0.0239$

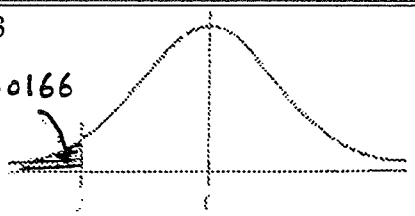
$= 0.9761$

نكشف عن هذا الناتج كشف عكسي

$z = 1.98$ تكون قيمته

$Z = 1.98$

43



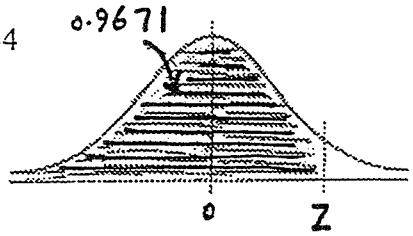
كشف عكسي مباشر

0.0166 عن

$z = -2.13$ تكون قيمته

$Z = -2.13$

44



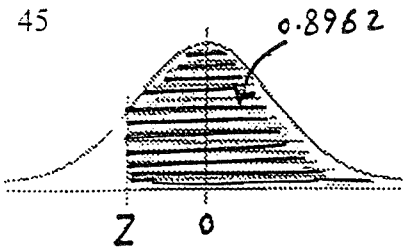
كشف عكسي مباشر

0.9671 عن

$z = 1.84$ تكون قيمته

$Z = 1.84$

45



∴ التظليل يمين

∴ تطرح المساحة المعطاة من 1

$1 - 0.8962$

$= 0.1038$

نكشف عن هذا الناتج كشف عكسي

$z = -1.26$ تكون قيمته

$Z = -1.26$

Applications of the Normal Distribution

The standard normal distribution

Is normal distribution with $\mu = 0$ *and* $\sigma = 1$

التوزيع الطبيعي المعياري

To solve problems by using the standard normal distribution, transform the original variable to a standard normal distribution variable by using the formula

- $z = \dfrac{\text{value} - \text{mean}}{\text{standard deviation}}$ or $z = \dfrac{x - \mu}{\sigma}$
- $P(X > X_0)$

$$= P\left(Z > \frac{x_0 - \mu}{\sigma}\right)$$

ملحوظة

عند حساب الاحتمال حول المتغير x الذي يتبع توزيع طبيعي. يحول إلى توزيع طبيعي معياري Z

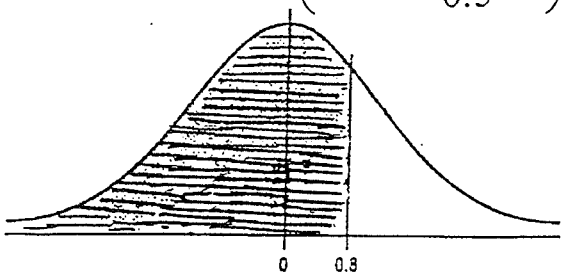
$$z = \frac{x - \mu}{\sigma}$$

Example:

The mean number of hours an American worker spends on the computer is 3.1 hours per workday. Assume the standard deviation is 0.5 hour. Find the percentage of workers who spend less than 3.5 hours on the computer. Assume the variable is normally distributed

Solution

$$P(x < 3.5) = p\left(z < \frac{3.5 - 3.1}{0.5}\right) = p(z < 0.8)$$



للتحويل إلى نسبة مئوية نضرب في ١٠٠ %

$= 0.7881 \rightarrow$

Therefore, 78.81 % of the workers spend less than 3.5 hours per workday on the computer

Example:

أفران الميكروويف

A survey found that people keep their microwave ovens an average of 3.2 years. The standard deviation is 0.56 year. If a person decides to buy a new microwave oven.

يملك

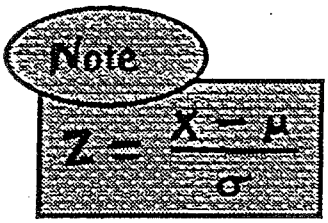
Find the probability that he or she has owned the old oven for the following amount of time. Assume the variable is normally distributed:

a. Less than 1.5 year's

b. Between 2 and 3 years

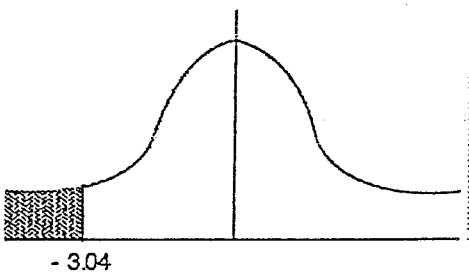
c. More than 3.2 years

d. What percent of microwave ovens would be replaced if a warranty (الضمان) of 18 months were given?



Solution

$\mu = 3.2$ $\sigma = 0.56$



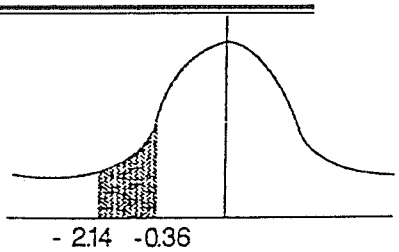
$$(a)\ P(x < 1.5) = p\left(z < \frac{1.5 - 3.2}{0.56}\right)$$

$$= P(z < -3.04)$$

$$= 0.0012$$

$$(b)\ P(2 < x < 3) = p\left(\frac{2-3.2}{0.56} < z < \frac{3-3.2}{0.56}\right)$$

$$= p(-2.14 < z < -0.36)$$



الحدان في جهه واحدة (طرح) بعد الكشف في الجدول (الأكبر – الأصغر)

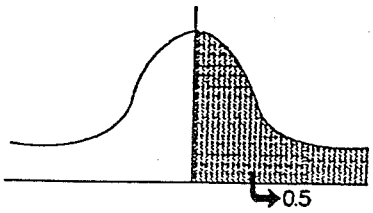
$0.3594 - 0.0162$

$= 0.3432$

(c) $p(x > 3.2) = p\left(z > \frac{3.2 - 3.2}{0.56}\right)$

$= p(z > 0)$

$= 0.5$



(d) 18 months $= \frac{18}{12} = 1.5$ years

Were μ and σ by years.

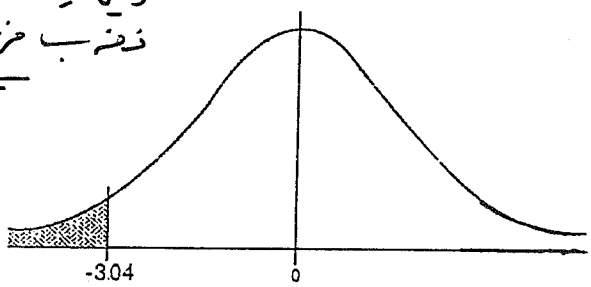
$p(x < 1.5) = p\left(z < \frac{1.5 - 3.2}{0.56}\right)$

$= p(z < -3.04)$

$= 0.0012$

$0.0012 \times 100\% = 0.12\%$ of the ovens must be replaced.

للتحويل إلى نسبة مئوية نضرب في ١٠٠ %



Example:

The average time for a mail carrier to cover his route (طريق) is 380 minutes, and the standard deviation is 16 minutes. If one of these trips is selected at random, find the probability that the carrier will have the following route time. Assume the variable is normally distributed.

a. At least 350 minutes

b. At most 395 minutes

Solution

$\mu = 380$ $\sigma = 16$

(a) At least (على الأقل) 350 minutes

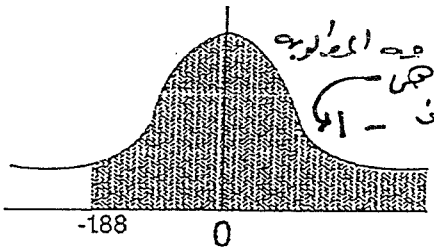
$$P(x \geq 350) = p\left(Z \geq \frac{350 - 380}{16}\right)$$

$$= p(z \geq -1.88)$$

$$= 1 - 0.0301$$

$$= 0.9699$$

التحليل يصير
المساحة المطلوبة = 1 - ناتج الكشف



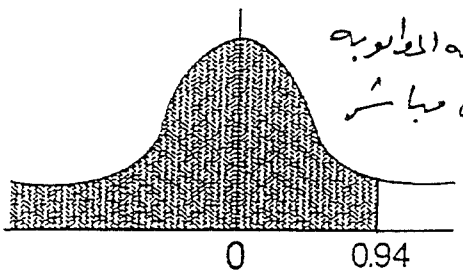
(b) At most (على الأكثر) 395 minutes

$$P(x \leq 395) = p\left(z \leq \frac{395 - 380}{16}\right)$$

$$= p(z \leq 0.94)$$

$$= 0.8264$$

التحليل يسار
المساحة المطلوبة
كشف مباشر



ALSAADI

Example:

ساعة يد

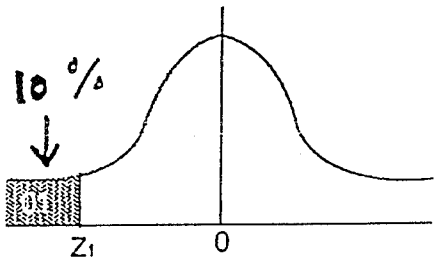
The mean lifetime of a wristwatch is 25 months, with a standard deviation of 5 months. If the distribution is normal.

For how many months should a guarantee ضمان be made if the manufacturer الصانع (صاحب المصنع) does not want to exchange more than 10% of the watches? Assume the variable is normally distributed.

Solution

$\mu = 25$ $\sigma = 5$

$P(z < z_1) = 0.1$



Therefore $z_1 = -1.28$ & $z_1 = \frac{x - 25}{\sigma}$

$$\frac{x - 25}{5} = -1.28$$

$$X - 25 = (-1.28)(5)$$

$$X = (-1.28)(5) + 25 = 18.6 \text{ month}$$

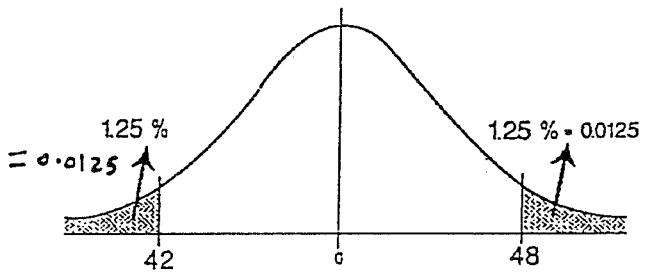
Example:

In a certain normal distribution, 1.25% of the area lies to the left of 42, and 1.25% of the area lies to the right of 48.

Find μ and σ.

Solution

$$\mu = \frac{42 + 48}{2} = \frac{90}{2} = 45$$



$$P(Z < z_1) = 0.0125$$

$$Z_1 = -2.24$$

$$\text{There for } z = \frac{x - \mu}{\sigma}$$

$$-2.24 = \frac{42 - 45}{\sigma}$$

$$\sigma = \frac{42 - 45}{-2.24} = 1.34$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998