

السعودي ALBAADI

نسخة جديدة منقحة

1433/32

16

Ch. 6 - Part 2

- The Central Limit Theorem
(Distribution of Sample Means)

STAT. 110

جمال السعدي
رياضيات - إحصاء

Best Wishes And Good Luck.

Ch. 6 Part. 2

نظريه التمام المركزيه The Central Limit Theorem

• في حالة : مجتمع أخذت منه عينة حجمها n . n : sample size taken from population

A sampling distribution of sample means is a distribution using the means computed from all possible random samples of a specific size taken from a population Properties of the Distribution of Sample Means

1. The mean of the sample means will be the same as the population mean.
2. The standard deviation of the sample means will be smaller than the standard deviation of the population, and it will be equal to the population standard deviation divided by the square root of the sample size.

$$1. z = \frac{x - \mu}{\sigma} \quad \text{Used to gain information about an individual data value}$$

في حالة فرد when the variable is normally distributed.

$$2. z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \quad \text{Used to gain information when applying the central}$$

في حالة عينه حجمها n limit theorem about a sample mean when the variable is normally distributed

Example:

The mean weight of 15-year-old males is 142 pounds, and the standard deviation is 12.3 pounds. If a sample of thirty-six 15-year-old males is selected, find the probability that the mean of the sample will be greater than 144.5 pounds. Assume the variable is normally distributed. Based on your answer, would you consider the group overweight?

Solution

$$\mu = 142, \sigma = 12.3 \quad \text{and} \quad n = 36$$

$$P(x' > 144.5)$$

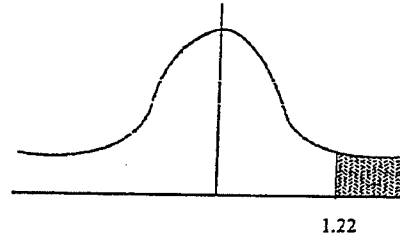
$$= P\left(z > \frac{144 - 142}{\frac{12.3}{\sqrt{36}}}\right)$$

$$= P(z > 1.22)$$

$$= 1 - 0.8888$$

$$= 0.1112$$

$$= 11.12\%$$



التفصيل
 :: المساحة المطلوبة (الأصمالة)
 ناتج البحث في الجدول = 1 - 0.8888
 عم 1.22
 ← الجدول موجود page 24

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- No: since the average weight is within 2 standard deviation of the mean.

Example:

The average age of chemical engineers is 37 years a standard deviation of 4 years. If an engineering firm employs 25 chemical engineers, find the probability that the average age of the group is greater than 38.2 years old. If this is the case, would it be safe to assume that the engineers in this group are generally much older than average?

Solution

$$\mu = 37, \quad \sigma = 4 \quad \text{and} \quad n = 25$$

$$P(\bar{x}' > 38.2)$$

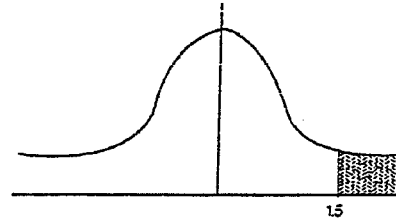
$$= p \left(z > \frac{38.2 - 37}{\frac{4}{\sqrt{25}}} \right)$$

$$= p(z > 1.5)$$

$$= 1 - 0.9332$$

$$= 0.0668$$

$$= 6.68\%$$



التفليل جيد
 - المساحة المطلوبة (الاحتمال)
 ناتج ذلك من الجدول
 1.5 = 1 -

Example:

The average annual salary in Pennsylvania was \$24,393 in 1992. Assume that salaries were normally distributed for a certain group of wage earners, and the standard deviation of this group was \$4362.

- a. Find the probability that a randomly selected individual ^{أشخاص} earned less than \$26,000. ^{مكسبه}
- b. Find the probability that, for a randomly selected sample of 25 individuals, the mean salary was less than \$26,000.
- c. Why is the probability for part b higher than the probability for part a.

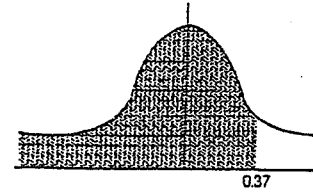
Solution

$$\mu = 24393, \quad \sigma = 4362$$

$$(a) p(x < 26000) = p\left(z < \frac{26000 - 24393}{4362}\right)$$

$$= p(z < 0.37)$$

$$= 0.6443$$



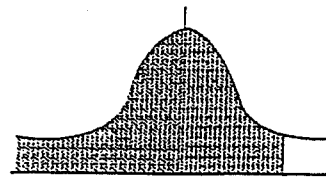
التوزيع يار
: كسب مباشر فاجدول

$$(b) \mu = 24393, \quad \sigma = 4362 \quad \text{and } n = 25$$

$$P(x' < 26000) = p\left(z < \frac{26000 - 24393}{\frac{4362}{\sqrt{25}}}\right)$$

$$= p(z < 1.84)$$

$$= 0.9671$$



التوزيع يار
: كسب مباشر فاجدول

(c) Sample means are less variable than individual data.

Example:

ضغظ الدم الانتباض

Assume that the mean systolic blood pressure of normal adults is 120 millimeters of mercury (mm Hg) and the standard deviation is 5.6. Assume the variable is normally distributed.

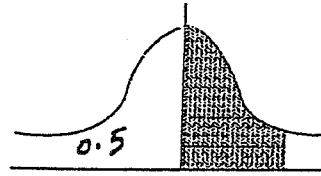
- (a) If an individual is selected, find the probability that the individual's pressure will be between 120 and 121.8 mm Hg.
- (b) If a sample of 30 adults is randomly selected, find the probability that the sample mean will be between 120 and 121.8 mm Hg.
- (c) Why is the answer to part a so much smaller than the answer to part b?

Solution

$$(a) \quad p(120 < x < 121.8) = p\left(\frac{120-120}{5.6} < z < \frac{121.8-120}{5.6}\right)$$

$$= 0.6255 - 0.5 = 0.1255$$

0.32 تكلفه 0.32 من الجدول ثم نطرح من ناتج الكسفة 0.5



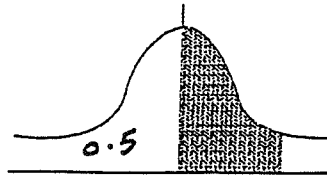
$$(b) \quad \mu = 120, \quad \sigma = 5.6 \quad \text{and} \quad n = 30$$

$$P(120 < \bar{x} < 121.8) = p\left(\frac{120-120}{\frac{5.6}{\sqrt{30}}} < z < \frac{121.8-120}{\frac{5.6}{\sqrt{30}}}\right)$$

$$= p(0 < z < 1.76) = 0.9608 - 0.5$$

$$= 0.4608$$

1.76 تكلفه 0.9608 من الجدول ثم نطرح من ناتج الكسفة 0.5



(c) Sample means are less variable than individual data.

A survey found that the microwave ovens have an average life of 3 years with a standard deviation of 0.5 year. Assume the variable is normally distributed. What percent of microwave ovens would be replaced if a warranty of 18 months were given?

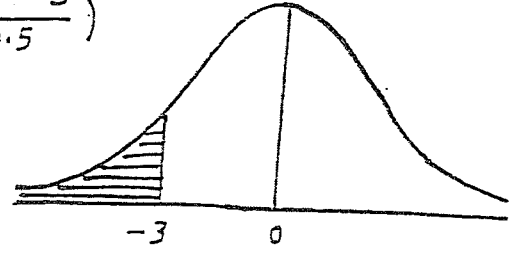
A) 0.13% B) 2.28% C) 10.56% D) 0.52%

$$18 \text{ months} = \frac{18}{12} = 1.5 \text{ years}$$

where μ and σ by years

$$P(X < 1.5) = P\left(Z < \frac{1.5 - 3}{0.5}\right)$$

$$= P(Z < -3)$$



* average $\mu = 3$

* $\sigma = 0.5$

$$= 0.0013$$

لإيجاد الـ Percent ضرب عن 100%

$$= 0.0013 \times 100\%$$

$$= 0.13\%$$

0.13%

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$$P(95 < X < a) = 0.6309$$

A) 115.5 B) 101.5 C) 84.5 D) 15.5

$$\mu = 100$$

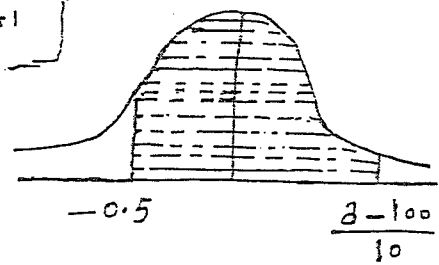
$$\sigma = 10$$

$$P(95 < X < a) = 0.6309$$

$$= P\left(\frac{95-100}{10} < Z < \frac{a-100}{10}\right) = 0.6309$$

$$= P(-0.5 < Z < \frac{a-100}{10}) = 0.6309$$

لو كان الحد من جهة واحدة تكون المساحة تكوناً أقل من 0.5
 المساحة 0.6309 أكبر من 0.5
 الحد من جهتين مختلفتين من خط المنتصف
 المساحة $\frac{a-100}{10}$ يقع من الجهة اليمنى
 المساحة 0.5 يقع من الجهة اليسرى



$$P\left(Z < \frac{a-100}{10}\right) - P(Z < -0.5) = 0.6309$$

نكتب من الجدول بالرجوع

$$P\left(Z < \frac{a-100}{10}\right) = 0.6309 + 0.3085$$

$$P\left(Z < \frac{a-100}{10}\right) = 0.9394 \leftarrow \text{كيف نكتبه}$$

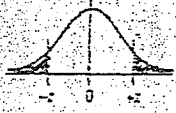
$$\frac{a-100}{10} = 1.55$$

$$Z = 1.55$$

$$a - 100 = (1.55)(10)$$

$$a = (1.55)(10) + 100 \Rightarrow \underline{\underline{a = 115.5}}$$

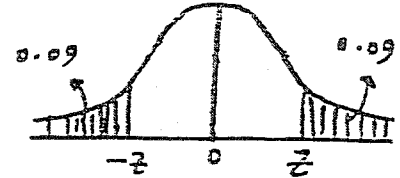
Find the value of z such that the shaded tail areas equals 18%



- A) 1.4 B) 1.34 C) 1.22 D) 1.47

مجموع الماحتين المظلتين على الأطراف = 0.18

$$\therefore \text{المساحة الواحدة} = \frac{0.18}{2} = 0.09$$



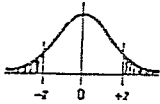
من كثف مع 0.09 كثف تكفي

أجد أنها أقرب إلى 0.0901

فتكون z -المقابل له -1.34

$$\Rightarrow \boxed{z = 1.34}$$

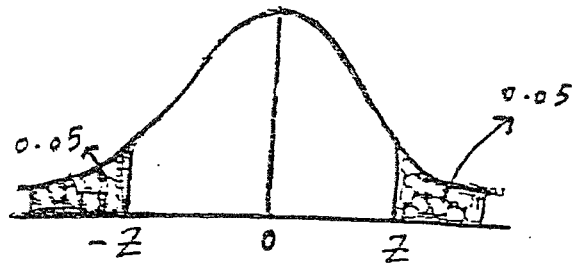
Find the value of z such that the shaded tail areas equals 0.10



- A) 1.64 B) 1.28 C) -1.96 D) 0.25

مجموع الماحتين المظلتين = 0.10

$$\therefore \text{المساحة الواحدة} = \frac{0.10}{2} = 0.05$$



من كثف مع 0.05 كثف تكفي

أجد أنها أقرب إلى 0.0505

فتكون z -المقابل له -1.64

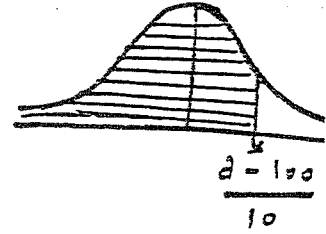
$$\Rightarrow \boxed{z = 1.64}$$

$$P(X < a) = 0.8665$$

A) 101.1 B) 113.3 C) 122.2 D) 111.1

$$\mu = 100 \quad \sigma = 10$$

$$P\left(Z < \frac{a - 100}{10}\right) = 0.8665$$



نكسوف في البرول كسوف كسما عم 0.8665
 تكونا يتهه الناتج هه قيهه $\frac{a - 100}{10}$

اما 1.1 تحت 0.1
 :- الناتج 1.11

$$\therefore \frac{a - 100}{10} = 1.11$$

$$\Rightarrow a - 100 = (1.11)(10)$$

$$\Rightarrow a = (1.11)(10) + 100$$

$$\Rightarrow a = 111.1$$

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Use the following to answer questions

Let X be a normally distributed random variable with mean 100 and a standard deviation 10. Use this information to find the value of a such that

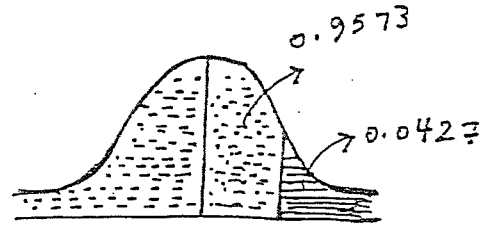
$$P(X > a) = 0.0427$$

A) 101.72 B) 17.2 C) 117.2 D) 82.8

$$P(X > a) = 0.0427$$

$$= P\left(Z > \frac{a - 100}{10}\right) = 0.0427$$

$$\therefore P\left(Z < \frac{a - 100}{10}\right) = 0.9573$$



نكتب كيف عكسها عن (من داخل الجدول) 0.9573 نجد أن 1.7 تحت 0.02

$$\therefore Z = 1.72 \rightarrow Z = \frac{a - \mu}{\sigma}$$

$$1.72 = \frac{a - 100}{10}$$

$$a - 100 = 17.2$$

$$a = 17.2 + 100$$

$$\Rightarrow a = 117.2$$

Use the following to answer questions

The time T_1 to travel from A to B through city center (road R_1) is normally distributed with mean μ_1 10 minutes and standard deviation 2 minutes.

The time T_2 to travel from A to B through a new ring road (road R_2) is normally distributed with mean μ_2 15 minutes and standard deviation 3 minutes.

You have 12 minutes to travel from A to B on an important appointment. Use these information to solve the following

$$\mu_1 = 10, \sigma_1 = 2 \quad \mu_2 = 15, \sigma_2 = 3$$

$$P(T_2 > 12)$$

- A) 0.6587 B) 0.8413 C) 0.3413 D) 0.1587

$$P(T_2 > 12) = P(Z_2 > \frac{12-15}{3}) = P(Z_2 > -1)$$

$$= 1 - 0.1587 = 0.8413$$

أكبر من ناقص الاكتفاء من الجدول

$$P(T_1 > 12)$$

- A) 0.1587 B) 0.6587 C) 0.3413 D) 0.8413

$$P(T_1 > 12) = P(Z_1 > \frac{12-10}{2}) = P(Z_1 > 1) = 0.1587$$

$$= 1 - 0.8413$$

أكبر من ناقص الاكتفاء من الجدول

Your correct decision is

- A) R_1 is better than R_2 C) Both R_1 and R_2 are the same
B) R_1 is not as good as R_2 D) R_2 is better than R_1

R_1 is better than R_2 ((لأنه وقت الوصول عن طريق R_1 أقل))
أفضل من

If the scores for a test have a mean of 70 and a standard deviation of 12, find the percentage of scores that will fall below 50. Assume the test scores are normally distributed.

- A) 35.54% B) 4.75% C) 42.07% D) 45.54%

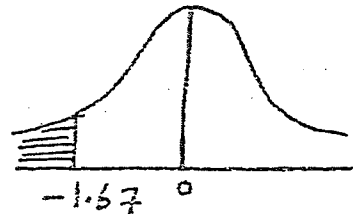
$$\mu = 70, \sigma = 12$$

$$P(X < 50)$$

$$= P(Z < \frac{50-70}{12}) = P(Z < -1.67)$$

كم في ميسر من جدول الموجب والسلب

$$= 0.0475$$



$$\text{The percentage} = 0.0475 \times 100\% = 4.75\%$$

Use the following to answer questions

Suppose that the monthly allowance, X , of a student in a given school is normally distributed with mean \$300 and standard deviation \$50.

$$\mu = 300 \quad \sigma = 50$$

The probability that the monthly allowance of a student selected at random is between \$275 and \$325 is

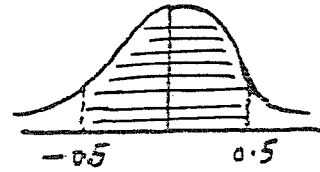
- A) 0.383 B) 0.617 C) 0.8085 D) 0.1915

$$P(275 < X < 325) = P\left(\frac{275-300}{50} < z < \frac{325-300}{50}\right)$$

$$= P(-0.5 < z < 0.5)$$

$$= P(z < 0.5) - P(z < -0.5)$$

$$= 0.6915 - 0.3085 = \underline{\underline{0.383}}$$



If a random sample of 9 students is selected randomly, find the probability that the mean allowance of the sample is between \$275 and \$325

- A) 0.134 B) 0.4332 C) 0.567 D) 0.8664

$$(\text{sample}) \rightarrow n = 9$$

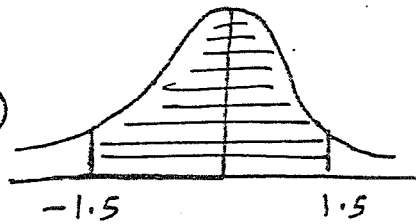
$$P(275 < X < 325) = P\left(\frac{275-300}{\frac{50}{\sqrt{9}}} < z < \frac{325-300}{\frac{50}{\sqrt{9}}}\right)$$

$$= P(-1.5 < z < 1.5)$$

$$= P(z < 1.5) - P(z < -1.5)$$

$$= 0.9332 - 0.0668$$

$$= \underline{\underline{0.8664}}$$



Use the following to answer questions

The monthly income, X , of a family in a given city is normally distributed with mean \$3000 and standard deviation \$500.

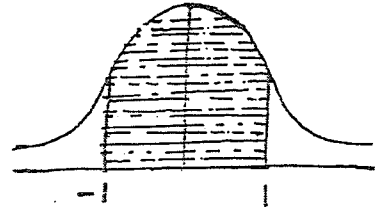
The probability that a person selected at random earns a monthly income between \$2500 and \$3500
A) 0.3413 B) 0.6826 C) 0.1587 D) 0.3174

$$P(2500 < X < 3500)$$

$$= P\left(\frac{2500 - 3000}{500} < Z < \frac{3500 - 3000}{500}\right)$$

$$= P(-1 < Z < 1)$$

الكثافة العددية 1 ، -1
من الجدول ثم طرح الناتجين الأكبر ناقص الأصغر



$$= 0.2413 - 0.1587 = \underline{\underline{0.6826}}$$

If a random sample of size 9 is selected at random, find the probability that the mean income of the sample is between \$2500 and \$3500

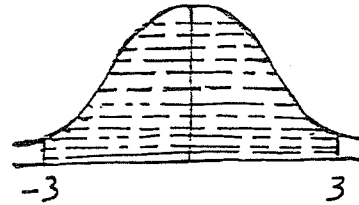
A) 0.0013 B) 0.4987 C) 0.9974 D) 0.5601

$$P(2500 < \bar{X} < 3500)$$

$$= P\left(\frac{2500 - 3000}{\frac{500}{\sqrt{9}}} < Z < \frac{3500 - 3000}{\frac{500}{\sqrt{9}}}\right)$$

$$= P(-3 < Z < 3)$$

الكثافة العددية 3 ، -3
من الجدول ثم طرح الناتجين الأكبر ناقص الأصغر



$$= 0.9987 - 0.0013 = 0.9974$$

Use the following to answer questions

The time T_1 to travel from A to B through city center (road R_1) is normally distributed with mean 20 minutes and standard deviation 5 minutes.

The time T_2 to travel from A to B through a new ring road (road R_2) is normally distributed with mean 15 minutes and standard deviation 8 minutes.

You have 17 minutes to travel from A to B on an important appointment. Using this information, solve the following

$$P(T_1 > 17)$$

- A) 0.2743 B) 0.7257 C) 0.2257 D) 0.7743

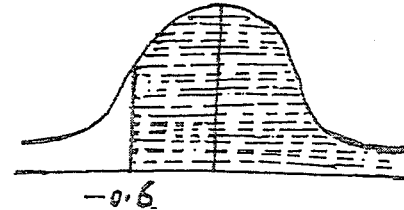
$$P(T_1 > 17)$$

$$= P\left(Z_1 > \frac{17 - 20}{5}\right)$$

$$= P(Z_1 > -0.6)$$

$$= 1 - 0.2743$$

$$= \boxed{0.7257}$$



∴ التظليل صحيح
∴ الاحتمال المطلوب
هو 1 ناقص الكيف فز الجداول بم -0.6

$$P(T_2 > 17)$$

- A) 0.5987 B) 0.0987 C) 0.4013 D) 0.9013

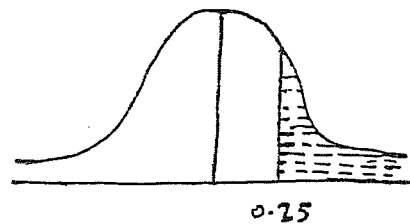
$$P(T_2 > 17)$$

$$= P\left(Z_2 > \frac{17 - 15}{8}\right)$$

$$= P(Z_2 > 0.25)$$

$$= 1 - 0.5987$$

$$= \boxed{0.4013}$$



∴ التظليل صحيح
∴ الاحتمال المطلوب
هو 1 ناقص الكيف فز الجداول بم 0.25

Your correct decision is

- A) R_2 is better than R_1
B) R_1 is better than R_2

- C) Both R_1 and R_2 are the same
D) R_2 is not as good as R_1

Use the following to answer questions

Let X be a normally distributed random variable with mean 100 and a standard deviation 10. Use this information to find the value of a such that

$$P(a < X < 111) = 0.6223$$

A) 97 B) 95 C) 98 D) 93

$$\mu = 100 \quad \sigma = 10$$

$$P(a < X < 111) = 0.6223$$

$$P\left(\frac{a-100}{10} < Z < \frac{111-100}{10}\right) = 0.6223$$

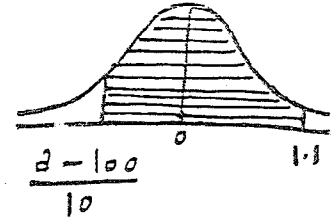
$$P\left(\frac{a-100}{10} < Z < 1.1\right) = 0.6223$$

لو كان الحدان في جهتي واحد وكانت المساحة بينهما أقل من 0.5

بـ المساحة 0.6223 أكبر من 0.5

∴ الحدان في جهتين مختلفتين من خط المنتصف.

أما أنه $\frac{a-100}{10}$ يقع في الجهة اليسرى
كأنه 1.1 يقع في الجهة اليمنى.



$$P(Z < 1.1) - P\left(Z < \frac{a-100}{10}\right) = 0.6223$$

$$0.8643 - P\left(Z < \frac{a-100}{10}\right) = 0.6223$$

$$0.8643 - 0.6223 = P\left(Z < \frac{a-100}{10}\right)$$

$$\therefore P\left(Z < \frac{a-100}{10}\right) = 0.242$$

كأنه عكسهم ←

تكون قيمه $Z = -0.7$

$$\Rightarrow \frac{a-100}{10} = -0.7$$

$$\Rightarrow a - 100 = (-0.7)(10)$$

$$a = (-0.7)(10) + 100 \Rightarrow \boxed{a = 93}$$

Use the following to answer questions

Let T_1 , the time to travel from A to B through an old road, be normally distributed with mean 22 minutes and standard deviation 5 minutes.

Let T_2 , the time to travel from A to B through a new road, be normally distributed with mean 16 minutes and standard deviation 7 minutes.

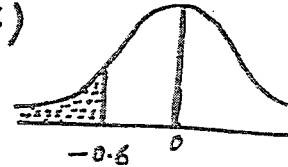
You have 19 minutes to travel from A to B on an important appointment. Using this information, solve the following

Your correct decision is that the new road is ... the old road.

A) same as B) not as good as C) worse than D) better than

** old road $\mu = 22$ $\sigma = 5$

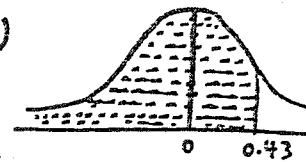
$$P(T_1 < 19) = P\left(z < \frac{19-22}{5}\right) = P(z < -0.6)$$



$$\therefore P(T_1 < 19) = \boxed{0.2743}$$

** New road $\mu = 16$ $\sigma = 7$

$$P(T_2 < 19) = P\left(z < \frac{19-16}{7}\right) = P(z < 0.43)$$



$$\therefore P(T_2 < 19) = \boxed{0.6664}$$

$$\therefore P(T_2) > P(T_1)$$

\therefore احتمال أنه يصل أسرع (في وقت أقل) عندما يستخدم الطريق الجديد

\therefore New road is better than old road

$P(T_1 < 19)$ الاجابة أعلر
A) 0.7743 B) 0.2743 C) 0.7257 D) 0.2257

$P(T_2 < 19)$ الاجابة أعلر
A) 0.4013 B) 0.9013 C) 0.0987 D) 0.6664

Use the following to answer questions

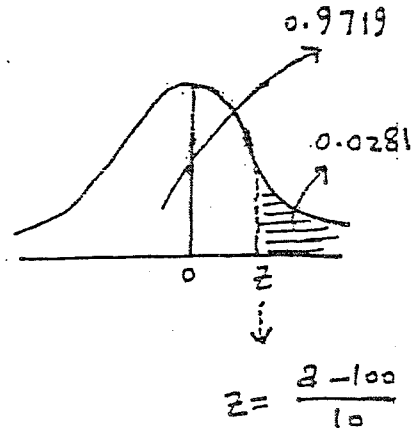
Let X be a normally distributed random variable with mean 100 and a standard deviation 10. Use this information to find the value of a such that

$P(X > a) = 0.0281$

- A) 1.91 B) 119.1 C) 80.9 D) 19.1

$\mu = 100$ $\sigma = 10$

$P(X > a) = 0.0281$



$P(Z > \frac{a-100}{10}) = 0.0281$

بالتكبير كسوف وكسر من الداخل للخارج

$1 - 0.0281 = 0.9719$ ← مس

$z = 1.91$

$1.91 = \frac{a-100}{10}$

$\Rightarrow a-100 = (1.91)(10)$

$a = (1.91)(10) + 100$

$a = 119.1$

$P(90 < X < a) = 0.6519$

- A) 90.0 B) 108.8 C) 110.0 D) 91.2

$P(90 < X < a) = 0.6519$

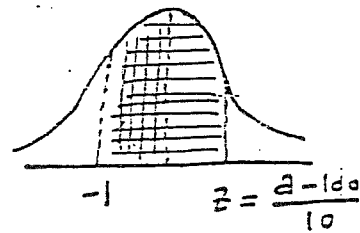
$P(\frac{90-100}{10} < Z < \frac{a-100}{10}) = 0.6519$

$P(-1 < Z < \frac{a-100}{10}) = 0.6519$

$P(Z < \frac{a-100}{10}) - P(Z < -1) = 0.6519$

$\therefore P(Z < \frac{a-100}{10}) = 0.8106$ ← كتف من اول الموجب

$\Rightarrow z = 0.88$ ← كتف من اول الموجب



$0.88 = \frac{a-100}{10}$

$a-100 = (0.88)(10)$

$a = (0.88)(10) + 100$

$a = 108.8$

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Use the following to answer questions

The weekly income, X , of a family in a given city is normally distributed with mean \$200 and standard deviation \$25.

The probability that a person selected at random earns a weekly income between \$150 and \$250

- A) 0.3174 B) 0.9542 C) 0.6826 D) 0.1587

$$\mu = 200$$

$$\sigma = 25$$

$$P(150 < X < 250)$$

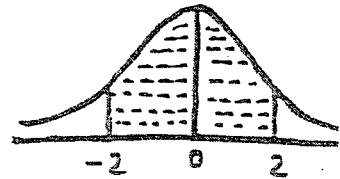
$$= P\left(\frac{150 - 200}{25} < Z < \frac{250 - 200}{25}\right)$$

$$= P(-2 < Z < 2)$$

الكشف عن التوزيع مع الرسم 2، 2 من الجدول
ثم طرح الأكبر ناقصاً الأصغر.

$$= 0.9772 - 0.0228$$

$$= \boxed{0.954}$$



If a random sample of size 4 is selected at random, find the probability that the mean income of the sample is between \$187.5 and \$212.5

- A) 0.4987 B) 0.6826 C) 0.0013 D) 0.9974

$$\text{sample size : } n = 4$$

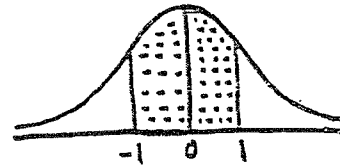
$$P(187.5 < X < 212.5)$$

$$= P\left(\frac{187.5 - 200}{\frac{25}{\sqrt{4}}} < Z < \frac{212.5 - 200}{\frac{25}{\sqrt{4}}}\right)$$

$$= P(-1 < Z < 1)$$

الكشف عن التوزيع مع الرسم 1، 1 من الجدول
ثم طرح الأكبر ناقصاً الأصغر.

$$= 0.8413 - 0.1587 = \boxed{0.6826}$$



To qualify for the medical school, the student must score in the top 10% on a general test. The test has a mean of 200 and a standard deviation of 20. Find the lowest possible score to qualify. Assume test scores are normally distributed.

- A) 276 B) 1.28 C) 25.6 D) 226

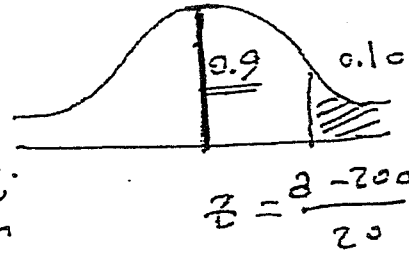
$$P(X > a) = 0.10$$

$$P\left(z > \frac{a-200}{20}\right) = 0.10$$

$$P\left(z < \frac{a-200}{20}\right) = 0.9$$

نكتبها كمنفك مع 0.9
نجدونها اقرب إلى 0.8997

$$z = 1.28 \leftarrow \text{التي تقابل } z$$



$$z = \frac{a-200}{20}$$

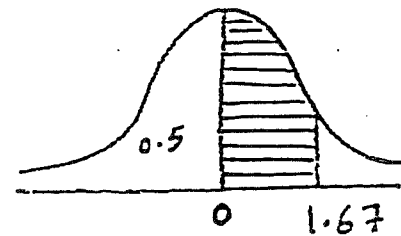
$$1.28 = \frac{a-200}{20}$$

$$a-200 = 25.6$$

$$a = 225.6$$

Find the probability for $P(0 < z < 1.67)$

- A) 0.4525 B) 0.4207 C) 0.3554 D) 0.4554



نكتبها مع 1.67 من جدول التوزيع والنائب

ثم نطرح من الناتج 0.5

$$= 0.9525 - 0.5 = 0.4525$$

When the distribution is positively skewed, the relationship of the mean, median, and mode will be ...

A) mode > median > mean.

C) mean < mode < median

B) median < mode < mean.

D) mean > median > mode.

- positive skew

mean > median > mode

اجابة السؤال

- Negative skew

mean < median < mode

شرح زياده

Symmetric skew

mean = median = mode

شرح زياده

A distribution of the means that are computed from all possible random samples of a specific size taken with replacement from a population." The previous statement is the definition of ...

A) central limit theorem B) empirical distribution C) sampling distribution D) sampling error

تعريف

Sampling distribution .

تعريف

- Sampling error is the difference between the sample measure and the corresponding population measure due to the fact that the sample is not a perfect representation of the population.

The standard deviation of a distribution is 20. If a sample of 225 is selected, what is the standard error of the mean?

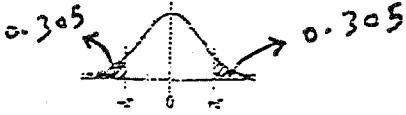
- A) $\frac{4}{45}$ B) $\frac{7}{5}$ C) $\frac{4}{3}$ D) $\frac{3}{4}$

$$\sigma = 20$$

$$n = 225$$

$$\text{standard error of the mean} = \frac{\sigma}{\sqrt{n}} = \frac{20}{\sqrt{225}} = \frac{4}{3}$$

Find the value of z such that the shaded tail areas equals 0.61



- A) 0.11 B) 0.305 C) 0.195 D) 0.51

المساحات المظلمة
تساوي مجموعها 0.61
∴ مساحة المنطقة الواحدة
= $\frac{0.61}{2} = 0.305$

$$0.3050$$

فكشفت عنها كيف تكسر
من داخل الجدول فتكون قيمة z المقابلة لها 0.51

$$\therefore z = 0.51$$

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Chapter Quiz

Determine whether each statement is true or false. If the statement is false, explain why.

- ① The total area under a normal distribution is infinite.
- ② The standard normal distribution is a continuous distribution.
- ③ All variables that are approximately normally distributed can be transformed to standard normal variables.
- ④ The z value corresponding to a number below the mean is always negative.
- ⑤ The area under the standard normal distribution to the left of $z = 0$ is negative.
- ⑥ The central limit theorem applies to means of samples selected from different populations.

Select the best answer.

- ⑦ The mean of the standard normal distribution is
 a. 0 b. 1 c. 100 d. variable
- ⑧ Approximately what percentage of normally distributed data values will fall within 1 standard deviation above or below the mean?
 a. 68% b. 95% c. 99.7% d. Variable
- ⑨ Which is not a property of the standard normal distribution?
 a. It's symmetric about the mean. b. It's uniform.
 c. It's bell-shaped. d. It's unimodal.

10) When a distribution is positively skewed, the relationship of the mean, median, and mode from left to right will be .

- a. Mean, median, mode
- b. Mode, median, mean
- c. Median, mode, mean
- d. Mean, mode, median

11) The standard deviation of all possible sample means equals

- a. The population standard deviation.
- b. The population standard deviation divided by the population mean.
- c. The population standard deviation divided by the square root of the sample size.
- d. The square root of the population standard deviation.

Complete the following statements with the best answer.

12) When one is using the standard normal distribution,

$$P(z < 0) = \underline{0.5}.$$

13) The difference between a sample mean and a population

mean is due to **Sampling error**.

14) The mean of the sample means equals **Population mean**.

15) The standard deviation of all possible sample means is called

Standard error of the mean.

إنتهى 6 ح. 6

كل الأمنيات بالنجاح والتوفيق

السعودي

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0543	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0803	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9915	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9935	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

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