مدونة المناهج السعودية

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الموقع التعليمي لجميع المراحل الدراسية

في المملكة العربية السعودية

# Operation on Functions

1️⃣ **Definition.**

2️⃣ **Composition.**

# 1 Operation on Functions

**DEFINITION 1** Operations on Functions

The **sum, difference, product,** and **quotient** of the functions $f$ and $g$ are the functions defined by

| | | |
|---|---|---|
| Sum function | $(f+g)(x) = f(x) + g(x)$ | D: $A \cap B$ |
| Difference function | $(f-g)(x) = f(x) - g(x)$ | D: $A \cap B$ |
| Product function | $(fg)(x) = f(x)g(x)$ | D: $A \cap B$ |
| Quotient function | $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad g(x) \neq 0$ | D: $\{x \in A \cap B, g(x) \neq 0\}$ |

Example 1: Let $f(x) = x^2 - 3$ and $g(x) = 2x + 5$, find $f+g$, $f-g$, $fg$, $f/g$ and their domain.

- $(f+g)(x) = f(x) + g(x)$

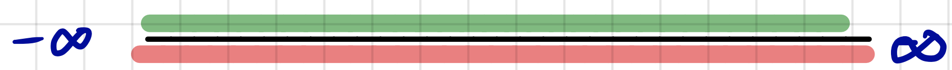
$$= x^2 - 3 + 2x + 5$$
$$= x^2 + 2x + 2$$

$\therefore D(f+g) = A \cap B = (-\infty, \infty)$

$A = D(f) = \mathbb{R} = (-\infty, \infty)$

$B = D(g) = \mathbb{R} = (-\infty, \infty)$

$A \cap B = (-\infty, \infty)$




- $(f-g)(x) = f(x) - g(x)$

$$= x^2 - 3 - (2x + 5)$$
$$= x^2 - 3 - 2x - 5$$
$$= x^2 - 2x - 8$$

$\therefore D(f-g) = A \cap B = (-\infty, \infty)$

- $(fg)(x) = f(x)\, g(x)$

$$= (x^2 - 3)(2x + 5)$$
$$= 2x^3 + 5x^2 - 6x - 15$$

$\therefore D(fg) = A \cap B = (-\infty, \infty)$

- $\left(\frac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$

$$= \frac{x^2-3}{2x+5}$$

$A = D(f) = \mathbb{R} = (-\infty, \infty)$

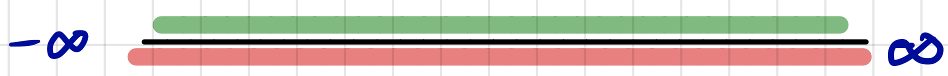
$B = D(g) = \mathbb{R} = (-\infty, \infty)$

$D(f/g) = \{x \in A \cap B,\ g(x) \neq 0\}$

$g(x) \neq 0 \Rightarrow 2x + 5 \neq 0$

$\Rightarrow 2x \neq -5$

$\Rightarrow x \neq \frac{-5}{2}$




$\therefore D(f/g) = \{x \in \mathbb{R},\ g(x) \neq -\frac{2}{3}\}$

$= \mathbb{R} - \{-\frac{2}{3}\}$

**Example 2:** Let $f(x) = \sqrt{4-x}$ and $g(x) = \sqrt{3+x}$, find $f+g$, $f-g$, $fg$, $f/g$ and their domain.

- $(f+g)(x) = f(x) + g(x)$

$$= \sqrt{4-x} + \sqrt{3+x}$$

$\therefore D(f+g)(x) = A \cap B$

$= [-3, 4]$

$A = D(f): 4 - x \geq 0$

$4 \geq x \Rightarrow x \leq 4$

$\therefore D(f) = (-\infty, 4]$

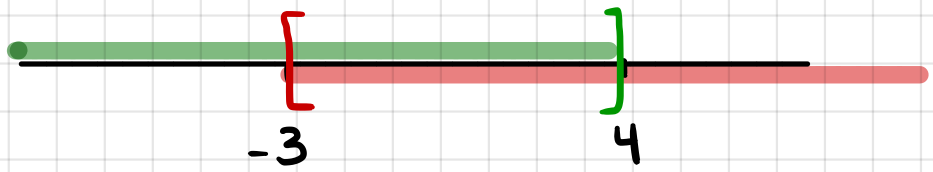
$B = D(g): 3 + x \geq 0$

$x \geq -3$

$\therefore D(g) = [-3, \infty)$

- $(f-g)(x) = f(x) - g(x)$

$$= \sqrt{4-x} - \sqrt{3+x}$$




$\therefore D(f-g)(x) = A \cap B$

$= [-3, 4]$

$$(fg)(x) = f(x)\, g(x)$$
$$= \sqrt{4-x}\,\sqrt{3+x}$$
$$= \sqrt{(4-x)(3+x)}$$
$$= \sqrt{12+4x-3x-x^2}$$
$$= \sqrt{12+x-x^2}$$

$$\therefore D(fg)=(x)= A \cap B = [-3,4]$$

- $$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$
$$= \frac{\sqrt{4-x}}{\sqrt{3+x}}$$
$$= \sqrt{\frac{4-x}{3+x}}$$

$A = D(f) : 4-x \geq 0$

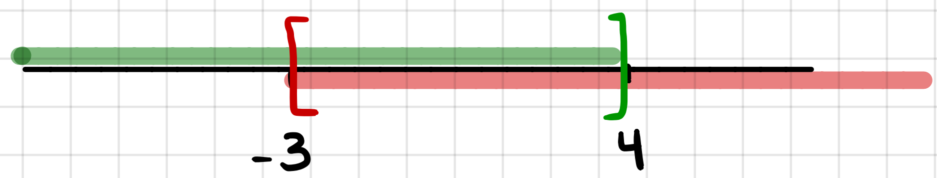
$4 \geq x \Rightarrow x \leq 4$

$\therefore D(f) = (-\infty, 4]$

$B = D(g) : 3+x \geq 0$

$x \geq -3$

$\therefore D(g) = [-3, \infty)$




$$D\left(\frac{f}{g}\right) = \{x \in A \cap B,\ g(x) \neq 0\}$$
$$= \{x \in [-3,4],\ 3+x \neq 0\}$$
$$= \{x \in [-3,4],\ x \neq -3\}$$
$$= (-3, 4]$$

**Example 3:** Let $f(x) = \dfrac{x}{x-1}$ and $g(x) = \dfrac{x-4}{x+3}$.

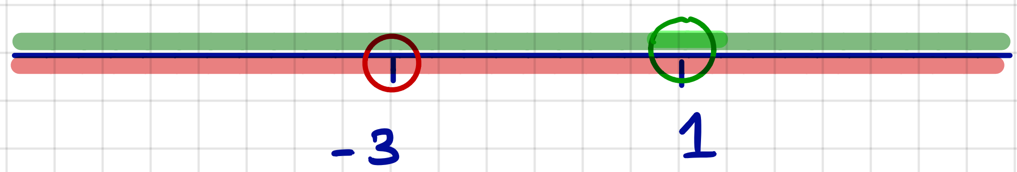
Find the function $\frac{f}{g}$ and find its domain

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\frac{x}{x-1}}{\frac{x-4}{x+3}}$$

$$= \frac{x}{x-1} \cdot \frac{x+3}{x-4}$$

$$= \frac{x(x+3)}{(x-1)(x-4)}$$

$A = D(f) = \mathbb{R} - \{1\}$ , $B = D(g) = \mathbb{R} - \{-3\}$




$A \cap B = \mathbb{R} - \{-3, 1\}$

$D\left(\frac{f}{g}\right) = \{x \in A \cap B , g(x) \neq 0\}$

$g(x) = (x-1)(x-4) \neq 0$

$\Rightarrow \quad x \neq 1 \quad \text{or} \quad x = 4$

$\therefore D\left(\frac{f}{g}\right) = \{x \in \mathbb{R} - \{-3, 1\} , g(x) \neq 1, 4\}$

$= \{x \in \mathbb{R} - \{-3, 1, 4\}\}$

# 2 Compostion

**DEFINITION 2** Composition

The **composition** of function $f$ with function $g$ is denoted by $f \circ g$ and is defined by

$$(f \circ g)(x) = f(g(x))$$

The domain of $f \circ g$ is the set of all real numbers $x$ in the domain of $g$ such that $g(x)$ is in the domain of $f$.

## How to find the domain of the composite function

**Step 1:** Find the domain of inside function. If there are restrictions on the domain, keep them.

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**Step 2:** Construct the composite function. find the domain of this new function. If there are restrictions on this domain, add them to the restrictions from step 1.

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في الخطوة ١

Example 1 : Find $f \circ g\ (x)$ and its domain for each of the following functions:

- $f(x) = x^2 + 2$ , $g(x) = \sqrt{3-x}$

$(f \circ g)(x) = f(g(x)) = (\sqrt{3-x})^2 + 2$

$= 3 - x + 2$

$= 5 - x$ $\boxed{\text{Domain} = \mathbb{R}}$ لايوجد قيد

$D(g): \ 3 - x \geq 0$

$\Rightarrow 3 \geq x$

$D(g) = (-\infty, 3]$

يوجد قيد

$\therefore D(f \circ g)(x) = (-\infty, 3]$

Example 2: (a) Find fog and gof and the domain of each,

where $f(x) = \frac{3x}{x-1}$ and $g(x) = \frac{2}{x}$

- $fog(x) = f(g(x)) = \frac{3\left(\frac{2}{x}\right)}{\left(\frac{2}{x}\right)-1}$

$\boxed{D(g) = R - \{0\}}$

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$$= \frac{\frac{6}{x}}{\frac{2-x}{x}} = \frac{6}{\cancel{x}} \cdot \frac{\cancel{x}}{2-x}$$

$$= \frac{6}{2-x}$$

$\boxed{\text{Domain}: R - \{2\}}$ يوجد قيد

$\therefore$ Domain fog : $R - \{0, 2\}$

- $gof(x) = g(f(x)) = \frac{2}{\left(\frac{3x}{x-1}\right)}$

$\boxed{R - \{1\}}$

$$= \frac{2}{3x} \cdot \frac{x-1}{1}$$

$$= \frac{2(x-1)}{3x}$$

$\boxed{\text{Domain}: R - \{0\}}$

$\therefore$ Domain gof : $R - \{0, 1\}$

(b) compute $(fog)(4)$ and $(gof)(3)$

$\because (fog)(x) = \frac{6}{2-x}$ من فقرة a

$\therefore (fog)(4) = \frac{6}{2-4} = \frac{6}{-2} = -3.$

$\because (gof)(x) = \frac{2(x-1)}{3x}$

$\therefore (gof)(2) = \frac{2(2-1)}{3 \cdot 2} = \frac{2}{6} = \frac{1}{3}$