$$\frac{\cos 2\theta + i \sin 2\theta}{1+\cos 2\theta + i \sin 2\theta}$$

$$= \frac{-2\sin^2\theta + 2i\sin\theta\cos\theta}{2\cos^2\theta + 2i\sin\theta\cos\theta}$$

$$= \frac{\cancel{2}\sin\theta(-\sin\theta + i\cos\theta)}{\cancel{2}\cos\theta(\cos\theta + i\sin\theta)}$$

$$= \frac{\sin\theta(i^2\sin\theta + i\cos\theta)}{\cos\theta(\cos\theta + i\sin\theta)}$$

$$= \frac{i\sin\theta\,\cancel{(i\sin\theta + \cos\theta)}}{\cos\theta\,\cancel{(\cos\theta + i\sin\theta)}}$$

$$= i\tan\theta$$

$$= \frac{2\sin^2\frac{\theta}{2} + 2i\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}$$

$$\div 2\sin\frac{\theta}{2} \neq 0$$

$$= \frac{\sin\frac{\theta}{2} + i\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}$$

$$= \frac{\sin\frac{\theta}{2}}{\sin\frac{\theta}{2}} + i\frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}$$

$$= 1 + i\cot\frac{\theta}{2}$$

$$\frac{z^2-1}{z^2+1} = i\tan\theta$$

$$z = \cos\theta + i\sin\theta$$

$$L_1 = \frac{\cos^2\theta - \sin^2\theta + 2i\sin\theta\cos\theta - 1}{\cos^2\theta - \sin^2\theta + 2i\sin\theta\cos\theta + 1}$$

$$= \frac{\cos 2\theta + i\sin 2\theta - 1}{\cos 2\theta + i\sin 2\theta + 1}$$

$$\frac{2}{1-z} = 1 + i\cot\frac{\theta}{2}$$

$$L_1 = \frac{2}{1-z} \qquad |z| = 1$$

$$z = \cos\theta + i\sin\theta$$

$$L_1 = \frac{2}{1-\cos\theta - i\sin\theta}$$

$$= \frac{2(1-\cos\theta + i\sin\theta)}{(1-\cos\theta)^2 + \sin^2\theta}$$

$$= \frac{2(1-\cos\theta + i\sin\theta)}{1 - 2\cos\theta + \cos^2\theta + \sin^2\theta}$$

$$= \frac{2(1-\cos\theta + i\sin\theta)}{2 - 2\cos\theta}$$

$$= \frac{1-\cos\theta + i\sin\theta}{1-\cos\theta}$$

تمرين: اكتب $\cos^3 x$ بدلالة مجموع نسب مثلثية لمضاعفات x واستنتج قيمة

احسب

$$\int_0^{\frac{\pi}{3}} \cos^3 x \, dx$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2} \Rightarrow \cos^3 x = \frac{1}{8}\left(e^{ix} + e^{-ix}\right)^3$$

$$\cos^3 x = \frac{1}{8}\left(e^{3ix} + 3e^{ix} + 3e^{-ix} + e^{-3ix}\right)$$

$$= \frac{1}{8}\left(e^{3ix} + e^{-3ix} + 3\left(e^{ix} + e^{-ix}\right)\right)$$

$$= \frac{1}{8}\left(2\cos 3x + 6\cos x\right) = \frac{1}{4}\cos 3x + \frac{3}{4}\cos x$$

$$\int_0^{\frac{\pi}{3}} \cos^3 x = \int_0^{\frac{\pi}{3}} \left(\frac{1}{4}\cos 3x + \frac{3}{4}\cos x\right) dx$$

$$= \left[\frac{1}{12}\sin 3x + \frac{3}{4}\sin x\right]_0^{\frac{\pi}{3}}$$

$$= \frac{1}{12}\sin\pi + \frac{3}{4}\sin\frac{\pi}{3} - 0 = \frac{3}{4}\cdot\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{8}$$

اثبت أن $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

مستخدماً دستور أويلر

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

$$\sin^3\theta = \frac{\left(e^{i\theta} - e^{-i\theta}\right)^3}{(2i)^3}$$

$$\sin^3\theta = \frac{-1}{8i}\left(e^{i\theta} - e^{-i\theta}\right)^3$$

$$= \frac{-1}{8i}\left(e^{3i\theta} - 3e^{i\theta} + 3e^{-i\theta} - e^{-i3\theta}\right)$$

$$= \frac{-1}{8i}\left(e^{3i\theta} - e^{-3i\theta} - 3\left(e^{i\theta} - e^{-i\theta}\right)\right)$$

$$= \frac{-1}{8i}\left(2i\sin 3\theta - 6i\sin\theta\right)$$

$$= \frac{-1}{4}\sin 3\theta + \frac{3}{4}\sin\theta$$

$$4\sin^3\theta = -\sin 3\theta + 3\sin\theta$$

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

$$Z = 1 + e^{2i\theta}$$

$$\theta \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$$

اكتب Z بالشكل الأسي

$$Z = e^{i\theta} e^{-i\theta} + e^{i\theta} \cdot e^{i\theta}$$

$$= \left(e^{-i\theta} + e^{i\theta}\right) e^{i\theta}$$

$$= 2\cos\theta \cdot e^{i\theta}$$

اثبت

$$\sin 2\theta = 2\cos\theta\sin\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

مستخدماً دستور ديموافر

$$(\cos\theta + i\sin\theta)^2 = \cos 2\theta + i\sin 2\theta$$

$$\cos^2\theta - \sin^2\theta + 2i\sin\theta\cos\theta = \cos 2\theta + i\sin 2\theta$$

بالمطابقة

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\sin 2\theta = 2\sin\theta \cdot \cos\theta$$

تمارين هامة في العقدية

1) عيّن مجموعة النقاط M الممثلة للعدد العقدي z

أ) $\dfrac{e^{i\frac{\pi}{3}}}{z} \in \mathbb{R}^*$

ب) $e^{i\frac{\pi}{3}} \cdot \bar{z} \in \mathbb{R}$

2) $z_A = 2 + i \qquad z_B = 3 + 2i$

$z_C = 1 + 2i$

اثبت أن ABC مثلث قائم في A.

3) M النقطة للعدد العقدي $z = x + iy$

اوجد المعادلة الديكارتية لمجموعة النقط

a) $|z - 2 + i| = |z - 1 - 3i|$

b) $(z + \bar{z})^2 + (7 - \bar{z})^2 = 4$

$z + \bar{z} = 2Re = 2x$

$z - \bar{z} = 2iIm = 2iy$

c) $|iz + 3 - 2i| = 5$

d) $z \cdot \bar{z} \leq 4$

المسوحة ضوئيا بـ CamScanner