Workshop Solutions to Sections 3.1 and 3.2

1) $$\lim_{x\to -2}(x^3-2x+1)=(-2)^3-2(-2)+1 = -8+4+1=-3$$

2) $$\lim_{x\to 2}(3x^2+x-4)=3(2)^2+(2)-4 = 12+2-4=10$$

3) $$\lim_{x\to 1}(x^2+3x-5)^3=((1)^2+3(1)-5)^3 = (1+3-5)^3=(-1)^3=-1$$

4) $$\lim_{x\to -2}(2x^3+3x^2+5)=2(-2)^3+3(-2)^2+5 = 2(-8)+3(4)+5 = -16+12+5=1$$

5) $$\lim_{x\to -2}\frac{x^2-2}{x-2}=\frac{(-2)^2-2}{(-2)-2}=\frac{4-2}{-2-2}=\frac{2}{-4}=-\frac{1}{2}$$

6) $$\lim_{x\to 2}\frac{x^3+5}{x^2+1}=\frac{(2)^3+5}{(2)^2+1}=\frac{8+5}{4+1}=\frac{13}{5}$$

7) $$\lim_{x\to 0}\frac{x^2+3x+5}{x^2-3}=\frac{(0)^2+3(0)+5}{(0)^2-3}=\frac{0+0+5}{0-3} =\frac{5}{-3}=-\frac{5}{3}$$

8) $$\lim_{x\to 1}\frac{x-1}{x^2+x-5}=\frac{(1)-1}{(1)^2+(1)-5}=\frac{1-1}{1+1-5}=\frac{0}{-3}=0$$

9) $$\lim_{x\to -1}\sqrt{x^3-10x+7}=\sqrt{(-1)^3-10(-1)+7} =\sqrt{-1+10+7}=\sqrt{16}=4$$

10) $$\lim_{x\to -1}\frac{1-(x+4)^{-2}}{x-2}=\frac{1-\left((-1)+4\right)^{-2}}{(-1)-2} =\frac{1-(-1+4)^{-2}}{-1-2}=\frac{1-(3)^{-2}}{-3}=\frac{1-\frac{1}{3^2}}{-3} =\frac{1-\frac{1}{9}}{-3}=\frac{\frac{8}{9}}{-3}=\frac{8}{9}\times\frac{1}{-3}=\frac{8}{-27}=-\frac{8}{27}$$

11) $$\lim_{x\to -1}\frac{x^3+2x}{8-2x}=\frac{(-1)^3+2(-1)}{8-2(-1)}=\frac{-1-2}{8+2}=\frac{-3}{10} =-\frac{3}{10}$$

12) $$\lim_{x\to 4}\frac{x^2-3x}{5+x}=\frac{(4)^2-3(4)}{5+(4)}=\frac{16-12}{5+4}=\frac{4}{9}$$

13) $$\lim_{x\to 4}\frac{x^2-4x}{5+x}=\frac{(4)^2-4(4)}{5+(4)}=\frac{16-16}{5+4}=\frac{0}{9}=0$$

15) $$\lim_{x\to 0}\frac{x^3-5x^2}{x^2}=\lim_{x\to 0}\frac{x^2(x-5)}{x^2} =\lim_{x\to 0}(x-5)=(0)-5=-5$$

14) $$\lim_{x\to 4}\frac{3^{-1}-(2x-5)^{-1}}{4-x}=\lim_{x\to 4}\frac{\frac{1}{3}-\frac{1}{2x-5}}{4-x} =\lim_{x\to 4}\frac{\frac{2x-5-3}{3(2x-5)}}{4-x} =\lim_{x\to 4}\frac{2x-8}{3(2x-5)(4-x)} =\lim_{x\to 4}\frac{2(x-4)}{3(2x-5)(4-x)} =\lim_{x\to 4}\frac{-2(4-x)}{3(2x-5)(4-x)}=\lim_{x\to 4}\frac{-2}{3(2x-5)} =\frac{-2}{3(2(4)-5)}=\frac{-2}{3(8-5)}=\frac{-2}{9}=-\frac{2}{9}$$

16) $$\lim_{x\to 6}\frac{x-6}{x^2-36}=\lim_{x\to 6}\frac{x-6}{(x-6)(x+6)}=\lim_{x\to 6}\frac{1}{x+6} =\frac{1}{(6)+6}=\frac{1}{12}$$

17) $$\lim_{x\to 6}\frac{x^2-36}{x-6}=\lim_{x\to 6}\frac{(x-6)(x+6)}{x-6}=\lim_{x\to 6}(x+6) =(6)+6=12$$

18) $$\lim_{x\to -6}\frac{x+6}{x^2-36}=\lim_{x\to -6}\frac{x+6}{(x-6)(x+6)}=\lim_{x\to -6}\frac{1}{x-6} =\frac{1}{(-6)-6}=\frac{1}{-12}=-\frac{1}{12}$$

19) $$\lim_{x\to 3}\frac{x^3-27}{x-3}=\lim_{x\to 3}\frac{(x-3)(x^2+3x+9)}{x-3} =\lim_{x\to 3}(x^2+3x+9)=(3)^2+3(3)+9 =9+9+9=27$$

20) $$\lim_{x\to 3}\frac{x-3}{x^3-27}=\lim_{x\to 3}\frac{x-3}{(x-3)(x^2+3x+9)} =\lim_{x\to 3}\frac{1}{x^2+3x+9}=\frac{1}{(3)^2+3(3)+9} =\frac{1}{9+9+9}=\frac{1}{27}$$

21) $$\lim_{x\to-2}\frac{x+2}{x^3+8}=\lim_{x\to-2}\frac{x+2}{(x+2)(x^2-2x+4)}$$
$$=\lim_{x\to-2}\frac{1}{x^2-2x+4}$$
$$=\frac{1}{(-2)^2-2(-2)+4}=\frac{1}{4+4+4}=\frac{1}{12}$$

22) $$\lim_{x\to-2}\frac{x^3+8}{x+2}=\lim_{x\to-2}\frac{(x+2)(x^2-2x+4)}{x+2}$$
$$=\lim_{x\to-2}(x^2-2x+4)=(-2)^2-2(-2)+4$$
$$=4+4+4=12$$

23) $$\lim_{x\to4}\frac{x^2-3x-4}{x-4}=\lim_{x\to4}\frac{(x-4)(x+1)}{x-4}=\lim_{x\to4}(x+1)$$
$$=(4)+1=5$$

24) $$\lim_{x\to3}\frac{x^2+4x-21}{x^2-8x+15}=\lim_{x\to3}\frac{(x+7)(x-3)}{(x-5)(x-3)}=\lim_{x\to3}\frac{x+7}{x-5}$$
$$=\frac{(3)+7}{(3)-5}=\frac{10}{-2}=-5$$

25) $$\lim_{x\to0}\frac{x}{1-(1-x)^2}=\lim_{x\to0}\frac{x}{1-(1-2x+x^2)}$$
$$=\lim_{x\to0}\frac{x}{1-1+2x-x^2}$$
$$=\lim_{x\to0}\frac{x}{2x-x^2}=\lim_{x\to0}\frac{x}{x(2-x)}$$
$$=\lim_{x\to0}\frac{1}{2-x}=\frac{1}{2-(0)}=\frac{1}{2}$$

26) $$\lim_{x\to2}\frac{\sqrt[3]{x+6}-2}{x-2}=\lim_{x\to2}\frac{\sqrt[3]{x+6}-2}{(x+6)-8}=\lim_{x\to2}\frac{\sqrt[3]{x+6}-2}{\left(\sqrt[3]{x+6}\right)^3-8}$$
$$=\lim_{x\to2}\frac{\sqrt[3]{x+6}-2}{\left(\sqrt[3]{x+6}-2\right)\left(\left(\sqrt[3]{x+6}\right)^2+2\sqrt[3]{x+6}+4\right)}$$
$$=\lim_{x\to2}\frac{1}{\left(\sqrt[3]{x+6}\right)^2+2\sqrt[3]{x+6}+4}$$
$$=\frac{1}{\left(\sqrt[3]{(2)+6}\right)^2+2\sqrt[3]{(2)+6}+4}=\frac{1}{4+4+4}=\frac{1}{12}$$

27) $$\lim_{x\to0}\frac{\sqrt{x+25}-5}{x}$$
$$=\lim_{x\to0}\left[\frac{\sqrt{x+25}-5}{x}\times\frac{\sqrt{x+25}+5}{\sqrt{x+25}+5}\right]$$
$$=\lim_{x\to0}\frac{(x+25)-25}{x\left(\sqrt{x+25}+5\right)}$$
$$=\lim_{x\to0}\frac{x}{x\left(\sqrt{x+25}+5\right)}$$
$$=\lim_{x\to0}\frac{1}{\sqrt{x+25}+5}=\frac{1}{\sqrt{(0)+25}+5}$$
$$=\frac{1}{5+5}=\frac{1}{10}$$

28) $$\lim_{x\to0}\frac{x}{\sqrt{x+25}-5}=\lim_{x\to0}\left[\frac{x}{\sqrt{x+25}-5}\times\frac{\sqrt{x+25}+5}{\sqrt{x+25}+5}\right]$$
$$=\lim_{x\to0}\frac{x\left(\sqrt{x+25}+5\right)}{(x+25)-25}$$
$$=\lim_{x\to0}\frac{x\left(\sqrt{x+25}+5\right)}{x}$$
$$=\lim_{x\to0}\left(\sqrt{x+25}+5\right)=\sqrt{(0)+25}+5$$
$$=5+5=10$$

29) $$\lim_{x\to2}\frac{x-2}{2-\sqrt{6-x}}=\lim_{x\to2}\left[\frac{x-2}{2-\sqrt{6-x}}\times\frac{2+\sqrt{6-x}}{2+\sqrt{6-x}}\right]$$
$$=\lim_{x\to2}\frac{(x-2)\left(2+\sqrt{6-x}\right)}{4-(6-x)}$$
$$=\lim_{x\to2}\frac{(x-2)\left(2+\sqrt{6-x}\right)}{4-6+x}$$
$$=\lim_{x\to2}\frac{(x-2)\left(2+\sqrt{6-x}\right)}{x-2}$$
$$=\lim_{x\to2}\left(2+\sqrt{6-x}\right)=2+\sqrt{6-(2)}$$
$$=2+2=4$$

30) $$\lim_{x\to2}\frac{2-\sqrt{6-x}}{x+2}=\frac{2-\sqrt{6-(2)}}{(2)+2}=\frac{2-2}{4}=0$$

31) $$\lim_{x\to3}\frac{1-\sqrt{x-2}}{2-\sqrt{x+1}}$$
$$=\lim_{x\to3}\left[\frac{1-\sqrt{x-2}}{2-\sqrt{x+1}}\times\frac{1+\sqrt{x-2}}{1+\sqrt{x-2}}\times\frac{2+\sqrt{x+1}}{2+\sqrt{x+1}}\right]$$
$$=\lim_{x\to3}\left[\frac{1-(x-2)}{4-(x+1)}\times\frac{2+\sqrt{x+1}}{1+\sqrt{x-2}}\right]$$
$$=\lim_{x\to3}\left[\frac{3-x}{3-x}\times\frac{2+\sqrt{x+1}}{1+\sqrt{x-2}}\right]$$
$$=\lim_{x\to3}\frac{2+\sqrt{x+1}}{1+\sqrt{x-2}}=\frac{2+\sqrt{(3)+1}}{1+\sqrt{(3)-2}}=\frac{2+2}{1+1}$$
$$=\frac{4}{2}=2$$

32) If $2x \le f(x) \le 3x^2 - 8$, then
$$\lim_{x\to 2} f(x) =$$
<u>Solution:</u>
$$\lim_{x\to 2} 2x = 2(2) = 4$$
and
$$\lim_{x\to 2}(3x^2 - 8) = 3(2)^2 - 8 = 12 - 8 = 4$$
It follows from the Sandwich Theorem that
$$\lim_{x\to 2} f(x) = 4$$

33) $\lim_{x\to 0}\left[x\cos\left(x+\frac{1}{x}\right)\right] =$

We know that the cosine of any angle is between -1 and 1. So,
$$-1 \le \cos\left(x+\frac{1}{x}\right) \le 1$$
Now, multiply throughout by x, we get
$$-x \le x\cos\left(x+\frac{1}{x}\right) \le x$$
But $\lim_{x\to 0} x = 0$ and $\lim_{x\to 0}(-x) = 0$.
It follows from the Sandwich Theorem that
$$\lim_{x\to 0}\left[x\cos\left(x+\frac{1}{x}\right)\right] = 0$$

34) $\lim_{x\to 0}\left[x\sin\left(\frac{1}{x}\right)\right] =$

We know that the sine of any angle is between -1 and 1. So,
$$-1 \le \sin\left(\frac{1}{x}\right) \le 1$$
Now, multiply throughout by x, we get
$$-x \le x\sin\left(\frac{1}{x}\right) \le x$$
But $\lim_{x\to 0} x = 0$ and $\lim_{x\to 0}(-x) = 0$.
It follows from the Sandwich Theorem that
$$\lim_{x\to 0}\left[x\sin\left(\frac{1}{x}\right)\right] = 0$$

35) If $\frac{x^2+1}{x-1} \le f(x) \le x - 1$, then
$$\lim_{x\to 0} f(x) =$$
<u>Solution:</u>
$$\lim_{x\to 0}\frac{x^2+1}{x-1} = \frac{(0)^2+1}{(0)-1} = \frac{1}{-1} = -1$$
and
$$\lim_{x\to 0}(x-1) = (0) - 1 = -1$$
It follows from the Sandwich Theorem that
$$\lim_{x\to 0} f(x) = -1$$

36) If $4(x-1) \le f(x) \le x^3 + x - 2$, then
$$\lim_{x\to 1} f(x) =$$
<u>Solution:</u>
$$\lim_{x\to 1}\big(4(x-1)\big) = 4\big((1)-1\big) = 4\times 0 = 0$$
and
$$\lim_{x\to 1}(x^3 + x - 2) = (1)^3 + (1) - 2 = 1 + 1 - 2 = 0$$
It follows from the Sandwich Theorem that
$$\lim_{x\to 1} f(x) = 0$$

37) If
$$\lim_{x\to 3}\frac{f(x)+4}{x-1} = 3,$$
then
$$\lim_{x\to 3} f(x) =$$
<u>Solution:</u>
$$\lim_{x\to 3}\frac{f(x)+4}{x-1} = \frac{\lim_{x\to 3}(f(x)+4)}{\lim_{x\to 3}(x-1)} = \frac{\lim_{x\to 3} f(x) + \lim_{x\to 3}(4)}{\lim_{x\to 3}(x) - \lim_{x\to 3}(1)}$$
$$= \frac{\lim_{x\to 3} f(x) + 4}{3-1} = \frac{\lim_{x\to 3} f(x) + 4}{2}$$
Now
$$\frac{\lim_{x\to 3} f(x) + 4}{2} = 3$$
$$\lim_{x\to 3} f(x) + 4 = 6 \quad \Leftrightarrow \quad \lim_{x\to 3} f(x) = 2$$

38) $\lim_{x\to 2}\frac{2^{-1}-(3x-4)^{-1}}{2-x}$

$$=\lim_{x\to 2}\frac{\frac{1}{2}-\frac{1}{3x-4}}{2-x}$$
$$=\lim_{x\to 2}\frac{\frac{3x-4-2}{2(3x-4)}}{2-x}$$
$$=\lim_{x\to 2}\frac{\frac{3x-6}{2(3x-4)}}{2-x}$$
$$=\lim_{x\to 2}\frac{\frac{3(x-2)}{2(3x-4)}}{2-x}$$
$$=\lim_{x\to 2}\frac{3(x-2)}{2(3x-4)(2-x)}$$
$$=\lim_{x\to 2}\frac{-3(2-x)}{2(3x-4)(2-x)}=\lim_{x\to 2}\frac{-3}{2(3x-4)}$$
$$=\frac{-3}{2(3(2)-4)}=\frac{-3}{2\times 2}=-\frac{3}{4}$$

39) $\lim_{x\to 0}\frac{(x+1)^3-1}{x}=\lim_{x\to 0}\frac{(x^3+3x^2+3x+1)-1}{x}$

$$=\lim_{x\to 0}\frac{x^3+3x^2+3x}{x}$$
$$=\lim_{x\to 0}\frac{x(x^2+3x+3)}{x}=\lim_{x\to 0}(x^2+3x+3)$$
$$=(0)^2+3(0)+3=3$$

40) If

$$\lim_{x\to 1}\frac{f(x)+3x}{x^2-5f(x)}=1,$$

then

$$\lim_{x\to 1}f(x)=$$

Solution:

$$\lim_{x\to 1}\frac{f(x)+3x}{x^2-5f(x)}=\frac{\lim_{x\to 1}(f(x)+3x)}{\lim_{x\to 1}(x^2-5f(x))}$$
$$=\frac{\lim_{x\to 1}f(x)+\lim_{x\to 1}(3x)}{\lim_{x\to 1}(x^2)-\lim_{x\to 1}(5f(x))}$$
$$=\frac{\lim_{x\to 1}f(x)+3(1)}{(1)^2-5\lim_{x\to 1}f(x)}=\frac{\lim_{x\to 1}f(x)+3}{1-5\lim_{x\to 1}f(x)}$$

Now

$$\frac{\lim_{x\to 1}f(x)+3}{1-5\lim_{x\to 1}f(x)}=1$$

$$\lim_{x\to 1}f(x)+3=(1)\left(1-5\lim_{x\to 1}f(x)\right)$$
$$\Leftrightarrow\quad \lim_{x\to 1}f(x)+3=1-5\lim_{x\to 1}f(x)$$
$$\Leftrightarrow\quad \lim_{x\to 1}f(x)+5\lim_{x\to 1}f(x)=1-3$$
$$\Leftrightarrow\quad 6\lim_{x\to 1}f(x)=-2$$
$$\Leftrightarrow\quad \lim_{x\to 1}f(x)=\frac{-2}{6}=-\frac{1}{3}$$

41) $\lim_{x\to 4}\frac{x^2-6x+8}{x^2+x-20}$

$$=\lim_{x\to 4}\frac{(x-2)(x-4)}{(x-4)(x+5)}$$
$$=\lim_{x\to 4}\frac{x-2}{x+5}=\frac{(4)-2}{(4)+5}=\frac{2}{9}$$

42) $\lim_{x\to -2}\frac{x^3+8}{x^2-x-6}$

$$=\lim_{x\to -2}\frac{(x+2)(x^2-2x+4)}{(x-3)(x+2)}$$
$$=\lim_{x\to -2}\frac{x^2-2x+4}{x-3}=\frac{(-2)^2-2(-2)+4}{(-2)-3}$$
$$=\frac{4+4+4}{-5}=\frac{12}{-5}=-\frac{12}{5}$$

43) $\lim_{x\to 1}\left[\frac{x^2-2}{x+4}+x^2-2x\right]=\frac{(1)^2-2}{(1)+4}+(1)^2-2(1)$

$$=\frac{1-2}{1+4}+1-2=\frac{-1}{5}-1=\frac{-1-5}{5}=-\frac{6}{5}$$

44) $$\lim_{x\to-2}\frac{4x^2+6x-4}{2x^2-8}$$
$$=\lim_{x\to-2}\frac{2(2x^2+3x-2)}{2(x^2-4)}$$
$$=\lim_{x\to-2}\frac{2x^2+3x-2}{x^2-4}$$
$$=\lim_{x\to-2}\frac{(2x-1)(x+2)}{(x-2)(x+2)}$$
$$=\lim_{x\to-2}\frac{2x-1}{x-2}=\frac{2(-2)-1}{(-2)-2}=\frac{-4-1}{-2-2}$$
$$=\frac{-5}{-4}=\frac{5}{4}$$

45) $$\lim_{x\to-1}\frac{x^2-2x-3}{x^5-x^3}$$
$$=\lim_{x\to-1}\frac{(x-3)(x+1)}{x^3(x^2-1)}$$
$$=\lim_{x\to-1}\frac{(x-3)(x+1)}{x^3(x-1)(x+1)}$$
$$=\lim_{x\to-1}\frac{x-3}{x^3(x-1)}=\frac{(-1)-3}{(-1)^3\big((-1)-1\big)}$$
$$=\frac{-1-3}{(-1)(-2)}=\frac{-4}{2}=-2$$

46) $$\lim_{x\to3}\frac{\sqrt{2x+1}(x^2-9)}{(2x+3)(x-3)}$$
$$=\lim_{x\to3}\frac{\sqrt{2x+1}(x-3)(x+3)}{(2x+3)(x-3)}$$
$$=\lim_{x\to3}\frac{\sqrt{2x+1}(x+3)}{2x+3}=\frac{\sqrt{2(3)+1}\big((3)+3\big)}{2(3)+3}$$
$$=\frac{6\sqrt{7}}{9}=\frac{2\sqrt{7}}{3}$$

47) $$\lim_{x\to1}\frac{\sqrt{3-2x}-1}{x-1}=\lim_{x\to1}\left[\frac{\sqrt{3-2x}-1}{x-1}\times\frac{\sqrt{3-2x}+1}{\sqrt{3-2x}+1}\right]$$
$$=\lim_{x\to1}\frac{(3-2x)-1}{(x-1)\big(\sqrt{3-2x}+1\big)}$$
$$=\lim_{x\to1}\frac{2-2x}{(x-1)\big(\sqrt{3-2x}+1\big)}$$
$$=\lim_{x\to1}\frac{2(1-x)}{(x-1)\big(\sqrt{3-2x}+1\big)}=$$
$$=\lim_{x\to1}\frac{-2(x-1)}{(x-1)\big(\sqrt{3-2x}+1\big)}=$$
$$=\lim_{x\to1}\frac{-2}{\sqrt{3-2x}+1}=\frac{-2}{\sqrt{3-2(1)}+1}$$
$$=\frac{-2}{\sqrt{3-2}+1}=\frac{-2}{2}=-1$$

48) $$\lim_{x\to0}\frac{(x+1)^2-1}{x}=\lim_{x\to0}\frac{(x^2+2x+1)-1}{x}$$
$$=\lim_{x\to0}\frac{x^2+2x}{x}=\lim_{x\to0}\frac{x(x+2)}{x}$$
$$=\lim_{x\to0}(x+2)=(0)+2=2$$

49) $$\lim_{x\to1}\frac{\sqrt{2x+2}-2}{\sqrt{3x-2}-1}$$
$$=\lim_{x\to1}\left[\frac{\sqrt{2x+2}-2}{\sqrt{3x-2}-1}\times\frac{\sqrt{2x+2}+2}{\sqrt{2x+2}+2}\times\frac{\sqrt{3x-2}+1}{\sqrt{3x-2}+1}\right]$$
$$=\lim_{x\to1}\left[\frac{(2x+2)-4}{(3x-2)-1}\times\frac{\sqrt{3x-2}+1}{\sqrt{2x+2}+2}\right]$$
$$=\lim_{x\to1}\left[\frac{2x-2}{3x-3}\times\frac{\sqrt{3x-2}+1}{\sqrt{2x+2}+2}\right]$$
$$=\lim_{x\to1}\left[\frac{2(x-1)}{3(x-1)}\times\frac{\sqrt{3x-2}+1}{\sqrt{2x+2}+2}\right]$$
$$=\lim_{x\to1}\left[\frac{2}{3}\times\frac{\sqrt{3x-2}+1}{\sqrt{2x+2}+2}\right]=\frac{2}{3}\times\frac{\sqrt{3(1)-2}+1}{\sqrt{2(1)+2}+2}$$
$$=\frac{2}{3}\times\frac{\sqrt{1}+1}{\sqrt{4}+2}=\frac{2}{3}\times\frac{2}{4}=\frac{1}{3}$$

50) $\lim_{x\to 2}\frac{3-\sqrt{2x+5}}{x-2}$

$$= \lim_{x\to 2}\left[\frac{3-\sqrt{2x+5}}{x-2}\times\frac{3+\sqrt{2x+5}}{3+\sqrt{2x+5}}\right]$$
$$= \lim_{x\to 2}\frac{9-(2x+5)}{(x-2)\left(3+\sqrt{2x+5}\right)}$$
$$= \lim_{x\to 2}\frac{4-2x}{(x-2)\left(3+\sqrt{2x+5}\right)}$$
$$= \lim_{x\to 2}\frac{2(2-x)}{(x-2)\left(3+\sqrt{2x+5}\right)}$$
$$= \lim_{x\to 2}\frac{-2(x-2)}{(x-2)\left(3+\sqrt{2x+5}\right)}$$
$$= \lim_{x\to 2}\frac{-2}{3+\sqrt{2x+5}} = \frac{-2}{3+\sqrt{2(2)+5}}$$
$$= \frac{-2}{3+\sqrt{9}} = \frac{-2}{6} = -\frac{1}{3}$$

51) $\lim_{x\to -1}\frac{x^2+3x+2}{x^2+1} = \frac{(-1)^2+3(-1)+2}{(-1)^2+1} = \frac{1-3+2}{1+1}$

$$= \frac{0}{2} = 0$$

52) If

$$\lim_{x\to k} f(x) = -\frac{1}{2}$$

and

$$\lim_{x\to k} g(x) = \frac{2}{3}$$

Then

$$\lim_{x\to k}\frac{f(x)}{g(x)} = \frac{-\frac{1}{2}}{\frac{2}{3}} = -\frac{1}{2}\times\frac{3}{2} = -\frac{3}{4}$$

53) $\lim_{x\to 0}\frac{\sqrt{x+4}-2}{x} = \lim_{x\to 0}\left[\frac{\sqrt{x+4}-2}{x}\times\frac{\sqrt{x+4}+2}{\sqrt{x+4}+2}\right]$

$$= \lim_{x\to 0}\frac{(x+4)-4}{x\left(\sqrt{x+4}+2\right)}$$
$$= \lim_{x\to 0}\frac{x}{x\left(\sqrt{x+4}+2\right)}$$
$$= \lim_{x\to 0}\frac{1}{\sqrt{x+4}+2} = \frac{1}{\sqrt{(0)+4}+2}$$
$$= \frac{1}{\sqrt{4}+2} = \frac{1}{4}$$

54) $\lim_{x\to -1}\frac{x^2-5x-6}{x+1} = \lim_{x\to -1}\frac{(x-6)(x+1)}{x+1} = \lim_{x\to -1}(x-6)$

$$= (-1)-6 = -7$$

55) $\lim_{x\to 0}\frac{(x+3)^{-1}-3^{-1}}{x} = \lim_{x\to 0}\frac{\frac{1}{x+3}-\frac{1}{3}}{x} = \lim_{x\to 0}\frac{\frac{3-(x+3)}{3(x+3)}}{x}$

$$= \lim_{x\to 0}\frac{-x}{3x(x+3)} = \lim_{x\to 0}\frac{-1}{3(x+3)}$$
$$= \frac{-1}{3\left((0)+3\right)} = \frac{-1}{9} = -\frac{1}{9}$$

56) If

$$\lim_{x\to 1} f(x) = 3$$

$$\lim_{x\to 1} g(x) = -4$$

and

$$\lim_{x\to 1} h(x) = -1$$

then

$$\lim_{x\to 1}\left[\frac{5f(x)}{2g(x)}+h(x)\right] = \frac{\lim_{x\to 1} 5f(x)}{\lim_{x\to 1} 2g(x)} + \lim_{x\to 1} h(x)$$
$$= \frac{5\lim_{x\to 1} f(x)}{2\lim_{x\to 1} g(x)} + \lim_{x\to 1} h(x)$$
$$= \frac{5(3)}{2(-4)} + (-1) = \frac{15}{-8} - 1 = -\frac{15}{8} - 1$$
$$= \frac{-15-8}{8} = -\frac{23}{8}$$

57) If

$$\lim_{x\to 1} g(x) = -4$$

and

$$\lim_{x\to 1} h(x) = -1$$

then

$$\lim_{x\to 1}\sqrt{g(x)h(x)} = \sqrt{\left[\lim_{x\to 1} g(x)\right]\left[\lim_{x\to 1} h(x)\right]} = \sqrt{(-4)(-1)}$$
$$= \sqrt{4} = 2$$

58) If

$$\lim_{x\to 1} f(x) = 3$$
$$\lim_{x\to 1} g(x) = -4$$

and

$$\lim_{x\to 1} h(x) = -1$$

then

$$\lim_{x\to 1}[2f(x)g(x)h(x)] = 2\left[\lim_{x\to 1} f(x)\right]\left[\lim_{x\to 1} g(x)\right]\left[\lim_{x\to 1} h(x)\right]$$
$$= 2(3)(-4)(-1) = 24$$