**Question 3**: Show that the function $f(x) = x^2 - 4$ satisfies Roll's Th. on $[-2,2]$ And find the values of $c$.

$f(x) = x^2 - 4$ , $f(x)$ polynomial $\Rightarrow$ $f(x)$ is continuous and differentiable everywhere.

1) $f(a) = f(-2) = (-2)^2 - 4 = 4 - 4 = 0$ ✓, 2) $f(b) = f(2) = 2^2 - 4 = 4 - 4 = 0$ ✓

$\Rightarrow$ $f(x)$ satisfied Rolls Th. $\therefore$ $f'(x) = 2x$ , $f'(c) = 0 \rightarrow 2c = 0$

$\rightarrow \boxed{c = 0 \in (-2,2)}$.

**Question 4:** Show that the function $f(x) = -x^2 + 4x$ satisfies Mean-Value Th. on $[-1,2]$ and find the values of c.

$f(x) = -x^2 + 4x$ , $f(x)$ polynomial $\Rightarrow$ $f(x)$ is continuous and differentiable everywhere.

$\rightarrow \exists c \in (-1,2);\ f'(c) = \dfrac{f(b) - f(a)}{b - a}$

1) $f(a) = f(-1) = -(-1)^2 + 4(-1) = -1 - 4 = \underline{\underline{-5}}$

2) $f(b) = f(2) = -(2)^2 + 4(2) = -4 + 8 = \underline{\underline{4}}$

3) $f'(x) = -2x + 4 \rightarrow f'(c) = -2c + 4$

$f'(c) = \dfrac{f(b) - f(a)}{b - a} \rightarrow -2c + 4 = \dfrac{4 - (-5)}{2 - (-1)}$

$\rightarrow -2c + 4 = \dfrac{4 + 5}{2 + 1} \rightarrow -2c + 4 = \dfrac{9}{3} = 3$

$\rightarrow -2c + 4 = 3 \rightarrow -2c = 3 - 4$

$\rightarrow -2c = -1 \rightarrow c = \dfrac{-1}{-2} \rightarrow \boxed{c = \tfrac{1}{2} \in (-1,2)}$

**Question 4:** Given $f(x) = 3x^2 - 12x$, find the interval where the function is decreasing or increasing and all relative extrema.

- increasing, decreasing

$f(x) = 3x^2 - 12x$

$f'(x) = 6x - 12$

$f'(x) = 0 \rightarrow 6x - 12 = 0$

$6x = 12$

$\rightarrow x = \dfrac{12}{6} \rightarrow \boxed{x = 2}$

- relative extrema

By $2^{nd}$ D.T

$\rightarrow f''(x) = 6$

$f''(2) = 6 > 0$

$\rightarrow$ relative minimum at $x = 2$

| Interval | $f'(x) = 6x - 12$ | Conclusion |
|---|---|---|
| $x < 2$ | $-$ | $f$ is decreasing on $(-\infty, 2]$ |
| $x > 2$ | $+$ | $f$ is increasing on $[2, \infty)$ |

- By $1^{st}$ D.T $\rightarrow$ $f'(x) < 0$ at $\underset{\text{left}}{x < 2}$ and $f'(x) > 0$ at $\underset{\text{right}}{x > 2}$

$-\ 2\ +$

$\Rightarrow$ relative minimum at $x = 2$.

**Question 1:** Find the equation of the tangent line of $f(x) = x^2$ at $x = 3$.

The equation of the tangent line: $y - f(x_0) = f'(x_0)(x - x_0)$

$x_0 = 3$ , $f(x_0) = f(3) = 3^2 = 9$ , $f'(x) = 2x$ , $f'(x_0) = f'(3) = 2(3) = 6$

$\therefore y - f(x_0) = f'(x_0)(x - x_0) \rightarrow y - 9 = 6(x - 3)$

$\rightarrow y - 9 = 6x - 18$

$\rightarrow y = 6x - 18 + 9 \rightarrow \boxed{y = 6x - 9}$

**Question 2:** Find $\frac{dy}{dx}$ of the following functions :

(1) $y = (2x + 1)(x + 5)$

$*(fg)' = fg' + gf'$ | $\cdot \frac{dy}{dx} = (2x+1)(1) + (x+5)(2)$
$= 2x + 1 + 2x + 10$
$= 4x + 11$

(2) $y = \frac{x-1}{x+1}$

$*\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$ | $\cdot \frac{dy}{dx} = \frac{(x+1)(1) - (x-1)(1)}{(x+1)^2} = \frac{x + 1 - x + 1}{(x+1)^2} = \frac{2}{(x+1)^2}$.

(3) $y = (x^2 + 5x + 7)^4$

$\frac{dy}{dx} = 4(x^2+5x+7)^3 \cdot (2x+5) = 4(2x+5)(x^2+5x+7)^3 = (8x+20)(x^2+5x+7)^3$.

(4) $y = 5t$ , $t = x^2$ (using chain rule)

$* \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} \rightarrow \frac{dy}{dx} = 5 \cdot 2x = 10x$ .

(5) $y = \sin(3x)$

$\frac{dy}{dx} = \cos(3x) \cdot (3) = 3\cos(3x)$.

(6) $x^2 + y^2 = 9$

$\frac{d}{dx}[x^2 + y^2] = \frac{d}{dx}[9] \rightarrow 2x + 2y\frac{dy}{dx} = 0 \rightarrow 2y\frac{dy}{dx} = -2x \rightarrow \frac{dy}{dx} = \frac{-2x}{2y}$

$\rightarrow \frac{dy}{dx} = \frac{-x}{y}$ .