

# Chapter Two

## **Fundamental of Mechanics**

# Vectors and Scalar Quantities

## Scalar

Scalar  $\equiv$  a quantity that can be described by **magnitude** only  
So, it is represented by just a **number**.

Examples: Speed, Mass, Temperature, Time

## Vectors



Vector  $\equiv$  a quantity that requires both **magnitude** and **direction**

Examples: Displacement, Velocity, Force, Acceleration, Weight

*Note the difference between  
displacement and distance*

# Resultant of Vectors

## Resultant:

- The sum of two or more vectors
  - For vectors in the same direction:
    - add arithmetically.

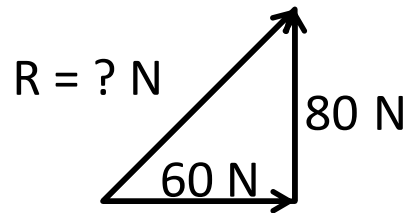
$$\begin{array}{c} F_1 = 6 \text{ N} \\ \longrightarrow \end{array} \quad \begin{array}{c} F_2 = 3 \text{ N} \\ \longrightarrow \end{array} = \begin{array}{c} R = 9 \text{ N} \\ \longrightarrow \end{array}$$

- For vectors in opposite directions:
  - subtract arithmetically.

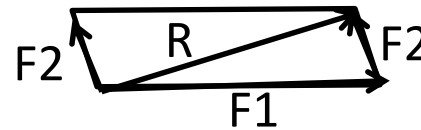
$$\begin{array}{c} F_1 = 6 \text{ N} \\ \longrightarrow \end{array} \quad \begin{array}{c} F_2 = 3 \text{ N} \\ \longleftarrow \end{array} = \begin{array}{c} R = 3 \text{ N} \\ \longrightarrow \end{array}$$

# Resultant Of Vectors

- Two vectors at right angles to each other:
  - use Pythagorean Theorem:  $R^2 = X^2 + Y^2$ .



- Two vectors that don't act in the same or opposite direction:
  - use Parallelogram rule.



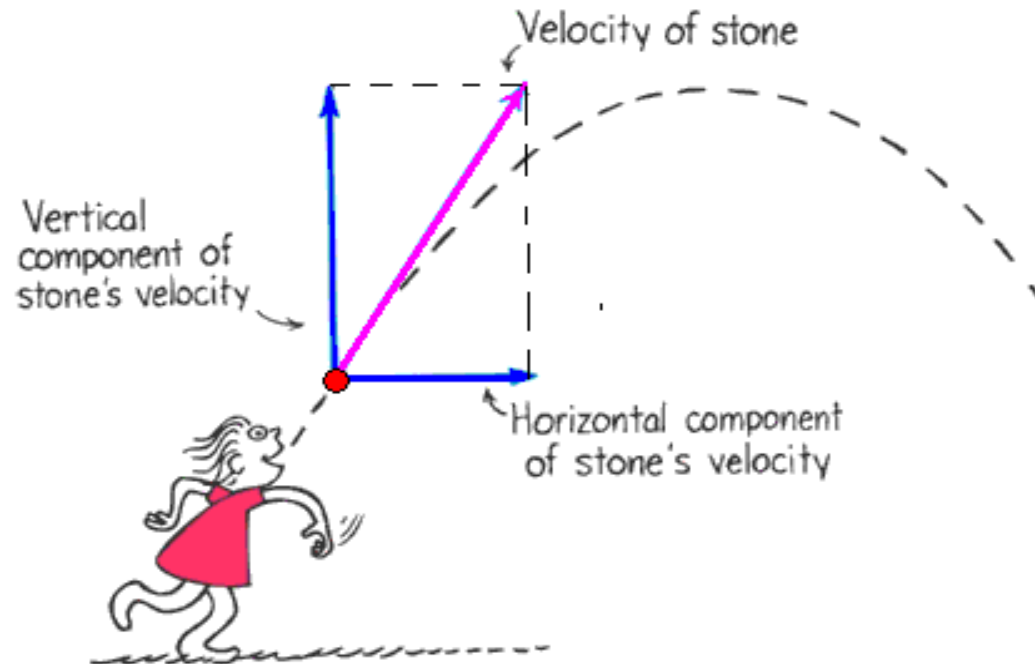
# *Parallelogram rule:* Finding the resultant geometrically



Generally applies for rectangular and nonrectangular vectors

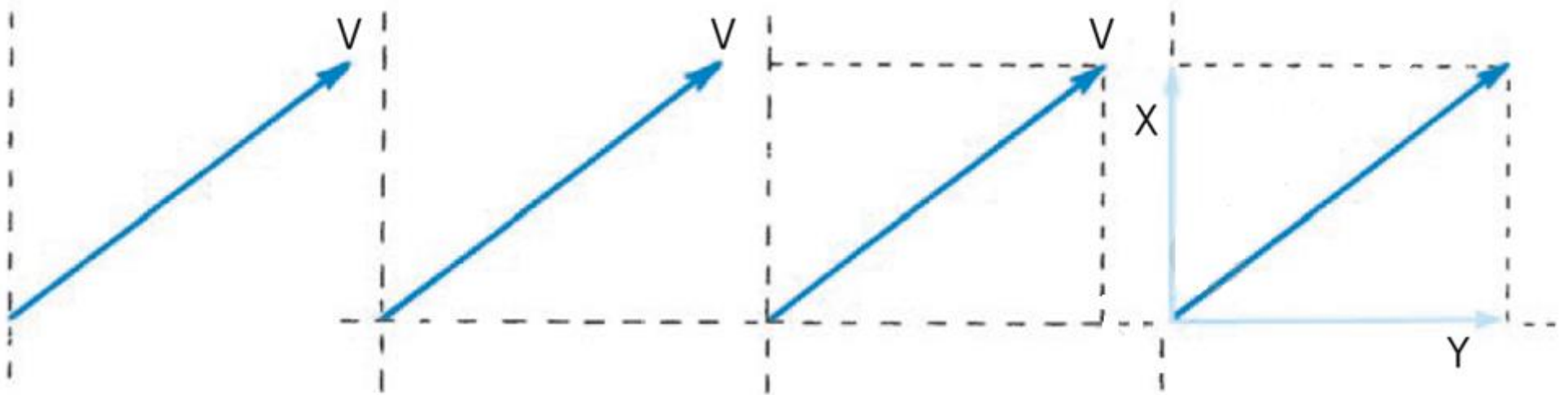
# Vector components

- Vertical and horizontal components of a vector are perpendicular to each other
- Determined by resolution.



# Finding vector components by *resolution*

Example:

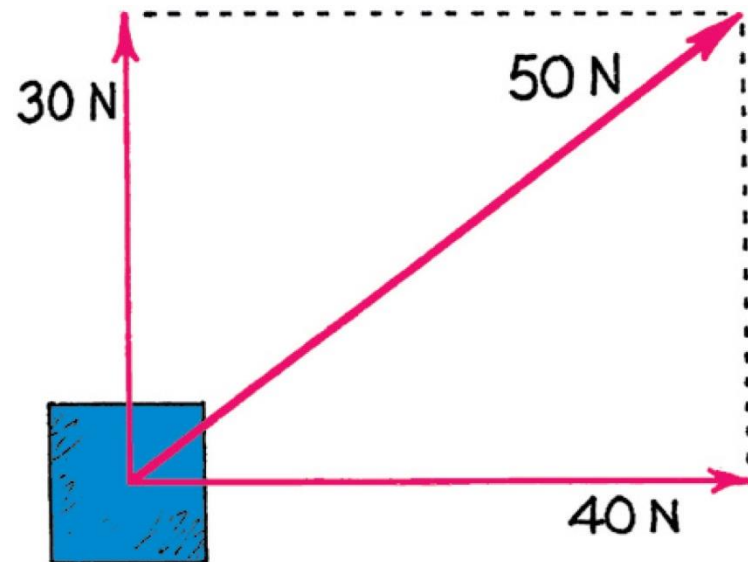


# Vectors

## CHECK YOUR UNDERSTANDING

Referring to the figure, which of the following are true statements?

- A. 50 N is the resultant of the 30- and 40-N vectors.
- B. The 30-N vector can be considered a component of the 50-N vector.
- C. The 40-N vector can be considered a component of the 50-N vector.
- D. All of the above are correct.

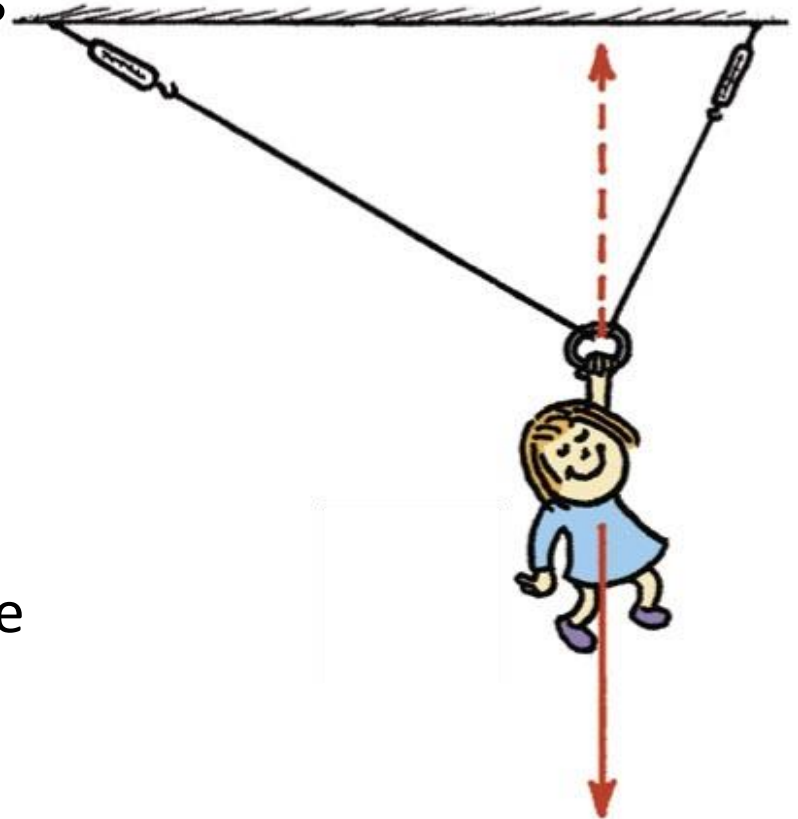




# Vectors

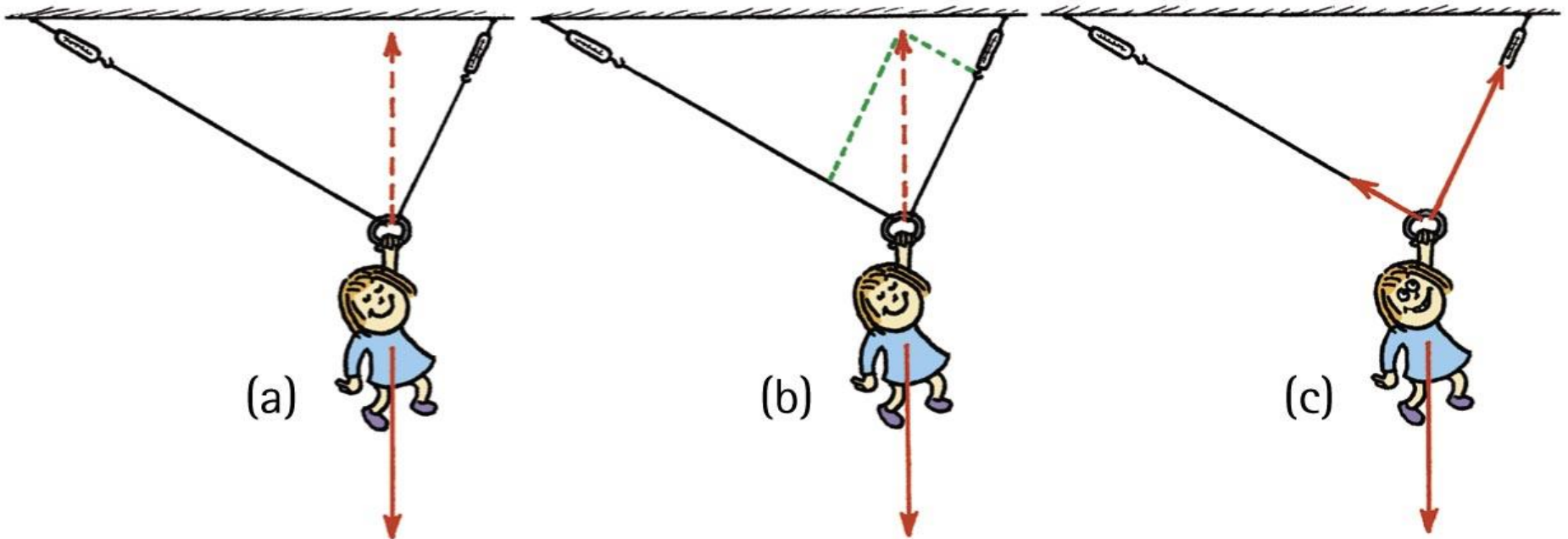
Nellie Newton hangs from a rope as shown.

- Which side has the greater tension?
- There are three forces acting on Nellie:
  - her weight,
  - a tension in the left-hand side of the rope,
  - and a tension in the right-hand side of the rope.



# Vectors

- Because of the different angles, different rope tensions will occur in each side.
- Nellie hangs in equilibrium, so her weight is supported by two rope tensions, adding vectorially to be equal and opposite to her weight.
- The parallelogram rule shows that the tension in the right-hand rope is greater than the tension in the left-hand rope.

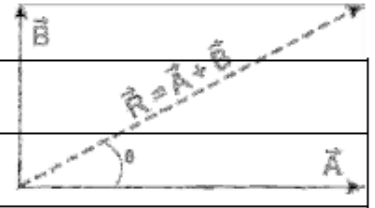


## EXAMPLES:

67. Adding two perpendicular vectors ( $\vec{A}$ ) and ( $\vec{B}$ ) gives a resultant ( $\vec{R}$ ) with magnitude:

A	$R = \sqrt{A^2 + B^2}$
B	$R = A^2 + B^2$

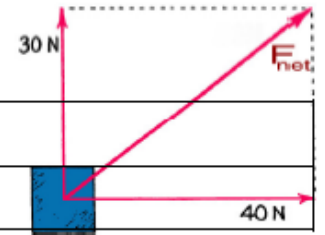
C	$R = \sqrt{A + B}$
D	$R = 1 / \sqrt{A^2 + B^2}$



68. Two perpendicular forces,  $F_1 = 40 \text{ N}$  and  $F_2 = 30 \text{ N}$ , act on a brick. The magnitude of the net force ( $F_{\text{net}}$ ) on the brick is:

A	70 N
B	50 N

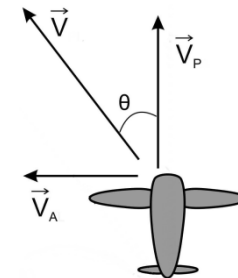
C	0 N
D	10 N



69. If an airplane heading north with speed  $v_p = 400 \text{ km/h}$  faces a westbound wind (رياح نحو الغرب) of speed  $v_A = 300 \text{ km/h}$ , the resultant velocity of the plane is:

- A. 500 km/h, north-west
- C. 500 km/h, north-east

- B. 700 km/h, north-east
- D. 700 km/h, north-west



# Linear Motion

## Definitions

**Speed**  $\equiv$  scalar quantity requiring magnitude only to describe how fast a body is.

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

### INSTANTANEOUS SPEED:

The speed at any instant of time

**Dimensions:** Length/Time ( $[L/T]$ ) ; **Units:** m/s, km/h, ft/min, etc ...

### Approximate Speeds in Different Units

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$$12 \text{ mi/h} = 20 \text{ km/h} = 6 \text{ m/s}$$

$$25 \text{ mi/h} = 40 \text{ km/h} = 11 \text{ m/s}$$

$$37 \text{ mi/h} = 60 \text{ km/h} = 17 \text{ m/s}$$

## AVERAGE SPEED

Average speed is defined as:

$$\text{average speed} = \frac{\text{total distance covered}}{\text{time interval}}$$

### EXAMPLES:

29. A horse gallops (يجري) a distance of 10 kilometers in 30 minutes. Its average speed is:

A	15 km/h
B	20 km/h

C	30 km/h
D	40 km/h

A car travelling from Medina to Mecca (400 Km) in 4 hours, its instantaneous speed at Sasco (midway) is:

A	100 Km/h
B	120 Km/h
C	140 Km/h
D	unknown

# Velocity

**Velocity**  $\equiv$  vector quantity requiring magnitude & direction. It describes *how fast* and in *what direction*.

## CONSTANT VELOCITY:

Means motion in straight line at a constant speed.

## CHANGING VELOCITY:

If *either* the speed or the direction (*or both*) changes, then the velocity changes.



# Acceleration

**Acceleration**  $\equiv$  Is the change in velocity per unit time.

$$\begin{aligned}\text{average acceleration} &= \frac{\text{change in velocity (or speed)}}{\text{elapsed time}} \\ &= \frac{\text{final velocity} - \text{initial velocity}}{\text{time}}\end{aligned}$$

$$a = \frac{\Delta v}{t} = \frac{v_f - v_i}{t}$$

$$\Delta v = at$$

**Dimensions:** Length/Time<sup>2</sup> ([L]/[T<sup>2</sup>]) ; **Units:** m/s<sup>2</sup>, km/h<sup>2</sup>, ft/min<sup>2</sup>, etc ...

## EXAMPLE:

A dragster starts from rest (velocity = 0 ft/s) and attains a speed of 150 ft/s in 10.0 s. Find its acceleration.

### Data:

$$\Delta v = 150 \text{ ft/s} - 0 \text{ ft/s} = 150 \text{ ft/s}$$

$$t = 10.0 \text{ s}$$

$$a = ?$$

### Basic Equation:

$$\Delta v = at$$

### Working Equation:

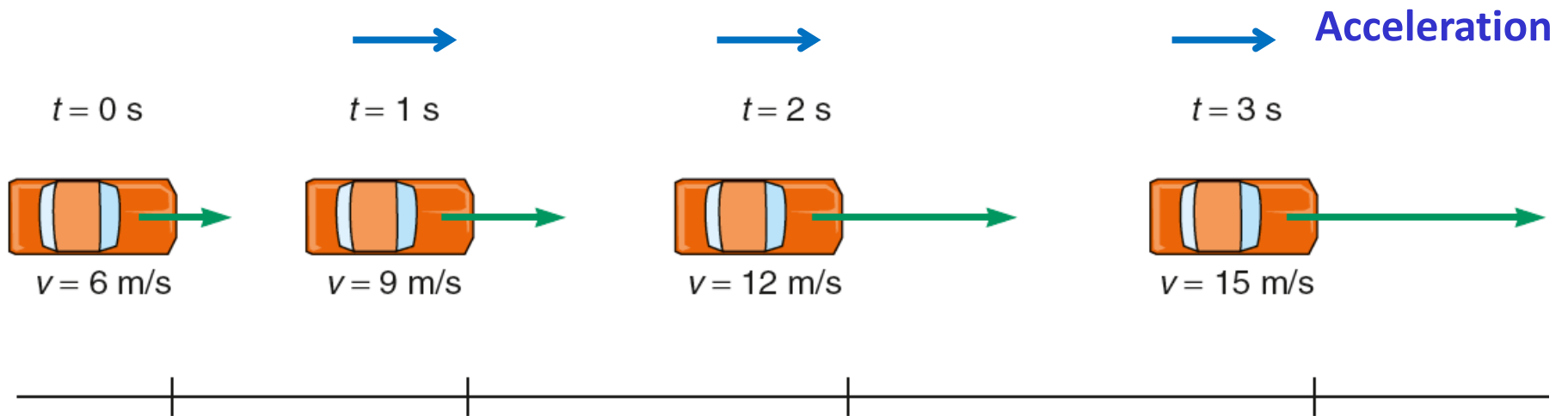
$$a = \frac{\Delta v}{t}$$

### Substitution:

$$\begin{aligned} a &= \frac{150 \text{ ft/s}}{10.0 \text{ s}} \\ &= 15.0 \frac{\text{ft/s}}{\text{s}} \text{ or } 15.0 \text{ feet per second per second} \end{aligned}$$



# Acceleration



This car is speeding up with a constant acceleration. Note how the distance covered and the velocity change during each time interval.

## EXAMPLE:

A car accelerates from 45 km/h to 80 km/h in 3.00 s. Find its acceleration (in m/s<sup>2</sup>).

### Data:

$$\Delta v = 80 \text{ km/h} - 45 \text{ km/h} = 35 \text{ km/h}$$

$$t = 3.00 \text{ s}$$

$$a = ?$$

### Basic Equation:

$$\Delta v = at$$

### Working Equation:

$$a = \frac{\Delta v}{t}$$

### Substitution:

$$\begin{aligned} a &= \frac{35 \text{ km/h}}{3.00 \text{ s}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ h}}{3600 \text{ s}} \\ &= 3.2 \text{ m/s}^2 \end{aligned}$$

Note the use of the conversion factors to change the units km/h/s to m/s<sup>2</sup>.

## EXAMPLE:

A plane accelerates at  $8.5 \text{ m/s}^2$  for  $4.5 \text{ s}$ . Find its increase in speed (in  $\text{m/s}$ ).

### Data:

$$a = 8.5 \text{ m/s}^2$$

$$t = 4.5 \text{ s}$$

$$\Delta v = ?$$

### Basic Equation:

$$\Delta v = at$$

**Working Equation:** Same

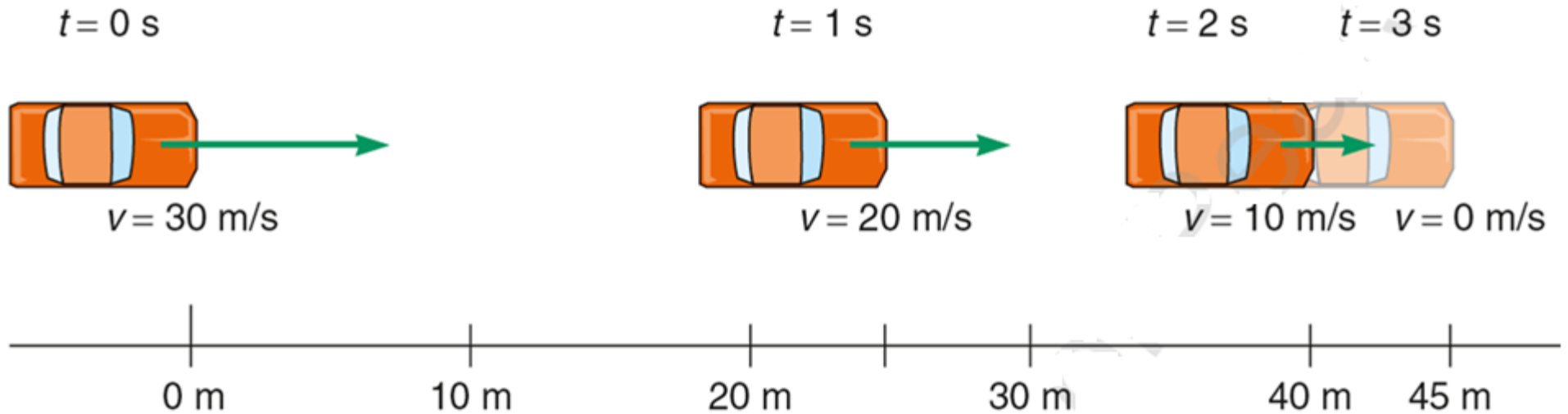
### Substitution:

$$\begin{aligned}\Delta v &= (8.5 \text{ m/s}^2)(4.5 \text{ s}) \\ &= 38 \text{ m/s}\end{aligned}$$

$$\boxed{\frac{\text{m}}{\text{s}^2} \times \text{s} = \frac{\text{m}}{\text{s}}}$$

# Deceleration

Deceleration ←



This car is slowing down with a constant acceleration of  $-10 \text{ m/s}^2$ . Note how the distance covered and the velocity change during each unit of time interval.

## EXAMPLE:

A driver steps off the gas pedal and coasts at rate of  $-3 \text{ m/s}^2$  for  $5.00 \text{ s}$ . Find the driver's new speed if originally travelling at a velocity of  $20.00 \text{ m/s}$ .

### Data:

$$a = -3.00 \text{ m/s}^2$$

$$t = 5.00 \text{ s}$$

$$v_i = 20.0 \text{ m/s}$$

The negative acceleration indicates that the acceleration is in opposite direction of the velocity; that is, the object is slowing down

### Basic Equation:

$$\Delta v = at$$

$$v_f - v_i = at$$

### Working Equation:

$$v_f = v_i + at$$

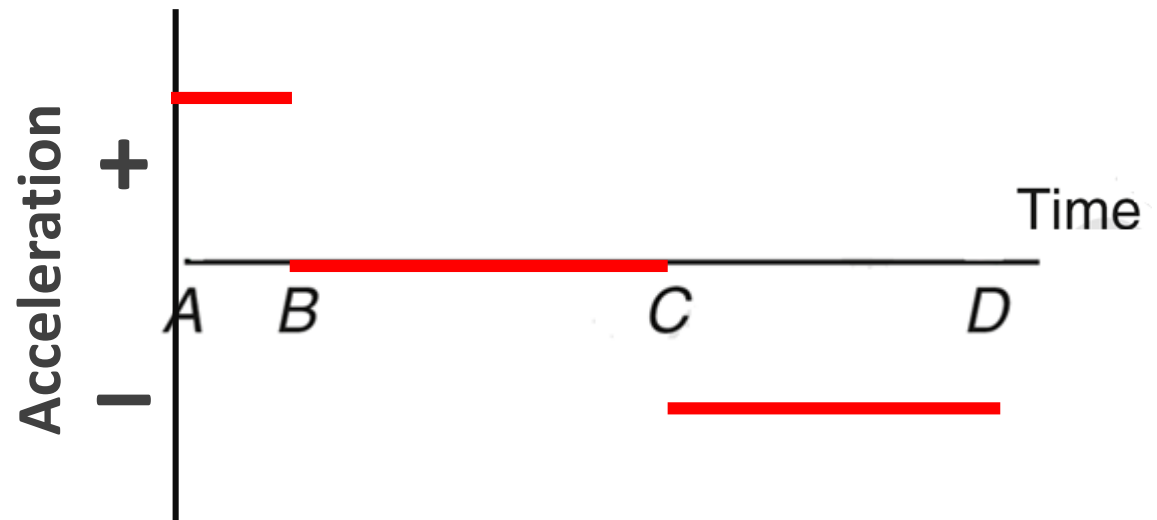
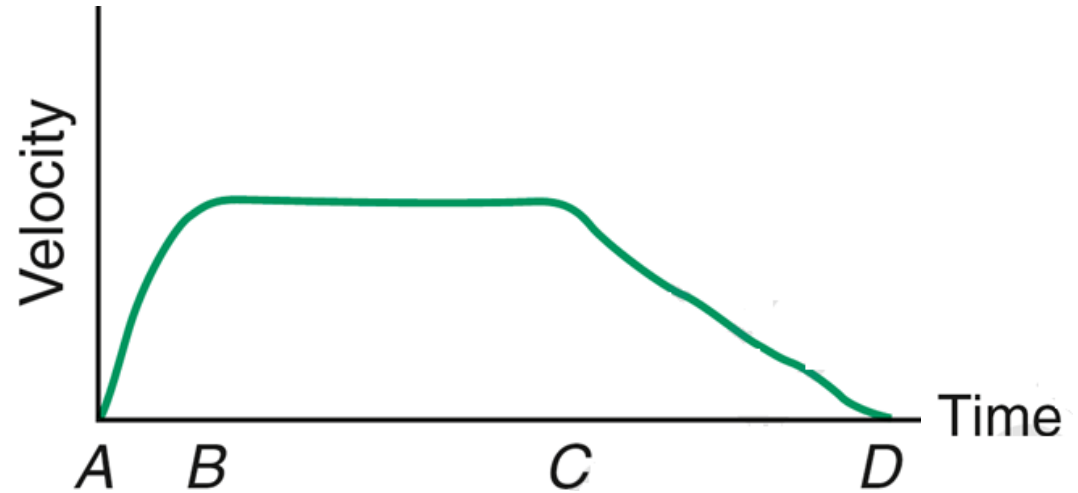
### Substitution:

$$\begin{aligned} v_f &= 20.0 \text{ m/s} + (-3.00 \text{ m/s}^2)(5.00 \text{ s}) \\ &= 5.0 \text{ m/s} \end{aligned}$$

# Acceleration as a vector: *geometrical representation*



Motion of a high-speed train going from one station to another. When the velocity increases, the acceleration is positive. When the velocity is constant, the acceleration is zero. When the velocity decreases, the acceleration is negative.



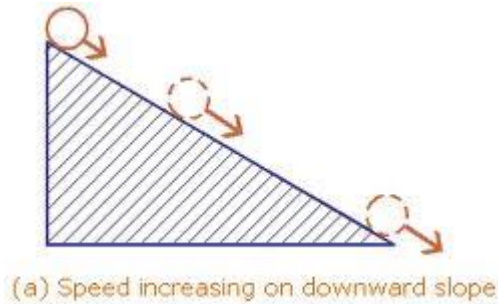
# Uniformly accelerated motion and free fall

Characterized by the constant acceleration  $\Rightarrow$  its direction & magnitude are unchanging.

## EXAMPLES:



Motion in straight lines



Rolling objects



Falling objects

# ACCELERATED MOTION:

Equations for motion in straight line with constant acceleration:

$$\begin{array}{lll} 1. v_{\text{avg}} = \frac{v_f + v_i}{2} & 3. v_f = v_i + at & 5. s = \frac{1}{2}(v_f + v_i)t \\ 2. a = \frac{v_f - v_i}{t} & 4. s = v_i t + \frac{1}{2}at^2 & 6. 2as = v_f^2 - v_i^2 \end{array}$$

where	$s$ = displacement	$v_{\text{avg}}$ = average velocity
	$v_f$ = final velocity	$a$ = constant acceleration
	$v_i$ = initial velocity	$t$ = time

**Displacement** is a vector pointing from the initial to the final position and with magnitude equals the shortest distance between the initial and final position



## EXAMPLE:

The average velocity of a rolling freight car is 2.00 m/s. How long does it take for the car to role 15.0 m?

### Data:

$$s = 15.0 \text{ m}$$

$$v_{\text{avg}} = 2.00 \text{ m/s}$$

$$t = ?$$

### Basic Equation:

$$s = v_{\text{avg}} t$$

### Working Equation:

$$t = \frac{s}{v_{\text{avg}}}$$

### Substitution:

$$\begin{aligned} t &= \frac{15.0 \text{ m}}{2.00 \text{ m/s}} \\ &= 7.50 \text{ s} \end{aligned}$$

$$\frac{\text{m}}{\text{m/s}} = \text{m} \div \frac{\text{m}}{\text{s}} = \text{m} \cdot \frac{\text{s}}{\text{m}} = \text{s}$$

## EXAMPLE:

A train slowing to a stop has an average acceleration of  $-3.00 \text{ m/s}^2$ . if its initial velocity is  $30.0 \text{ m/s}$ , how far does it travel in  $4.00 \text{ s}$ ?

Solution:

Data:

$$a = -3.00 \text{ m/s}^2$$

$$v_i = 30.0 \text{ m/s}$$

$$t = 4.00 \text{ s}$$

$$S = ?$$

Basic Equation:

$$S = v_i t + \frac{1}{2} a t^2$$






Substitution:

$$\begin{aligned} S &= (30.0) \times (4.00) + \frac{1}{2} \times (-3.00) \times (4.00) \\ &= 120 - 24.0 \text{ m} \\ &= 96 \text{ m} \end{aligned}$$

# Free Fall

- When acceleration  $a = g = 9.8 \text{ m/s}^2$
- Acceleration is  $g$  when *air resistance negligible*.
- Acceleration depends on force (weight) and inertia.

A ball falls with constant acceleration  $a = g = 9.82 \text{ m/s}^2$  with the speed and the distance traveled calculated at the given times. Because the ball was dropped,  $v_i = 0$  and the formulas for  $v_f$  and  $s$  are shown simplified. Note how the velocity and the distance traveled increase during each successive time interval.

<u>Time</u>	<u>Distance Traveled</u>		<u>Speed</u>
$t = 0$	$s = \frac{1}{2}at^2$ 0 m		$v_f = at$ 0 m/s
$t = 1.00 \text{ s}$	$s = \frac{1}{2}(9.80 \text{ m/s}^2)(1.00 \text{ s})^2$ $s = 4.90 \text{ m}$		$v_f = (9.80 \text{ m/s}^2)(1.00 \text{ s})$ $v_f = 9.80 \text{ m/s}$
$t = 2.00 \text{ s}$	$s = \frac{1}{2}(9.80 \text{ m/s}^2)(2.00 \text{ s})^2$ $s = 19.6 \text{ m}$		$v_f = (9.80 \text{ m/s}^2)(2.00 \text{ s})$ $v_f = 19.6 \text{ m/s}$
$t = 3.00 \text{ s}$	$s = \frac{1}{2}(9.80 \text{ m/s}^2)(3.00 \text{ s})^2$ $s = 44.1 \text{ m}$		$v_f = (9.80 \text{ m/s}^2)(3.00 \text{ s})$ $v_f = 29.4 \text{ m/s}$
$t = 4.00 \text{ s}$	$s = \frac{1}{2}(9.80 \text{ m/s}^2)(4.00 \text{ s})^2$ $s = 78.4 \text{ m}$		$v_f = (9.80 \text{ m/s}^2)(4.00 \text{ s})$ $v_f = 39.2 \text{ m/s}$

# Non-Free Fall

When acceleration of fall is less than  $g$ , non-free fall

- occurs when air resistance is non-negligible.
- depends on two things:
  - speed and
  - frontal surface area.

## Terminal speed

- occurs when acceleration terminates (when air resistance equals weight and net force is zero).

## Terminal velocity

- same as terminal speed, with direction implied or specified.

## EXAMPLE:

A rock is thrown straight down from a cliff with an initial velocity of 10.0 ft/s. Its final velocity when it strikes the water below is 31.0 ft/s. The acceleration due to gravity is 32.2 ft/s<sup>2</sup>. How long is the rock in flight?

### Data:

$$v_i = 10.0 \text{ ft/s}$$

$$a = 32.2 \text{ ft/s}^2$$

$$v_f = 31.0 \text{ ft/s}$$

$$t = ?$$

Note the importance of listing all the data as an aid to finding the basic equation.

### Basic Equation:

$$v_f = v_i + at \quad \text{or} \quad a = \frac{v_f - v_i}{t} \quad (\text{two forms of the same equation})$$

### Working Equation:

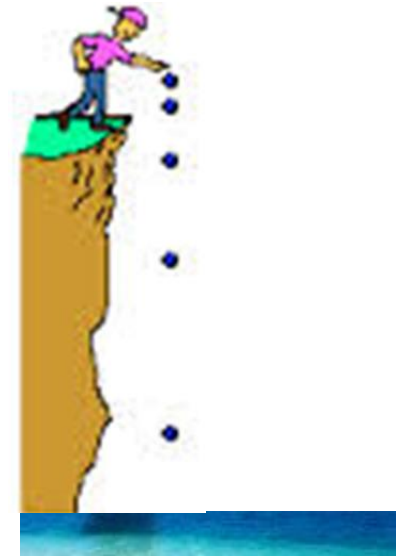
$$t = \frac{v_f - v_i}{a}$$

### Substitution:

$$t = \frac{31.0 \text{ ft/s} - 10.0 \text{ ft/s}}{32.2 \text{ ft/s}^2}$$

$$= \frac{21.0 \text{ ft/s}}{32.2 \text{ ft/s}^2}$$

$$= 0.655 \text{ s}$$



$$\frac{\text{ft/s}}{\text{ft/s}^2} = \frac{\text{ft}}{\text{s}} \div \frac{\text{ft}}{\text{s}^2} = \frac{\text{ft}}{\text{s}} \cdot \frac{\text{s}^2}{\text{ft}} = \text{s}$$

# Upward motion

When an object is thrown vertically upward, its speed is uniformly decreased by the force of gravity until it stops for an instant at its peak before falling back to the ground.

## EXAMPLE:

A ball is thrown vertically upward with initial speed 1 m/s, determine the time for it to reach the highest altitude.

$$v_f = v_i + a t$$

$$v_f = 0 \text{ m/s}$$

$$v_i = 1 \text{ m/s}$$

$$a = -g = -9.8 \text{ m/s}^2$$

$$t = ?$$

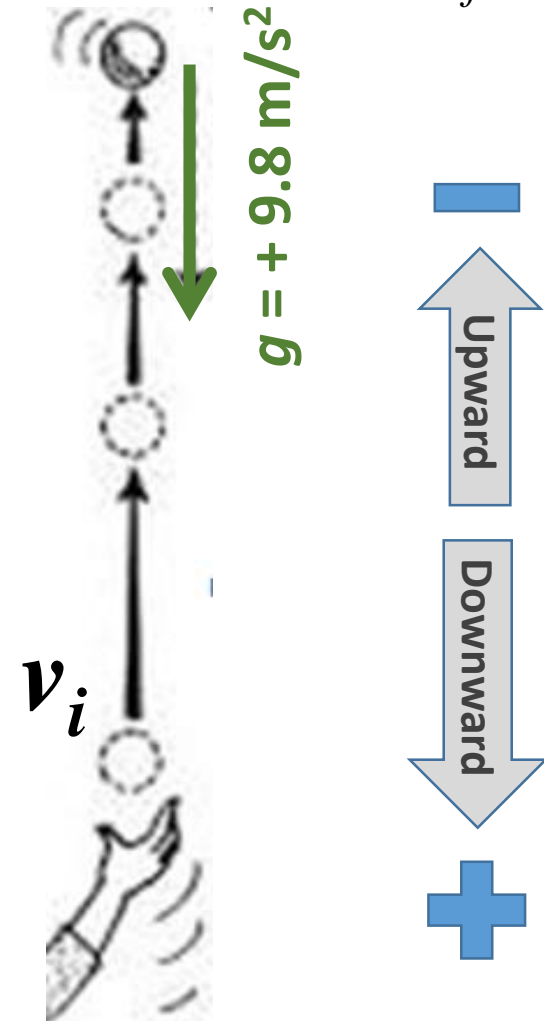
$$t = [v_f - v_i] / a$$

$$= [0 - 1 \text{ m/s}] / (-9.8 \text{ m/s}^2)$$

$$= 0.1 \text{ s}$$

Final velocity at top of the path,  $v_f = 0$

For upward motion take  $g = -9.8 \text{ m/s}^2$  (negative)  
and for downward motion take  $g = 9.8 \text{ m/s}^2$  (positive)



# Laws of Motion

The force:

- Is a vector (has magnitude and direction).
- Is any push or pull.
- Tends to change the state of motion of an object.
- Tends to produce acceleration in the direction of its application.
  - But, for instance, opposite and equal forces cancel each other, resulting in zero acceleration
- SI unit of force is Newton (N)
  - Conversion factor SI  $\leftrightarrow$  British system:  $4.45 \text{ N} = 1 \text{ lb}$



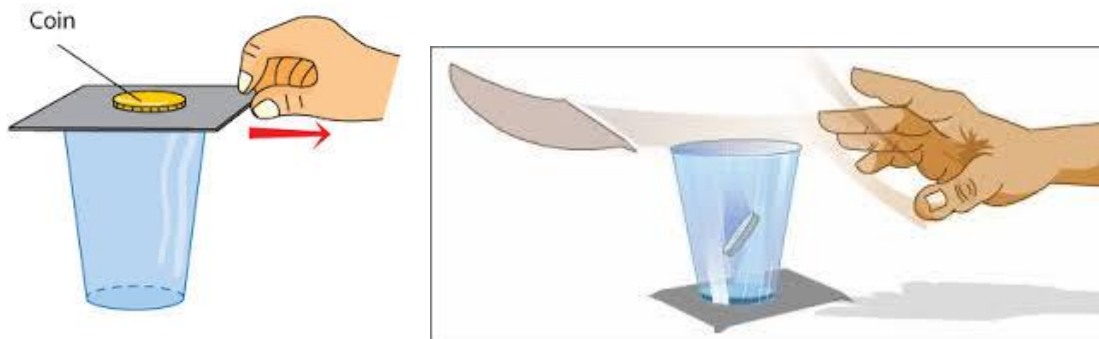
**FIGURE 2.13**

The pushing force and the frictional force cancel each other out, resulting in zero acceleration.

# First Law (Law of Inertia)

Inertia:

- Inertia is a property of matter to resist changes in motion.
- depends on the amount of matter in an object (its *mass*).
- is related to the Newton first law of motion which is also called the *law of inertia*: **a body at state of rest (speed = 0) or motion with *constant velocity* (constant speed in straight line) tends to remain at this state unless acted upon by an unbalanced force.**



The coin tends to remain at rest.

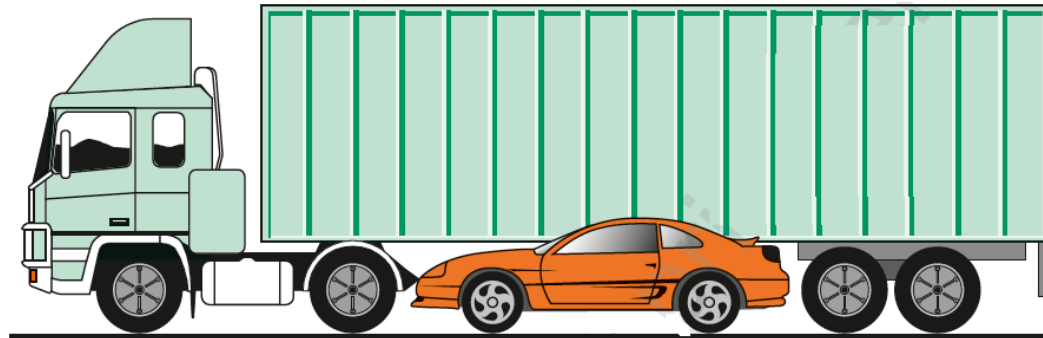


The car tends to continue moving.



# Mass:

- is a measure of the inertia:
  - *The greater the mass of a body the greater is its resistance to motion*



A larger body (in mass) has a greater resistance to a change in its motion than does a smaller one.

- SI unit is: Kilogram (kg):
  - $1 \text{ kg} = 0.0685 \text{ slug}$

# Second Law (Law of acceleration)

Newton second law (the law of acceleration):

$$\mathbf{F = m a}$$

$F \equiv$  the total force.

$m \equiv$  mass.

$a \equiv$  acceleration.

$\Rightarrow$  SI unit of force = Newton (N)

$\Rightarrow$  From Newton 2<sup>nd</sup> law:  $1 \text{ N} = 1 \text{ kg m/s}^2$

or in British system:  $1 \text{ lb} = 1 \text{ slug ft/s}^2$ .

In other metric system:  $1 \text{ dyne} = 1 \text{ g cm/s}^2$ .

## EXAMPLE:

What force is necessary to produce an acceleration of  $6.00 \text{ m/s}^2$  on a mass of  $5.00 \text{ kg}$ ?

### Data:

$$m = 5.00 \text{ kg}$$

$$a = 6.00 \text{ m/s}^2$$

$$F = ?$$

### Basic Equation:

$$F = ma$$

**Working Equation:** Same

### Substitution:

$$\begin{aligned} F &= (5.00 \text{ kg})(6.00 \text{ m/s}^2) \\ &= 30.0 \text{ kg m/s}^2 \\ &= 30.0 \text{ N} \quad (1 \text{ N} = 1 \text{ kg m/s}^2) \end{aligned}$$

## EXAMPLE:

What force is necessary to produce an acceleration of  $2.00 \text{ ft/s}^2$  on a mass of  $3.00$  slugs?

### Data:

$$m = 3.00 \text{ slugs}$$

$$a = 2.00 \text{ ft/s}^2$$

$$F = ?$$

### Basic Equation:

$$F = ma$$

**Working Equation:** Same

### Substitution:

$$\begin{aligned} F &= (3.00 \text{ slugs})(2.00 \text{ ft/s}^2) \\ &= 6.00 \text{ slug ft/s}^2 \\ &= 6.00 \text{ lb} \quad (1 \text{ lb} = 1 \text{ slug ft/s}^2) \end{aligned}$$



# Gravity and weight

## Weight:

- The force on an object due to gravity
- Scientific unit of force is the newton (N)
- Free fall  $\Rightarrow$  acceleration due to gravity =  $g = 9.8 \text{ m/s}^2$ . ( $g = 32.2 \text{ ft/s}^2$ , British system).
- Newton second law:  $F = m a$ , for free fall,  $a = g$ ,  $F = F_w \Rightarrow$

$$F_w = mg$$

where  $F_w =$  weight

$m =$  mass

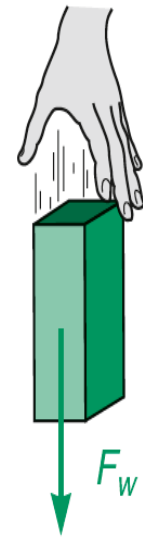
$g =$  acceleration due to gravity

$g = 9.80 \text{ m/s}^2$  (earth, metric)

$g = 32.2 \text{ ft/s}^2$  (earth, U.S.)



(a) The upward force of the hand equals the downward force of the weight.



(b) The downward force of the weight is now greater.

## EXAMPLE:

Find the weight of 5.00 kg.

### Data:

$$m = 5.00 \text{ kg}$$

$$g = 9.80 \text{ m/s}^2$$

$$F_w = ?$$

### Basic Equation:

$$F_w = mg$$

### Working Equation: Same

### Substitution:

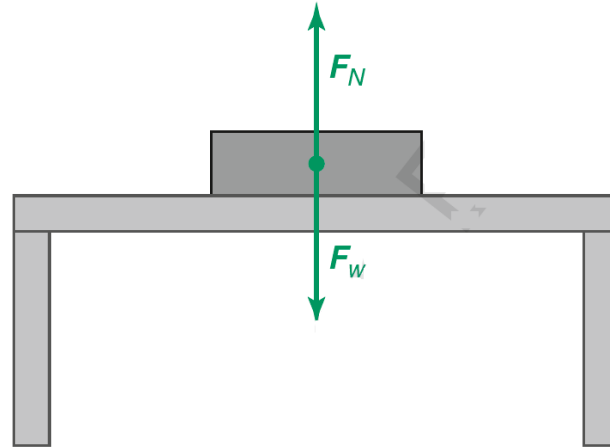
$$F_w = (5.00 \text{ kg})(9.80 \text{ m/s}^2)$$

$$= 49.0 \text{ kg m/s}^2$$

$$= 49.0 \text{ N} \quad (1 \text{ N} = 1 \text{ kg m/s}^2)$$

# WEIGHT VERSUS NORMAL FORCE

When an object is in contact with a surface, a force is exerted on that object by the surface. This force, called a **normal force**, is *perpendicular to the contact surface*.



Magnitude of  $F_N$  = magnitude of  $F_w$

## EXAMPLE:

14. The normal force on a 2-kg book lying on a level table is:

A	1 N
B	2 N

C	10 N
D	20 N

# MASS VERSUS WEIGHT

EXAMPLE: Astronaut mass =  $m = 75.0$  kg



*Near the earth's surface:*

- The acceleration due to gravity =  $g = 9.80$  m/s<sup>2</sup>

- **The weight =**

$$F_w = m g = (75.0 \text{ kg}) (9.80 \text{ m/s}^2) = \mathbf{735 \text{ N.}}$$



*Near the moon's surface:*

- The acceleration due to gravity =  $g = 1.63$  m/s<sup>2</sup>

- **The weight =**

$$F_w = m g = (75.0 \text{ kg}) (1.63 \text{ m/s}^2) = \mathbf{122 \text{ N.}}$$

*So mass remains the same, but the weight varies according to the gravitational pull  $\Rightarrow$  mass is a fundamental quantity.*



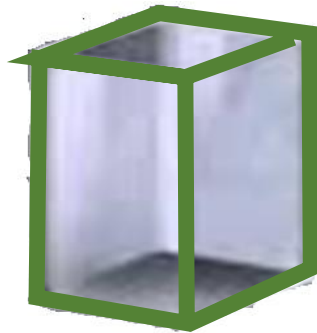
# MASS VERSUS VOLUME

## *Mass:*

- The amount of inertia or material in an object.
- Units: **kg**

## *Volume:*

- Measures the space occupied by an object.
- Units: **[Length]<sup>3</sup> ≡ m<sup>3</sup>, cm<sup>3</sup>, Liter (L), ft<sup>3</sup>, ...**



Air

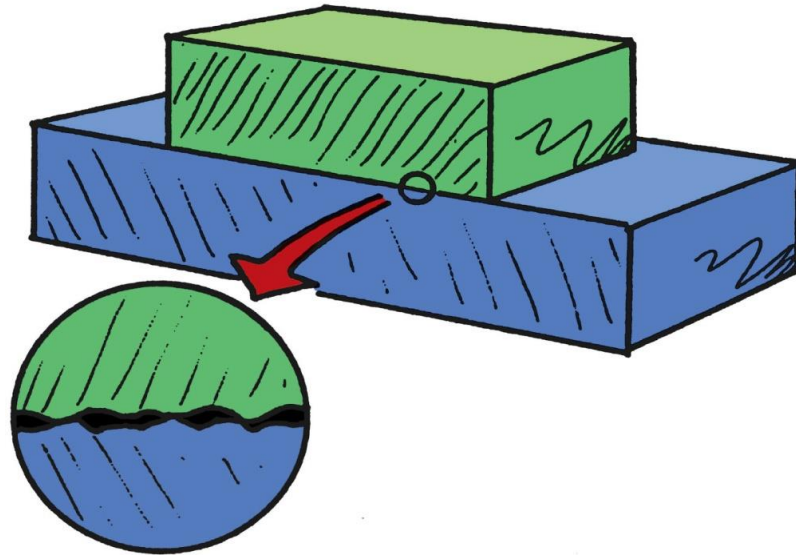


Lead

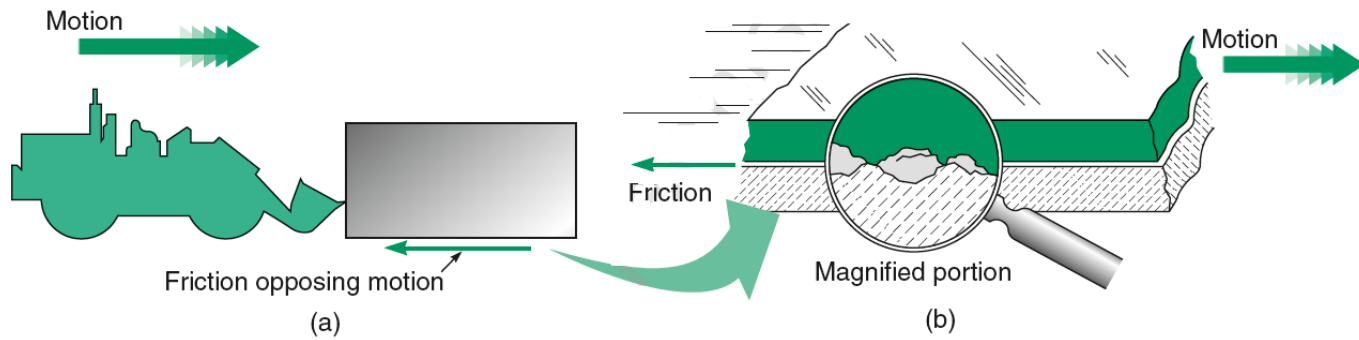
Same *volumes* but different *masses*

# Friction

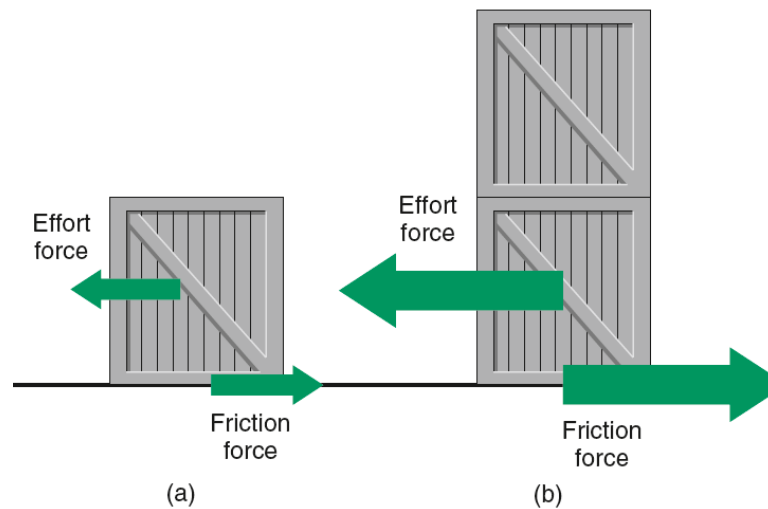
- is a force that resists the relative motion of two objects in contact.
- depends on the kinds of material and how much they are pressed together.
- is due to tiny surface bumps and to “stickiness” of the atoms on a material’s surface.



Example: Friction between a crate on a smooth wooden floor is less than that on a rough floor.



Friction resists motion of objects in contact with each other.



Friction increases as the force between the surfaces increases.

## Friction equation

The characteristics of friction can be described by the following equation:

$$F_f = \mu F_N$$

where  $F_f$  = frictional force

$F_N$  = normal force (force perpendicular to the contact surface)

$\mu$  = coefficient of friction

Higher  $\mu \Rightarrow$  two rough surfaces; smaller  $\mu \Rightarrow$  two smooth surfaces (*not too smooth*)

- *Friction is a force that always acts parallel to the surface in contact and opposite to the direction of motion.*
- *Friction increases as the force between the surfaces increases.*

# Friction

Static friction:

The two surfaces are at rest relative to each other

$$= \mu_s F_N$$

*kinetic friction:*

The two surfaces are in relative motion

$$= \mu_k F_N$$

## Coefficients of Friction ( $\mu$ )

Material	Static Friction	Kinetic Friction
Hardwood on hardwood	0.40	0.25
Steel on concrete	<b>0.7</b>	0.30
Aluminum on aluminum	1.9	<b>1.4</b>
Rubber on dry concrete	2.0	1.0
Rubber on wet concrete	1.5	0.97

$\Rightarrow$  Static friction > Kinetic friction

# Friction Reduction

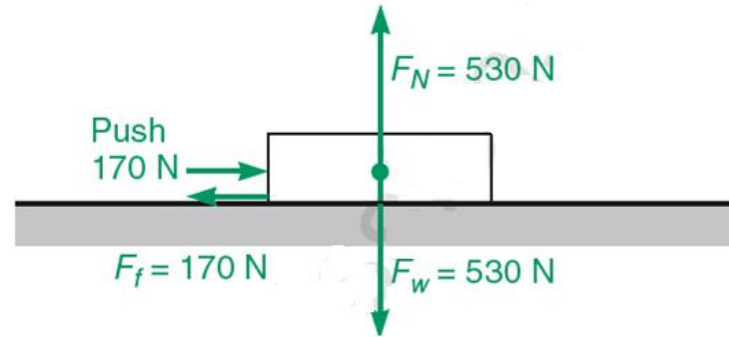
In general, to reduce kinetic friction:

1. Use smoother surfaces.
2. Use lubrication to provide a thin film between surfaces.
3. Use Teflon to greatly reduce friction between surfaces when an oil lubricant is not desirable, such as in electric motors.
4. Substitute rolling friction for sliding friction.

## EXAMPLE

A force of 170 N is needed to keep a 530-N wooden box sliding on a wooden floor. What is the coefficient of kinetic friction?

**Sketch:**



**Data:**

$$F_f = 170 \text{ N}$$

$$F_N = 530 \text{ N}$$

$$\mu = ?$$

**Basic Equation:**

$$F_f = \mu F_N$$

**Working Equation:**

$$\mu = \frac{F_f}{F_N}$$

**Substitution:**

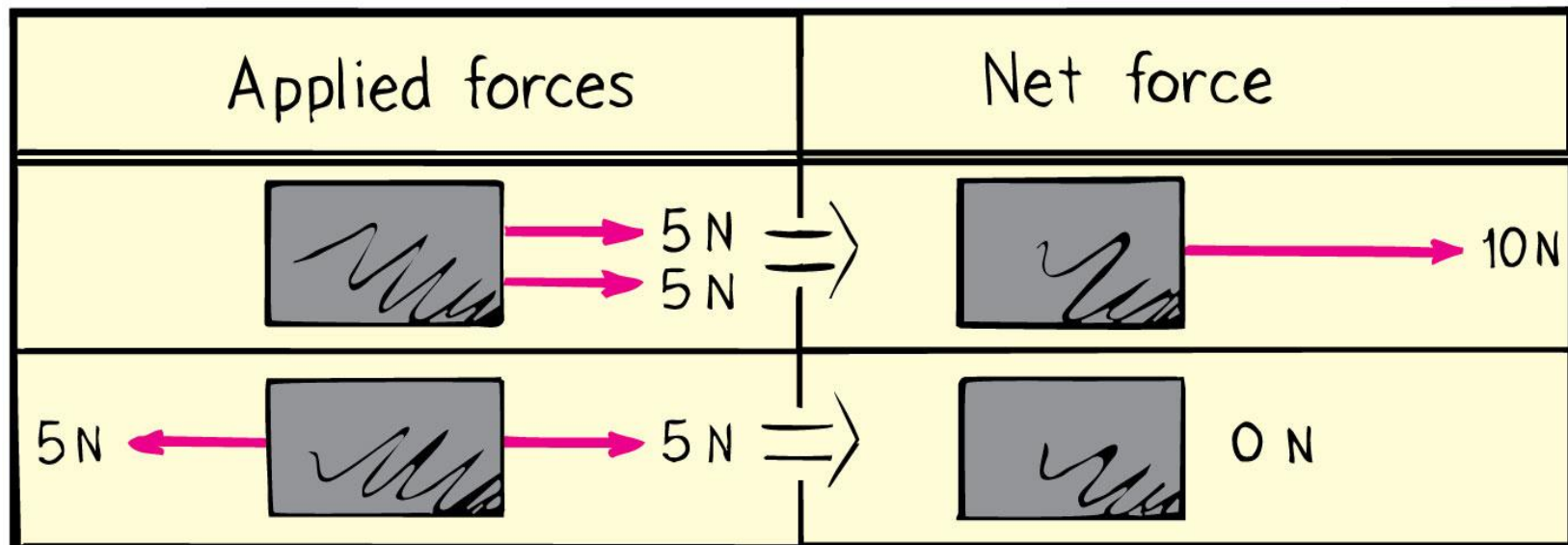
$$\begin{aligned}\mu &= \frac{170 \text{ N}}{530 \text{ N}} \\ &= 0.32\end{aligned}$$

Note that  $\mu$  does not have a unit because the force units always cancel.

## ■ Total Forces in One Dimension

The **total**, or net, **force** acting on an object is the *resultant* of all the forces.

**Example:** If you pull on a box with 10 N and a friend pulls oppositely with 5 N, the net force is 5 N in the direction you are pulling.

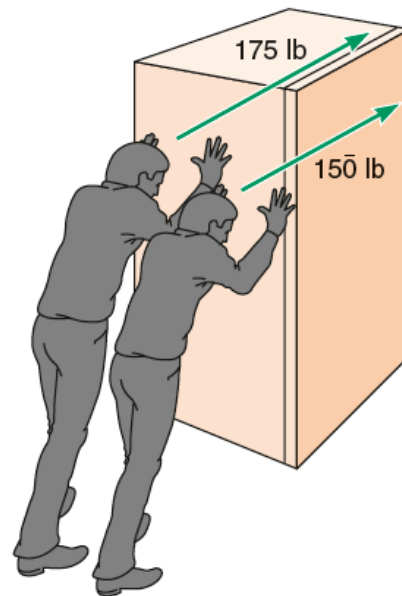




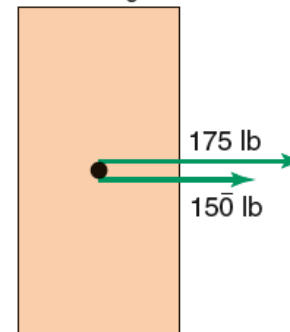
## EXAMPLE

Two workers push in the same direction (to the right) on a crate. The force exerted by one worker is  $150\bar{\text{lb}}$ . The force exerted by the other is  $175\text{ lb}$ . Find the net force exerted.

Sketch:



Force diagram



Both forces act in the same direction, so the total force is the sum of the two.

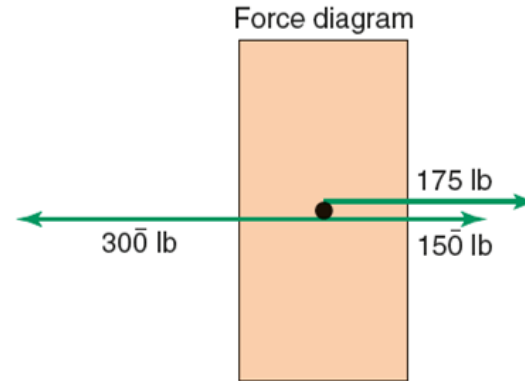
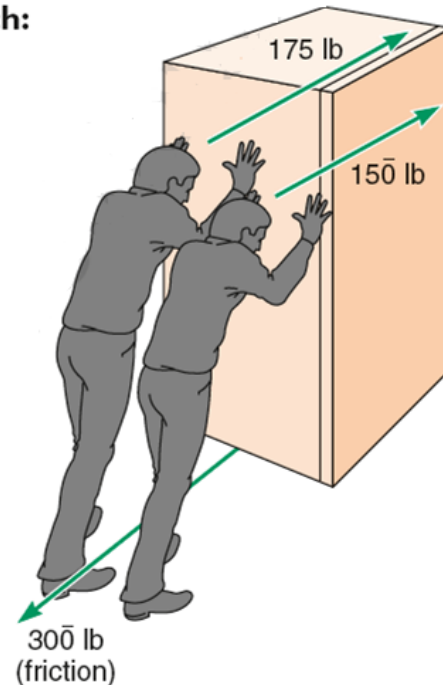
**Note:** The Greek letter  $\Sigma$  (sigma) means "sum of."

$$\begin{aligned}\Sigma \mathbf{F} &= 175\text{ lb} + 150\bar{\text{lb}} \\ &= 325\text{ lb to the right}\end{aligned}$$

## EXAMPLE:

The same two workers push the crate to the right, and the motion is opposed by a static frictional force of  $300\text{ lb}$ . Find the net force.

Sketch:



The workers push in one direction and static friction pushes in the opposite direction, so we add the forces exerted by the workers and subtract the frictional force.

$$\begin{aligned}\Sigma \mathbf{F} &= 175\text{ lb} + 150\text{ lb} - 300\text{ lb} \\ &= 25\text{ lb to the right}\end{aligned}$$

## EXAMPLE:

The crate has a mass of 5.00 slugs. What is its acceleration when the workers are pushing against the frictional force?

**Data:**

$$F = \Sigma \mathbf{F} = 175 \text{ lb} + 150 \text{ lb} - 300 \text{ lb} = 25 \text{ lb to the right}$$

$$m = 5.00 \text{ slugs}$$

$$a = ?$$

**Basic Equation:**

$$F = ma$$

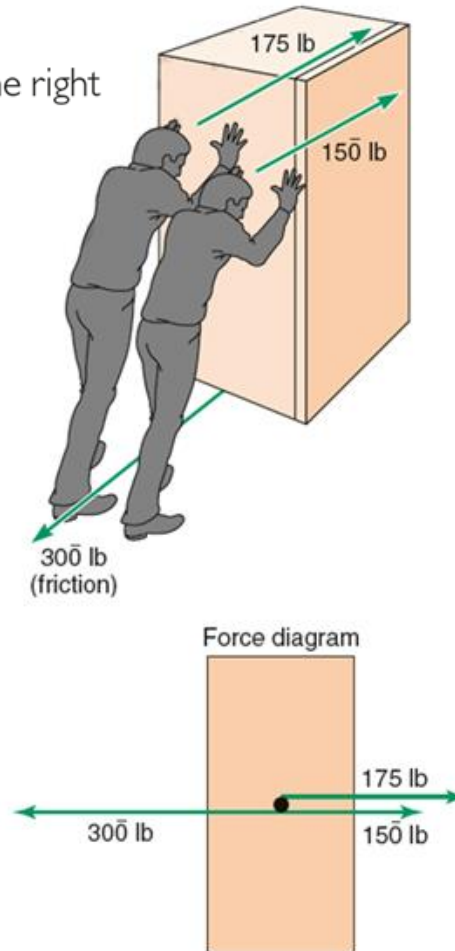
**Working Equation:**

$$a = \frac{F}{m}$$

**Substitution:**

$$\begin{aligned} a &= \frac{25 \text{ lb}}{5.00 \text{ slugs}} \\ &= 5.0 \frac{\text{lb}}{\text{slugs}} \times \frac{1 \text{ slug ft/s}^2}{1 \text{ lb}} \\ &= 5.0 \text{ ft/s}^2 \end{aligned}$$

**Note:** We use a conversion factor to obtain acceleration units.



## EXAMPLE

---

Two workers push in the same direction on a large pallet. The force exerted by one worker is 645 N. The force exerted by the other worker is 755 N. The motion is opposed by a frictional force of 1175 N. Find the net force.

$$\begin{aligned}\Sigma \mathbf{F} &= 645 \text{ N} + 755 \text{ N} - 1175 \text{ N} \\ &= 225 \text{ N}\end{aligned}$$

# Third Law of Motion (Action and Reaction)

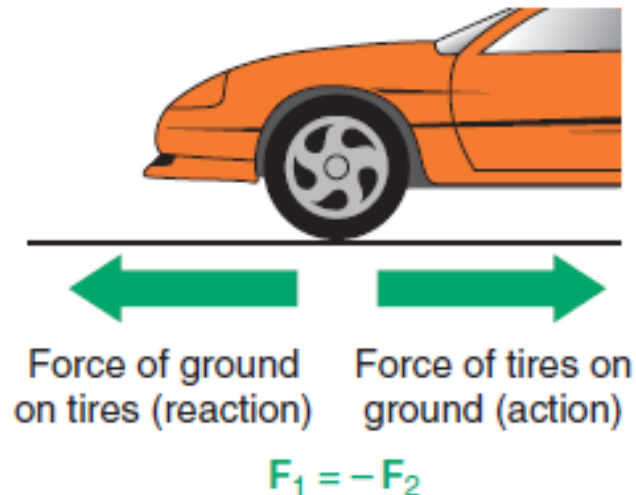
Action and reaction forces

- one force is called the action force; the other force is called the reaction force.
- are co-pairs of a single interaction.
- neither force exists without the other.
- are equal in strength and opposite in direction.
- **always act on *different* objects.**

# Law of Action and Reaction

The third law of motion, the *law of action and reaction*, can be stated as follows: **To every action there is always an opposed equal reaction.**

**Example:** Tires of car push back against the road while the road pushes the tires forward.



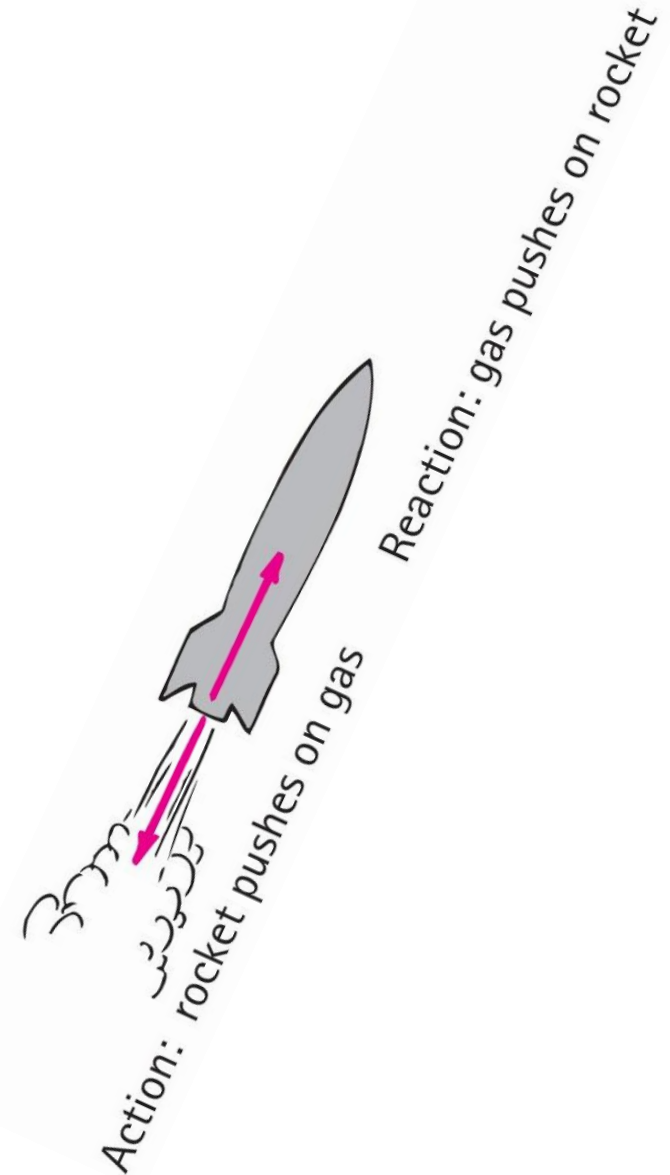
# Newton's Third Law of Motion

Simple rule to identify action and reaction

- Identify the interaction—one thing interacts with another
  - Action: Object A exerts a force on object B.
  - Reaction: Object B exerts a force on object A.

Example: Action—rocket (object A) exerts force on gas (object B).

Reaction—gas (object B) exerts force on rocket (object A).



# Work

(when the angle between the force and the direction of motion i.e.  $\theta = 0^\circ$ )

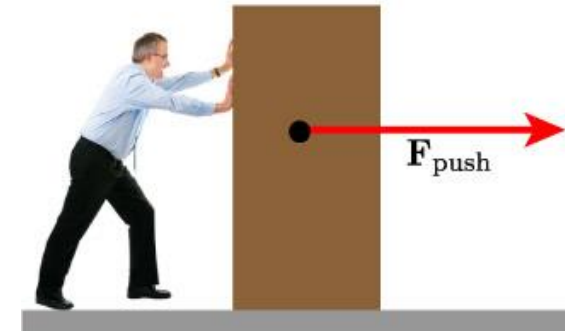
*Work is the product of the force in the direction of the motion and the displacement.*

$$W = Fs$$

where  $W =$  work

$F =$  force applied *in the direction of the motion*

$s =$  displacement



Work is a transferred energy during the motion (displacement).

Two things occur whenever work is done:

- application of force
- movement of something by that force



# Work

## CHECK YOUR NEIGHBOR

If you push against a stationary brick wall for several minutes, you do no work

- A. on the wall.
- B. at all.
- C. Both of the above.
- D. None of the above.



*Work is directly proportional to both applied force and displacement*

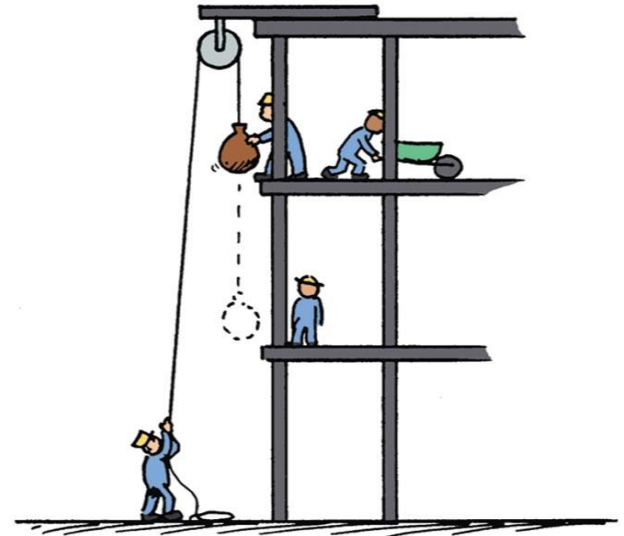
Examples:

- Twice as much work is done in lifting 2 loads 1 story high versus lifting 1 load the same vertical distance.

Reason: force needed to lift twice the load is twice as much.

- Twice as much work is done in lifting a load 2 stories instead of 1 story.

Reason: distance is twice as great.



# Units of work

Since the work = force x displacement, the unit then:  
= newton x meter (N m)

SI system:

$$1 \text{ N m} = 1 \text{ joule} = 1 \text{ J}$$

British system (or U.S. system)

$$\begin{aligned} \text{work} &= \text{force} \times \text{displacement} \\ &= \text{pounds} \times \text{feet} = \text{ft lb} \end{aligned}$$

## EXAMPLE

Find the amount of work done by a worker lifting 225 N of bricks to a height of 1.75 m as shown in Figure 2.28.

### Data:

$$F = 225 \text{ N}$$

$$s = 1.75 \text{ m}$$

$$W = ?$$

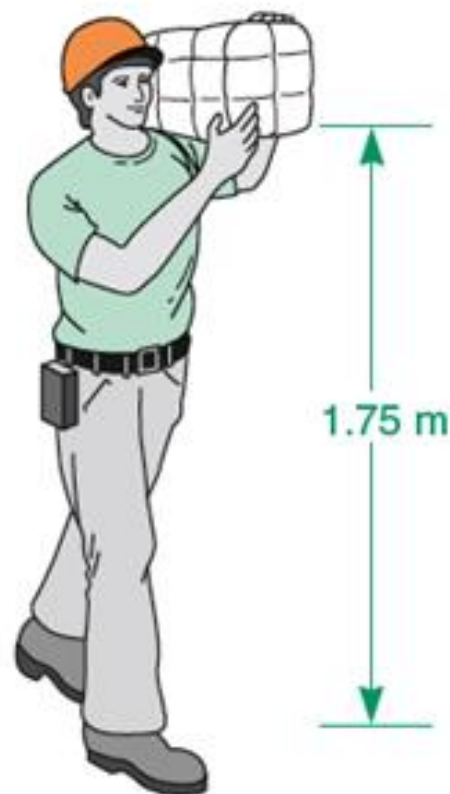
### Basic Equation:

$$W = Fs$$

### Working Equation: Same

### Substitution

$$\begin{aligned} W &= (225 \text{ N})(1.75 \text{ m}) \\ &= 394 \text{ N m or } 394 \text{ J} \end{aligned}$$



## EXAMPLE

A worker pushes a 350-lb cart a distance of 30 ft by exerting a constant force of 40 lb as shown in Figure 2.29. How much work does the person do?

**Data:**

$$F = 40 \text{ lb}$$

$$s = 30 \text{ ft}$$

$$W = ?$$

**Basic Equation:**

$$W = Fs$$

**Working Equation:** Same

**Substitution:**

$$\begin{aligned} W &= (40 \text{ lb})(30 \text{ ft}) \\ &= 1200 \text{ ft lb} \end{aligned}$$

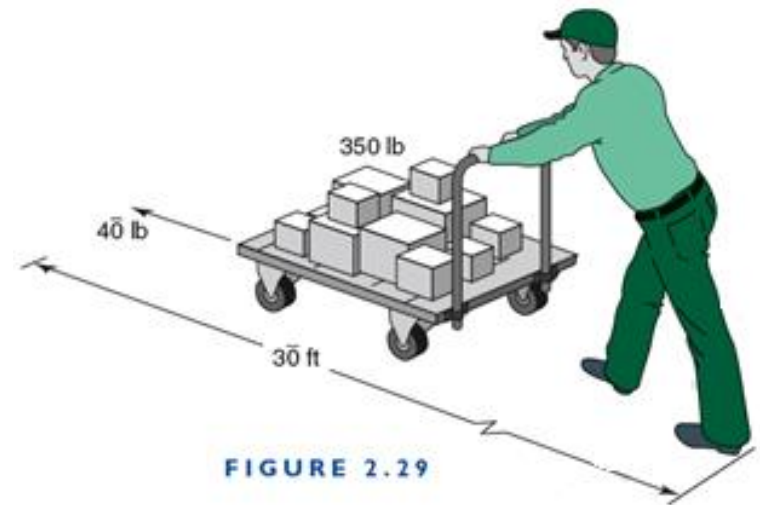
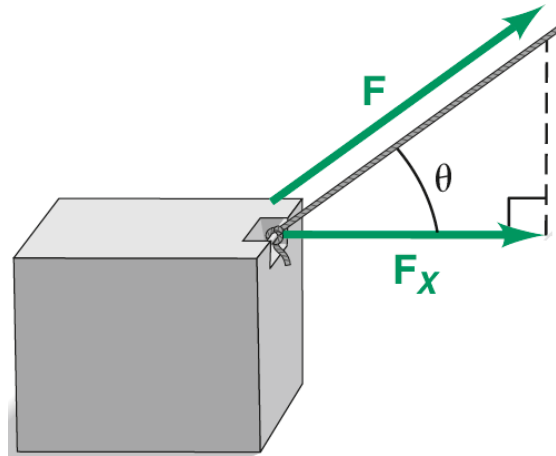


FIGURE 2.29

# Work done by a force not in the direction of motion



$$\cos \theta = \frac{\text{side adjacent to } \theta}{\text{hypotenuse}} = \frac{|\mathbf{F}_x|}{|\mathbf{F}|}$$

$$W = F s \cos \theta$$

$W$  = the work done

$F$  = the applied force

$s$  = the displacement

$\theta$  = the angle between the applied force  
and the direction of the motion

**Note:** Work by force perpendicular ( $\theta = 90^\circ$ ) to the direction of motion is zero. E.g. work by the weight = 0 J in previous example

## EXAMPLE

A person pulls a sled along level ground a distance of 15.0 m by exerting a constant force of 215 N at an angle of  $30.0^\circ$  with the ground (Figure 2.31). How much work does he do?

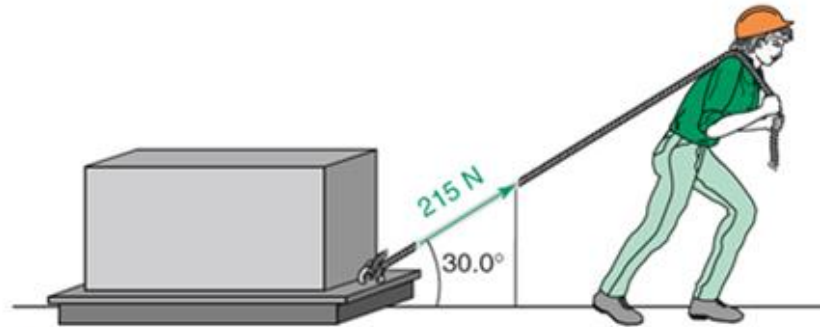


FIGURE 2.31

**Data:**

$$F = 215 \text{ N}$$

$$s = 15.0 \text{ m}$$

$$\theta = 30.0^\circ$$

$$W = ?$$

**Basic Equation:**

$$W = Fs \cos \theta$$

**Working Equation: Same**

**Substitution:**

$$W = (215 \text{ N})(15.0 \text{ m}) \cos 30.0^\circ$$

$$= 2790 \text{ N m}$$

$$= 2790 \text{ J} \quad (1 \text{ N m} = 1 \text{ J})$$

## EXAMPLE

Junaid and Sami use a push mower to mow a lawn. Junaid, who is taller, pushes at a constant force of 33.1 N on the handle at an angle of  $55.0^\circ$  with the ground. Sami, who is shorter, pushes at a constant force of 23.2 N on the handle at an angle of  $35.0^\circ$  with the ground. Assume they each push the mower 3000 m. Who does more work and by how much?

### Data:

$$F = 33.1 \text{ N}$$

$$s = 3000 \text{ m}$$

$$\theta = 55.0^\circ$$

$$W = ?$$

### Basic Equation:

$$W = Fs \cos \theta$$

### Working Equation: Same

### Substitution:

$$\begin{aligned} W &= (33.1 \text{ N})(3000 \text{ m}) \cos 55.0^\circ \\ &= 57,000 \text{ N m} \\ &= 57,000 \text{ J} \quad (1 \text{ N m} = 1 \text{ J}) \end{aligned}$$

$$F = 23.2 \text{ N}$$

$$s = 3000 \text{ m}$$

$$\theta = 35.0^\circ$$

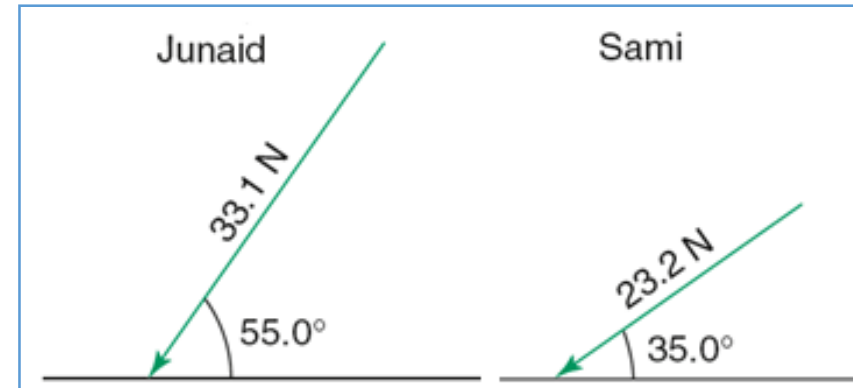
$$W = ?$$

$$W = Fs \cos \theta$$

### Same

$$\begin{aligned} W &= (23.2 \text{ N})(3000 \text{ m}) \cos 35.0^\circ \\ &= 57,000 \text{ N m} \\ &= 57,000 \text{ J} \end{aligned}$$

### Sketch:



They do the same amount of work. However, Junaid must exert more energy because he pushes into the ground more than Sami, who pushes more in the direction of the motion.



# Power

**Power** is *the rate of doing work*;

$$P = \frac{W}{t}$$

$P$  = power

$W$  = work

$t$  = time

# Power

The units of power are familiar to most of us. In the metric system, the unit of power is the *watt*:

$$P = \frac{W}{t} = \frac{Fs}{t} = \frac{N\ m}{s} = \frac{J}{s} = \text{watt}$$

Power is often expressed in kilowatts and megawatts:

$$1000 \text{ watts (W)} = 1 \text{ kilowatt (kW)}$$

$$1,000,000 \text{ watts} = 1 \text{ megawatt (MW)}$$

In the U.S. system, the unit of power is either ft lb/s or horsepower:

$$P = \frac{W}{t} = \frac{Fs}{t} = \frac{\text{ft lb}}{s}$$

*Horsepower* (hp) is a unit defined by **James Watt**:

$$1 \text{ horsepower (hp)} = 550 \text{ ft lb/s} = 33,000 \text{ ft lb/min}$$

## EXAMPLE:

A freight elevator with operator weighs 5000 N. If it is raised to a height of 15.0 m in 10.0 s, how much power is developed?

### Data:

$$F = 5000 \text{ N}$$

$$s = 15.0 \text{ m}$$

$$t = 10.0 \text{ s}$$

$$P = ?$$

### Basic Equations:

$$P = \frac{W}{t} \quad \text{and} \quad W = Fs$$

### Working Equation:

$$P = \frac{Fs}{t}$$

### Substitution:

$$\begin{aligned} P &= \frac{(5000 \text{ N})(15.0 \text{ m})}{10.0 \text{ s}} \\ &= 7500 \text{ N m/s} \end{aligned}$$

## EXAMPLE 2.28

The mass of a large steel wrecking ball is 2000 kg. What power is used to raise it to a height of 40.0 m if the work is done in 20.0 s?

### Data:

$$m = 2000 \text{ kg} \quad s = 40.0 \text{ m} \quad t = 20.0 \text{ s} \quad P = ?$$

### Basic Equations:

$$P = \frac{W}{t} \text{ and } W = Fs$$

### Working Equation:

$$P = \frac{Fs}{t}$$

**Substitution:** Note that we cannot directly substitute into the working equation because our data are given in terms of *mass* and we must find *force* to substitute in  $P = Fs/t$ . The force is the weight of the ball:

$$F = mg = (2000 \text{ kg})(9.80 \text{ m/s}^2) = 19,600 \text{ kg m/s}^2 = 19,600 \text{ N}$$

Then

$$\begin{aligned} P &= \frac{Fs}{t} = \frac{(19,600 \text{ N})(40.0 \text{ m})}{20.0 \text{ s}} \\ &= 39,200 \text{ N m/s} \\ &= 39,200 \text{ W} \quad \text{or} \quad 39.2 \text{ kW} \end{aligned}$$

## EXAMPLE: 2.31

A pump is needed to lift 1500 L of water per minute a distance of 45.0 m. What power, in kW, must the pump be able to deliver? (1 L of water has a mass of 1 kg.)

**Data:**  $m = 1500 \text{ L} \times \frac{1 \text{ kg}}{1 \text{ L}} = 1500 \text{ kg}$        $s = 45.0 \text{ m}$        $t = 1 \text{ min} = 60.0 \text{ s}$   
 $g = 9.80 \text{ m/s}^2$        $P = ?$

**Basic Equations:**

$$P = \frac{W}{t}, \quad W = Fs, \quad \text{and} \quad F = mg, \quad \text{or} \quad P = \frac{mgs}{t}$$

**Working Equation:**

$$P = \frac{mgs}{t}$$

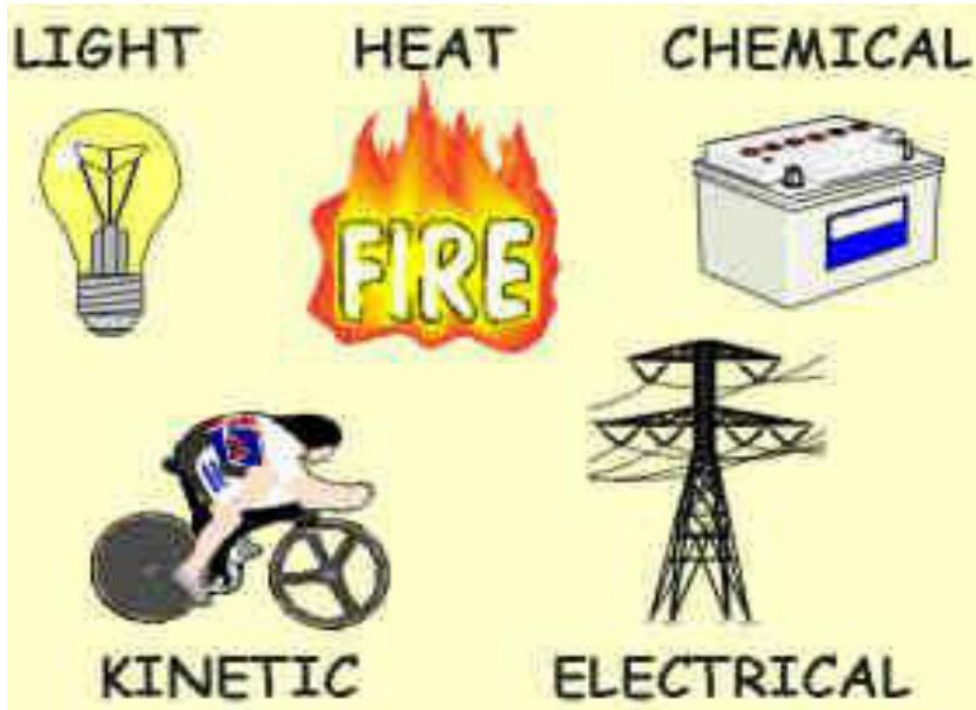
**Substitution:**

$$\begin{aligned} P &= \frac{(1500 \text{ kg})(9.80 \text{ m/s}^2)(45.0 \text{ m})}{60.0 \text{ s}} \\ &= 1.10 \times 10^4 \text{ kg m}^2/\text{s}^3 \quad \left( 1 \text{ W} = \frac{1 \text{ J}}{\text{s}} = \frac{1 \text{ N m}}{\text{s}} = \frac{1 (\text{kg m/s}^2)(\text{m})}{\text{s}} = 1 \text{ kg m}^2/\text{s}^3 \right) \\ &= 1.10 \times 10^4 \cancel{\text{W}} \times \frac{1 \text{ KW}}{10^3 \cancel{\text{W}}} \\ &= 11.0 \text{ KW} \end{aligned}$$

# Energy

**Energy** is defined as the ability to do work.

**Forms of energy:**



Renewable energies

## Units of energy:

*SI system:*      Joule (J)

*U.S. system:*    ft lb

# Mechanical Energy

- The mechanical energy of a body or a system is due to its position, its motion, or its internal structure.

There are two forms of mechanical energy:

- Potential energy
- Kinetic energy



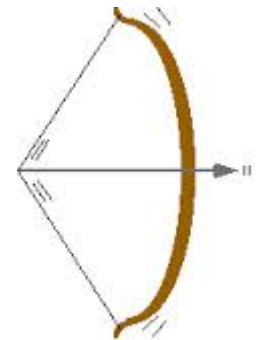
# Potential Energy

- **Potential energy** is the stored energy of a body due to its *internal characteristics* or its *position*.

1. **Internal potential energy** is determined by the nature or condition of the substance;

## Example:

- A stretched bow has stored energy that can do work on an arrow.
- A stretched rubber band of a slingshot has stored energy and is capable of doing work.



# Potential Energy

2. **Gravitational potential energy** is determined by the position of an object relative to a particular reference level.

## **Example:**

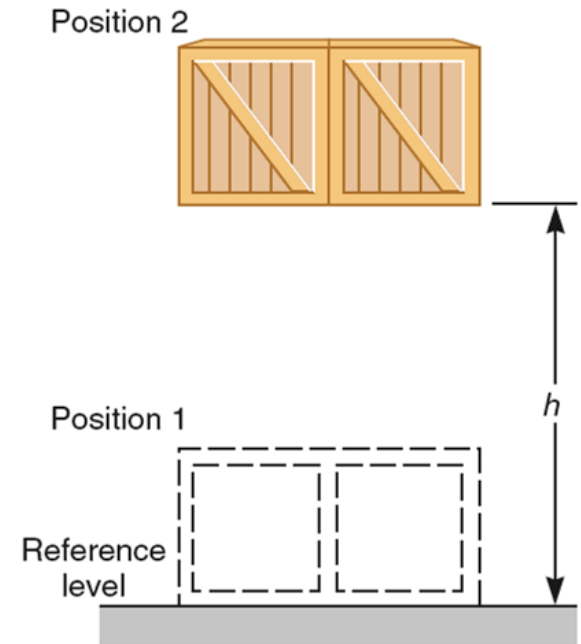
- water in an elevated reservoir
- raised ram of a pile driver

# Gravitational potential energy

- Equal to the work done (force required to move it upward  $\times$  the vertical distance moved against gravity) in lifting it
- In equation form:

$$E_p = m g h$$

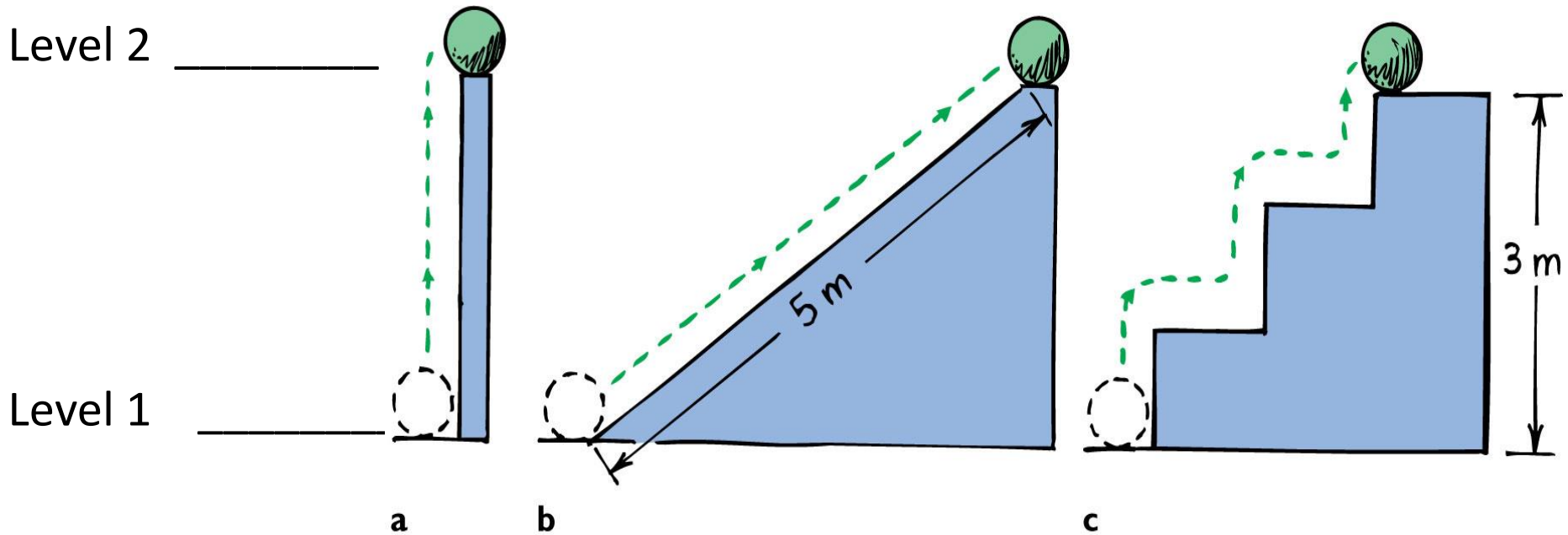
where  $E_p$  = potential energy  
 $m$  = mass  
 $g = 9.80 \text{ m/s}^2$  or  $32.2 \text{ ft/s}^2$   
 $h$  = height above reference level



**FIGURE 2.33**

Work done in raising the crate gives it potential energy.

Example: A 10-N ball is lifted from level 1 to level 2 by three means as shown in the figure below. Which case has a greater potential energy?

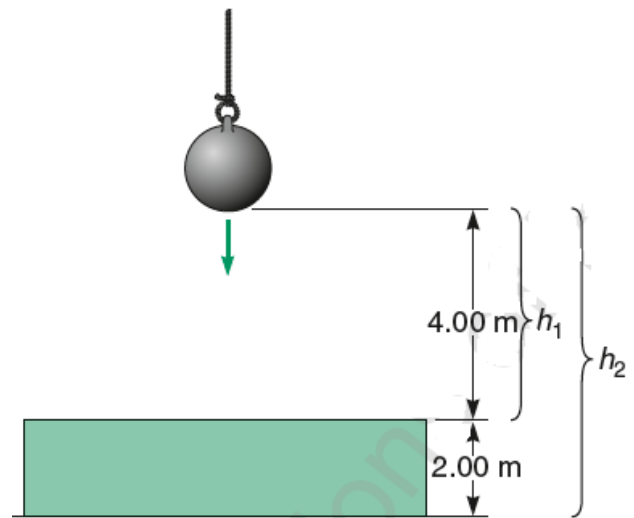


A	Potential Energy in (a) is greater
B	Potential Energy in (b) is greater
C	Potential Energy in (c) is greater
D	Potential Energy is same in the three cases.

## EXAMPLE 2.32

A wrecking ball of mass  $200\text{ kg}$  is poised  $4.00\text{ m}$  above a concrete platform whose top is  $2.00\text{ m}$  above the ground. (a) With respect to the platform, what is the potential energy of the ball? (b) With respect to the ground, what is the potential energy of the ball?

**Sketch:**



**Data:**

$$m = 200\text{ kg} \quad h_1 = 4.00\text{ m} \quad h_2 = 6.00\text{ m} \quad E_p = ?$$

**Basic Equation:**

$$E_p = mgh$$

**Working Equation:** Same

**(a) Substitution:**

$$\begin{aligned} E_p &= (200\text{ kg})(9.80\text{ m/s}^2)(4.00\text{ m}) \\ &= 7840 \frac{\text{kg m}^2}{\text{s}^2} \times \frac{1\text{ J}}{\text{kg m}^2/\text{s}^2} \quad [1\text{ J}] = 1\text{ Nm} = 1(\text{kg m/s}^2)(\text{m}) = 1\text{ kg m}^2/\text{s}^2 \end{aligned}$$

$= 7840\text{ J}$  (which also indicates the amount of work done by gravity on a falling object)

**(b) Substitution:**

$$E_p = (200\text{ kg})(9.80\text{ m/s}^2)(6.00\text{ m}) = 11,800 \frac{\text{kg m}^2}{\text{s}^2} \times \frac{1\text{ J}}{\text{kg m}^2/\text{s}^2} = 11,800\text{ J}$$

# Kinetic Energy

- Energy of motion
- Kinetic energy is due to the mass and the velocity of a moving object
- is given by the formula:

$$E_k = \frac{1}{2} m v^2$$

where  $E_k$  = kinetic energy  
 $m$  = mass of moving object  
 $v$  = velocity of moving object

- If object speed is doubled  $\Rightarrow$  kinetic energy is quadrupled.

# Kinetic Energy

Kinetic energy and work of a moving object

- Equal to the work required to bring it from rest to that speed, or the work the object can do while being brought to rest. In other words, if all the work is transferred into kinetic energy then:

Total work = net force  $\times$  displacement = kinetic energy,

or

$$F \cdot s = \frac{1}{2}mv^2$$

## EXAMPLE 2.33

A pile driver with mass  $10,000 \text{ kg}$  strikes a pile with velocity  $10.0 \text{ m/s}$ . (a) What is the kinetic energy of the driver as it strikes the pile? (b) If the pile is driven  $20.0 \text{ cm}$  into the ground, what force is applied to the pile by the driver as it strikes the pile? Assume that all the kinetic energy of the driver is converted to work.

**Data:**  $m = 1.00 \times 10^4 \text{ kg}$        $v = 10.0 \text{ m/s}$   
 $s = 20.0 \text{ cm} = 0.200 \text{ m}$        $F = ?$

(a) **Basic Equation:**

$$E_k = \frac{1}{2}mv^2$$

**Working Equation:** Same

**Substitution:**

$$\begin{aligned} E_k &= \frac{1}{2}(1.00 \times 10^4 \text{ kg})(10.0 \text{ m/s})^2 \\ &= 5.00 \times 10^5 \frac{\text{kg m}^2}{\text{s}^2} \times \frac{1 \text{ J}}{\text{kg m}^2/\text{s}^2} \quad [1 \text{ J} = 1 \text{ N m} = 1 (\text{kg m/s}^2)(\text{m}) = 1 \text{ kg m}^2/\text{s}^2] \\ &= 5.00 \times 10^5 \text{ J} \quad \text{or} \quad 500 \text{ kJ} \end{aligned}$$

(b) **Basic Equation:**

$$E_k = W = Fs$$

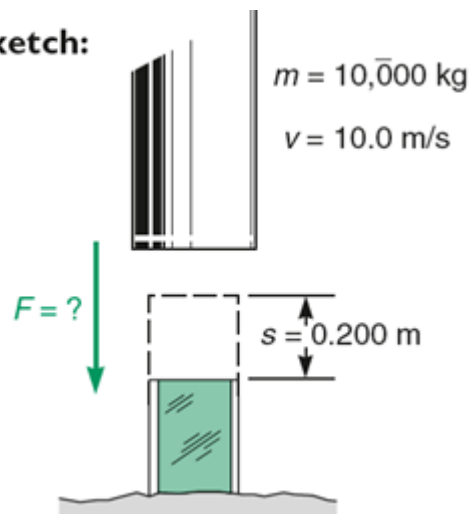
**Working Equation:**

$$F = \frac{E_k}{s} \quad [\text{Use } E_k \text{ from part (a).}]$$

**Substitution:**

$$\begin{aligned} F &= \frac{5.00 \times 10^5 \text{ J}}{0.200 \text{ m}} \times \frac{1 \text{ N m}}{1 \text{ J}} \quad (1 \text{ J} = 1 \text{ N m}) \\ &= 2.50 \times 10^6 \text{ N} \end{aligned}$$

**Sketch:**





## EXAMPLE 2.34

A 60.0-g bullet is fired from a gun with 3150 J of kinetic energy. Find its velocity.

### Data:

$$E_k = 3150 \text{ J}$$

$$m = 60.0 \text{ g} = 0.0600 \text{ kg}$$

$$v = ?$$

### Basic Equation:

$$E_k = \frac{1}{2}mv^2$$

### Working Equation:

$$v = \sqrt{\frac{2E_k}{m}}$$

### Substitution:

$$v = \sqrt{\frac{2(3150 \text{ J})}{0.0600 \text{ kg}}} \times \frac{1 \text{ kg m}^2/\text{s}^2}{1 \text{ J}}$$

$$= 324 \text{ m/s}$$

$$[1 \text{ J}] = 1 \text{ N m} = 1 (\text{kg m/s}^2)(\text{m}) = 1 \text{ kg m}^2/\text{s}^2$$

# Conservation of Energy

Law of conservation of energy:

Energy cannot be created or destroyed; it may be transformed from one form into another, but the total amount of energy never changes.

# Conservation of Mechanical Energy

$$\text{Mechanical Energy} = \text{Potential Energy} + \text{Kinetic Energy}$$



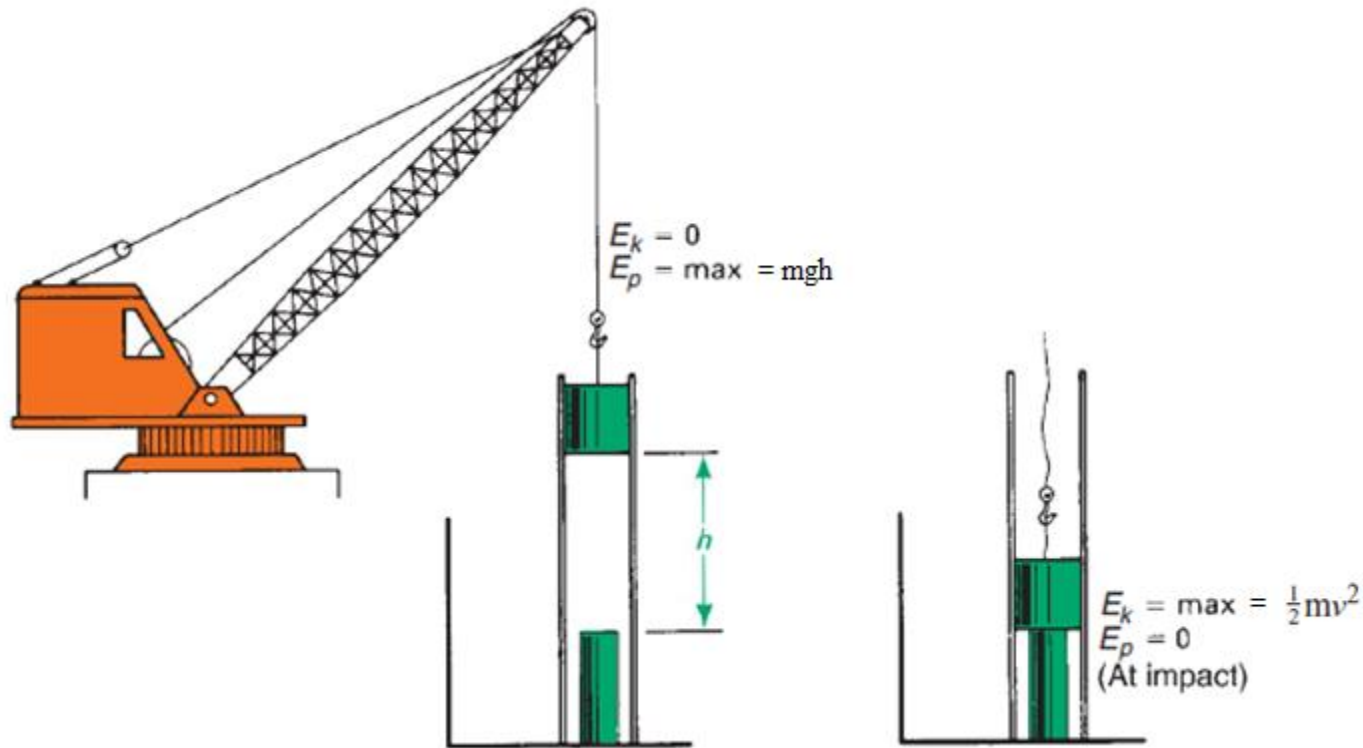
## Law of Conservation of Mechanical Energy

The sum of the kinetic energy and the potential energy in a system is constant if no resistant forces do work.

Consider the system of a bow and arrow. In drawing the bow, we do work on the system and give it potential energy. When the bowstring is released, most of the potential energy is transferred to the arrow as kinetic energy and some as heat to the bow.

# Conservation of Mechanical Energy

**Example:** Energy transforms without net loss or net gain in the operation of a pile driver.



- conservation of mechanical energy  $\Rightarrow \max E_p = \max E_k$   
 $mgh = \frac{1}{2}mv^2$
- Solving for the velocity  $\Rightarrow v = \sqrt{2gh}$

### Example 2.35

A pile driver falls freely from a height of 3.50 m above a pile. What is its velocity as it hits the pile?

Solution

Data:

$$h = 3.50 \text{ m}$$

$$g = 9.8 \text{ m/s}^2$$

$$v = ?$$

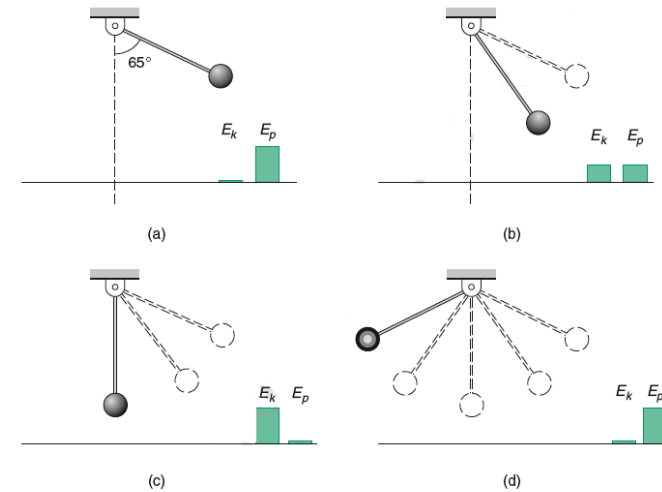
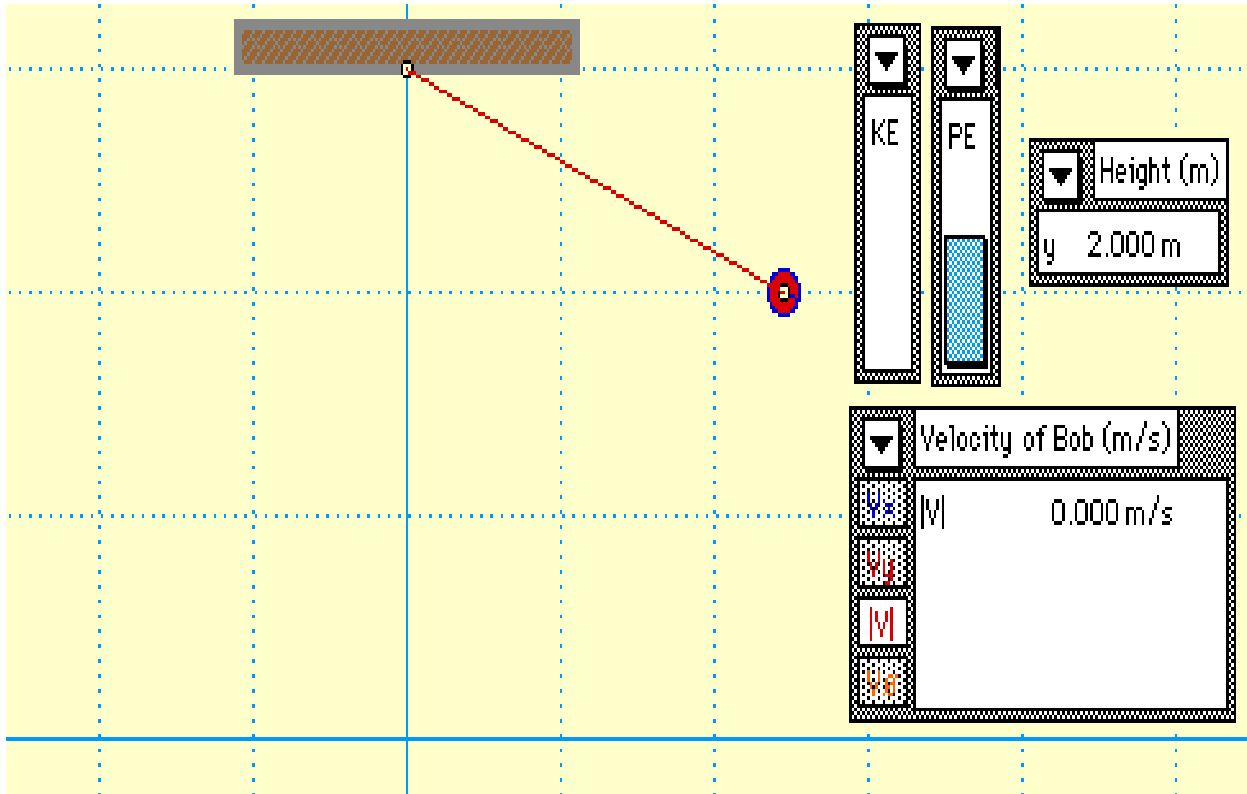
Basic Equation:

$$v = \sqrt{2gh}$$

Substitution:

$$\begin{aligned} v &= \sqrt{2 \times 9.80 \times 3.50} \\ &= 8.28 \text{ m/s} \end{aligned}$$

# Conservation of Mechanical Energy



Kinetic and potential energy changes in a pendulum

***When no resistance forces, the pendulum will swing forever.***