

2.4 Derivatives of Trigonometric Functions

$f(x)$	$f'(x)$
$\sin u$	$u' \cos u$
$\cos u$	$-u' \sin u$
$\tan u$	$u' \sec^2 u$
$\cot u$	$-u' \csc^2 u$
$\sec u$	$u' \sec u \cdot \tan u$
$\csc u$	$-u' \csc u \cdot \cot u$

Exercise :

(a) If $f(x) = \sin 3x$, find $f'(x)$

Solution

$$f'(x) = 3 \cos 3x$$

(b) If $f(x) = \cos x^2$, find $f'(x)$

Solution

$$f'(x) = -2x \sin x^2$$

(c) If $f(x) = \sec 5x^3$, find $f'(x)$

Solution

$$f'(x) = 15x^2 \sec 5x^3 \cdot \tan 5x^3$$

Example 1 : Find $D_x(3 \sin x - 2 \cos x)$

Solution

$$D_x(3 \sin x - 2 \cos x) = 3 \cos x + 2 \sin x$$

Example 2 : Find the equation of the tangent line to the graph of $y = 3 \sin x$ at the point $(\pi, 0)$

Solution

➤ Equation of tangent line :

$$y - y_1 = y'(x - x_1)$$

Thus:

$$y' = 3 \cos x = 3 \cos \pi = -3$$

∴ Equation of tangent line:

$$y - 0 = -3(x - \pi)$$

$$y = -3x + 3\pi$$

Example 3 : Find $D_x(x^2 \sin x)$

Solution

$$D_x(x^2 \sin x) = 2x \sin x + x^2 \cos x$$

Example 4: Find $\frac{d}{dx} \left(\frac{1+\sin x}{\cos x} \right)$

Solution

$$\begin{aligned} \frac{d}{dx} \left(\frac{1 + \sin x}{\cos x} \right) &= \frac{[(\cos x)(\cos x)] - [(1 + \sin x)(-\sin x)]}{\cos^2 x} \\ &= \frac{[\cos^2 x] - [-\sin x - \sin^2 x]}{\cos^2 x} = \frac{\cos^2 x + \sin x + \sin^2 x}{\cos^2 x} \\ &= \frac{1 + \sin x}{\cos^2 x} \end{aligned}$$