

FINAL EXAM, Semester 2, 1438-1439 h

Math 219 — Calculus II

Time allowed: 2 hours,

Max marks: 50

ابدأ كل سؤال في صفحة جديدة.

➤ Question 1. (5×2 + 3 = 13 marks)

1 Find the derivative

(a) $y = \ln(\cos x)$, (b) $y = \sin^{-1}(e^x)$, (c) $y = \sec^{-1}(x^2)$, (d) $y = \log_{10}(\sinh x)$,
(e) $y = 3^{\tan^2 x}$.

2 Use logarithmic differentiation to find the derivative of $y = (1+x^2)^x$.

➤ Question 2. (6×3 = 18 marks)

1 Use substitution rule to evaluate

(a) $I_1 = \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$,

(b) $I_2 = \int \frac{\cos x \sin x}{9 + \cos^2 x} dx$.

2 Use integration by parts to evaluate

(a) $I_3 = \int x \cdot e^{3x} dx$,

(b) $I_4 = \int \tan^{-1}(3x) dx$.

3 Evaluate the trigonometric integrals

(a) $I_5 = \int \sin^2(x) dx$,

(b) $I_6 = \int \tan^4 x dx$.

➤ Question 3. (4 × 3 = 12 marks)

1 Use trigonometric substitutions to evaluate

(a) $I_7 = \int \frac{x^3}{\sqrt{9-x^2}} dx$ (set $x = 3 \sin \theta$),

(b) $I_8 = \int \frac{1}{x^2 \sqrt{x^2-1}} dx$ (set $x = \sec \theta$).

2 Use partial fraction decomposition to evaluate $I_9 = \int \frac{4x^2+1}{x(2x-1)^2} dx$.

➤ Question 4. (3 + 4 = 7 marks)

1 The region bounded by the graph of $y = \frac{2}{\sqrt{x}}$, the x-axis and the lines $x = 1$, $x = 4$ is revolved about the x-axis. Find the volume of the resulting solid.

2 Find the arc length of the curve $y = \ln(\sec x)$ from $A(0, 0)$ to $B\left(\frac{\pi}{4}, \ln(\sqrt{2})\right)$, $\left[\sec\left(\frac{\pi}{4}\right) = \sqrt{2}, \tan\left(\frac{\pi}{4}\right) = 1\right]$.

مع تمنياتنا بالنجاح، أسرة رياض ٢١٩