مدونة المناهج السعودية

https://eduschool40.blog

الموقع التعليمي لجميع المراحل الدراسية

في المملكة العربية السعودية

Math 100

Tabuk University

Wada Altiary

خطة مقرر رياضيات ١

Relating Absolute Value and Distance

DEFINITION 1 Absolute Value

$$|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

$|-3| \boxed{= -(-3)} = 3$

$|4| = 4$

[*Note:* $-x$ is positive if x is negative.]

Example: Write without the absolute value:

(A) $|\pi - 3| = \pi - 3$

(B) $|3 - \pi| = -(3 - \pi) = \pi - 3$

Note:

$\pi = 3.14$ So

$3.14 - 3 = 0.14$

positive

Remark: $|b - a| = |a - b|$

DEFINITION 2 Distance Between Points *A* and *B*

Let A and B be two points on a real number line with coordinates a and b, respectively. The **distance between *A* and *B*** is given by

$$d(A, B) = |b - a|$$

This distance is also called the **length of the line segment** joining A and B.

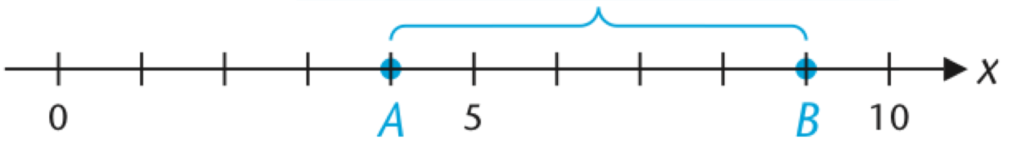
Example: Find the distance between given points

(A) $a = 4,\ b = 9$ (B) $a = 9,\ b = 4$ (C) $a = 0,\ b = 6$

Solution:

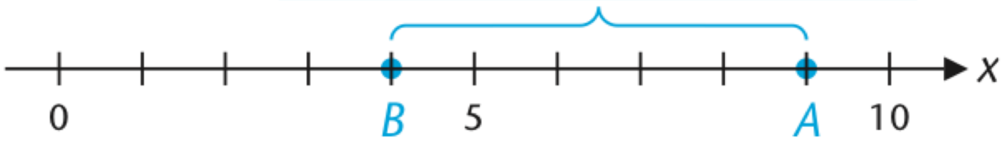
$d(A, B) = |9 - 4| = |5| = 5$

(A)



$d(A, B) = |4 - 9| = |-5| = 5$

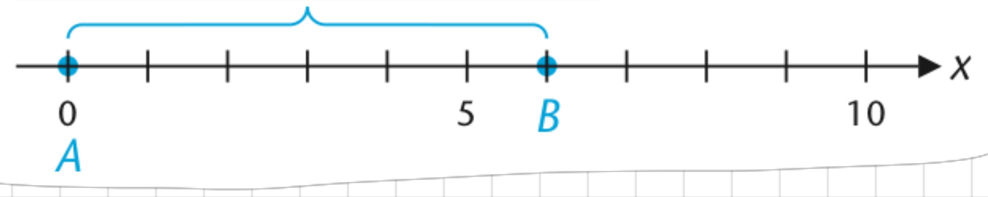
(B)



Remark:

$d(A, B) = d(B, A)$

(C) $d(A, B) = |6 - 0| = |6| = 6$



Remark:

$d(O, B) = |b - 0| = |b|$

نقطة الأصل

Example: Express each verbal statement as an absolute value equation or inequality.

(A) x is 4 units from 2.

(B) y is less than 3 units form -5.

(C) t is no more than 5 units from 7.

(D) w is no less than 2 units form -1.

SOLUTIONS

(A) $\boxed{d(x, 2) =}\ |x - 2| = 4$

(B) $\boxed{d(y, -5) =}\ |y + 5| < 3$

(C) $\boxed{d(t, 7) =}\ |t - 7| \le 5$

(D) $\boxed{d(w, -1) =}\ |w + 1| \ge 2$

Solving Absolute Value Equations and Inequalities

Steps for Solving Absolute Value Equation:

- Isolate the absolute value
- Analyze the equation " Is it possible to solve?"
- Solve the equation
- Check your answer

ملاحظة: إذا كانت المعادلة تساوي عدد سالب فالمعادلة مستحيلة الحل

Example: Solve the following Equations

1) $|x-3| = 5$

Step 1 : ✅

Step 2 : ✅

Step 3 :

$x-3 = 5$ or $-(x-3) = 5$

بتطبيق تعريف الداله المطلقه

$x = 5+3$ or $-x+3 = 5$

$x = 8$ or $-x = 5-3$

$-x = 2$

$x = -2$

Step 4:

$x = 8$

$|8-3| = 5$

$|5| = 5$

$5 = 5$

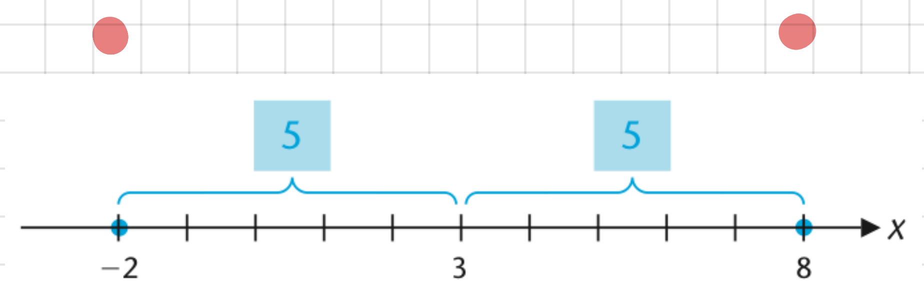
$x =$

$\therefore x = \{-2, 8\}$

يسمى هذا النوع من الأقواس رمز المجموعة

Set notation

التمثيل البياني:

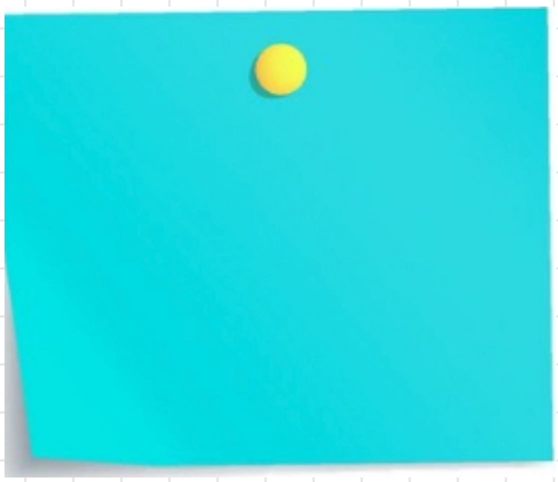


2) $|3x-7|+7=2$

Step 1: $|3x-7| = 2-7$

$|3x-7| = -5$

Step 2: No Solution or $\emptyset$



3) $|3x-7|+7=9$

Step 1: $|3x-7| = 9-7$

$|3x-7| = 2$

Step 2: ✅

Step 3: $3x-7=2$ or $-(3x-7)=2$

$3x=2+7$ or $-3x+7=2$

$3x=9$ or $-3x=2-7$

$x=3$ or $-3x=-5$

$x=5/3$

Step 4:

$x=3$

$|3.3-7| = 2$

$|9-7| = 2$

$|2| = 2$

2

$x=5/3$

$|\cancel{3}.\frac{5}{\cancel{3}}-7| = 2$

$|5-7| = 2$

$|-2| = 2$

$2 = 2$

Steps for Solving Absolute Value Inequalities:

- Isolate the absolute value
- Analyze the Inequality " Is it possible to solve?"
- Solve the absolute value inequality
- Check your answer

ملاحظة: اذا كانت المتراجحة اقل من الصفر تكون مستحيلة الحل

Example: Solve the following Inequalities

1) $|x-3| < 5$

Step 1: ✅

Step 2: ✅

Step 3: $x-3 < 5$ and $-(x-3) < 5$

$x < 5+3$ and $-x+3 < 5$

$x < 8$ and $-x < 5-3$

$-x < 2$

$x > -2$

ملاحظة: عند ضرب المتراجحة بعدد سالب نعكس إشارة المتراجحة

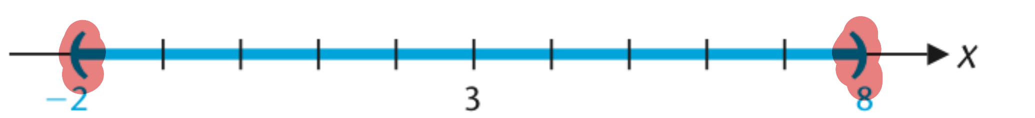
Step 4:

$x < 8$	$x > -2$
$\lvert 7-3 \rvert < 5$	$\lvert -1-3 \rvert > -2$
$\lvert 4 \rvert < 5$	$\lvert -4 \rvert > -2$
$4 < 5$ works!	$4 > -2$ works!

$\therefore x = (-2, 8)$

يسمى هذا النوع من الأقواس رمز الفترة

Interval notation



جميع الأعداد مابين ٨ و ـ٢ تحقق المتراجحة

2) $0 < |x-3| < 5$

$0 < |x-3|$

or

$|x-3| > 0$

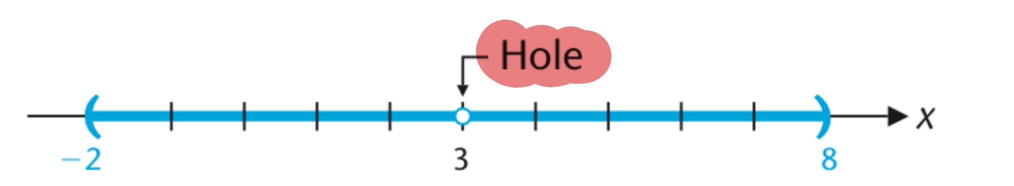
If $x = 3$ then

$|3-3| > 0$

$|0| > 0$

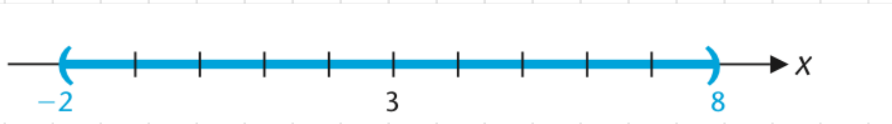
$0 > 0$ does not work

So $x = (-2, 3) \cup (3, 8)$



$|x-3| < 5$

تم حلها في المثال السابق
وكانت النتيجة كالتالي



$x = (-2, 8)$

H.W: Solve

1) $0 < |x+2| < 6$

2) $|x+2| \geq 0$

3) $|x-3| > 5$

Step 1 : ✅

Step 2: ✅

Step 3: $x-3 > 5$ or $-(x-3) < 5$

$x > 5+3$ or $-x+3 < 5$

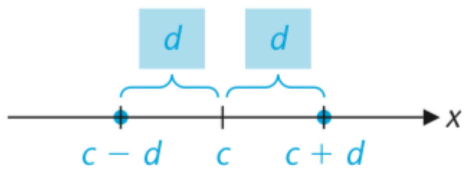
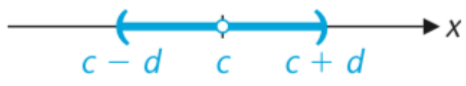
$x > 8$ or $-x < 5-3$

$-x < 2$

$x < 2$

Step 4:

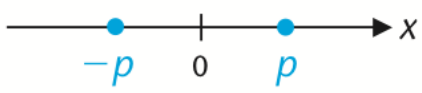
$\therefore x = (-\infty, -2) \cup (8, \infty)$

Form ($d > 0$)	Geometric interpretation	Solution	Graph
$\lvert x - c \rvert = d$	Distance between x and c is equal to d.	$\{c - d, c + d\}$ → Set notaition	
$\lvert x - c \rvert < d$	Distance between x and c is less than d.	$(c - d, c + d)$ Interval notaition.	
$0 < \lvert x - c \rvert < d$	Distance between x and c is less than d, but $x \neq c$.	$(c - d, c) \cup (c, c + d)$	
$\lvert x - c \rvert > d$	Distance between x and c is greater than d.	$(-\infty, c - d) \cup (c + d, \infty)$	

› THEOREM 2 Properties of Equations and Inequalities Involving $|x|$

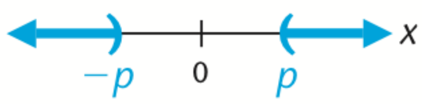
For $p > 0$:

1. $|x| = p$ is equivalent to $x = p$ or $x = -p$.



2. $|x| < p$ is equivalent to $-p < x < p$.

3. $|x| > p$ is equivalent to $x < -p$ or $x > p$.



› THEOREM 3 Properties of Equations and Inequalities Involving $|ax + b|$

For $p > 0$:

1. $|ax + b| = p$ is equivalent to $ax + b = p$ or $ax + b = -p$.
2. $|ax + b| < p$ is equivalent to $-p < ax + b < p$.
3. $|ax + b| > p$ is equivalent to $ax + b < -p$ or $ax + b > p$.

Continuous: Solving Absolute Value Problems

Example: Solve each equation or inequality

A) $|3x+5| = 4$ B) $|x| < 5$

C) $|2x-1| < 3$ D) $|7-3x| \leq 2$

Solution: Step 1 and step 2 are done.

Step 3:

By applying definition ← $\boxed{(A).\ |3x+5| = 4}$ → By applying theorem3

By applying definition:

$3x+5=4$ or $-(3x+5)=4$

$3x = 4-5$ or $-3x-5=4$

$3x=-1$ or $-3x=9$

$x=-\frac{1}{3}$ or $x=-3$

Step 4: ✅ check!!

$\therefore\ x = \{-\frac{1}{3}, -3\}$

By applying theorem3:

$3x+5=4$ or $3x+5=-4$

$3x=4-5$ or $3x=-9$

or $x=-3$

$\boxed{(B)\ |x| < 5}$

$x<5$ and $-x<5$

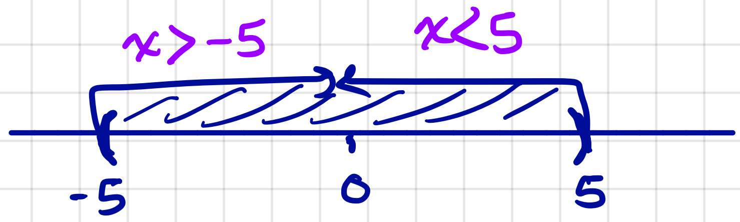
$x>-5$

$-5 < x < 5$

$\therefore\ x = (-5, 5)$

Step 4: ✅ check!

$\therefore\ x = (-5, 5)$



$$\boxed{(C)\quad |2x-1| < 3}$$

$2x-1 < 3$ and $-(2x-1) < 3$

$2x < 4$ and $-2x+1 < 3$

$x < 2$ and $-2x < 2$

$-x < 1$

$x > -1$

step 4: ✅

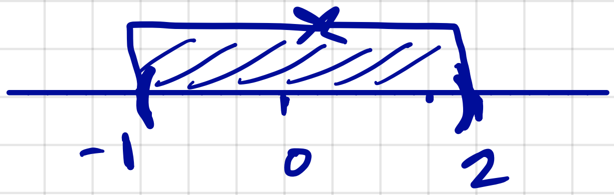
$\therefore x = (-1, 2)$

$-3 < 2x-1 < 3$

$-3+1 < 2x < 3+1$

$-2 < 2x < 4$

$-1 < x < 2$



$$\boxed{D)\ |7-3x| \leq 2}$$

H.W

$$\begin{array}{ccccc} -7 & & -7 & & -7 \\ -2 & \leq & 7-3x & \leq & 2 \end{array}$$

$-9 \leq -3x \leq -5$

$3 \geq x \geq \frac{5}{3}$

$\frac{5}{3} \leq x \leq 3$

$\therefore x = [\frac{5}{3}, 3]$

ماذا لاحظتي في
B,C,D

Example: Solve the following:

(A) $|x| > 3$

$x > 3 \quad \text{or} \quad x < -3$

$(-\infty, -3) \cup (3, \infty)$

(B) $|2x - 1| \geq 3$

$2x - 1 \geq 3 \quad \text{or} \quad 2x - 1 \leq -3$

$2x \geq 3 + 1 \quad \text{or} \quad 2x \leq -3 + 1$

$2x \geq 4 \quad \text{or} \quad 2x \leq -2$

$x \geq 2 \quad \text{or} \quad x \leq -1$

$\therefore \; x = (-\infty, -1] \cup [2, \infty)$

ماذا لاحظتي؟

C) $|7 - 3x| > 2$

$7 - 3x > 2 \quad \text{or} \quad 7 - 3x < -2$

$-3x > 2 - 7 \quad \text{or} \quad -3x < -2 - 7$

$-3x > -5 \quad \text{or} \quad -3x < -9$

$x < \frac{5}{3} \quad \text{or} \quad x > 3$

$\therefore \; x = (-\infty, \frac{5}{3}) \cup (3, \infty)$

Example: Solve $|x+4| = 3x - 8$

$$x+4 = 3x-8 \quad \text{or} \quad -(x+4) = 3x-8$$
$$4+8 = 3x-x \quad \text{or} \quad -x-4 = 3x-8$$
$$12 = 2x \quad \text{or} \quad -4+8 = 3x+x$$
$$6 = x \quad \text{or} \quad 4 = 4x$$
$$1 = x$$

ملاحظه : في هذة المسألة لايمكن تطبيق نظرية خصائص القيمة المطلقة وذلك لوجود x في الطرف الاخر وهذا يعني لا نعلم ما اذا كانت قيمة x موجبة او سالبة.

Check:

$x = 6$
$$|6+4| = 3(6)-8$$
$$|10| = 18-8$$
$$10 = 10 \checkmark$$

$x = 1$
$$|1+4| = 3(1)-8$$
$$|5| = -5$$
$$5 \neq -5$$

H.W: Solve $|3x-4| = x+5$

$\therefore x = \{6\}$

Absolute Value and Radical Inequalities

Definition: For any real number

$$\sqrt{x^2} = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

For example:

$$\sqrt{(2)^2} = \sqrt{(-2)^2} = \sqrt{4} = 2$$

$$\boxed{\text{Remark: } \sqrt{x^2} = |x|}$$

Example: Solve $\sqrt{(x-2)^2} \leq 5$

Solution:

$$|x-2| \leq 5$$

$$-5 \leq x-2 \leq 5$$

$$-5+2 \leq x \leq 5+2$$

$$-3 \leq x \leq 7$$

$\therefore \; x = [-3, 7]$

H.W: Solve
$\sqrt{(x+2)^2} < 3$

ملاحظة: الأسئلة (هاي
لايت اخضر) متعلقة
بدرس الأعداد المركبه

Unit 2: Complex Numbers

تشمل هذه الوحدة على المواضيع التالية:

تعرف الإعداد المركبة

Complex Number

DEFINITION 1 Complex Number

A **complex number** is a number of the form

$$a + bi \qquad \textbf{Standard Form}$$

where a and b are real numbers and i is called the **imaginary unit**.

Some examples of complex numbers are

$$3 - 2i \qquad \tfrac{1}{2} + 5i \qquad 2 - \tfrac{1}{3}i$$
$$0 + 3i \qquad 5 + 0i \qquad 0 + 0i$$

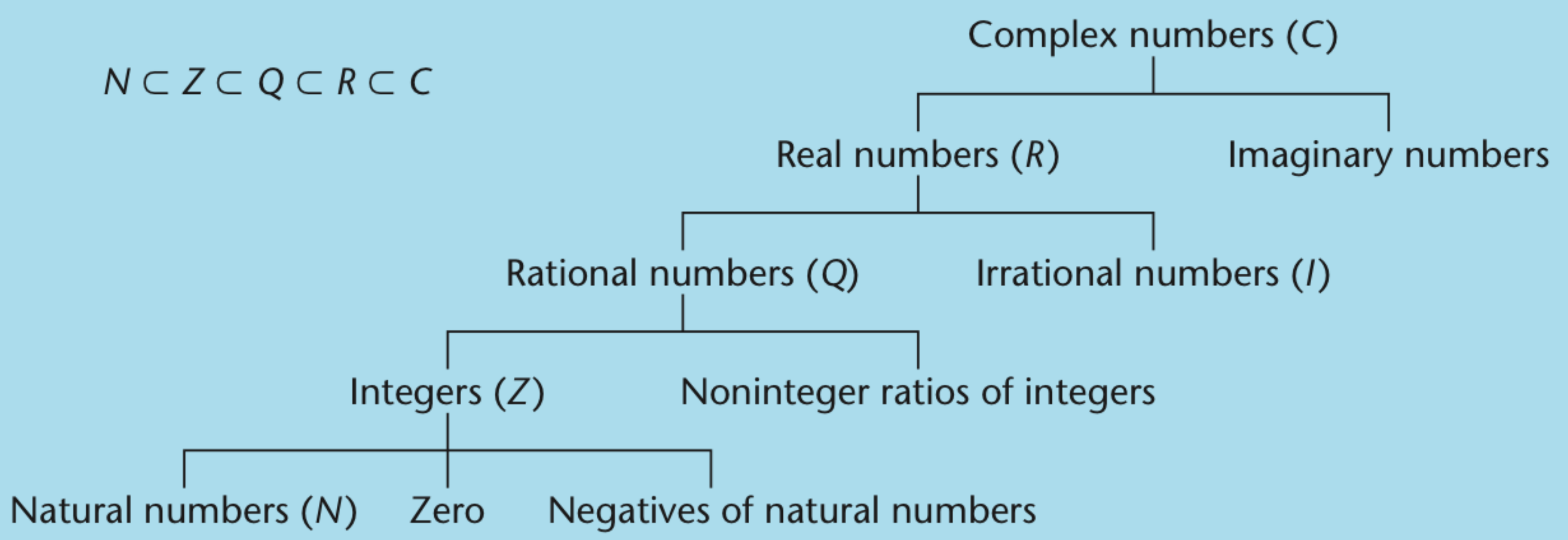
The notation $3 - 2i$ is shorthand for $3 + (-2)i$.

DEFINITION 2 Special Terms

i		**Imaginary Unit**
$a + bi$	a and b real numbers	**Complex Number**
$a + bi$	$b \neq 0$	**Imaginary Number**
$0 + bi = bi$	$b \neq 0$	**Pure Imaginary Number**
bi		**Imaginary Part of $a + bi$**
$a + 0i = a$		**Real Number**
a		**Real Part of $a + bi$**
$0 = 0 + 0i$		**Zero**
$a - bi$		**Conjugate of $a + bi$**

The relationship of the complex number system to the other number systems:

$$N \subset Z \subset Q \subset R \subset C$$



Example1:

Identify the real part, the imaginary part, and the conjugate of each of the following numbers:

(A) $3 - 2i$ (B) $2 + 5i$ (C) $7i$ (D) 6

Real Part	Imaginary part	Conjugate	ملاحظات
3	$-2i$	$3+2i$	يأخذ الجزء التخيلي بإشارته
2	$5i$	$2-5i$	
0	$7i$	$-7i$	العدد تخيلي إذن يكون له مرافق
6	0	6	لأن العدد حقيقي والمرافق يكون في الجزء التخيلي

Operations with Complex Number

› **DEFINITION 3** Equality and Basic Operations

1. **Equality:** $a + bi = c + di$ if and only if $a = c$ and $b = d$
2. **Addition:** $(a + bi) + (c + di) = (a + c) + (b + d)i$
3. **Multiplication:** $(a + bi)(c + di) = (ac - bd) + (ad + bc)i$

Example 2: **Carry out each operation and express the answer in stand form**

(A) $(2-3i)+(6+2i)$ (B) $(-5+4i)+(0+0i)$

(C) $(7-3i)-(6+2i)$ (D) $(-2+7i)+(2-7i)$

Solution

(A) $(2-3i)+(6+2i) = 2-3i+6+2i$

$= (2+6)+(-3+2)i$

$= 8-i$

(B) $(-5+4i)+(0+0i) = -5+4i+0+0i$

$= -5+4i$

(C) $(7-3i)-(6+2i) = 7-3i-6-2i$

$= (7-6)+(-3-2)i$

$= 1-5i$

(D) $(-2+7i)+(2-7i) = -2+7i+2-7i = 0$

Example 3: **Carry out each operation and express the answer in standard form**

(A) $(2-3i)(6+2i)$ (B) $1(3-5i)$

(C) $i(1+i)$ (D) $(3+4i)(3-4i)$.

Solution:

(A) $(2-3i)(6+2i) = 12+4i-18i-6i^2$

$= 12-14i-6(-1)$

$= 12-14i+6$

$= 18-14i$

(B) $1(3-5i) = 3-5i$

فقط تعديل للشكل

(C) $i(1+i) = i+i^2 = i-1 = -1+i$

(D) $(3+4i)(3-4i) = 9 \quad 12i \; -12i - 16i^2$

$= 9-16(-1)$

$= 9+16 = 25$

THEOREM 1 Product of a Complex Number and Its Conjugate

$$(a + bi)(a - bi) = a^2 + b^2 \quad \text{A real number}$$

عدد مرافقه

For example: $(3+4i)(3-4i) = 3^2 + 4^2 = 9+16 = 25$ real!

معنى النظريه أنة عند الضرب في العدد ومرافقه نستطيع مباشره ان نربع a و b ونجمعهم دون الحاجة لتطبيق خطوات الضرب

Remarks

For any complex number $a + bi$,

$$1(a + bi) = (a + bi)1 = a + bi$$

$\frac{1}{a + bi}$ is the **reciprocal** of $a + bi$ $\quad a + bi \neq 0$

or multiplicative inverse

المعكوس الضربي

Example4: Reciprocals and Quotients

Write each expression in standard form:

(A) $\frac{1}{2 + 3i}$ (B) $\frac{7 - 3i}{1 + i}$

كي نكتب المعكوس في الصورة القياسية للأعداد المركبة لابد أن نضرب البسط والمقام في مرافق المقام

Solution:

$$\frac{1}{2 + 3i} = \frac{1}{2 + 3i} \cdot \frac{2 - 3i}{2 - 3i} = \frac{2 - 3i}{4 - 9i^2} = \frac{2 - 3i}{4 + 9}$$

$$= \frac{2 - 3i}{13} = \frac{2}{13} - \frac{3}{13}i$$

CHECK

$$(2 + 3i)\left(\frac{2}{13} - \frac{3}{13}i\right) = \frac{4}{13} - \frac{6}{13}i + \frac{6}{13}i - \frac{9}{13}i^2$$

$$= \frac{4}{13} + \frac{9}{13} = 1$$

(B) $\dfrac{7-3i}{1+i} = \dfrac{7-3i}{1+i} \cdot \dfrac{1-i}{1-i} = \boxed{\dfrac{7-7i-3i+3i^2}{1-i^2}}$

$$= \frac{4-10i}{2} = 2-5i$$

CHECK $(1+i)(2-5i) = 2-5i+2i-5i^2 = 7-3i$

Natural number powers of i take on particularly simple forms:

i	$i^5 = i^4 \cdot i = (1)i = i$
$i^2 = -1$	$i^6 = i^4 \cdot i^2 = 1(-1) = -1$
$i^3 = i^2 \cdot i = (-1)i = -i$	$i^7 = i^4 \cdot i^3 = 1(-i) = -i$
$i^4 = i^2 \cdot i^2 = (-1)(-1) = 1$	$i^8 = i^4 \cdot i^4 = 1 \cdot 1 = 1$

نلاحظ بان بعد الاس ٤
تتكرر النتائج وهذا يعني
أن قوى i تكون دوريه
بعد الأس ٤

Example 5: Evaluate each the following:

طريقة الحل : نقسم الاس
على ٤ ونأخذ الباقي

(A) $i^{17} = i^{1} = i$ $\quad$ .s $\quad 17 = 4\times4+1$

$i^{24} = i^{0} = 1$ $\quad 24 = 4\times6+0$

$i^{38} = i^{2} = -1$ $\quad 38 = 4\times9+2$

$i^{47} = i^{3} = -$ $\quad 47 = 4\times11+3$

Relating Complex Numbers and Radicals

$\sqrt{-a} = \sqrt{-1}\sqrt{a} \quad i\sqrt{a}$

› **DEFINITION 4** Principal Square Root of a Negative Real Number

The **principal square root of a negative real number,** denoted by $\sqrt{-a}$, where a is positive, is defined by

$$\sqrt{-a} = i\sqrt{a} \qquad \sqrt{-3} = i\sqrt{3} \qquad \sqrt{-9} = i\sqrt{9} = 3i$$

The other square root of $-a$, $a > 0$, is $-\sqrt{-a} = -i\sqrt{a}$.

$\sqrt{-9} = \sqrt{9}\,i = 3i$

Complex Numbers and Radicals

Write in standard form:

(A) $\sqrt{-4}$ (B) $4 + \sqrt{-5}$ (C) $\dfrac{-3 - \sqrt{-5}}{2}$ (D) $\dfrac{1}{1 - \sqrt{-9}}$

SOLUTIONS

(A) $\sqrt{-4} = i\sqrt{4} = 2i$ (B) $4 + \sqrt{-5} = 4 + i\sqrt{5}$

(C) $\dfrac{-3 - \sqrt{-5}}{2} = \dfrac{-3 - i\sqrt{5}}{2} = -\dfrac{3}{2} - \dfrac{\sqrt{5}}{2}i$

(D) $\dfrac{1}{1 - \sqrt{-9}} = \dfrac{1}{1 - 3i} = \dfrac{1 \cdot (1 + 3i)}{(1 - 3i) \cdot (1 + 3i)}$

$$= \frac{1 + 3i}{1 - 9i^2} = \frac{1 + 3i}{10} = \frac{1}{10} + \frac{3}{10}i$$

ملاحظة:
الأسئلة (هاي
لايت اصفر)
متعلقة بدرس
الأعداد المركبه

Try to solve it

› Solving Equations Involving Complex Numbers

Equations Involving Complex Numbers

(A) Solve for real numbers x and y:

$(3x + 2) + (2y - 4)i = -4 + 6i$

(B) Solve for complex number z:

$(3 + 2i)z - 3 + 6i = 8 - 4i$

SOLUTIONS

(A) Equate the real and imaginary parts of each side of the equation to form two equations:

Real Parts	**Imaginary Parts**
$3x + 2 = -4$	$2y - 4 = 6$
$3x = -6$	$2y = 10$
$x = -2$	$y = 5$

(B) $(3 + 2i)z - 3 + 6i = 8 - 4i$ — Add $3 - 6i$ to both sides.

$(3 + 2i)z = 11 - 10i$ — Divide both sides by $3 + 2i$.

$z = \dfrac{11 - 10i}{3 + 2i}$ — Multiply numerator and denominator by $3 - 2i$.

$= \dfrac{(11 - 10i)(3 - 2i)}{(3 + 2i)(3 - 2i)}$ — Simplify.

$= \dfrac{13 - 52i}{13}$

$= 1 - 4i$

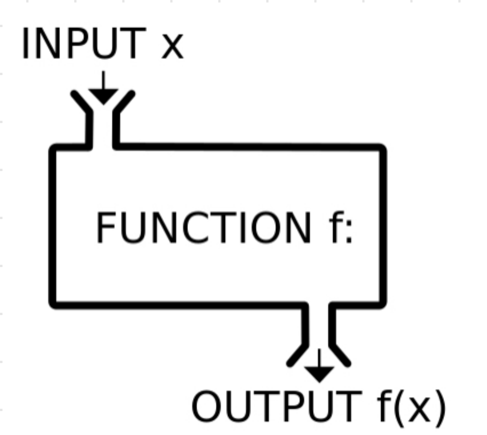
Functions

1. Definitions
2. Ways to representing functions
3. Evaluating functions
4. Domain of functions

Functions

1 What is a function?

A function is any mapp that takes an input and one out put.



2 A functions may be defined by:

- Arrow Diagram
- Set of ordered pairs
- An Equations
- Graph

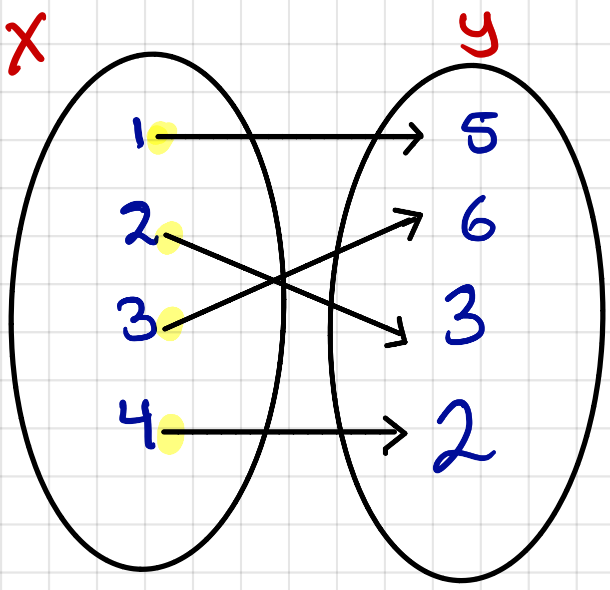
Functions

Functions Defined by Arrow Diagram

To be function: For each element in the first set there correspond one and only one element in the second test

Domain: First set

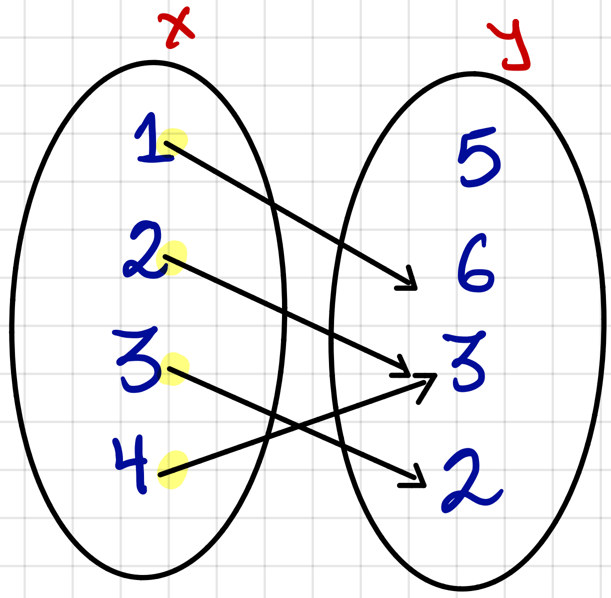
Range: Second Set



Function: Yes

Domain: {1,2,3,4}

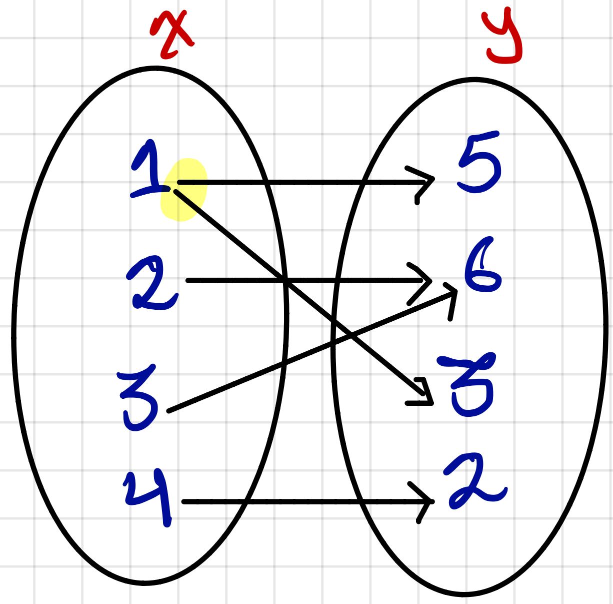
Range: {5,6,3,2}



Function: Yes

Domain: {1,2,3,4}

Range: {6,3,2}



Function: No

Domain:

Range:

Functions Defined by Set of Ordered Pairs

To be function: No ordered pairs have the same first component and different second component.

Domain: First component

Range: Second component

Determine whether each set specifies a function. If it does, then state the domain and range.

(A) $S = \{(1, 4), (2, 3), (3, 2), (4, 3), (5, 4)\}$

(B) $T = \{(1, 4), (2, 3), (3, 2), (2, 4), (1, 5)\}$

A)

Function: Yes

Domain: $\{1,2,3,4,5\}$

Range $\{2,3,4\}$

B)

Function: No

Domain:

Range

Functions Defined by an Equations

To be function: For each value of independent variable x there correspond exactly one value of dependent variable y.

Domain: Set of all possible real x-value which will make the function "work" or "defined"

Range: Set of all y-value corresponding to domain value.

Example:

$y = x^2 + 2x$

x	y
-2	0
-1	-1
0	0
1	3
2	8

Function: Yes

$y = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

Function: Yes

$y = \sqrt{x}$

$x = y^2$

x	y
4	-2
1	-1
0	0
1	1
4	2

Function: No

Note: It is very easy to determine whether an equation defines a function or not if we have the graph of the equation.

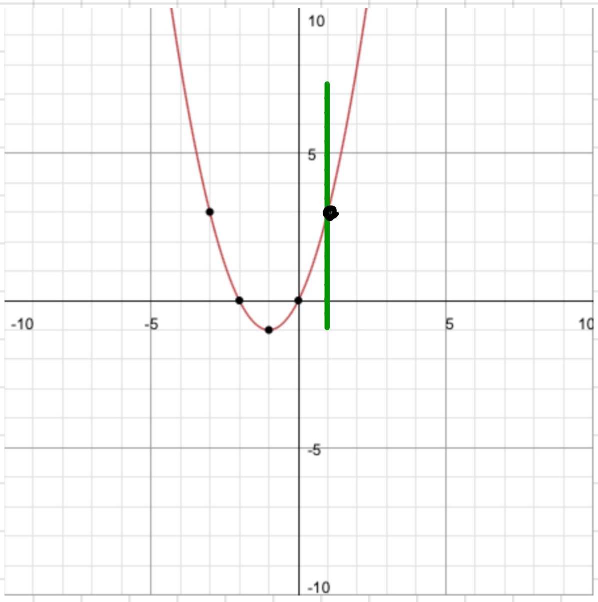
Functions Defined by Graph

To be function: Vertical Line Test (VLT):

Function: if each VL pass through at most one point on graph.

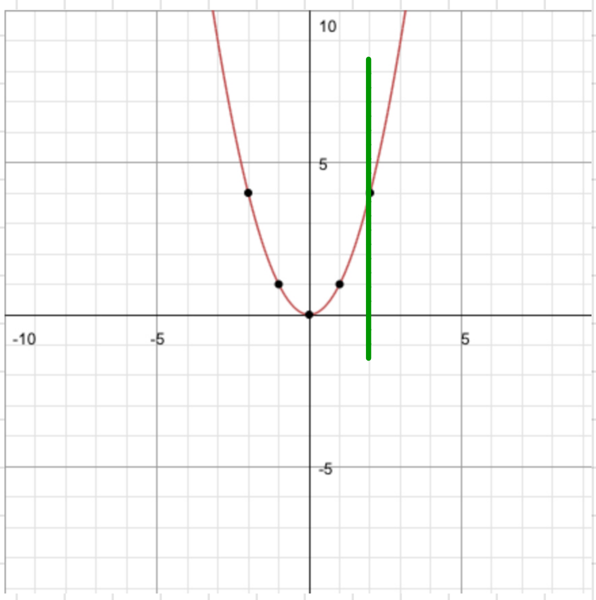
Not function: if any VL pass through two or more points on the graph.

$y = x^2 + 2x$



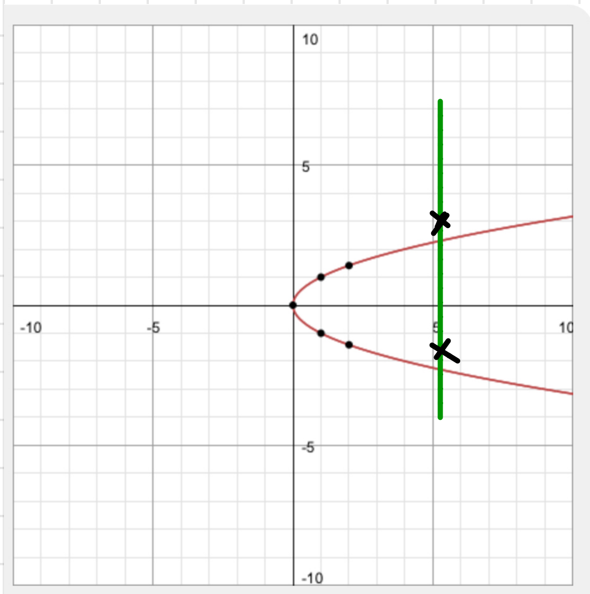
Function by VLT

$y = x^2$



Function by VLT

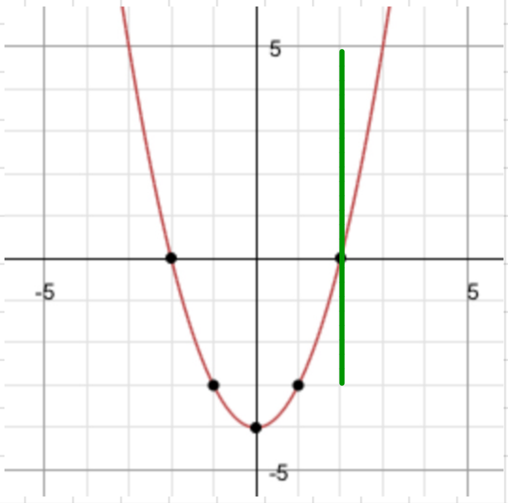
$x = y^2$



Not function by VLT

Example: **Determine if each equation defines a function with independent variable x**

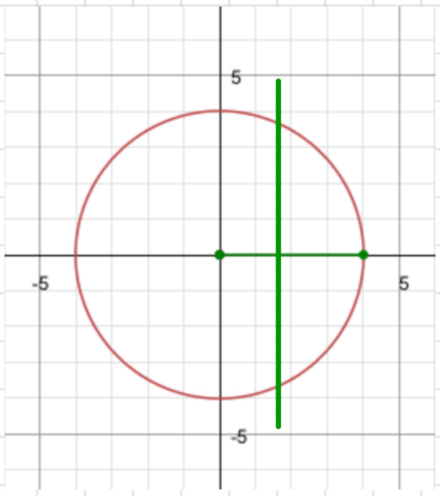
A) $y = x^2 + 4$



B) $x^2 + y^2 = 16$

$y^2 = 16 - x^2$

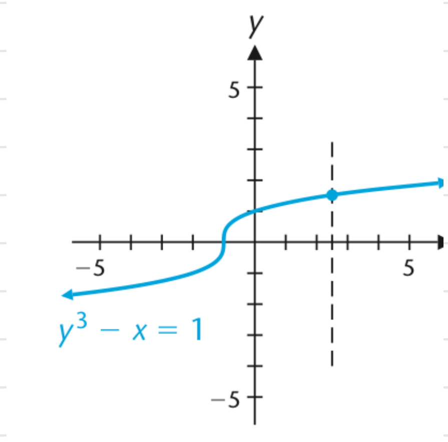
$y = \pm\sqrt{16 - x^2}$



C) $y^3 - x = 1$

$y^3 = 1 + x$

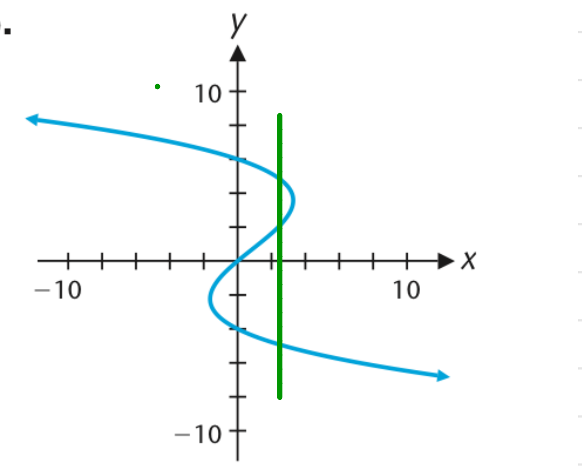
$y = \sqrt[3]{1 + x}$



(a)

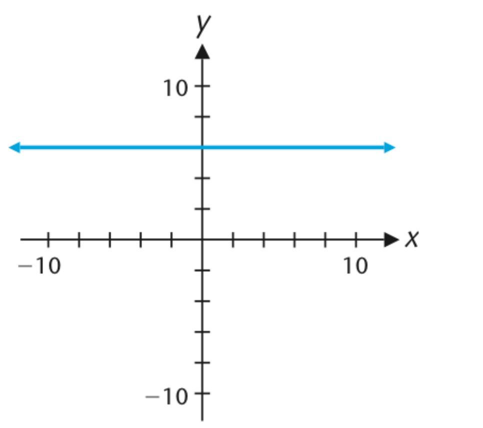
Example: **Determine if each graph defines a function**

15.



Not function

16.

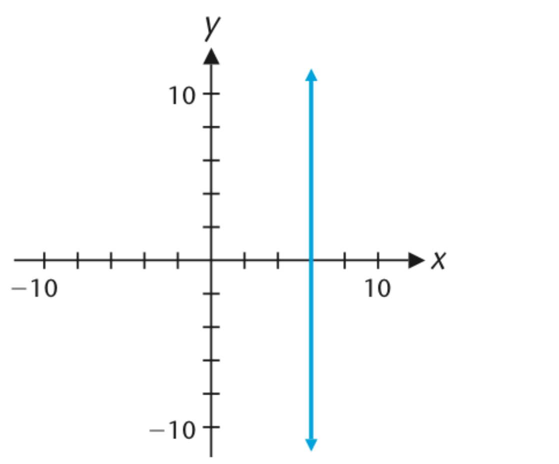


Function

Domain = R

Range = 3

17.



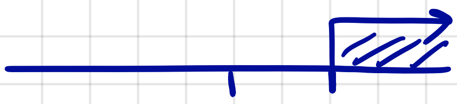
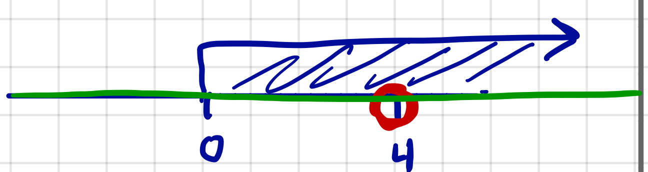
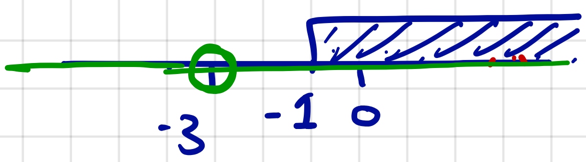
Not function

ملاحظة: سيتم دراسة إيجاد المجال والمدى من التمثيل البياني في الدرس اللا حق.

3 Finding the domain of Functions

Polynomial	Fraction only	Square root only
$f(x)= x^2+2x+1$ **Domain=** All real numbers $\mathbb{R}$ or $(-\infty, \infty)$	$f(x)= \dfrac{2}{x-4}$ **Domain=** bottom expression $\neq 0$ R-{أصفار المقام}	$f(x)= \sqrt{x+1}$ **Domain=** expression under root ≥ 0
square root on bottom	**Square root on bottom on x only**	**square root on top**
$f(x)= \dfrac{5}{\sqrt{x+1}}$ **Domain=** expression under root > 0	$f(x)= \dfrac{x}{\sqrt{x}-2}$ • $x \geq 0$ • Bottom expression $\neq 0$ • Take intersection	$f(x)= \dfrac{\sqrt{x+1}}{x^2-4}$ • under root ≥ 0 • Bottom $\neq 0$ • Take intersection.

Example

Polynomial	Fraction only	Square root only
$f(x)= 16+3x-x^2$ Domain $= \mathbb{R} = (-\infty, \infty)$	$f(x)= \dfrac{15}{x-3}$ $x-3 \neq 0 \Rightarrow x \neq 3$ $\therefore$ Domain $= \mathbb{R}-\{3\}$ or $(-\infty, 3) \cup (3, \infty)$	$f(x)= \sqrt{x-3}$ $x-3 \geq 0$ $x \geq 3$ $\therefore$ Domain $= [3, \infty)$
square root on bottom	**Square root on bottom on x only**	**square root on top**
$f(x)= \dfrac{x}{\sqrt{x-2}}$ $x-2 > 0 \Rightarrow x > 2$  Domain $= (2, \infty)$	$f(x)= \dfrac{x}{\sqrt{x}-2}$ • $x \geq 0$ • $\sqrt{x}-2 \neq 0 \Rightarrow \sqrt{x} \neq 2$ $\Rightarrow x \neq 4$ •  Domain $= [0, 4) \cup (4, \infty)$	$f(x)= \dfrac{\sqrt{x+1}}{x+3}$ • $x+1 \geq 0 \Rightarrow x \geq -1$ • $x+3 \neq 0 \Rightarrow x \neq -3$  Domain $= [-1, \infty)$

Example: **Find the domain of each of the following function**

$f(x) = x^2 + 16$

Domain $= \mathbb{R} = (-\infty, \infty)$

$f(x) = \dfrac{x}{x^2+16}$

$x^2 + 16 = 0$. There is no such x.

$\therefore$ Domain $= \mathbb{R} = (-\infty, \infty)$

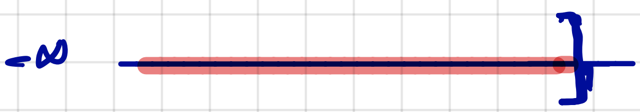
$g(x) = \sqrt{10-2x}$

$10 - 2x \geq 0 \Rightarrow 10 \geq 2x$

$\Rightarrow 5 \geq x$

$x \leq 5$

$\therefore$ Domain $= (-\infty, 5]$



$h(x) = \dfrac{x}{x^3+27}$

$x^3 + 27 \neq 0$

$\Rightarrow x^3 \neq -27 \Rightarrow x \neq \sqrt[3]{-27}$

$\Rightarrow x \neq -3$

$\therefore$ Domain $= \mathbb{R} - \{-3\}$

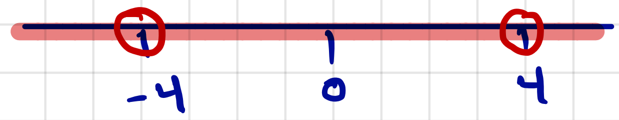
$(-\infty, -3) \cup (-3, \infty)$

$f(x) = \dfrac{x}{x^2-16}$

$x^2 - 16 \neq 0$

$(x-4)(x+4) \neq 0$

$x \neq 4$ or $x \neq -4$



$\therefore$ Domain $\mathbb{R}$, $x \neq \mp 4$

or $\mathbb{R} - \{+4, -4\}$

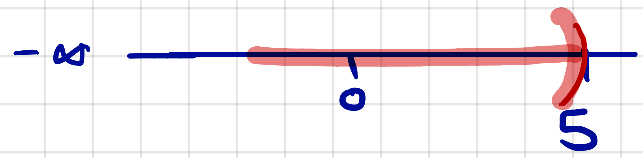
$(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$

$g(x) = \dfrac{2}{\sqrt{10-2x}}$

$10 - 2x > 0 \Rightarrow 10 > 2x$

$5 > x$ or $x < 5$

. Domain $= (-\infty, 5)$



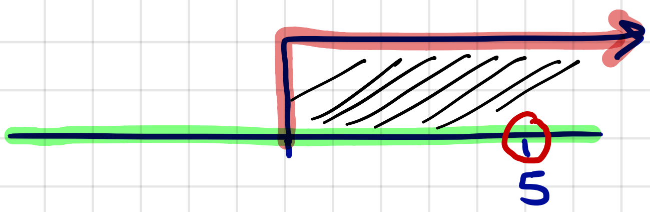
$g(x) = \dfrac{2}{10 - \sqrt{2x}}$

• $2x \geq 0 \Rightarrow x \geq 0$

• $10 - \sqrt{2x} \neq 0$

$10 \neq \sqrt{2x}$

$5 \neq x$



Domain $= [0, 5) \cup (5, \infty)$

4 Evaluating Function

(A) Find $f(6)$, $f(a)$, and $f(6+a)$ for $f(x) = \dfrac{15}{x-3}$.

(B) Find $g(7)$, $g(h)$, and $g(7+h)$ for $g(x) = 16 + 3x - x^2$.

(C) Find $k(9)$, $4k(a)$, and $k(4a)$ for $k(x) = \dfrac{2}{\sqrt{x}-2}$.

SOLUTIONS

(A) $$f(6) = \frac{15}{6-3} = \frac{15}{3} = 5$$

$$f(a) = \frac{15}{a-3}$$

$$f(6+a) = \frac{15}{6+a-3} = \frac{15}{3+a}$$

(B) $$g(7) = 16 + 3(7) - (7)^2 = 16 + 21 - 49 = -12$$

$$g(h) = 16 + 3h - h^2$$

$$g(7+h) = 16 + 3(7+h) - (7+h)^2$$ Remove the first set of parentheses and square the binomial.

$$= 16 + 21 + 3h - (49 + 14h + h^2)$$ Combine like terms and remove the parentheses.

$$= 37 + 3h - 49 - 14h - h^2$$ Combine like terms.

$$= -12 - 11h - h^2$$

(C) $$k(9) = \frac{2}{\sqrt{9}-2} = \frac{2}{3-2} = 2$$ $\sqrt{9} = 3$, not ± 3.

$$4k(a) = 4\frac{2}{\sqrt{a}-2} = \frac{8}{\sqrt{a}-2}$$

$$k(4a) = \frac{2}{\sqrt{4a}-2}$$ $\sqrt{4a} = \sqrt{4}\sqrt{a} = 2\sqrt{a}$.

$$= \frac{2}{2\sqrt{a}-2}$$ Divide numerator and denominator by 2.

$$= \frac{1}{\sqrt{a}-1}$$

Evaluating and Simplifying a Difference Quotient

For $f(x) = x^2 + 4x + 5$, find and simplify:

(A) $f(x + h)$ (B) $f(x + h) - f(x)$ (C) $\dfrac{f(x + h) - f(x)}{h}, h \neq 0$

SOLUTIONS

(A) To find $f(x + h)$, we replace x with $x + h$ everywhere it appears in the equation that defines f and simplify:

$$\begin{aligned} f(x + h) &= (x + h)^2 + 4(x + h) + 5 \\ &= x^2 + 2xh + h^2 + 4x + 4h + 5 \end{aligned}$$

(B) Using the result of part A, we get

$$\begin{aligned} f(x + h) - f(x) &= x^2 + 2xh + h^2 + 4x + 4h + 5 - (x^2 + 4x + 5) \\ &= x^2 + 2xh + h^2 + 4x + 4h + 5 - x^2 - 4x - 5 \\ &= 2xh + h^2 + 4h \end{aligned}$$

(C) $$\begin{aligned} \frac{f(x + h) - f(x)}{h} &= \frac{2xh + h^2 + 4h}{h} = \frac{\cancel{h}(2x + h + 4)}{\cancel{h}} \\ &= 2x + h + 4 \end{aligned}$$

الدرس التالي
graphing)
(function

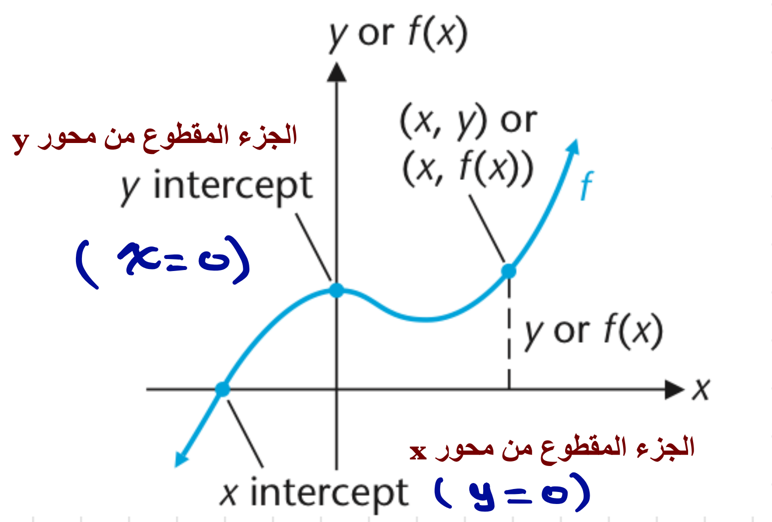
Scan me

Graphing Functions

1. Intercepts of a function
2. Finding the domain &Range from a graph
3. Identifying increasing & decreasing function
4. Linear Function
5. Piecewise Functions

Graphing Function

1️⃣ Intercepts of a Function



Example: **find the domain, x intercept, y intercept of** $f(x) = \frac{4-3x}{2x+5}$

Solution:

$2x+5 = 0 \Rightarrow 2x = -5$

$\Rightarrow x = -\frac{5}{2}$

$\therefore$ Domain $= R - \{-\frac{5}{2}\}$

$= (-\infty, -\frac{5}{2}) \cup (-\frac{5}{2}, \infty)$

- x-intercept ($y=0$)

$0 = 4 - 3x$

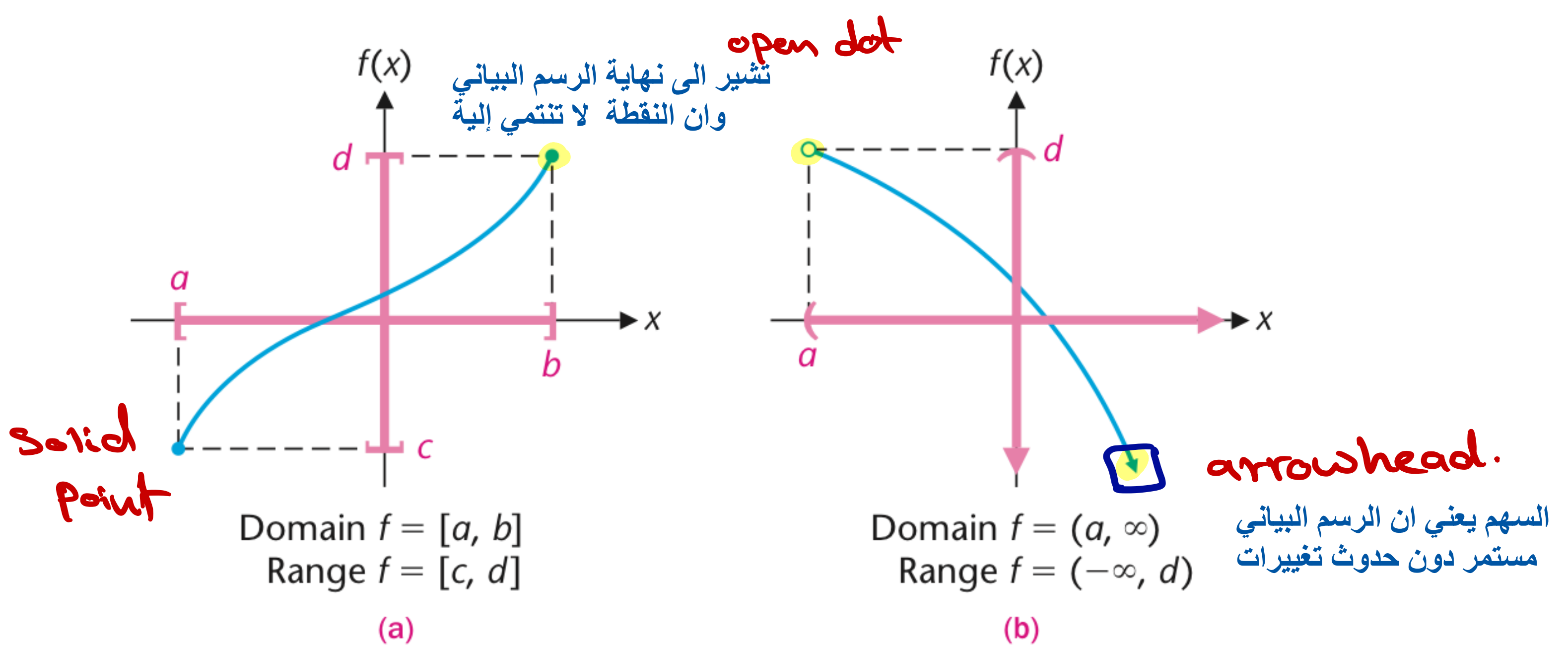
$3x = 4$

$\Rightarrow x = \frac{4}{3}$

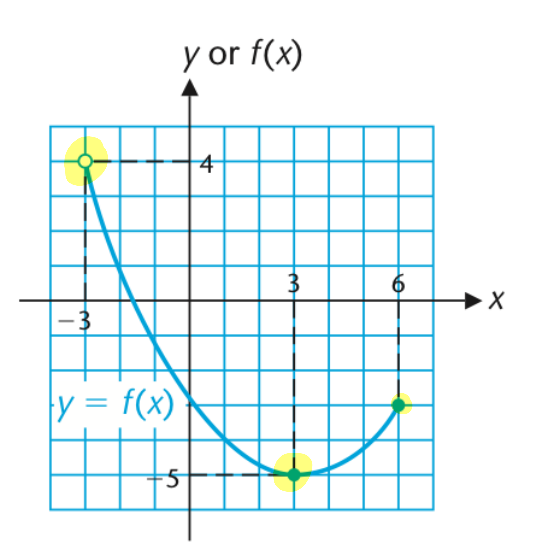
- y-intercept ($x=0$)

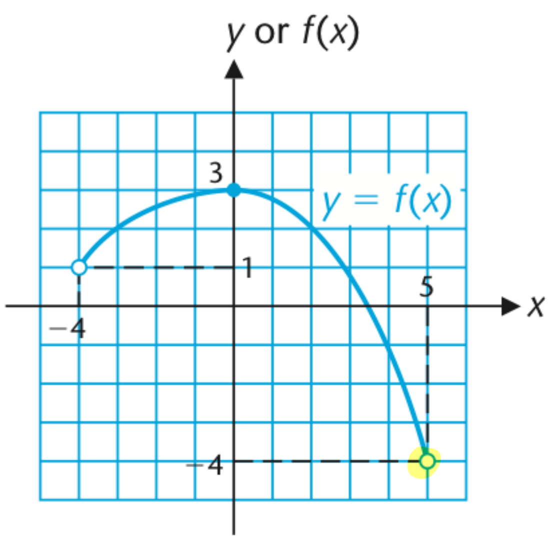
$f(0) = \frac{4-3(0)}{2(0)+5} = \frac{4}{5}$

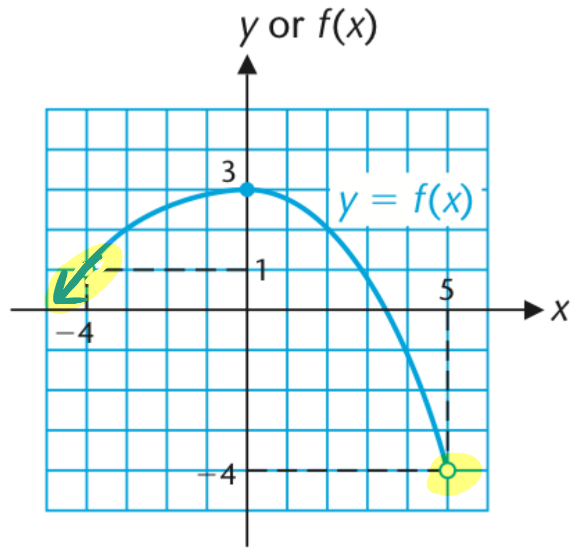
2 Finding the Domain and Range from the Graph



Example: **Find the domain and range for each graph**







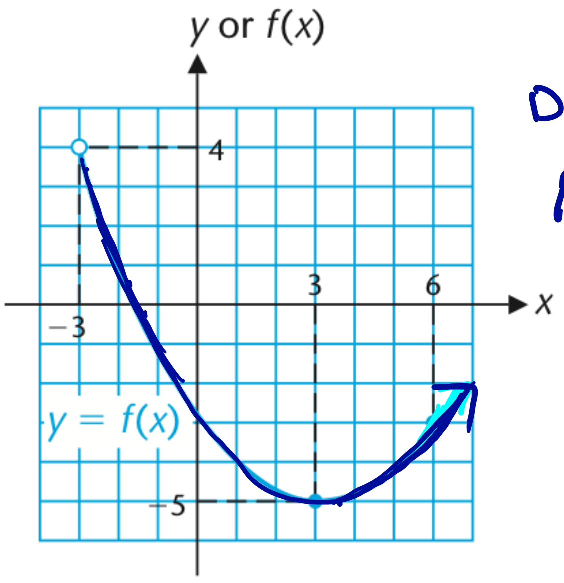
Domain = (-3, 6]
Range = [-5, 4)
f(3) = -5

Domain = (-4, 5)
Range = (-4, 3]
f(0) = 3

Domain = (-∞, 5)
Range = (-4, 3]
f(3) = 0

HW: **Find the domain and range for the following graph**

Find f(1), f(3), f(5).



Domain = (-3, ∞)

Range [-5, 4)

f(3) = -5

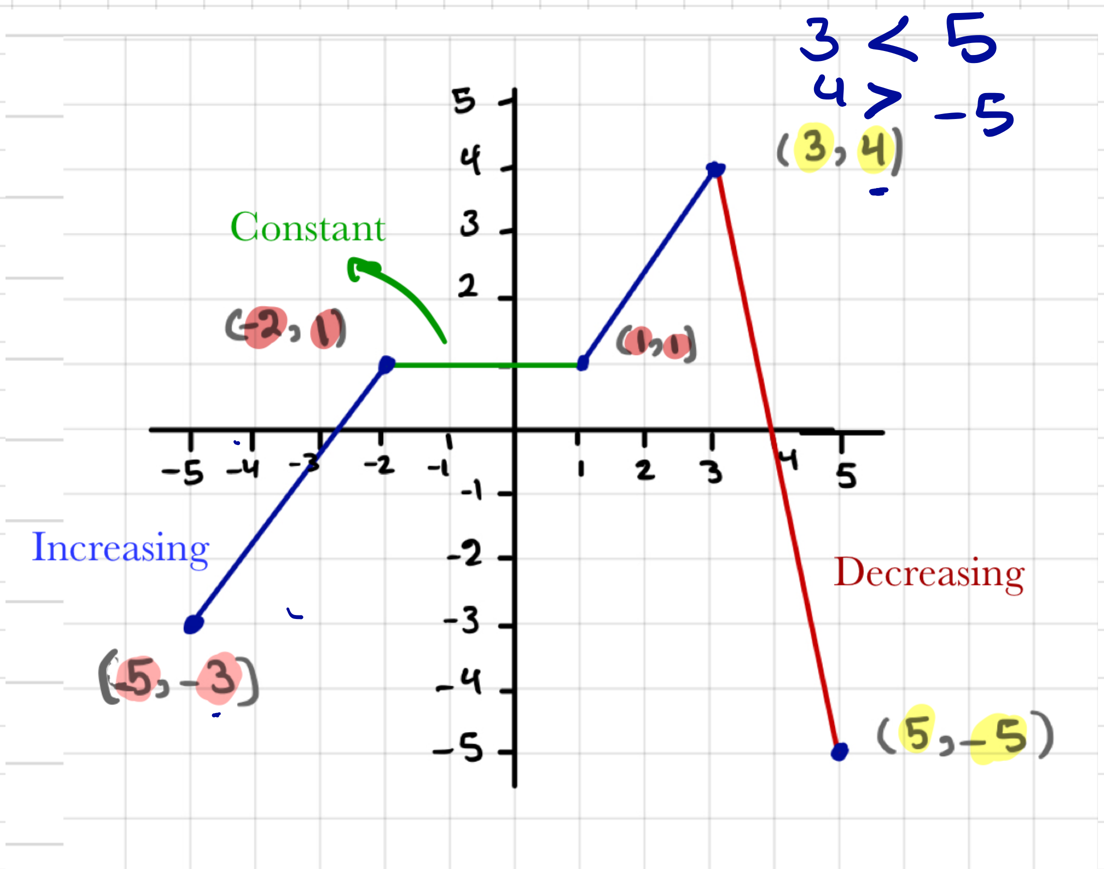
f(6) = -3

f(4) =

تمارين : إيجاد المدى
والمجال من التمثيل البياني

SCAN ME

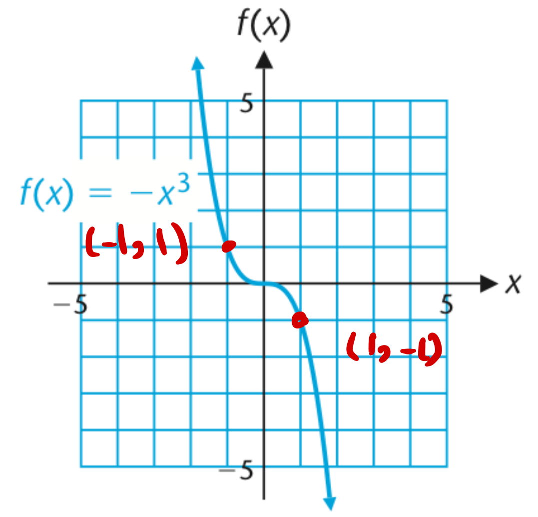
3 Identifying increasing and decreasing function.



Increasing: $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$

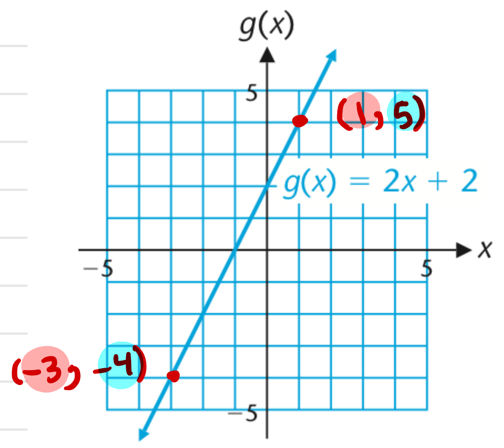
Decreasing: $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$

Constant: $x_1 < x_2 \Rightarrow f(x_1) = f(x_2)$.

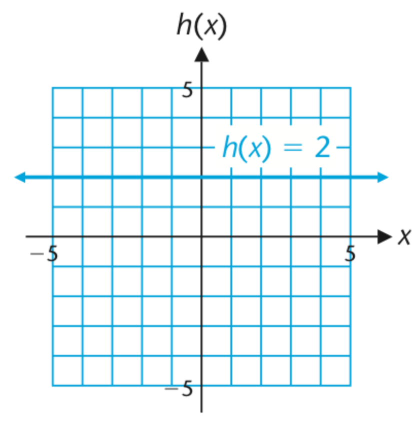


Decreasing on (−∞, ∞)

(a)

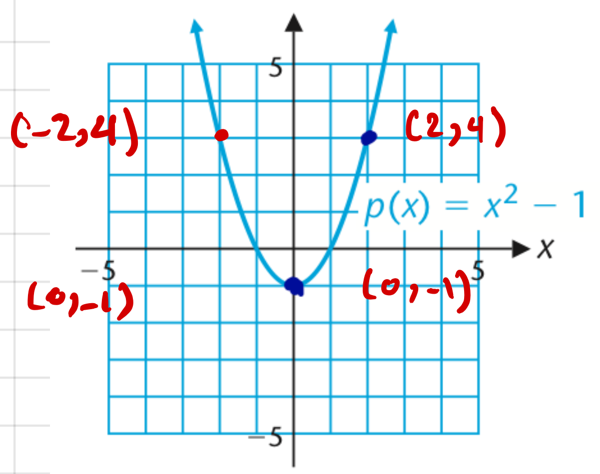


Increasing on (−∞, ∞)



Constant on (−∞, ∞)

(c)



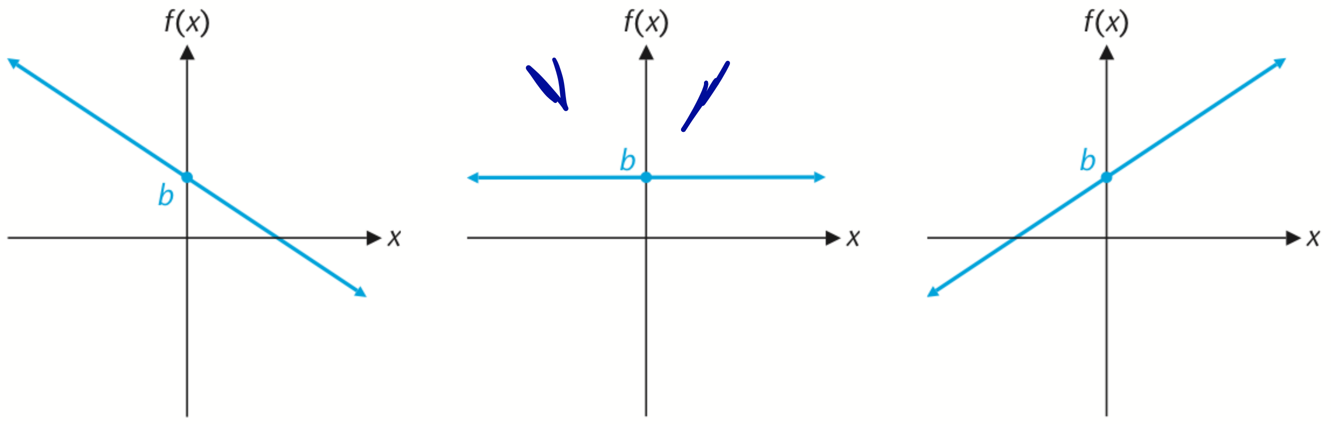
Decreasing on (−∞, 0]
Increasing on [0, ∞)

(d)

4 Linear Function

GRAPH PROPERTIES OF $f(x) = mx + b$

The graph of a linear function is a line with slope m and y intercept b.



$m < 0$	$m = 0$	$m > 0$
Decreasing on $(-\infty, \infty)$	Constant on $(-\infty, \infty)$	Increasing on $(-\infty, \infty)$
Domain: $(-\infty, \infty)$	Domain: $(-\infty, \infty)$	Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$	Range: $\{b\}$	Range: $(-\infty, \infty)$

5 Piecewise-Defined Function

Functions whose definitions involve more than one expression are called Piecwise- defined functions

Example:

The function f is defined by

$$f(x) = \begin{cases} 4x + 11 & \text{if } x < -2 \\ 3 & \text{if } -2 \le x \le 1 \\ -\frac{1}{2}x + \frac{7}{2} & \text{if } x > 1 \end{cases}$$

(A) Find $f(-3)$, $f(-2)$, $f(1)$, and $f(3)$.

(B) Graph f.

(C) Find the domain, range, and intervals where f is increasing, decreasing, or constant.

Piecewise-Defined Function

SOLUTIONS

(A) For $x < -2$, $f(x) = 4x + 11$, so

$$f(-3) = 4(-3) + 11 = -1$$

For $-2 \le x \le 1$, $f(x) = 3$, so

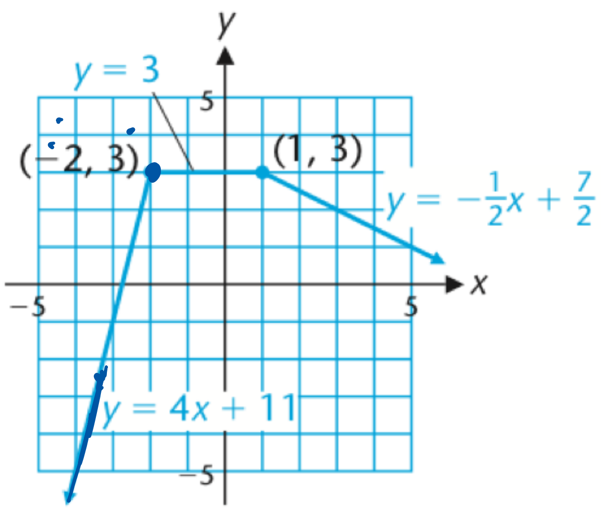
$$f(-2) = 3 \qquad \text{and} \qquad f(1) = 3$$

For $x > 1$, $f(x) = -\frac{1}{2}x + \frac{7}{2}$, so

$$f(3) = -\tfrac{1}{2}(3) + \tfrac{7}{2} = 2$$

(B) To graph f, we graph each expression in the definition of f over the appropriate interval. That is, we graph

$$\begin{array}{ll} y = 4x + 11 & \text{for } x < -2 \\ y = 3 & \text{for } -2 \le x \le 1 \\ y = -\frac{1}{2}x + \frac{7}{2} & \text{for } x > 1 \end{array}$$



(C) Domain of f: $(-\infty, -2) \cup [-2, 1] \cup (1, \infty) = (-\infty, \infty)$

Range : $(-\infty, 3]$

F increasing on $(-\infty, -2)$

decreasing on $(1, \infty)$

Constant on $[-2, 1]$

Even and odd Function

Algebraically: A function is

Even : if $f(-x) = f(x)$

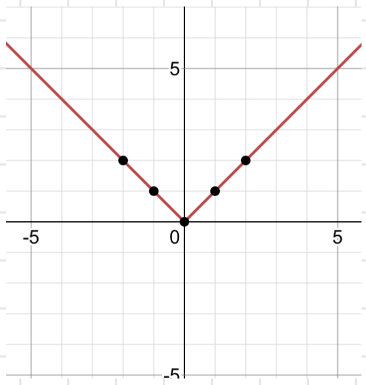
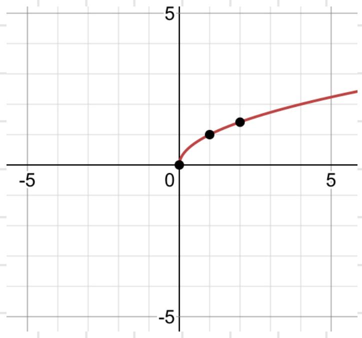
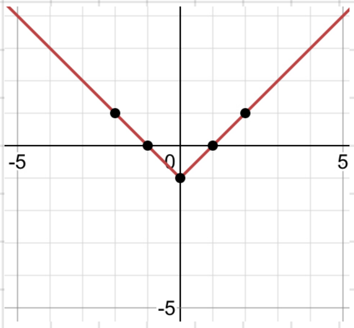
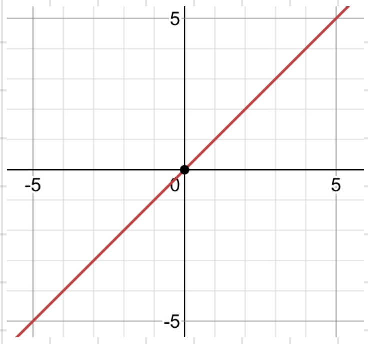
odd : if $f(-x) = -f(x)$.

Graphically:

Even function: symmetric with respect to y axis.

Odd function: symmetric with respect to origin.

Example	Solution	Comments.
$f(x) = x^2 + 1$	$f(-x) = (-x)^2 + 1$ $= x^2 + 1$ $= f(x)$ $\therefore f(x)$ is even	إذا كانت جميع أسس المتغير x زوجية فإن الدالة المعطاه زوجيه ملاحظة: الحد الثابت يعتبر زوجي لانه عبارة عن $1.x^0$ والصفر زوجي .
$f(x) = x^3 + x$	$f(-x) = (-x)^3 + (-x)$ $= -x^3 - x$ $= -(x^3 + x)$ $= -f(x)$ $\therefore f(x)$ is odd	إذا كانت جميع أسس المتغير x فردية ولا تحتوي على عدد ثابت فإن الدالة المعطاة فرديه.
$f(x) = x^4 + 3x.$	$f(-x) = (-x)^4 + 3(-x)$ $= x^4 - 3x$ $\neq f(x)$ $-f(x) = -x^4 - 3x$ $\neq f(x)$ Niether	إذا كانت أسس المتغير في الدالة المعطاة زوجي وفردي فإن الدالة لا زوجية ولا فردية.

Examples	Solutions	Comments.
$f(x) = \lvert x \rvert$	$f(-x) = \lvert -x \rvert = \lvert x \rvert = f(x)$ Even	
$f(x) = \sqrt{x}$	$f(-x) = \sqrt{-x} \neq f(x)$ $-f(x) = -\sqrt{x} \neq f(x)$ Neither	
$f(x) = \lvert x \rvert - 1$	$f(-x) = \lvert -x \rvert - 1$ $= \lvert x \rvert - 1$ $= f(x)$ Even.	
$f(x) = -x$	$f(-x) = -(-x) = -f(x)$ odd	

Remark:

$E \pm E = E$

$O \pm O = O$

$E \pm O =$ Nither

$E \times E = E$

$O \times O = E$

$E \times O = O$

$E / E = E$

$O / O = E$

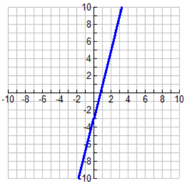
$E / O = O$

Even, Odd, or Neither Worksheet

____

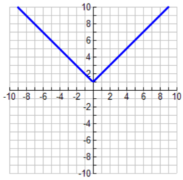
Determine whether the following functions are even, odd, or neither.

1. f(x)= 4x – 3



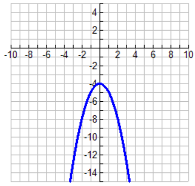
Nither

2. f(x)= $|x| + 1$



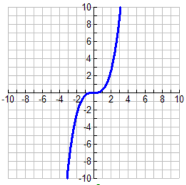
Even

3. f(x)= $-x^2 - 4$



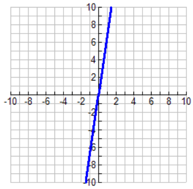
Even

4. f(x)= $\frac{1}{3}x^3$



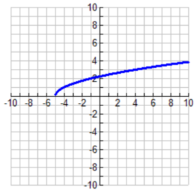
odd

5. f(x)= 7x



odd

6. f(x)= $\sqrt{x+5}$



Nither

Even

7. f(x)= $3x^2$

$f(-x) = 3(-x)^2$

$= 3x^2$

$= f(x)$

Even

odd Even

8. f(x) = $x^3 - 2$

$f(-x) = (-x)^3 - 2$

$= -x^3 - 2$

$\neq f(x)$

$-f(x) = -(x^3 - 2)$

$= -x^3 + 2$

$\neq f(x)$

Neither

odd Even

9. f(x)= 3x + 4

$f(-x) = 3(-x) + 4$

$= -3x + 4$

$\neq f(x)$

$-f(x) = -(3x + 4)$

$= -3x - 4$

$\neq f(x).$

Neither.

Even Even

10. $f(x) = x^2 - 5$

$f(-x) = (-x)^2 - 5$

$= x^2 - 5$

$= f(x)$

Even

odd even

11. $f(x) = 10x + 5$

$f(-x) = 10(-x) + 5$

$= -10x - 5$

$\neq f(x)$

$-f(x) = -(10x + 5)$

$= -10x - 5$

$\neq f(x)$

Niether

12. $f(x) = 2(x+1)^2$

$f(x) = 2(x^2 + 2x + 2)$

even odd

$= 2x^2 + 4x + 4$

$f(-x) = 2(-x^2) + 4(-x) + 4$

$= 2x^2 - 4x + 4$

$\neq f(x)$

$-f(x) = -(2x^2 + 4x + 4)$

$= -2x^2 - 4x - 4$

$\neq f(x)$ Nither

Multiple Choice Questions

1)- whic of the following function is neither even nor odd.

a) $f(x) = 3$ b) $f(x) = x$ c) $x - 1$ d) $f(x) = |x|$

2)- which of the following function is an odd function.

a) $f(x) = 3x^5$ b) $f(x) = x^2$ c) $f(x) = x^4$ d) $f(x) = 2x^8$

3)- The function $f(x) = 5$ is an even function.

a)- True b)- False.

4)- The function $f(x) = \dfrac{\overset{\text{odd}}{x}}{\underset{\text{even}}{x^2 - 1}}$ is $\quad \frac{0}{E} = 0$

a) even b)- odd c)- Nither

Quadratic Functions

1. Definition and properties
2. How to convert from vertex form to standard and vise verse.
3. Find the equation from Given properties.
4. Solving quadratic inequalities

1

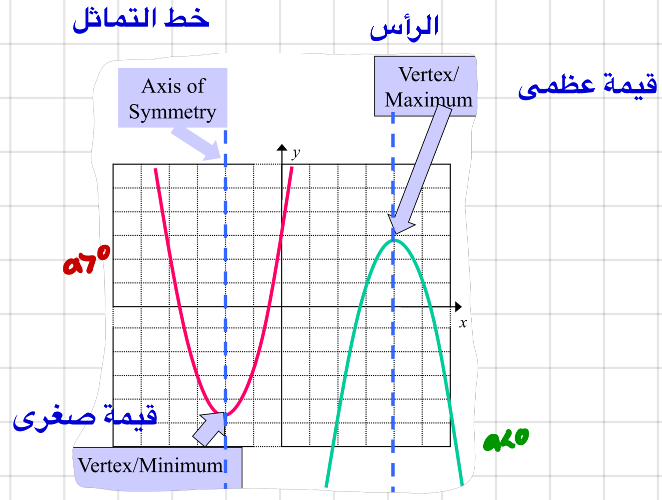
Quadratic Functions

Quadratic function: Any functions that contains an x^2 term.

Standard Form : $f(x) = ax^2 + bx + c \quad , \quad a \neq 0$

Vertex Form : $f(x) = a(x-h)^2 + k \quad , \quad a \neq 0$

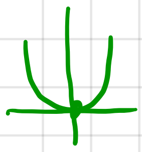
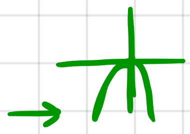
- The graph of a quadratic function is called **Parabola.** (U shape)



Type of Form / Properties	Vertex Form	General Form
	$f(x) = a(x-h)^2 + k$	$ax^2 + bx + c$
Vertex	(h, k)	$\left(-\frac{b}{2a}, f\left(\frac{-b}{2a}\right)\right)$
Axis of Symmetry	$x = h$	$x = \frac{-b}{2a}$
Domain	$R = (-\infty, \infty)$	$R = (-\infty, \infty)$
Range	$(-\infty, k]$ if $a<0$ $[k, \infty)$ if $a>0$	$\left(-\infty, f\left(\frac{-b}{2a}\right)\right]$, $a<0$ $\left[f\left(\frac{-b}{2a}\right), \infty\right)$, $a>0$
Open (Up or down)	up , $a>0$ down , $a<0$	up , $a>0$ down , $a<0$
Max/Min	$a<0$ max $a>0$ min $\Big\} = k$	$a<0$ max $a>0$ min $\Big\} = f\left(\frac{-b}{2a}\right)$
Increasing and Decreasing Intervals	$(-\infty, h)$, (h, ∞)	$\left(-\infty, \frac{-b}{2a}\right)$ $\left(\frac{-b}{2a}, \infty\right)$

Quadratic Functions

Vertex Form

Properties \ Form	$f(x) = 2(x-2)^2 - 4$	$f(x) = -(x-2)^2 + 6$
Vertex	$(2, -4)$	$(2, 6)$
Domain	$\mathbb{R}$	$\mathbb{R}$
Range	$[-4, \infty)$	$(-\infty, 6]$
Axis of symmetry	$x = h = 2$	$x = 2$
Open (up or down)	up	Down
Max /Min Value	Min value: -4	max value: 6
Increasing or decreasing interval	Inc on $(2, \infty)$ Dec on $(-\infty, 2)$ 	Inc $(-\infty, 2]$ Dec on $[2, \infty)$ 

General Form

$$f(x) = 2x^2 - 8x + 4$$

vertex: $x = \frac{-b}{2a} = \frac{-(-8)}{2.2} = \frac{8}{4} = 2$

$f(\frac{-b}{2a}) = f(2) = 2(2)^2 - 8(2) + 4$

$= 2.4 - 16 + 4 = -4$

$\therefore$ vertex $(2, -4)$

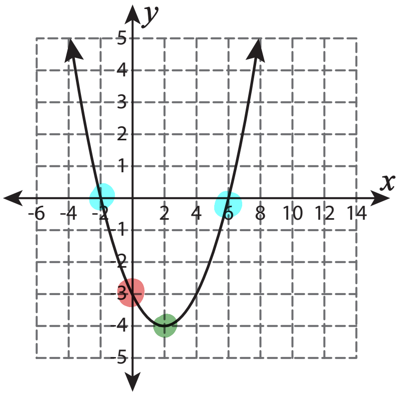
- Domain: $\mathbb{R}$
- Range: $[-4, \infty)$
- Axis: $x = 2$
- open: up.
- Inc on $[2, \infty)$
  dec on $(-\infty, 2]$

Properties of Quadratic Function

Sheet 1

Find the properties of each quadratic function.

1)



Domain : **Real Numbers**

Range : **{ y is real : $y \geq -4$ }**

x-intercepts : **(–2, 0) and (6, 0)**

y-intercept : **(0, –3)**

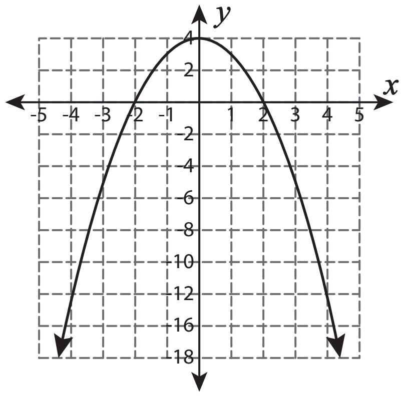
Vertex : **(2, –4)**

Minimum value : **$y = -4$** or k=-4

Axis of symmetry : **$x = 2$** or h= 2

Open up or down : **Up**

2)



Domain : **Real Numbers**

Range : **{ y is real : $y \leq 4$ }**

x-intercepts : **(–2, 0) and (2, 0)**

y-intercept : **(0, 4)**

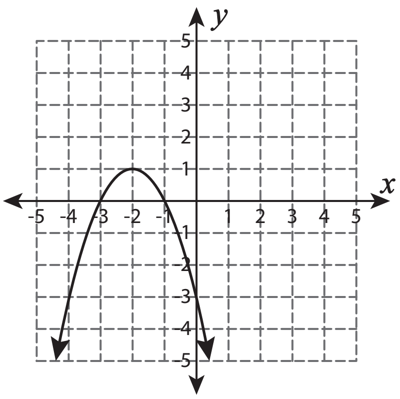
Vertex : **(0, 4)**

Maximum value : **$y = 4$**

Axis of symmetry : **$x = 0$**

Open up or down : **Down**

3)



Domain : **Real Numbers**

Range : **{ y is real : $y \leq 1$ }**

x-intercepts : **(–3, 0) and (–1, 0)**

y-intercept : **(0, –3)**

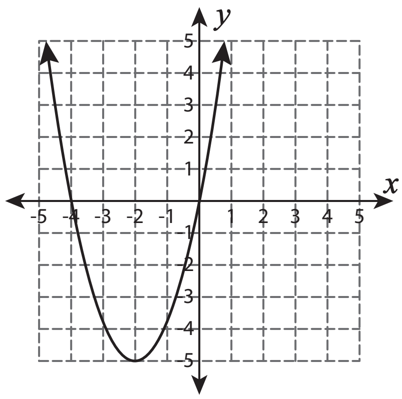
Vertex : **(–2, 1)**

Maximum value : **$y = 1$**

Axis of symmetry : **$x = -2$**

Open up or down : **Down**

4)



Domain : **Real Numbers**

Range : **{ y is real : $y \geq -5$ }**

x-intercepts : **(–4, 0) and (0, 0)**

y-intercept : **(0, 0)**

Vertex : **(–2, –5)**

Minimum value : **$y = -5$**

Axis of symmetry : **$x = -2$**

Open up or down : **Up**

2 How to convert from standard form to vertex form

Example : Convert the following quadratic equations from standard form to vertex form

- $F(x) = 3x^2 - 18x + 5$

$a = 3$, $b = -18$

$x = \frac{-b}{2a} = \frac{-(-18)}{2 \cdot 3} = \frac{18}{6} = 3$

$F(3) = 3(3)^2 - 18(3) + 5 = -22$

$\therefore$ vertex : $(3, -22)$

$f(x) = a(x-h)^2 + k$

$= 3(x-3)^2 - 22$

How to convert from vertex form to standard form

Example : Convert the following quadratic equations from vertex form to standard form.

- $f(x) = (x-4)^2 - 1$

$= (x^2 - 8x + 16) - 1 = x^2 - 8x + 15$

- $F(x) = 2(x+3)^2 - 3$

$= 2(x^2 + 6x + 9) - 3$

$= 2x^2 + 12x + 18 - 3$

$= 2x^2 + 12x + 15$

Find the equation of a quadratic function that satisfy the given properties

Properties	**Equation**
• vertex : $(3, -2)$ • x intercept : 4 $(4, 0)$	$f(x) = a(x-3)^2 - 2$ x intercept 4 $\Rightarrow f(4) = 0$ $\Rightarrow a(4-3)^2 - 2 = 0$ $\Rightarrow a - 2 = 0 \Rightarrow a = 2$ $\therefore f(x) = 2(x-3)^2 - 2$
• vertex : $(4, -2)$ • y intercept : 2 $(0, 2)$	$f(x) = a(x-4)^2 - 2$ y intercept $f(0) = 2$ $\Rightarrow a(0-4)^2 - 2 = 2$ $\Rightarrow 16a = 2 + 2$ $\Rightarrow 16a = 4 \Rightarrow a = \frac{4}{16} = \frac{1}{4}$ $\therefore f(x) = \frac{1}{4}(x-4)^2 - 2$
• vertex : $(-3, -4)$ • addtional point $(1, 60)$	$f(x) = a(x+3)^2 - 4$ $(1, 60) \Rightarrow f(1) = 60$ $\Rightarrow a(1+3)^2 - 4 = 60$ $\Rightarrow 16a = 64$ $\Rightarrow a = 4$ $\therefore f(x) = 4(x+3)^2 - 4$

لحل أي معادلة من الدرجة الثانيه

$$x = \frac{-b \pm \sqrt{4ac - b^2}}{2a}$$

3 Solving Quadratic Inequalities

Solve : $x^2 - x > 12$

$x^2 - x - 12 > 0$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$\Rightarrow \quad x = 4 \quad$ or $\quad x = -3$

$(-\infty, -3) \cup (4, \infty)$

Solve: $x^2 - 4x \geq 14$

$x^2 - 4x - 14 \geq 0$

$x^2 - 4x - 14 = 0$

$$x = \frac{-(-4) \pm \sqrt{4(1)(-14) - (-4)^2}}{2(1)}$$

$$= \frac{4 \pm \sqrt{72}}{2} = \frac{4 \pm 6\sqrt{2}}{2}$$

$= \quad 2 + 3\sqrt{2} \quad$ and $\quad 2 - 3\sqrt{2}$

$(-\infty, 2-3\sqrt{2}) \cup (2+3\sqrt{2}, \infty)$

Operation on Functions

1️⃣ **Definition.**

2️⃣ **Composition.**

1

Operation on Functions

› DEFINITION 1 Operations on Functions

The **sum, difference, product,** and **quotient** of the functions f and g are the functions defined by

Sum function	$(f+g)(x) = f(x) + g(x)$	D: $A \cap B$
Difference function	$(f-g)(x) = f(x) - g(x)$	D: $A \cap B$
Product function	$(fg)(x) = f(x)g(x)$	D: $A \cap B$
Quotient function	$\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} \quad g(x) \neq 0$	D: $\{x \in A \cap B,\ g(x) \neq 0\}$

Example 1: Let $f(x) = x^2 - 3$ and $g(x) = 2x + 5$, find $f+g$, $f-g$, fg, f/g and their domain.

- $(f+g)(x) = f(x) + g(x)$

$$= x^2 - 3 + 2x + 5$$

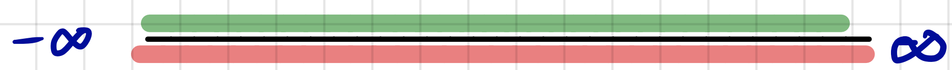
$$= x^2 + 2x + 2$$

$\therefore D(f+g) = A \cap B = (-\infty, \infty)$

$A = D(f) = \mathbb{R} = (-\infty, \infty)$

$B = D(g) = \mathbb{R} = (-\infty, \infty)$

$A \cap B = (-\infty, \infty)$



- $(f-g)(x) = f(x) - g(x)$

$$= x^2 - 3 - (2x + 5)$$

$$= x^2 - 3 - 2x - 5$$

$$= x^2 - 2x - 8$$

$\therefore D(f-g) = A \cap B = (-\infty, \infty)$

- $(fg)(x) = f(x)\, g(x)$

$$= (x^2 - 3)(2x + 5)$$

$$= 2x^3 + 5x^2 - 6x - 15$$

$\therefore D(fg) = A \cap B = (-\infty, \infty)$

- $\left(\frac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$

$$= \frac{x^2-3}{2x+5}$$

$g(x) \neq 0 \Rightarrow 2x+5 \neq 0$

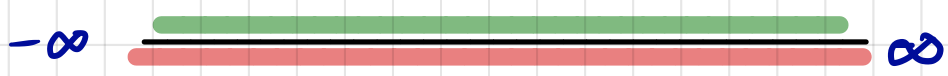
$\Rightarrow 2x \neq -5$

$\Rightarrow x \neq \frac{-5}{2}$

$A = D(f) = \mathbb{R} = (-\infty, \infty)$

$B = D(g) = \mathbb{R} = (-\infty, \infty)$

$D(f/g) = \{x \in A \cap B,\ g(x) \neq 0\}$



$\therefore D(f/g) = \{x \in \mathbb{R},\ g(x) \neq -\frac{2}{3}\}$

$= \mathbb{R} - \{-\frac{2}{3}\}$

Example 2: Let $f(x) = \sqrt{4-x}$ and $g(x) = \sqrt{3+x}$, find $f+g$, $f-g$, fg, f/g and their domain.

- $(f+g)(x) = f(x) + g(x)$

$= \sqrt{4-x} + \sqrt{3+x}$

$\therefore D(f+g)(x) = A \cap B$

$= [-3, 4]$

$A = D(f):\ 4-x \geq 0$

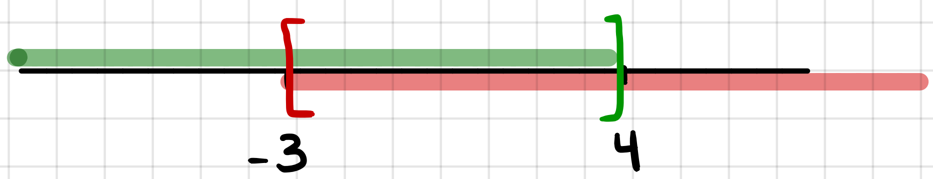
$4 \geq x \Rightarrow x \leq 4$

$\therefore D(f) = (-\infty, 4]$

$B = D(g):\ 3+x \geq 0$

$x \geq -3$

$\therefore D(g) = [-3, \infty)$



- $(f-g)(x) = f(x) - g(x)$

$= \sqrt{4-x} - \sqrt{3+x}$

$\therefore D(f-g)(x) = A \cap B$

$= [-3, 4]$

$(fg)(x) = f(x)\, g(x)$

$= \sqrt{4-x}\,\sqrt{3+x}$

$= \sqrt{(4-x)(3+x)}$

$= \sqrt{12+4x-3x-x^2}$

$= \sqrt{12+x-x^2}$

$\therefore D(fg)=(x)= A \cap B = [-3, 4]$

$A = D(f): 4-x \geq 0$

$4 \geq x \Rightarrow x \leq 4$

$\therefore D(f) = (-\infty, 4]$

$B = D(g): 3+x \geq 0$

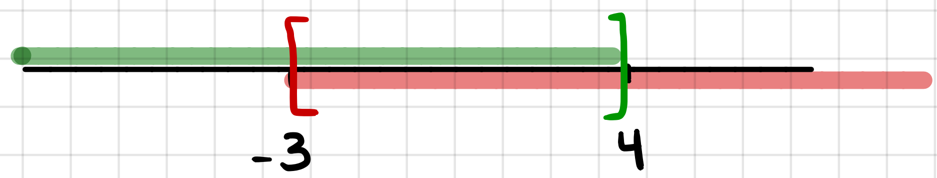
$x \geq -3$

$\therefore D(g) = [-3, \infty)$

- $\left(\frac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$

$= \dfrac{\sqrt{4-x}}{\sqrt{3+x}}$

$= \sqrt{\dfrac{4-x}{3+x}}$



$D\left(\frac{f}{g}\right) = \{x \in A \cap B,\ g(x) \neq 0\}$

$= \{x \in [-3, 4],\ 3+x \neq 0\}$

$= \{x \in [-3, 4],\ x \neq -3\}$

$= (-3, 4]$

Example 3: Let $f(x) = \dfrac{x}{x-1}$ and $g(x) = \dfrac{x-4}{x+3}$.

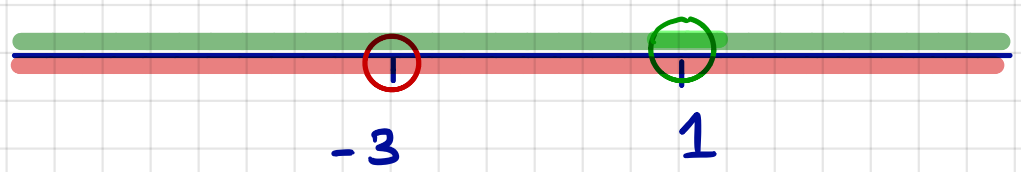
Find the function $\frac{f}{g}$ and find its domain

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\dfrac{x}{x-1}}{\dfrac{x-4}{x+3}}$$

$$= \frac{x}{x-1} \cdot \frac{x+3}{x-4}$$

$$= \frac{x\,(x+3)}{(x-1)\,(x-4)}$$

$A = D(f) = \mathbb{R} - \{1\}$, $B = D(g) = \mathbb{R} - \{-3\}$



$A \cap B = \mathbb{R} - \{-3, 1\}$

$D\left(\frac{f}{g}\right) = \{x \in A \cap B \,,\, g(x) \neq 0\}$

$g(x) = (x-1)(x-4) \neq 0$

$\Rightarrow \quad x \neq 1 \quad$ or $\quad x = 4$

$\therefore \; D\left(\frac{f}{g}\right) = \{x \in \mathbb{R} - \{-3, 1\} \,,\, g(x) \neq 1, 4\}$

$= \{x \in \mathbb{R} - \{-3, 1, 4\}\}$

2 Compostion

DEFINITION 2 Composition

The **composition** of function f with function g is denoted by $f \circ g$ and is defined by

$$(f \circ g)(x) = f(g(x))$$

The domain of $f \circ g$ is the set of all real numbers x in the domain of g such that $g(x)$ is in the domain of f.

How to find the domain of the composite function

Step 1: Find the domain of inside function. If there are restrictions on the domain, keep them.

نوجد مجال الدله الداخلية
ولوكان هناك قيود يتم حفظها

Step 2: Construct the composite function. find the domain of this new function. If there are restrictions on this domain, add them to the restrictions from step 1.

نقوم بعملية التحصيل المطلوبة
ونوجد مجال الدالة الناتجة عن
عملية التحصيل . لو كان هناك
قيود يتم إضافته للقيود الموجوده
في الخطوة ١

Example 1 : Find $f \circ g\,(x)$ and its domain for each of the following functions:

- $f(x) = x^2 + 2$, $g(x) = \sqrt{3-x}$

$(f \circ g)(x) = f(g(x)) = (\sqrt{3-x})^2 + 2$

$= 3 - x + 2$

$= 5 - x$ $\quad \boxed{\text{Domain} = \mathbb{R}}$ لايوجد قيد

$D(g)$: $3 - x \geq 0$

$\Rightarrow 3 \geq x$

$D(g) = (-\infty, 3]$

يوجد قيد

$\therefore D(f \circ g)(x) = (-\infty, 3]$

Example 2: (a) Find $f \circ g$ and $g \circ f$ and the domain of each,

where $f(x) = \frac{3x}{x-1}$ and $g(x) = \frac{2}{x}$

- $f \circ g(x) = f(g(x)) = \frac{3\left(\frac{2}{x}\right)}{\left(\frac{2}{x}\right)-1}$

$\boxed{D(g) = \mathbb{R} - \{0\}}$

يوجد قيد

$$= \frac{\frac{6}{x}}{\frac{2-x}{x}} = \frac{6}{\cancel{x}} \cdot \frac{\cancel{x}}{2-x}$$

$$= \frac{6}{2-x}$$

$\boxed{\text{Domain: } \mathbb{R} - \{2\}}$ يوجد قيد

$\therefore$ Domain $f \circ g$: $\mathbb{R} - \{0, 2\}$

- $g \circ f(x) = g(f(x)) = \frac{2}{\left(\frac{3x}{x-1}\right)}$

$\boxed{\mathbb{R} - \{1\}}$

$$= \frac{2}{3x} \cdot \frac{x-1}{1}$$

$$= \frac{2(x-1)}{3x}$$

$\boxed{\text{Domain: } \mathbb{R} - \{0\}}$

$\therefore$ Domain $g \circ f$: $\mathbb{R} - \{0, 1\}$

(b) compute $(f \circ g)(4)$ and $(g \circ f)(3)$

$\because (f \circ g)(x) = \frac{6}{2-x}$ من فقرة a

$\therefore (f \circ g)(4) = \frac{6}{2-4} = \frac{6}{-2} = -3$.

$\because (g \circ f)(x) = \frac{2(x-1)}{3x}$

$\therefore (g \circ f)(2) = \frac{2(2-1)}{3 \cdot 2} = \frac{2}{6} = \frac{1}{3}$

1

Inverse Functions

One to one function : A one-to-one function is a function where each input (x-value) has a unique output (y-value)

Example : **Determine if each the following function is one to one**

$f = \{(7, 3), (8, -5), (-2, 11), (-6, 4)\}$ is one-to-one

$h = \{(-3, 8), (-11, -9), (5, 4), (6, -9)\}$ is not one-to-one

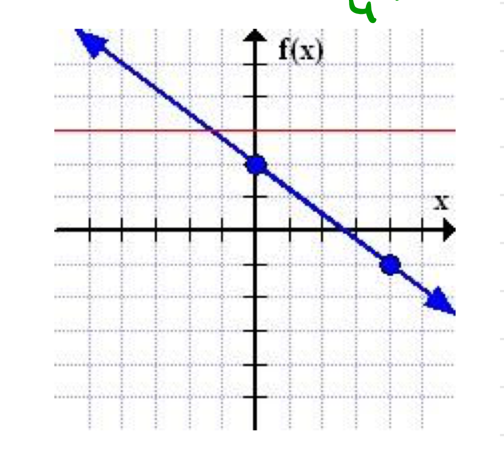
Is the Function a One-to-One Function?

Horizontal Line Test (HLT):

One-to-one: if each HL pass through at most one point on graph.

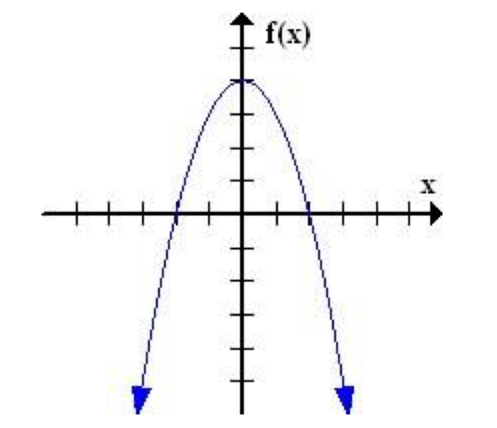
Example Determine if the function $f(x) = -\frac{3}{4}x + 2$ is a one-to-one function.

$f(x) = -\frac{3}{4}x+2$



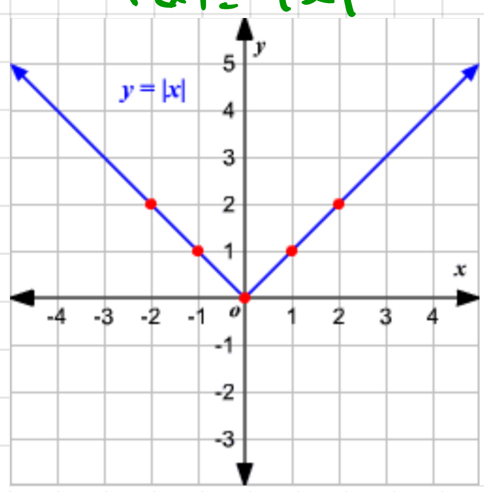
one-to one

$f(x) = -x^4+4$



not one-to-one

$f(x) = |x|$



not one-to-one

2 Finding the inverse of a function

A) Inverse of order pairs function

If f is a one-to-one $\Rightarrow f^{-1} = \{(y,x) : (x,y) \text{ is in } f\}$

If f is not one-to-one $\Rightarrow f^{-1}$ does not exist.

Example: For each of the following function find f^{-1}.

$f = \{(-3,9), (0,0), (3,9)\}$

f is not one-to-one, f^{-1} does not exist.

$\{\{(2,1), (4,2) (9,3)\}$

$f = \{(1,2), (2,4), (3,9)\}$

f is one-to-one, $f^{-1} = \{(2,1), (4,2), (9,3)\}$

Domain $f^{-1} = \{2, 4, 9\}$ = Range f.

Range $f^{-1} = \{1, 2, 4\}$ = Domain f.

B) Inverse of the equation function

- **Method 1:**

Step 1: Change f(x) to y.

Step 2: Switch x and y.

Step 3: Solve for y.

Step 4: Change y back to $f^{-1}(x)$.

$f(x) = 2x - 5$

$y = 2x - 5$

$x = 2y - 5$

$x + 5 = 2y$

$y = \dfrac{x+5}{2}$

$f^{-1}(x) = \dfrac{x+5}{2}$

- Method 2 :

$F(x) = 3x + 2$

١ـ نحول كل عملية ضرب لقسمة وكل عملية جمع لطرح والعكس.
٢ـ نعكس الترتيب

$$\begin{array}{cccc}
x & & (x-2)/3 & \to f^{-1}(x) \\
\downarrow & \times 3 & \uparrow & \div 3 \\
3x & & x-2 & \\
\downarrow & +2 & \uparrow & -2 \\
3x+2 & & x &
\end{array}$$

$$f^{-1}(x) = \frac{x-2}{3}.$$

Remark:	Domain of f^{-1} = Range of f.
	Range of f^{-1} = Domain of f.

Example: Find f^{-1} for $f(x) = \sqrt{x-1}$

Method 1:

$y = \sqrt{x-1}$

$x = \sqrt{y-1}$

$x^2 = y-1$

$x^2 + 1 = y$

$\therefore f^{-1}(x) = x^2 + 1$

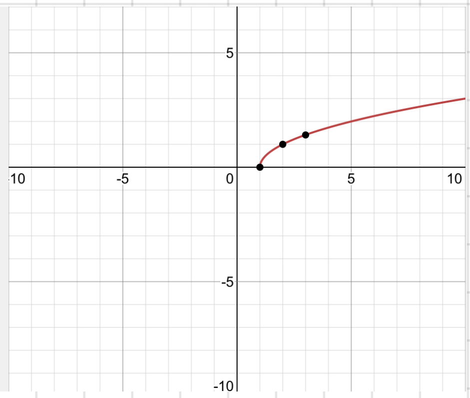
- Domain f^{-1} = Rang f.

$= [0, \infty)$

Method 2:

$$\begin{array}{cccc}
x & & x^2+1 & \to f^{-1}(x) \\
\downarrow & -1 & \uparrow & +1 \\
x-1 & & x^2 & \\
\downarrow & \text{Squar root} & \uparrow & \text{square} \\
\sqrt{x-1} & & x &
\end{array}$$

$\therefore f^{-1}(x) = x^2 + 1$



3 Deciding If Two Functions are Inverses

Remark:
- If f^{-1} exists then

$$f(f^{-1}(x)) = x \quad \text{and} \quad f^{-1}(f(x)) = x$$

- If $f(g(x)) = x$ and $g(f(x)) = x$ then f and g are inverses to each other.

Example: Are two function inverses

$$f(x) = 3x - 7 \qquad g(x) = \frac{x+7}{3}$$

- $f(g(x)) = \cancel{3}\left(\frac{x+7}{\cancel{3}}\right) - 7$

$$= x + 7 - 7 = x$$

- $g(f(x)) = \frac{3x \cancel{-7} \cancel{+7}}{3}$

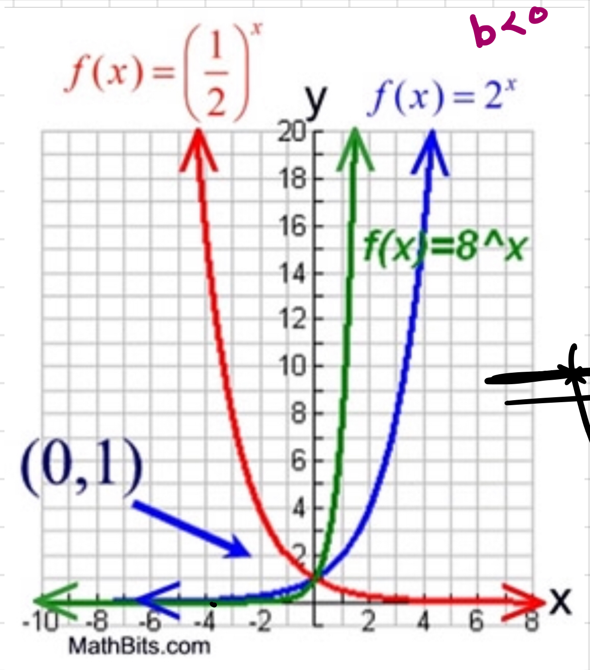
$$= \frac{3x}{3} = x$$

$\therefore$ f and g are inverses.

Exponential and logarithmic Function

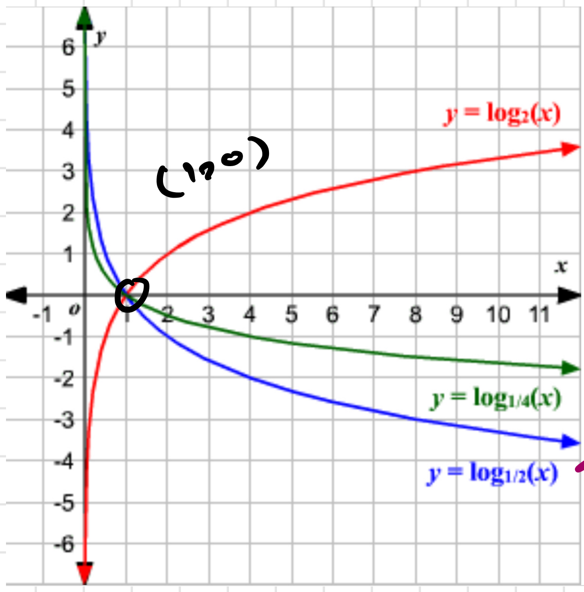
1

Exponential function



b<0

Logarithmic function



(1,0)

- Domain = R = $(-\infty, \infty)$
- Range = $(0, \infty)$
- F(x) pass through (0,1
- F(x) is 1_1
- IF :
- b>0 F(x) is increasing
- b<0 F(x) is decreasing

Domain = $(0, \infty)$

Range = R= $(-\infty, \infty)$

F(x) pass through (1,0)

F(x) is 1_1

IF :

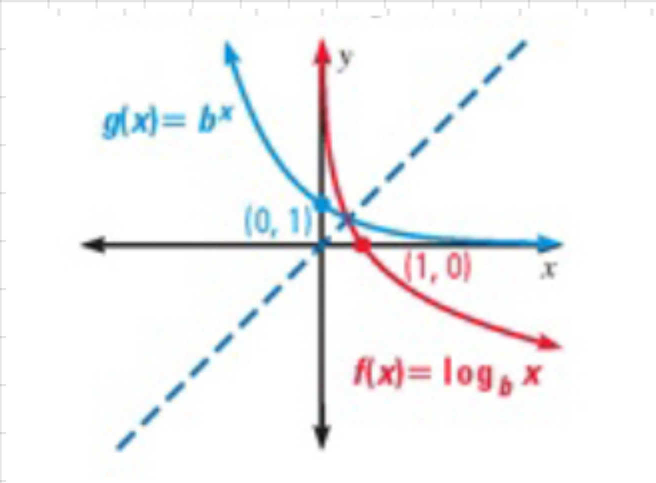
b>0 F(x) is increasing

b<0 F(x ... reasing

العلاقة بين
الدالة الأسية
واللوغاريتمية

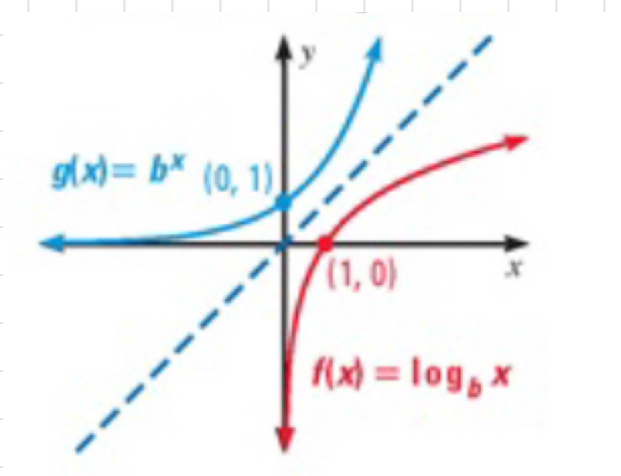
$y = b^x \iff \log_b$

الدالة اللوغاريتمية
هي معكوس
الدالة الأسية



(1,0)

(0,1)



2 Properties of Exp. and Log. Functions

خواص الدالة الأسية واللوغاريتميه

Exponential Function

Remark:

Base b: $y = b^x$

Base e: $y = e^x$

properties:

1. $a^x a^y = a^{x+y}$
2. $(a^x)^y = a^{xy}$
3. $(ab)^x = a^x b^x$
4. $\frac{a^x}{a^y} = a^{x-y}$
5. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
6. $a^{-x} = \frac{1}{a^x}$

8. $a^{\frac{1}{n}} = \sqrt[n]{a}$

$8^{\frac{1}{3}} = \sqrt[3]{8}$

9. $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

Equation properties:

$a^x = a^y \Leftrightarrow x = y$

$a^x = b^x \Leftrightarrow a = b$

Logarithmic Function

Remark:

Base b: $y = \log_b x$

Base e: $y = \log_e x$, $y = \ln x$

Base 10: $y = \log_{10} x$, $y = \log x$

Properties:

Base b

1. $\log_b xy = \log_b x + \log_b y$
2. $\log_b \frac{x}{y} = \log_b x - \log_b y$
3. $\log_b x^y = y \log_b x$

Base e

$\ln xy = \ln x + \ln y$

$\ln \frac{x}{y} = \ln x - \ln y$

$\ln x^y = y \ln x$

Equation properties:

$\log_b x = \log_b y \Leftrightarrow x = y$

usefull propeties:

$\log_b b = 1$ $\qquad$ $\ln e = 1$

$\log_b 1 = 0$ $\qquad$ $\ln 1 = 0$

Inverse properties:

1. $\log_b b^x = x$ $\qquad$ $\ln e^x = x$
2. $b^{\log_b x} = x$ $\qquad$ $e^{\ln x} = x$

يساوي

$\log_3 9 = 2$

اس

$y = b^x \Leftrightarrow \log_b y = x$

Log. Form	Exp. Form
$\log_3 81 = 4$	$3^4 = 81$
$\log_4 \frac{1}{64} = -3$	$4^{-3} = \frac{1}{64}$
$\log_x y = z$	$x^z = y$
$\log_3 1 = 0$	$3^0 = 1$
$\ln 1 = 0$	$e^0 = 1$
$\log 100 = 2$	$10^2 = 100$
$\log_7 7 = 1$	$7^1 = 7$

Exp. Form	Log. Form
$10^3 = 1000$	$\log_{10} 1000 = 3$
$3^{-4} = \frac{1}{81}$	$\log_3 \frac{1}{81} = -4$
$4^{-2} = 1/16$	$\log_4 \frac{1}{16} = -2$
$(\frac{1}{2})^{-5} = 32$	$\log_{\frac{1}{2}} 32 = -5$
$(\frac{1}{3})^{-3} = 27$	$\log_{1/3} 27 = -3$
$\sqrt{x} = y$	$\log_x y = \frac{1}{2}$
$8^2 = 64$	$\log_8 64 = 2$

Evaluate the following :

$\log_4 4 = 1$

$\log_e 1 = 0$

$b^{\log_b 3} = 3$

$\log_e e^{2x+1} = 2x+1$

$\log_5 1 = 0$

$\log_{10} 0.01$

$= \log_{10} 10^{-2}$

$= -2$

$= (4^2)^{\log_4 8}$

$= 4^{2\log_4 8}$

$= 4^{\log_4 8^2}$

$= 8^2 = 64$

3 How to solve Exp. and Log. Function Equations

$|3x+5|+2=6$

1. Exponintial Function

1. Isolate the exponintial expression نفصل الدالة الاسيه
2. we will have two possible cases. يكون لدينا حالتين

Case 1 نفس الأساس

same base
or
can be written to
have the same base

How to solve

1. Apply Exponintial ruls.
2. Solve for x

Case 2 أساس مختلف

Not the same base

How to solve

1. Take log of both sides نأخذ اللوغاريتم للطرفين
2. Apply logs properties نطبق خصائص اللوغاريتم
3. Solve for x

2. Logarithmic Function

log or ln

case 1 كل حد يحتوي على log او ln

Every term has the
word log or ln

How to solve:

1. use properties of log نستخدم خصائص اللوغاريتم
   to condens logs in to one term
   نستخدم خصائص اللوغاريتم كي نختصره إلى حد واحد
2. Cancel log from both sides نحذف اللوغاريتم من الطرفين
3. Solve for x.

Case 2 ليس كل حد يحتوي على log او ln

Not Every term has the
word log or ln

How to solve:

1. Isolat the log expression.
2. use properties of log to condens log in one term.
3. change from log to Exp. form.
4. Solve for x.

$$y=b^{x} \Leftrightarrow \log_b y = x$$

Examples on Exponential Equation

Example: Solve the following Equation:

1. $3^x + 4 = 13$

$3^x = 13 - 4$ فصلنا الدالة الأسية

$3^x = 9$ حصلنا على حالة إمكانية اعادة كتابة الطرف الثاني ليصبح نفس أساس الدالة الأسية

$3^x = 3^2$ تم إعادة الكتابة

$\Rightarrow x = 2$ طبقنا خصائص الدالة الأسيه

2. $3^x + 6 = 9$

$3^x = 9 - 6$

$3^x = 3$

$\Rightarrow x = 1$

3. $3^x - 2 = 12$

$3^x = 12 + 2$ فصلنا الدالة الأسية

$3^x = 14$ حصلنا على حالة عدم إمكانية اعادة كتابة الطرف الثاني ليصبح نفس أساس الدالة الأسية

$\log 3^x = \log 14$ نأخذ اللوغاريتم للطرفين

$x \log 3 = \log 14$ نطبق خصائص اللوغاريتم

$x = \dfrac{\log 14}{\log 3}$ نحل المعادلة بالنسبة ل x

4. $5^x = 5^2$

$x = 2$

6. $2^x = 7$

$\log 2^x = \log 7$

$x \log 2 = \log 7$

$x = \dfrac{\log 7}{\log 2}$

5. $4^{x+2} = 64$

$4^{x+2} = 4^3$

$\Rightarrow x + 2 = 3$

$\Rightarrow x = 3 - 2$

$\Rightarrow x = 1$

7. $6^x = 7^x$

Examples on Logarithmic Equation

case 1:

Solve for x:

1. $\log x - \log 6 = 2\log 4$

$\log\left(\frac{x}{6}\right) = \log 4^2$

$\frac{x}{6} = 16$

$\Rightarrow x = 16 \cdot 6 = 96$

2. $\log_7 3 + \log_7 x = \log_7 32$

$\log_7 (3x) = \log_7 32$

$3x = 32$

$\Rightarrow x = \frac{32}{3} = 10.6$

3. $\log_2 2x = \log_2 100$

$2x = 100$

$x = 50$

4. $\ln(x+4) = \ln 7$

$x+4 = 7$

$x = 7-4$

$x = 3$

$e^2 = 3x$

Case 2:

1. $-6 + \ln 3x = 0$

$\ln 3x = 6$

$3x = e^6$

$x = \frac{e^6}{3} = 134.47$

2. $\log(3x+1) = 2$

$10^2 = 3x+1$

$100 = 3x+1$

$99 = 3x$

$x = 33$

3. $2\log_6 4x = 0 \longrightarrow \log_6 4x = 0$

$6^0 = 4x$

$1 = 4x$

$x = \frac{1}{4}$

4. $2\ln 3x = 4$

$\ln 3x = 2$

$e^2 = 3x$

$x = \frac{e^2}{3} = 2.463$

Find the value of y :

1. $\log_5 25 = y$

$5^y = 25 \Rightarrow 5^y = 5^2$

$\Rightarrow y = 2$

2. $\log_5 1 = y$

$5^y = 1 \Rightarrow y = 0$

3. $\log_y 32 = 5$

$y^5 = 32$

$y^5 = 2^5$

$\Rightarrow y = 2$

4. $\log_3 1 = y$

$3^y = 1 \rightarrow y = 0$

5. $\log_2 8 = y$

$2^y = 8$

$2^y = 2^3$

$\Rightarrow y = 3$

6. $\log_9 y = -\frac{1}{2}$

$9^{-\frac{1}{2}} = y$

$\frac{1}{\sqrt{9}} = y$

$\frac{1}{3} = y$

7. $\log_{16} 4 = y$

$16^y = 4$

$(2^4)^y = 2^2$

$2^{4y} = 2^2$

$\Rightarrow 4y = 2$

$\Rightarrow y = \frac{1}{2}$

8. $\log_7 \frac{1}{7} = y$

$7^y = \frac{1}{7}$

$\Rightarrow y = -1$

9. $\log_4 \frac{1}{8} = y$

$4^y = \frac{1}{8}$

$4^y = 8^{-1}$

$(2^2)^y = (2^3)^{-1}$

$2^{2y} = 2^{-3}$

$2y = -3$

$y = \frac{-3}{2}$

10. $\log_2 \frac{1}{8} = y$

$2^y = \frac{1}{8}$

$2^y = \frac{1}{2^3}$

$2^y = 2^{-3}$

$y = -3$

11. $\log_3 \frac{1}{9} = y$

$3^y = \frac{1}{9}$

$3^y = \frac{1}{3^2}$

$3^y = 3^{-2}$

$\Rightarrow y = -2$

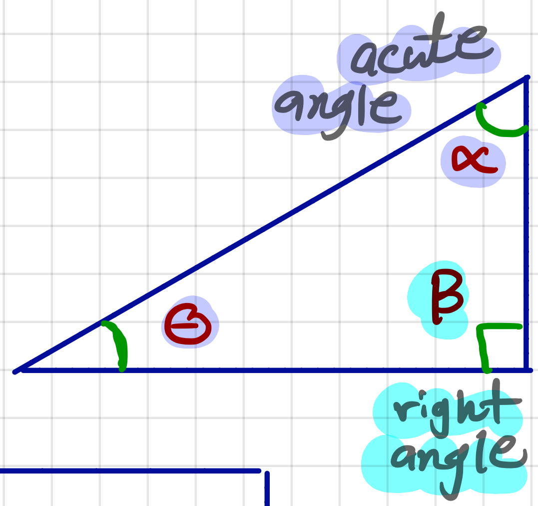
ملاحظات

* جميع الأمثلة هنا على الحالة الثانية من معادلات اللوغاريتم

* الخطوة ١ و٢ متحققة هنا لذلك ننتقل مباشره إلى الخطوة ٣ و ٤
وهي التحويل من ال log الى Exp ونحل لإيجاد المتغير المطلوب

Solving Right Tringles

Right Triangle: one angle is 90°
and tow angle are acute.
اقل من ٩٠ درجة



ملاحظة : لحل أي مسألة في **right triangle** نحتاج فقط لمعرفه شيئين رئيسيين:

1- الكسور المثلثية والتي لها نوعين كسور مثلثيه أساسية وكسور عكسية.

٢- نظرية فيثاغورس وهي عبارة عن مربع طول الوتر يساوي مجموع مربعي طولي الضلعين الآخرين

الكسور المثلثية

Trigonometric Ratios :

كسور أساسية

Basic Ratios

$\sin\theta = \dfrac{opp}{hyp}$

$\cos\theta = \dfrac{adj}{hyp}$

$\tan\theta = \dfrac{opp}{adj}$

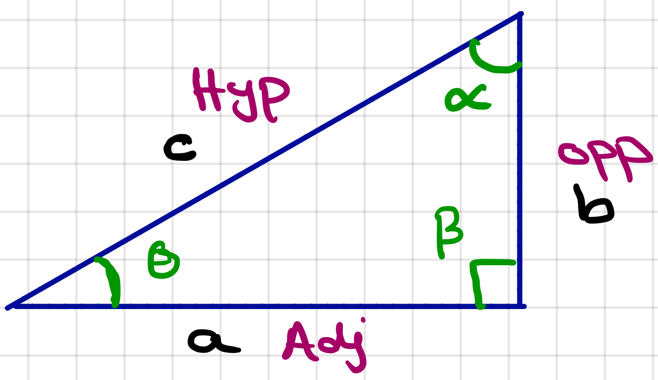
كسور عكسيه

Reciprocal Trig. Ratios

$\csc\theta = \dfrac{hyp}{opp}$

$\sec\theta = \dfrac{hyp}{adj}$

$\cot\theta = \dfrac{adj}{opp}$



PythagoreanTheorem

$a^2 + b^2 = c^2$

ملاحظة : هناك نوعين من المسآ ئل على حل المثلثات اليمنى :

١ـ معطى زاوية واحده وضلع واحد

How to solve

✓ لإيجاد الزاويه نكتب

الزاوية المطلوبه = $90° -$ الزاوية المعطاه

✓ لإيجاد الضلعين الآخرين

نختار المناسب من الكسور المثلثيه

وهي التي يكون فيها مجهول واحد.

٢ ـ معطى ضلعين فقط.

How to solve:

✓ لإيجاد الزاويه θ

نختار من الكسور المثلثيه المناسب

والتي يكون فيها ضلعين معلومين

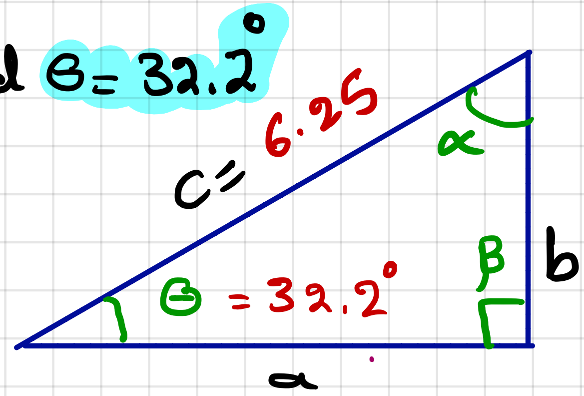
كي نستطيع حساب $\theta = \left(\frac{\text{النسبة}}{\text{المثلثية}}\right)^{-1}$

لإيجاد الزاوية α :

الزاوية المطلوبه = $90 -$ الزاويه التي اوجدناها سابقا

Given an Angle and a Side

Solve the right tringle with $c = 6.25$ and $\theta = 32.2°$



solve for α :

$$\alpha = 90° - 32.2 = 57.8°$$

solve for a :

$$\cos\theta = \frac{a}{c}$$

$$\cos 32.2° = \frac{a}{6.25}$$

$$\therefore a = \cos 32.2° \times 6.25$$

$$= 5.29 \text{ feet}.$$

Angle	sides
$\theta = 32.2°$	$a = ?$
$\alpha = ?$	$b = ?$
$\beta = 90°$	$c = 6.25$

solve for b :

$$\sin\theta = \frac{b}{c}$$

$$\sin 32.2° = \frac{b}{6.25}$$

$$\therefore b = \sin 32.2 \times 6.25$$

$$= 3.33 \text{ feet}.$$

ملاحظه : في هذا المثال استبعدنا

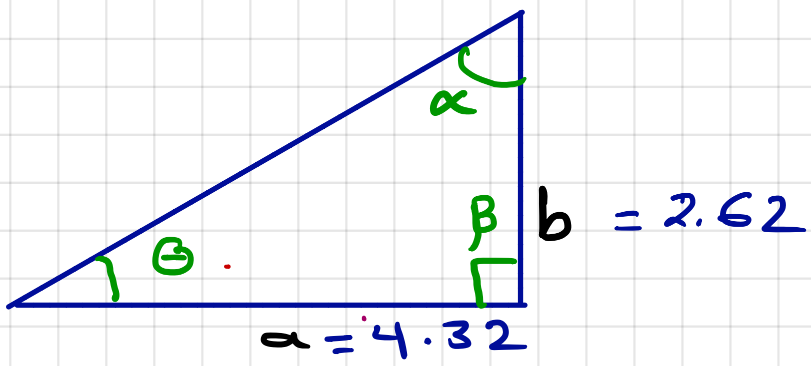
tan لانها عبارة عن $\tan\theta = \frac{b}{a}$

وكلا الضلعين مجهولين !!

Given tow sided

Solve the right tringle with $a = 4.32$ cm and $b = 2.62$ cm. Compute the angle measure to the nearst $10'$.

Angles	Sides
$\theta = ?$	$a = 4.32$
$\alpha = ?$	$b = 2.62$
$\beta = 90^\circ$	$C = ?$



solve for c :

$$\therefore \; c^2 = a^2 + b^2$$

$$c = \sqrt{(4.32)^2 + (2.62)^2} = 5.05 \text{ cm}$$

solve for θ :

$$\tan\theta = \frac{b}{a} = \frac{2.62}{4.32}$$

$$\theta = \tan^{-1}\left(\frac{2.62}{4.32}\right) = 31.2^\circ \quad \text{or}$$

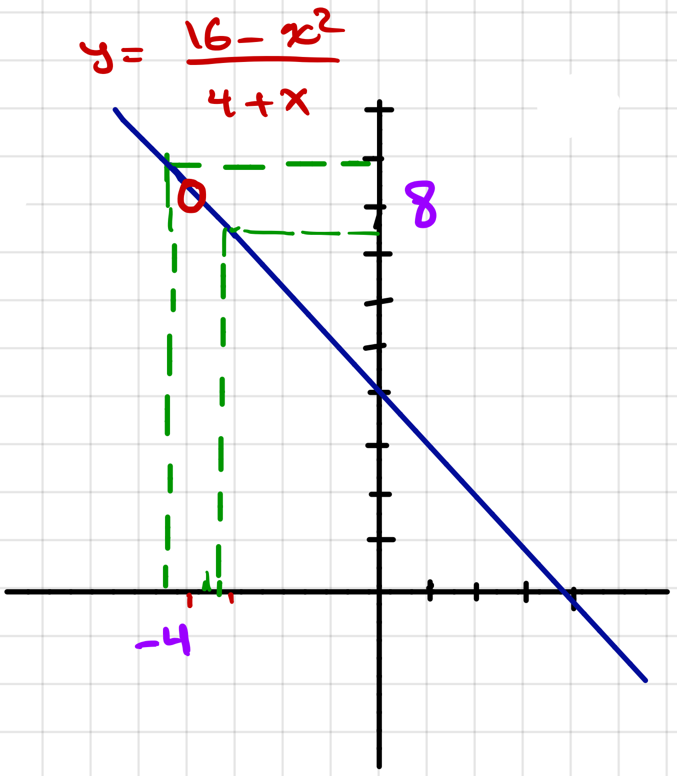
$$= 31^\circ 10' \quad (0.2 \times 60 = 12' \simeq 10')$$

solve for α :

$$\alpha = 90^\circ - 31.2^\circ = 58.8^\circ \quad \text{or}$$

$$= 58^\circ 50' \quad (0.8 \times 60 = 48 \simeq 50')$$

Introduction to the limit



$$f(x) = \frac{16-x^2}{x+4}$$

x	-3.9	-3.99	-3.999
$f(x)$	7.9	7.99	7.999

x	-4.1	-4.01	-4.001
$f(x)$	8.1	8.01	8.001

نلاحظ هنا بإن الداله غير معرفه عند -٤
ولكن عندما تقترب x من -٤ تقترب النتيجه
من ٨ . نسمي ٨ هي نهاية الداله عندما تقترب
x من -٤.

* نلاحظ من الرسم بإن الداله على يمين -٤
وعلى يسارها تقترب من ٨ .

* الدالة غير معرفه عند -٤ ونمثل ذلك على الرسم
بوضع دائرة مفتوحة.

Properties of Limit:

- $\lim\limits_{x\to a} [f(x) \pm g(x)] = \lim\limits_{x\to a} f(x) \pm \lim\limits_{x\to a} g(x)$
- $\lim\limits_{x\to a} f(x)g(x) = \lim\limits_{x\to a} f(x) . \lim\limits_{x\to a} g(x)$
- $\lim\limits_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)}$
- $\lim\limits_{x\to a} c = c$ For example : $\lim\limits_{x\to 3} 5 = 5$
- $\lim\limits_{x\to a} c f(x) = c \lim\limits_{x\to a} f(x)$

$$\lim_{x \to a} f(x)$$

Polynomial Functions

Way to solve

✓ Substituation Method

Rational Functions

Way to solve

طريقة التعويض ✓ Substituation Method

طريقة التحليل ✓ Factoring Method

طريقة الضرب في مرافق المقام ✓ Conjugate Method.

$\Big\}$ $\frac{0}{0}$ case

طريقة اختبار احد أطراف النهايات ✓ Examin the one sided limit.

$\Big\}$ $\frac{\text{عدد}}{0}$

Examples	Solution	Comments
$\lim_{x\to 2}(3x+5)$	$\lim_{x\to 2}(3x+5) = 2.3+5 = 11$	دالة كثيرة حدود بالتعويض المباشر
$\lim_{x\to -1}(x^3+5x^2-7)$	$\lim_{x\to -1}(x^3+5x^2-7)$ $=(-1)^3+5(-1)^2-7$ $=-3$	
$\lim_{x\to 5}\frac{2x^2+3}{x-x^2}$	$\lim_{x\to 5}\frac{2x^2+3}{x-x^2}$ $=\frac{2(5)^2+3}{5-25} = -\frac{53}{20}$	دالة كسرية :- دائمًا نبدأ بالتعويض المباشر وفي حالة الحصول على عدد يكون هو النهاية أما إذا حصلنا على كميات غير معرفة مثل $\frac{0}{0}$ أو $\frac{\text{عدد}}{0}$ نحل بالطرق الأخرى.
$\lim_{x\to 3}\frac{x-3x^2}{5+x}$	$\lim_{x\to 3}\frac{x-3x^2}{5+x}$ $=\frac{3-3(3)^2}{5+3}$ $=\frac{3-27}{8}$ $=\frac{-24}{8}=-3$	

Examples	Solutions	Comments.
$\lim\limits_{x\to -3} \frac{x+3}{x^2-9}$	$\lim\limits_{x\to -3} \frac{x+3}{x^2-9}$ $= \frac{-3+3}{(-3)^2-9} = \frac{0}{0}$ $\therefore \lim\limits_{x\to -3} \frac{x+3}{x^2-9}$ $= \lim\limits_{x\to -3} \frac{\cancel{x+3}}{(x-3)\cancel{(x+3)}}$ $= \lim\limits_{x\to -3} \frac{1}{x-3}$ $= \frac{1}{-3-3} = -\frac{1}{6}$	* عند التعويض المباشر في الدالة حصلنا على الكمية الغير معرفة $\frac{0}{0}$ لذلك نوجد limit بطريقة أخرى * استخدمنا هنا طريقة التحليل - (factoring) * $a^2-b^2 = (a-b)(a+b)$
$\lim\limits_{x\to 4} \frac{x-4}{\sqrt{x}-2}$	$\lim\limits_{x\to 4} \frac{x-4}{\sqrt{x}-2}$ $= \frac{4-4}{\sqrt{4}-2} = \frac{0}{0}$ $\lim\limits_{x\to 4} \frac{x-4}{\sqrt{x}-2} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2}$ $= \lim\limits_{x\to 4} \frac{\cancel{(x-4)}(\sqrt{x}+2)}{\cancel{x-4}}$ $= \lim\limits_{x\to 4} \sqrt{x}+2$ $= \sqrt{4}+2 = 4$	* حصلنا على $\frac{0}{0}$ عند وجود جذر في الدالة نستخدم إذاً المرافق

Examples	Solutions	Comments.
$\lim\limits_{x \to -3} \dfrac{2x}{(x+3)^2}$	$\lim\limits_{x \to -3} \dfrac{2x}{(x+3)^2}$ $= \dfrac{2(-3)}{(-3+3)^2} = \dfrac{-6}{0}$ $\lim\limits_{x \to -3^-} \dfrac{2x}{(x+3)^2} = -\infty$ $\lim\limits_{x \to -3^+} \dfrac{2x}{(x+3)^2} = -\infty$ $\therefore \lim\limits_{x \to -3} \dfrac{2x}{(x+3)^2} = -\infty$	عند التعويض المباشر في الدالة الكسرية حصلنا على $\frac{\text{عدد}}{0}$. في هذه الحالة نختبر ال limit من جهة يمين العدد ويسارة . * النأتج المحتملة لهذا النوع من النهايات $+\infty$ ، $-\infty$ أو DNE * كيف نحصل على النتيجة؟! بالحل على الدالة في المقام والتعويض فيها بأعداد عشوائية عن يمين العدد الذي تؤول إليه x أو يسارة حسب النهاية التي ندرسها إذا كانت سالبة فإن $\lim = \infty$ موجبة فإن $\lim = -\infty$
$\lim\limits_{x \to 6} \dfrac{-5}{2x-12}$	$\lim\limits_{x \to 6} \dfrac{-5}{2x-12}$ $= \dfrac{-5}{2.6-12} = \dfrac{-5}{0}$ $\lim\limits_{x \to 6^+} \dfrac{-5}{2x-12} = -\infty$ $\lim\limits_{x \to 6^-} \dfrac{-5}{2x-12} = \infty$ $\neq$ $\therefore \lim\limits_{x \to 6} \dfrac{-5}{2x-12}$ DNE	$x \to -3$ الدالة في المقام $(x+3)^2$ وعند دراسة $\lim\limits_{x \to -3^-}$ نأخذ عدد يسار -3 مثلاً -4 $(-4+3)^2 = 1$ (موجب) $\therefore \lim\limits_{x \to -3^-} = -\infty$ وهكذا

$$\lim_{x\to\pm\infty} f(x)$$

Polynomial Function	Rational Function

Polynomial Function

نأخذ الحد الذي له الأس الأعلى ثم نطبق التالي :-

$$\lim_{x\to\infty} x^n = \infty$$

$$\lim_{x\to-\infty} x^n = \infty \text{ or } -\infty$$

يعتمد على n :-

إذا كانت n زوجية $\longleftarrow \infty$

إذا كانت n فردية $\longleftarrow -\infty$

* الأخذ بالاعتبار بإشارة المتغير x.

Rational Function

يوجد ٣ طرق لإيجاد النهاية :-

١- نقسم جميع حدود البسط والمقام على أعلى أس للمتغير x في المقام.

٢- نقارن درجة البسط والمقام :

• درجة البسط > درجة المقام $\longleftarrow \pm\infty$

• درجة البسط = درجة المقام :-

$$\frac{\text{معامل أكبر أس في البسط}}{\text{معامل أكبر أس في المقام}}$$

• درجة السبط < درجة المقام $\longleftarrow 0$

٣- نأخذ الحد الذي له الأس الأعلى في البسط والمقام ثم نكمل الحل على الحالة.

Example : Find the following

$$\lim_{x\to\infty} (7 - 3x - 2x^2)$$

$$= \lim_{x\to\infty} -2x^2 = -\infty$$

$$\lim_{x\to\infty} (11 - 2x^2 - 4x^3)$$

$$= \lim_{x\to\infty} -4x^3 = -\infty$$

$$\lim_{x\to\infty} 4x^3 = \infty$$

$$\lim_{x\to-\infty} 4x^3 = -\infty$$

Example: Find the following:

1. $\lim\limits_{x\to\infty} \dfrac{2x+3}{x^2+1}$

Method 1:

$$\lim_{x\to\infty} \frac{\frac{2x}{x^2}+\frac{3}{x^2}}{\frac{x^2}{x^2}+\frac{1}{x^2}}$$

$$= \lim_{x\to\infty} \frac{\frac{2}{x}+\frac{3}{x^2}}{1+\frac{1}{x^2}}$$

$$= \frac{\cancel{\frac{2}{\infty}}+\cancel{\frac{3}{\infty^2}}}{1+\cancel{\frac{1}{\infty^2}}}$$

$$= \frac{0}{1} = 0$$

Method 2:

∵ درجة البسط < درجة المقام

$$\lim_{x\to\infty} \frac{2x+3}{x^2+1} = 0$$

Method 3:

$$\lim_{x\to\infty} \frac{2x+3}{x^2+1}$$

$$= \lim_{x\to\infty} \frac{2x}{x^2}$$

$$= \lim_{x\to\infty} \frac{2}{x}$$

$$= \frac{2}{\infty} = 0$$

2. $\lim\limits_{x\to\infty} \dfrac{3x^3+2}{5x^2-1}$

Method 1:

$$\lim_{x\to\infty} \frac{\frac{3x^3}{x^2}+\frac{2}{x^2}}{\frac{5x^2}{x^2}-\frac{1}{x^2}}$$

$$= \lim_{x\to\infty} \frac{3x+\frac{2}{x^2}}{5-\frac{1}{x^2}}$$

$$= \frac{3(\infty)+\cancel{\frac{2}{\infty^2}}}{5-\cancel{\frac{1}{\infty^2}}}$$

$$= \frac{3}{5}(\infty) = \infty$$

Method 2:

∵ درجة البسط > درجة المقام

∴ الحل $\pm\infty$

$$\lim_{x\to\infty} \frac{3(\infty)^3+2}{5(\infty)^2-1} = \infty$$

Method 3:

$$\lim_{x\to\infty} \frac{3x^3+2}{5x^2-1}$$

$$= \lim_{x\to\infty} \frac{3x^3}{5x^2}$$

$$= \lim_{x\to\infty} \frac{3}{5}x$$

$$= \frac{3}{5}(\infty) = \infty$$

3. $\lim\limits_{x \to -\infty} \dfrac{5x^2}{x+3}$

Method 1:	Method 2:	Method 3:
$\lim\limits_{x \to -\infty} \dfrac{5x^2}{x+3}$ $= \lim\limits_{x \to -\infty} \dfrac{\frac{5x^2}{x}}{\frac{x}{x} + \frac{3}{x}}$ $= \lim\limits_{x \to -\infty} \dfrac{5x}{1 + \frac{3}{x}}$ $= \dfrac{5(-\infty)}{1 + \cancel{\frac{3}{-\infty}}}$ $= 5(-\infty) = -\infty$	∵ درجة البسط > درجة المقام ∴ limit = $\pm\infty$ نحدد الإشارة على حسب limit $\therefore \lim\limits_{x \to -\infty} \dfrac{5x^2}{x+3} = -\infty$	$\lim\limits_{x \to -\infty} \dfrac{5x^2}{x+3}$ $= \lim\limits_{x \to -\infty} \dfrac{5x^2}{x}$ $= \lim\limits_{x \to -\infty} 5x$ $= 5(-\infty) = -\infty$

4. $\lim\limits_{x \to \infty} \dfrac{3-5x}{3x-1}$

Method 1:	Method 2:	Method 3:
	∵ درجة البسط = درجة المقام $\therefore \lim\limits_{x \to \infty} \dfrac{3-5x}{3x-1} = \dfrac{-5}{3}$	

More Examples

Find the following :

1. $\lim\limits_{x \to -\infty} \dfrac{8x^2+3x}{2x^2-1}$

2. $\lim\limits_{x \to -3} \dfrac{x^2+x-6}{x+3}$

3. $\lim\limits_{x \to -\infty} (2x^2-9)$

4. $\lim\limits_{x \to -\infty} (-x^3-x+6)$

Solution :

1. $\lim\limits_{x \to -\infty} \dfrac{8x^2+3x}{2x^2-1} = \lim\limits_{x \to -\infty} \dfrac{8\cancel{x^2}}{2\cancel{x^2}}$

$= \lim\limits_{x \to -\infty} 4 = 4.$

2. $\lim\limits_{x \to -3} \dfrac{x^2+x-6}{x+3} = \dfrac{0}{0}$

$\lim\limits_{x \to -3} \dfrac{\cancel{(x+3)}(x-2)}{\cancel{(x+3)}}$

$= \lim\limits_{x \to -3} x-2 = -5$

3. $\lim\limits_{x \to -\infty} 2x^2-9 = \lim\limits_{x \to -\infty} 2x^2 = \infty$

4. $\lim\limits_{x \to -\infty} -x^3-x+6 = \infty$