CHAPTER 3

Data Description

In chapter 2 showed how you can gain useful information from raw data by organizing them into a frequency distribution and then presenting the data by using various graphs. This chapter shows the statistical methods that can be used to summarize data. The most familiar of these methods is the finding of averages.

- *The average means the center of the distribution or the most typical case. Measures of average are also called measures of central tendency and include the mean, median, mode, and midrange.*
- *The measures that determine the spread of the data values are called measures of variation, or measures of dispersion. These measures include the range, variance, and standard deviation.*
- *The measures of position tell where a specific data value falls within the data set or its relative position in comparison with other data values. The most common position measures are percentiles, deciles, and quartiles.*
- ***The measures of central tendency, variation, and position explained in this chapter are part of what is called traditional statistics**.*

In section 3–4 shows the techniques of what is called exploratory data analysis. These techniques include the boxplot and the five-number summary.

Measures of Central Tendency:

Definition:

- *A statistic is a characteristic or measure obtained by using the data values from a sample.*
- *A parameter is a characteristic or measure obtained by using all the data values from aspecific population.*

*** The Mean*

The mean is the sum of the values, divided by the total number of values. The symbol represents the sample mean. where n represents the total number of values in the sample.

$$\bar{X} = \frac{X_1 + X_2 + X_3 + \cdot \cdot \cdot + X_n}{n} = \frac{\Sigma X}{n}$$

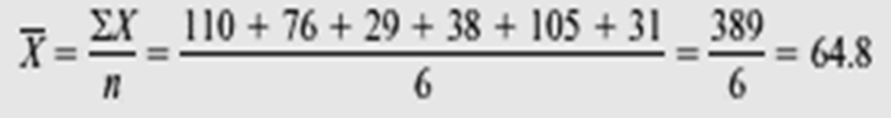
For a population, the Greek letter m (mu) is used for the mean. where N represents the total number of values in the population.

$$\mu = \frac{X_1 + X_2 + X_3 + \cdot \cdot \cdot + X_N}{N} = \frac{\Sigma X}{N}$$

The data show the number of patients in a sample of six hospitals who acquired an infection while hospitalized. Find the mean.

110   76   29   38   105   31

Solution:



$$\overline{X} = \frac{\Sigma X}{n} = \frac{110 + 76 + 29 + 38 + 105 + 31}{6} = \frac{389}{6} = 64.8$$

The mean of the number of hospital infections for the six hospitals is 64.8.

** The Median

The median is the midpoint of the data array. The symbol for the median is MD.

Steps in computing the median of a data array:

***Step 1**: Arrange the data in order.*

***Step 2**: Select the middle point.*

The number of rooms in the seven hotels in downtown Pittsburgh is 713, 300, 618, 595, 311, 401, and 292. Find the median.

Solution

Step 1 Arrange the data in order.

292, 300, 311, 401, 595, 618, 713

Step 2 Select the middle value.

292, 300, 311, 401, 595, 618, 713

↑

Median

Hence, the median is 401 rooms.

Six customers purchased these numbers of magazines: 1, 7, 3, 2, 3, 4. Find the median.

Solution

1, 2, 3, 3, 4, 7

↑

$MD = (3+3)/2 = 3$

Hence, the median number of magazines purchased is 3.

** The Mode

The value that occurs most often in a data set is called the mode.

Types of mode:

1- A data set that has only one value that occurs with the greatest frequency is said to be unimodal.

2- If a data set has two values that occur with the same greatest frequency, both values are considered to be the mode and the data set is said to be bimodal.

3- If a data set has more than two values that occur with the same greatest frequency, each value is used as the mode, and the data set is said to be multimodal.

4-When no data value occurs more than once, the data set is said to have no mode.

The variables are classified as qualitative or quantitative, and can be classified by the types of scales are used: nominal, ordinal, interval, and ratio.

The data show the number of licensed nuclear reactors in the United States for a recent 15-year period. Find the mode.

104 104 104 104 104

107 109 109 109 110

109 111 112 111 109

Solution

Since the values 104 and 109 both occur 5 times, the modes are 104 and 109. The data set is said to be bimodal.

** The Midrange

The midrange is defined as the sum of the lowest and highest values in the data set, divided by 2. The symbol MR is used for the midrange.

$$MR = \frac{\text{lowest value - highest value}}{2}$$

In the last two winter seasons, the city of Brownsville, Minnesota, reported these numbers of water-line breaks per month. Find the midrange.

2, 3, 6, 8, 4, 1

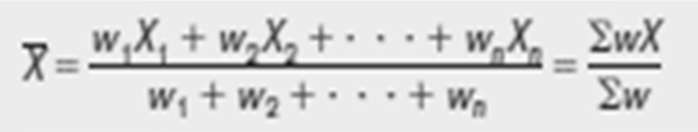
Solution

$$MR = \frac{1 - 8}{2} = 4.5$$

Hence, the midrange is 4.5.

*** The Weighted Mean*

The type of mean that considers an additional factor is called the weighted mean, and it is used when the values are not all equally represented.



$$\overline{X} = \frac{w_1X_1 + w_2X_2 + \cdot \cdot \cdot + w_nX_n}{w_1 + w_2 + \cdot \cdot \cdot + w_n} = \frac{\Sigma wX}{\Sigma w}$$

Find the weighted mean of a variable X by multiplying each value by its corresponding weight and dividing the sum of the products by the sum of the weights. where w1, w2, . . . , wn are the weights and X1, X2, . . . , Xn are the values

A student received an A in English Composition I (3 credits), a C in Introduction to Psychology (3 credits), a B in Biology I (4 credits), and a D in Physical Education (2 credits). Assuming A = 4 grade points, B = 3 grade points, C = 2 grade points, D = 1 grade point, and F = 0 grade points, find the student's grade point average.

Solution

Course	*Credits (w)*	*Grade (X)*
English I	*3*	*A (4 points)*
Psychology	*3*	*C (2 points)*
Biology I	*4*	*B (3 points)*
Physical	*2*	*D (1 point)*

$$\overline{X} = \frac{\Sigma wX}{\Sigma w} = \frac{3 \cdot 4 + 3 \cdot 2 + 4 \cdot 3 + 2 \cdot 1}{3 + 3 + 4 + 2} = \frac{32}{12} = 2.7$$

The grade point average is 2.7.

Properties and Uses of Central Tendency

The Mean

1. The mean is found by using all the values of the data.

4. The mean for the data set is unique and not necessarily one of the data values.

6. The mean is affected by extremely high or low values, called outliers, and may not be the appropriate average to use in these situations.

The Median

1. The median is used to find the center or middle value of a data set.

2. The median is used when it is necessary to find out whether the data values fall into the upper half or lower half of the distribution.

4. The median is affected less than the mean by extremely high or extremely low values.

The Mode

1. The mode is used when the most typical case is desired.

2. The mode is the easiest average to compute.

3. The mode can be used when the data are nominal or categorical, such as religious preference, gender, or political affiliation.

4. The mode is not always unique. A data set can have more than one mode, or the mode may not exist for a data set.

The Midrange

1. The midrange is easy to compute.

2. The midrange gives the midpoint.

3. The midrange is affected by extremely high or low values in a data set.

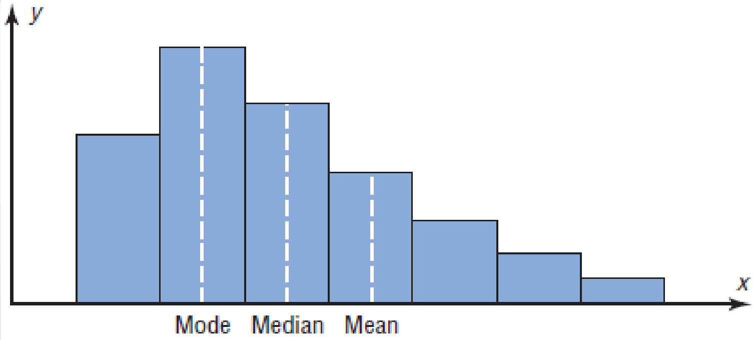
Distribution Shapes

Frequency distributions can assume many shapes:

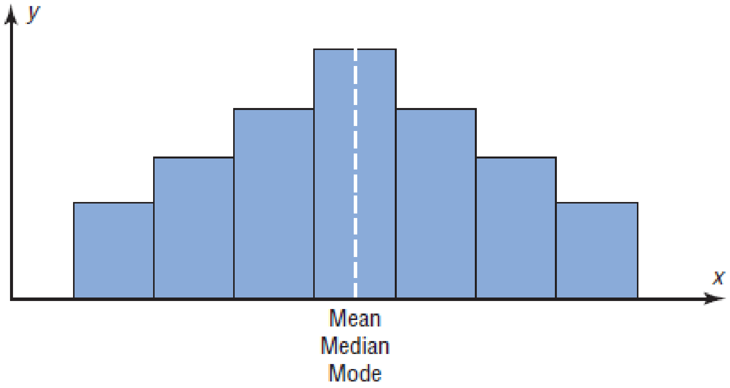
1) In a ***positively skewed or right-skewed distribution****, the majority of the data values fall to the left of the mean and cluster at the lower end of the distribution. Also, the mean is to the right of the median, and the mode is to the left of the median.*

2) In a ***symmetric distribution****, the data values are evenly distributed on both sides of the mean. In addition, when the distribution is unimodal, the mean, median, and mode are the same and are at the center of the distribution.*

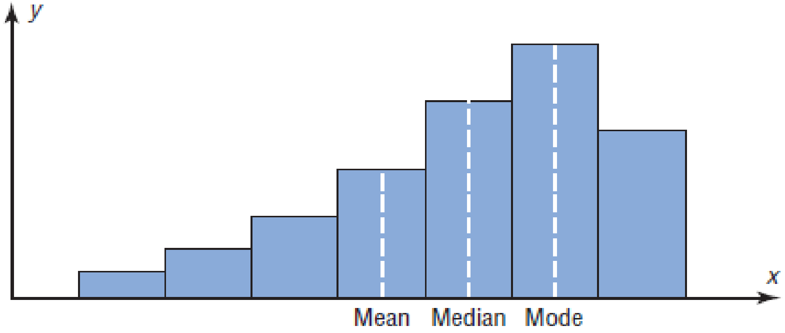
3) In a ***negatively skewed or left-skewed****, the data values fall to the right of the mean., the mean is to the left of the median, and the mode is to the right of the median.*



(a) Positively skewed or right-skewed



(b) Symmetric



(c) Negatively skewed or left-skewed

In statistics, to describe the data set accurately, statisticians must know more than the measures of central tendency

Measures of Variation:

For the spread or variability of a data set, three measures are commonly used: range, variance, and standard deviation. Each measure will be discussed in this section.

Range

The range is the highest value minus the lowest value. The symbol R is used for the range.

$$R = \text{highest value} - \text{lowest value}$$

Find the ranges for the paints:

(a) Brand A

10 20 30 35 40 50 60

(b) Brand B

25 30 35 40 45

Solution

For brand A, the range is

R = 60 - 10 = 50 months

For brand B, the range is

R = 45 - 25 = 20 months

The range for brand A shows that 50 months separate the largest data value from the smallest data value. For brand B, 20 months separate the largest data value from the smallest data value, which is less than one-half of brand A's range.

Population Variance and Standard Deviation

(Just the symbol & Formula)

To have a more meaningful statistic to measure the variability, statisticians use measures called the variance and standard deviation.

Population Variance:

The variance is the average of the squares of the distance each value is from the mean. The symbol for the population variance is σ^2 *(s is the Greek lowercase letter sigma). The formula for the population variance is*

$$\sigma^2 = \frac{\Sigma(X - \mu)^2}{N}$$

where

X : individual value

m : population mean

N : population size

standard deviation :

The standard deviation is the square root of the variance. The symbol for the population standard deviation is σ *. The corresponding formula for the population standard deviation is:*

$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{\Sigma(X-\mu)^2}{N}}$$

Sample Variance and Standard Deviation

Sample Variance:

The formula for the sample variance, denoted by S^2*, is*

$$s^2 = \frac{n(\Sigma X^2) - (\Sigma X)^2}{n(n-1)}$$

where

n: sample size

Standard Deviation :

The standard deviation is the square root of the variance. The symbol for the population standard deviation is S*. The corresponding formula for the population standard deviation is:*

$$s = \sqrt{\frac{n(\Sigma X^2) - (\Sigma X)^2}{n(n-1)}}$$

Find the sample variance and standard deviation for the amount of European auto sales for a sample of 6 years shown. The data are in millions of dollars.

11.2, 11.9, 12.0, 12.8, 13.4, 14.3

Solution

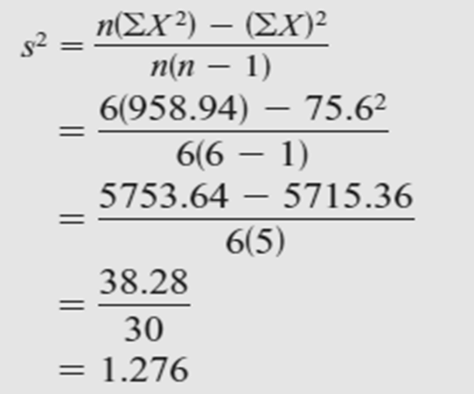
Step 1 Find the sum of the values.

$\sum X$ *= 11.2 + 11.9 + 12.0 + 12.8 + 13.4 + 14.3 + 75.6*

Step 2 Square each value and find the sum.

$\sum X^2$ *= 11.22 + 11.92 + 12.02 + 12.82 + 13.42 + 14.32 + 958.94*

Step 3 Substitute in the formulas and solve.



$$\begin{aligned} s^2 &= \frac{n(\Sigma X^2) - (\Sigma X)^2}{n(n-1)} \\ &= \frac{6(958.94) - 75.6^2}{6(6-1)} \\ &= \frac{5753.64 - 5715.36}{6(5)} \\ &= \frac{38.28}{30} \\ &= 1.276 \end{aligned}$$

The variance is 1.28 rounded.

***** Hence, the sample standard deviation is***

S = $\sqrt{1.28}$ = *1.13.*

Coefficient of Variation:

Whenever two samples have the same units of measure, the variance and standard deviation for each can be compared directly. But the statistic using coefficient of variation to compare the standard deviations when the units are different.

The ***coefficient of variation****, denoted by CVar, is the standard deviation divided by the mean. The result is expressed as a percentage.*

For samples, the

$$CVar = \frac{S}{m} \times 100\%$$

Ex: The mean of the number of sales of cars over a 3-month period is 87, and the standard deviation is 5. The mean of the commissions is $5225, and the standard deviation is $773. Compare the variations of the two.

Solution:

The coefficients of variation are

$$CVar = \frac{773}{5225} \times 100\% = 14.8\% \quad \text{commissions}$$

$$CVar = \frac{5}{87} \times 100\% = 5.7\% \text{ sales}$$

Since the coefficient of variation is larger for commissions, the commissions are more variable than the sales.

Measures of Position

In addition to measures of central tendency and measures of variation, there are measures of position or location. These measures include standard scores, percentiles, deciles, and quartiles. They are used to locate the relative position of a data value in the data set.

Standard Scores:

A z score or standard score for a value is obtained by subtracting the mean from the value and dividing the result by the standard deviation. The symbol for a standard score is z. The formula for samples is:

$$z = \frac{value\ - mean}{\text{standard deviation}} = \frac{X\ -\ \bar{X}}{S}$$

The z score represents the number of standard deviations that a data value falls above or below the mean.

EX: A student scored 65 on a calculus test that had a mean of 50 and a standard deviation of 10; she scored 30 on a history test with a mean of 25 and a standard deviation of 5. Compare her relative positions on the two tests.

Solution:

First, find the z scores. For calculus the z score is

$$z = \frac{65 - 50}{10} = 1.5$$

For history the z score is

$$z = \frac{30 - 25}{5} = 1$$

Since the z score for calculus is larger, her relative position in the calculus class is higher than her relative position in the history class.

When all data for a variable are transformed into z scores, the resulting distribution will have a mean of 0 and a standard deviation of 1. A z score, then, is actually the number of standard deviations each value is from the mean for a specific distribution. In the last example the calculus score of 65 was actually 1.5 standard deviations above the mean of 50.

Percentiles:

Percentiles are position measures used in educational and health-related fields to indicate the position of an individual in a group.

*** Percentiles** *divide the data set into 100 equal groups.*

Percentile Formula:

(1) *The percentile corresponding to a given value X is computed by using the following formula:*

$$\text{Percentile} = \frac{\text{number of values below } X \; + 0.5}{\text{total number of values}} \; * \; 100$$

(2) *The steps for finding a value corresponding to a given percentile:*

Step 1 *Arrange the data in order from lowest to highest.*

Step 2 *Substitute into the formula*

$$C = \frac{n \times p}{100}$$ *where: n: total number of values, p: percentile*

Step 3

(A) *If c is not a whole number, round up to the next whole number. Starting at the lowest value, count over to the number that corresponds to the rounded-up value.*

(B) *If c is a whole number, use the value halfway between the cth and (c 1)st values when counting up from the lowest value.*

EX: A teacher gives a 20-point test to 10 students. The scores are shown here. Find the percentile rank of a score of 12 and find the value corresponding to the 25th percentile

18, 15, 12, 6, 8, 2, 3, 5, 20, 10

Solution**:**

* Arrange the data in order from lowest to highest.

2, 3, 5, 6, 8, 10, 12, 15, 18, 20

Then substitute into the formula.

$$\text{Percentile} = \frac{6+0.5}{10} * 100 = 65\text{th}$$

** Substitute in the formula $C = \frac{10 \times 25}{100} = 2.5$

If c is not a whole number, round it up to the next whole number; in this case,

c = 3. Start at the lowest value and count over to the third value, which is 5. Hence,

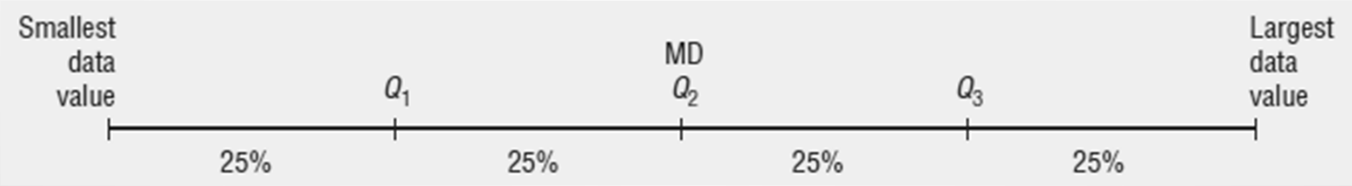
the value 5 corresponds

to the 25th percentile.

Quartiles:

Quartiles *divide the distribution into four groups, separated by Q1, Q2, Q3.*

Note that Q1 is the same as the 25th percentile; Q2 is the same as the 50th percentile, or the median; Q3 corresponds to the 75th percentile, as shown:



However, an easier method for finding quartiles is found in this Procedure Table:

1. Arrange the data in order from lowest to highest.

2. Find the median of the data values (Q2).

3. Find the median of the data values that fall bellow Q2 (Q1).

4. Find the median of the data values that fall above Q2 (Q3).

EX: Find Q1, Q2, and Q3 for the data set 15, 13, 6, 5, 12, 50, 22, 18.

Solution

Step 1 Arrange the data in order.

5, 6, 12, 13, 15, 18, 22, 50

Step 2 Find the median (Q2).

5, 6, 12, 13, 15, 18, 22, 50

↑

MD = (13 +15)/2 = 14

Step 3 Find the median of the data values less than 14.

5, 6, 12, 13

↑

Q1 = (6+12)/2 = 9

Step 4 Find the median of the data values greater than 14.

15, 18, 22, 50

↑

Q3 = (18+22)/2 =20

Outlier :

An outlier is an extremely high or an extremely low data value when compared with the rest of the data values.

- *Outliers can be identified as follows:*

1. Arrange the data in order and find Q1 and Q3.

2. Find the interquartile range: IQR=Q3-Q1.

3. The values that are smaller than Q1-(1.5)(IQR) or larger than Q3+(1.5)(IQR) are called outliers.

EX: Check the following data set for outliers. (using the last example)

Solution

Step 1 Arrange the data in order.

5, 6, 12, 13, 15, 18, 22, 50

Step 2 Find Q1 and Q3.

Q1 = 9 and Q3 = 20.

2. Find the interquartile range: IQR=Q3-Q1 = 20 – 9 = 11

3. The values that are smaller than 9-(1.5)(11) = -7.5 or larger

than 20+(1.5)(11) = 36.5 are called outliers. The value 50 is outside this interval;

hence, it can be considered an outlier.

Exploratory Data Analysis:

In exploratory data analysis (EDA), the data are represented graphically using a boxplot (sometimes called a box-and-whisker plot). The purpose of exploratory data analysis is to examine data to find out what information can be discovered about the data such as the center and the spread

**** The Five-Number Summary and Boxplots***

A boxplot can be used to graphically represent the data set. These plots involve five specific values:

1. The lowest value of the data set (i.e., minimum)

2. Q1

3. The median

4. Q3

5. The highest value of the data set (i.e., maximum)

These values are called a five-number summary of the data set.

Information obtained from a boxplot

* Using the box:

- If the median is near the center of the box, the distribution is approximately symmetric.

- If the median is to the left of the box, the distribution is positively skewed.

- If the median is to the right of the box, the distribution is negatively skewed.

* Using the lines:

- If the lines are about the same length, the distribution is approximately symmetric.

- If the right line is taller, the distribution is positively skewed.

- If the left line is taller, the distribution is negatively skewed.

Information obtained from a boxplot

** Using the box:*

- If the median is near the center of the box, the distribution is approximately symmetric.

- If the median is to the left of the box, the distribution is positively skewed.

- If the median is to the right of the box, the distribution is negatively skewed.

** Using the lines:*

- If the lines are about the same length, the distribution is approximately symmetric.

- If the right line is taller, the distribution is positively skewed.

- If the left line is taller, the distribution is negatively skewed.