

(1+2+3+4+5=15 marks)

Question 1: Let  $[\Omega, \mathcal{A}, P]$  be the probability space of rolling a fair die one time, and

$X$  is a random variable on  $[\Omega, \mathcal{A}, P]$  defined as follow:

$$X : \Omega = \left\{ \underset{\omega_1}{1}, \underset{\omega_2}{2}, \underset{\omega_3}{3}, \underset{\omega_4}{4}, \underset{\omega_5}{5}, \underset{\omega_6}{6} \right\} \longrightarrow \mathbb{R}$$

$$\omega \mapsto X(\omega) = \begin{cases} -1 & \text{for } \omega = \omega_1, \omega_2 \\ 0 & \text{for } \omega = \omega_3, \omega_4 \\ 1 & \text{for } \omega = \omega_5, \omega_6 \end{cases}$$

Then:

- What type is this random variable  $X$ ?
- What is the name of this random variable  $X$ ?
- Draw the graphical representation of this random variable  $X$ .
- Determine the distribution function of  $X$ .
- Calculate the mathematical expectation (the mean) of  $X$ .

Answers:

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(2+2+4+3+3+1=15 marks)

**Question 2:** Let  $X$  be a random variable with density function  $f_X$  given by the following relation:

$$f_X(x) = \begin{cases} \alpha x & \text{for } 0 \leq x \leq 1 \\ 0 & \text{Otherwise} \end{cases}$$

Where  $\alpha$  is a constant. Then:

- a) Determine the value of  $\alpha$ .
- b) Draw the graph of  $f_X$ .
- c) Calculate the following probabilities:

d-1)  $P(-0.25 < X \leq 0.25)$

d-2)  $P(X > 1.75)$

d-3)  $P(X = 0.5)$

**Answers:**

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