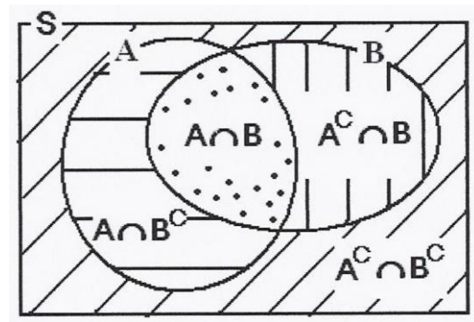
Probability

Definitions and Theorems:

$*\ 0 \le P(A) \le 1$

$*\ P(S) = 1$

$*\ P(\emptyset) = 0$



1- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
2- $P(A|B) = P(A \cap B)/P(B)$
3- $P(A \cap B) = P(A) \times P(B)\ \ (if\ A\ \&\ B\ are\ independent.)$
4- $P(A \cap B) = 0\ \ (if\ A\ \&\ B\ are\ disjoint.)$
5- $P(A^c) = 1 - P(A) \qquad ; P(A^c) = P(\bar{A})$

Question 1:

Suppose that we have: $P(A) = 0.4$, $P(B) = 0.5$, $P(A \cap B) = 0.2$

1. *The probability* $P(A \cup B)$ *is:* $\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.5 - 0.2 = 0.7}$

A)	<u>0.7</u>	B)	0.4	C)	0.5	D)	0

2. *The probability* $P(A \cap B^C)$ *is:* $\boxed{P(A \cap B^c) = P(A) - P(A \cap B) = 0.4 - 0.2 = 0.2}$

A)	0.51	B)	<u>0.20</u>	C)	0.40	D)	0.60

3. *The probability* $P(A|B)$ *is:* $\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.5} = 0.4}$

A)	0.51	B)	<u>0.40</u>	C)	0.20	D)	0.30

4. *The events A and B are:* $\boxed{P(A \cap B) \stackrel{?}{=} P(A) \times P(B) \Rightarrow 0.2 = 0.4 \times 0.5}$

A)	disjoint	B)	dependent	C)	equal	D)	<u>Independent</u>

<u>Question 2:</u>

If the events A, B we have: P(A) = 0.2, P(B) = 0.5 and P(A ∩ B) = 0.1 , then:

1) *The events A , B are :*

$$\boxed{P(A\cap B) \stackrel{?}{=} P(A)\times P(B) \Longrightarrow 0.1 = 0.2\times 0.5}$$

(a)Dependents (b) both are empties (c) Disjoints <u>(d) Independents</u>

2) *The probability of A or B is:*

$$\boxed{P(A\cup B) = P(A) + P(B) - P(A\cap B) = 0.2 + 0.5 - 0.1 = 0.6}$$

(a) 0.5 (b) 5.0 (c) 0.2 <u>(d) 0.6</u> (e) None is correct

3) *If P(A) = 0.3, P(B) = 0.4 and that A and B are disjoint, then P(A∪B) =*

$$\boxed{P(A\cup B) = P(A) + P(B) - 0 = 0.2 + 0.5 - 0 = 0.7}$$

<u>(a) 0.7</u> (b) 0.12 (c) 0.6 (d) 0.1 (e) None

4) *If P(A) = 0.2 and P(B | A) =0.4 , then P(A ∩ B) =*

$$\boxed{P(B|A) = \frac{P(A\cap B)}{P(A)} \Longrightarrow 0.4 = \frac{P(A\cap B)}{0.2} \Longrightarrow P(A\cap B) = 0.2\times 0.4 = 0.08}$$

(a) 0.6 <u>(b) 0.08</u> (c) 0.5 (d) 2.0 (e) None is correct

5) *Suppose that the probability a patient smokes is 0.20. If the probability that the patient smokes and has a lung cancer is 0.15, then the probability that the patient has a lung cancer given that the patient smokes is*

$$\boxed{\begin{array}{c} P(S) = 0.20 \quad P(S\cap C) = 0.15 \quad P(C|S) = ? \\ P(C|S) = \frac{P(C\cap S)}{P(S)} = \frac{0.15}{0.20} = 0.75 \end{array}}$$

(a) 0.25 (b) 0.2 <u>(c) 0.75</u> (d) 1.33 (e) None is correct

<u>Question 3:</u>

Suppose that we have two events A and B such that,

$$P(A) = 0.4, P(B) = 0.5, P(A \cap B) = 0.2.$$

[1] $P(A \cup B)$: $\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.5 - 0.2 = 0.7}$

A	0.1	**B**	0.9	**C**	<u>0.7</u>	**D**	0.8

[2] $P(A^c \cap B)$: $\boxed{P(A^c \cap B) = P(B) - P(A \cap B) = 0.5 - 0.2 = 0.3}$

A	<u>0.3</u>	**B**	0.2	**C**	0.5	**D**	0.7

[3] $P(A^c \cap B^c)$: $\boxed{P(A^c \cap B^c) = 1 - P(A \cup B) = 1 - 0.7 = 0.3}$

A	0.7	**B**	0.2	**C**	0.5	**D**	<u>0.3</u>

[4] $P(A^c)$: $\boxed{P(A^c) = 1 - P(A) = 1 - 0.4 = 0.6}$

A	0.4	**B**	<u>0.6</u>	**C**	0.1	**D**	0.9

[5] $P(A^c \mid B)$: $\boxed{P(A^c \mid B) = \frac{P(A^c \cap B)}{P(B)} = \frac{0.3}{0.5} = 0.6}$

A	0.4	**B**	<u>0.6</u>	**C**	0.1	**D**	0.9

[6] $P(B \mid A)$: $\boxed{P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{0.2}{0.4} = 0.5}$

A	0.4	**B**	0.6	**C**	0.3	**D**	<u>0.5</u>

[7] The events A and B are …. $\boxed{P(A \cap B) \stackrel{?}{=} P(A) \times P(B) \Rightarrow 0.2 = 0.4 \times 0.5}$

A	Exhaustive	**B**	Disjoint	**C**	<u>Independent</u>	**D**	Dependent

Question 4:

Following table shows 80 patients classified by sex and blood group.

Sex	*Blood Group*		
	A	*B*	*O*
Male (M)	*25*	*17*	*15*
Female (F)	*11*	*9*	*3*

1) *The probability that a patient selected randomly is a male and has blood group A is*

   (a) 25/36 *(b) 25/57* <u>*(c) 25/80*</u> *(d) 52/80* *(e) None*

2) *The probability that a patient selected randomly is a female is*

   (a) 6/80 *(b) 40/80* *(c) 22/80* <u>*(d) 23/80*</u> *(e) None*

3) *In a certain population, 4% have cancer, 20% are smokers and 2% are both smokers and have cancer. If a person is chosen at random from the population, find the probability that the person chosen is a smoker or has cancer.*

$$P(C) = 0.04 \quad P(S) = 0.20 \quad P(S \cap C) = 0.02 \quad P(S \cup C) = ?$$
$$P(S \cup C) = P(C) + P(S) - P(S \cap C)$$
$$P(S \cup C) = 0.04 + 0.20 - 0.02 = 0.22$$

(a) 0.02 *(b) 0.24* *(c) 0.2* <u>*(d) 0.22*</u> *(e) None*

Question 5:

Gender	*Diabetics (D)*	*Not Diabetic (D^c)*	*TOTAL*
Male (M)	*72*	*288*	*360*
Female (F)	*48*	*192*	*240*
TOTAL	*120*	*480*	*600*

Consider the information given in the table above. A person is selected randomly

1. *The probability that the person found is male and diabetic is:*

$$\boxed{P(M \cap D) = \frac{72}{600} = 0.12}$$

(A) 72	***(B) 0.12***	*(C) 0.60*	*(D) 0.67*

2. *The probability that the person found is male or diabetic is:*

$$\boxed{P(M \cup D) = P(M) + P(D) - P(M \cap D) = \frac{360}{600} + \frac{120}{600} - \frac{72}{600} = \frac{408}{600}}$$

(A) 0.12	***(B) 0.68***	*(C) 0.60*	*(D) 0.97*

3. *The probability that the person found is female is:*

$$\boxed{P(F) = \frac{240}{600} = 0.4}$$

(A) 0.24	*(B) 0.12*	***(C) 0.40***	*(D) 0.5*

4. *Suppose we know the person found is a male, the probability that he is diabetic, is:*

$$\boxed{P(D|M) = \frac{P(M \cap D)}{P(M)} = \frac{72/600}{360/600} = \frac{72}{360} = 0.2}$$

(A) 0.2	*(B) 0.12*	*(C) 0.40*	*(D) 0.68*

5. *The events M and D are:*

$$\boxed{P(M \cap D) = P(M) \times P(D) \Longrightarrow \frac{72}{600} = \frac{360}{600} \times \frac{120}{600}}$$

(A) Disjoint	***(B) Independent***	*(C) mutually exclusive*	*(D) Dependent*

Question 6:

A group of people is classified by the amount of fruits eaten and the health status:

Health Status \ Fruits Eaten	Few (F)	Some (S)	Many (M)	Total
Poor (B)	80	35	20	135
Good (G)	25	110	45	180
Excellent (E)	15	95	75	185
Total	120	240	140	500

If one of these people is randomly chosen give:

5. *The event "(eats few fruits) and (has good health)", is defined as.*

A)	$F \cup G^c$	B)	<u>$F \cap G$</u>	C)	$F \cup E$	D)	$S \cup E$

6. $P(B \cup M) =$

A)	<u>0.51</u>	B)	0.0.28	C)	0.27	D)	0.04

7. $P(G \cap S) =$

A)	0.48	B)	0.36	C)	<u>0.22</u>	D)	0.62

8. $P(E^c) =$

A)	<u>0.63</u>	B)	0.37	C)	0.50	D)	1

9. $P(G \mid S) =$

A)	0.6111	B)	0.2200	C)	<u>0.4583</u>	D)	0.36

10. $P(M \mid E) =$

A)	0.6111	B)	0.2200	C)	<u>0.405</u>	D)	0.36

Question 7:

The following table classifies a sample of individuals according to gender and period (in years) attendance in the college:

College Attended	Gender		
	Male	Female	Total
None	12	41	53
Two Years	14	63	77
Three Years	9	49	58
Four Years	7	50	57
Total	42	203	245

Suppose we select an individual at random, then:

1.	*The probability that the individual is male is:*						
(A)	0.8286	(B)	0.1714	(C)	0.0490	(D)	0.2857
2.	*The probability that the individual did not attend college (None) and female is:*						
(A)	0.0241	(B)	0.0490	(C)	0.1673	(D)	0.2163
3.	*The probability that the individual has three year or two year college attendance is:*						
(A)	0.551	(B)	0.0939	(C)	0.4571	(D)	0
4.	*If we pick an individual at random and found that he had three year college attendance, the probability that the individual is male is:*						
(A)	0.0367	(B)	0.2143	(C)	0.1552	(D)	0.1714
5.	*The probability that the individual is not a four year college attendance is:*						
(A)	0.7673	(B)	0.2327	(C)	0.0286	(D)	0.1429
6.	*The probability that the individual is a two year college attendance or male is:*						
(A)	0.0571	(B)	0.8858	(C)	0.2571	(D)	0.4286
7.	*The events: the individual is a four year college attendance and male are:*						
(A)	Mutually exclusive	(B)	Independent	(C)	Dependent	(D)	None of these

Question 8:

	Blood pressure		
	Low (L)	*Medium* (M)	*High* (H)
Has obesity (B)	*50*	*150*	*300*
Does not have obesity ($\bar{B}$)	*250*	*240*	*110*

If an individual is selected at random from this group, then the probability that he/she

1.has obesity or has medium blood pressure is equal to

A) 0.442 *B) 0.50* *C) 0.725* ***D) 0.673***

2.has low blood pressure given that he/she has obesity is equal to

A) 0.90 ***B) 0.1*** *C) 0.66* *D) 0.44*

Bayes' Theorem, Screening Tests, Sensitivity, Specificity, and Predictive Value Positive and Negative

	Disease		
Test Result	**Present (D)**	**Absent ($\bar{D}$)**	**Total**
Positive (T)	a	b	a + b = n(T)
Negative ($\bar{T}$)	c	d	c + d = n($\bar{T}$)
Total	a + c = n(D)	b + d = n($\bar{D}$)	n

1. The probability of false positive result:

$$P(T \mid \bar{D}) = \frac{n(T \cap \bar{D})}{n(\bar{D})} = \frac{b}{b+d}$$

2. The probability of false negative result:

$$P(\bar{T} \mid D) = \frac{n(\bar{T} \cap D)}{n(D)} = \frac{c}{a+c}$$

3. The sensitivity of the screening test:

$$P(T \mid D) = \frac{n(T \cap D)}{n(D)} = \frac{a}{a+c}$$

4. The specificity of the screening test:

$$P(\bar{T} \mid \bar{D}) = \frac{n(\bar{T} \cap \bar{D})}{n(\bar{D})} = \frac{d}{b+d}$$

- ***The predictive value positive:***

$$P(D|T) = \frac{P(D\cap T)}{P(T)} = \frac{P(T|D)P(D)}{P(T|D)P(D)+P(T|\bar{D})P(\bar{D})}$$

$$= \frac{(seneitivity)\ P\left(D_{given}\right)}{(seneitivity)\ P\left(D_{given}\right) + (false\ +ve)P\left(\bar{D}_{given}\right)}$$

- ***The predictive value negative:***

$$P(\bar{D}|\bar{T}) = \frac{P(\bar{T}\cap\bar{D})}{P(\bar{T})} = \frac{P(\bar{T}\cap\bar{D})P(\bar{D})}{P(\bar{T}\cap\bar{D})P(\bar{D})+P(\bar{T}|D)P(D)}$$

$$= \frac{(specificity)\ P\left(\bar{D}_{given}\right)}{(specificity)\ P\left(\bar{D}_{given}\right) + (false - ve)P\left(D_{given}\right)}$$

Question 1:

The following table shows the results of a screening test:

	Disease confirmed (D)	*Disease not confirmed ($\overline{D}$)*
Positive test (T)	*38*	*10*
Negative test ($\overline{T}$)	*5*	*18*

1. *The probability of false positive of the test is:* $\boxed{\frac{10}{28} = 0.3571}$

A)	<u>*0.3571*</u>	B)	*0.2083*	C)	*0.7916*	D)	*0.2173*

2. *The probability of false negative of the test is:* $\boxed{\frac{5}{43} = 0.1163}$

A)	*0.3571*	B)	*0.7826*	C)	*0.2173*	D)	<u>*0.1163*</u>

3. *The sensitivity value of the test is:* $\boxed{\frac{38}{43} = 0.8837}$

A)	*0.2173*	B)	<u>*0.8837*</u>	C)	*0.6429*	D)	*0.3571*

4. *The specificity value of the test is:* $\boxed{\frac{18}{28} = 0.6429}$

A)	<u>*0.6429*</u>	B)	*0.3571*	C)	*0.2173*	D)	*0.2535*

Suppose it is known that the rate of the disease is 0.113,

5. *The predictive value positive of a symptom is:* $\boxed{1 - 0.113 = 0.887}$

$$\boxed{= \frac{(seneitivity)\ P(D_{given})}{(seneitivity)\ P(D_{given}) + (false\ +ve) P(\overline{D}_{given})} = \frac{0.8837 \times 0.113}{0.8837 \times 0.113 + 0.3571 \times 0.887} = 0.2397}$$

A)	*0.9797*	B)		*0.5714*	C)	<u>*0.2397*</u>	D)	*0.34591*

6. *The predictive value negative of a symptom is:*

$$\boxed{= \frac{(specificity)\ P(\overline{D}_{given})}{(specificity)\ P(\overline{D}_{given}) + (false - ve) P(D_{given})} = \frac{0.6429 \times 0.887}{0.6429 \times 0.887 + 0.1163 \times 0.113} = 0.9772}$$

A)	<u>*0.9775*</u>	B)	*0.5714*	C)	*0.2397*	D)	*0.34591*

Question 2:

It is known that 40% of the population is diabetic. 330 persons who were diabetics went through a test where the test confirmed the disease for 288 persons. Among 270 healthy persons, test showed high sugar level for 22 persons. The information obtained is given in the table below.

Test	*Diabetics (D)*	*Not Diabetic (D^c)*	*TOTAL*
Positive (T)	*288*	*72*	*360*
Negative ($\bar{T}$)	*42*	*198*	*240*
TOTAL	*330*	*270*	*600*

1. *The sensitivity of the test is:* $\boxed{\frac{288}{330} = 0.873}$

(A) <u>0.873</u>	*(B) 0.480*	*(C) 0.733*	*(D) 0.33*

2. *The specificity of the test is:* $\boxed{\frac{198}{270} = 0.733}$

(A) 0.873	*(B) 0.330*	*(C) 0.48*	***(D) <u>0.733</u>***

3. *The probability of false positive is:* $\boxed{\frac{72}{270} = 0.267}$

(A) 0.1549	*(B) 0.127*	*(C) 0.713*	***(D) <u>0.267</u>***

4. *The predictive probability positive for the disease is:*

$$\boxed{= \frac{(seneitivity)\ P(D_{given})}{(seneitivity)\ P(D_{given}) + (false\ +ve)P(\bar{D}_{given})} = \frac{0.873 \times 0.40}{0.873 \times 0.40 + 0.267 \times 0.60} = 0.686}$$

(A) <u>0.686</u>	*(B) 0.800*	*(C) 0.480*	*(D) 0.873*

Question 3:

The following table shows the results of a screening test evaluation in which a random sample of 700 subjects with the disease and an independent random sample of 1300 subjects without the disease participated:

Disease ***Test result***	***Present***	***Absent***
Positive	***500***	***100***
Negative	***200***	***1200***

1) The sensitivity value of the test is: $\boxed{\frac{500}{700} = 0.7143}$

(A) 0.2649	*(B)* ***0.7143***	*(C) 0.7538*	*(D) 0.923*

2) The specificity value of the test is: $\boxed{\frac{1200}{1300} = 0.923}$

(A) 0.1	*(B) 0.7143*	*(C) 0.9943*	*(D)* ***0.923***

3) The probability of false positive of the test is: $\boxed{\frac{100}{1300} = 0.0769}$

(A) 0.0583	*(B) 0.2462*	*(C)* ***0.0769***	*(D) 0.2649*

4) If the rate of the disease in the general population is 0.002, then the predictive value positive of the test is:

$$\boxed{\begin{aligned} &= \frac{(seneitivity)\ P(D_{given})}{(seneitivity)\ P(D_{given}) + (false\ +ve)P(\overline{D}_{given})} \\ &= \frac{0.7143 \times 0.002}{0.7143 \times 0.002 + 0.0769 \times 0.998} = 0.01827 \end{aligned}}$$

(A) 0.9748	*(B)* ***0.01827***	*(C) 0.002*	*(D) 0.0252*

Question 4:

In a study of high blood pressure, 188 persons found positive, of a sample of 200 persons with the disease subjected to a screening test. While, 27 persons found positive, of an independent sample of 300 persons without the disease subjected to the same screening test. That is,

	High Blood Pressure		
Test Result	*Yes* D	*No* $\bar{D}$	*Total*
Positive T	*188*	*27*	*215*
Negative $\bar{T}$	*12*	*273*	*285*
Total	*200*	*300*	*500*

[1] Given that a person has the disease, the probability of a positive test result, that is, the "sensitivity" of this test is:

A	*0.49*	**B**	***0.94***	**C**	*0.35*	**D**	*0.55*

[2] Given that a person does not have the disease, the probability of a negative test result, that is, the "specificity" of this test is:

A	***0.91***	**B**	*0.75*	**C**	*0.63*	**D**	*0.49*

[3] The "false negative" results when a test indicates a negative status given that the true status is positive is:

A	*0.01*	**B**	*0.15*	**C**	*0.21*	**D**	***0.06***

[4] The "false positive" results when a test indicates a positive status given that the true status is negative is:

A	*0.16*	**B**	*0.31*	**C**	***0.09***	**D**	*0.02*

Assuming that 15% of the population under study is known to be with high blood pressure.

[5] Given a positive screening test, what is the probability that the person has the disease? That is, the "predictive value positive" is:

A	*0.22*	**B**	***0.65***	**C**	*0.93*	**D**	*0.70*

[6] Given a negative screening test result, what is the probability that the person does not have the disease? That is, the "predictive value negative" is:

A	*0.258*	**B**	*0.778*	**C**	***0.988***	**D**	*0.338*

Question 5:

Suppose that the ministry of health intends to check the reliability of the central Diabetic Lab in Riyadh. A sample person with Diabetic disease (D) and another without the disease ($\overline{D}$) had the Lab tests and the results are given below:

	Present (D)	*Absence* ($\overline{D}$)
Positive (T)	*950*	*40*
Negative ($\overline{T}$)	*25*	*640*

Then:

1.	*The probability of false negative result is:*						
(A)	**0.0256**	(B)	0.9412	(C)	0.9744	(D)	0.0588
2.	*The probability of false positive result is:*						
(A)	0.0256	(B)	0.9412	(C)	0.9744	(D)	**0.0588**
3.	*The sensitivity of the test is:*						
(A)	0.0256	(B)	0.9412	(C)	**0.9744**	(D)	0.0588
4.	*The specificity of the test is:*						
(A)	0.0256	(B)	**0.9412**	(C)	0.9744	(D)	0.0588

Assume that the true percentage of Diabetic patients in Riyadh is 25%. Then

5.	*The predictive value positive of the test is:*						
(A)	**0.847**	(B)	0.924	(C)	0.991	(D)	0.695
6.	*The predictive value negative of the test is:*						
(A)	0.195	(B)	0.982	(C)	0.847	(D)	**0.991**

Question 6:

A Fecal Occult Blood Screen Outcome Test is applied for 875 patients with bowel cancer. The same test was applied for another sample of 925 without bowel cancer. Obtained results are shown in the following table:

	Present Disease (D)	*Absent Disease* $(\bar{D})$
Test Positive (T)	*850*	*10*
Test Negative $(\bar{T})$	*25*	*915*

1. The sensitivity of the test is

A) 0.85 **B) <u>0.971</u>** *C) 0.915* *D) 0.988*

2. The specificity of the test is

A) 0.850 *B) 0.250* *C) 0.915* **D) <u>0.989</u>**

3. The probability of false positive is

A) 0.989 **B) <u>0.011</u>** *C) 0.250* *D) 0.915*

4. The probability of false negative is

A) 0.250 *B) 0.971* **C) <u>0.029</u>** *D) 0.10*

5. If the rate of the disease in the general population is equal to 15% then the predictive value positive of the test is

A) <u>0.941</u> *B) 0.995* *C) 0.674* *D) 0.150*

More Exercises

Question 1:

Givens:

$$\boldsymbol{P(A) = 0.5}, \quad \boldsymbol{P(B) = 0.4}, \quad \boldsymbol{P(C \cap A^c) = 0.6},$$
$$\boldsymbol{P(C \cap A) = 0.2}, \quad \boldsymbol{P(A \cup B) = 0.9}$$

(a) ***What is the probability of $\boldsymbol{P(C)}$:***

$$P(C) = P(C \cap A^c) + P(C \cap A) = 0.6 + 0.2 = 0.8$$

(b) ***What is the probability of $\boldsymbol{P(A \cap B)}$:***

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ \Rightarrow \quad 0.9 &= 0.5 \quad + \quad 0.4 - P(A \cap B) \\ P(A \cap B) &= 0 \end{aligned}$$

(c) ***What is the probability of $\boldsymbol{P(C \mid A)}$:***

$$P(C \mid A) = \frac{P(C \cap A)}{P(A)} = \frac{0.2}{0.5} = 0.4$$

(d) ***What is the probability of $\boldsymbol{P(B^c \cap A^c)}$***:

$$P(B^c \cap A^c) = 1 - P(B \cup A) = 1 - 0.9 = 0.1$$

Question 2:

Givens:

$$\boldsymbol{P(B) = 0.3}, \quad \boldsymbol{P(A \mid B) = 0.4}$$

$\boldsymbol{Then\ find\ P(A \cap B)} = ?$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$
$$\Rightarrow 0.4 = \frac{P(A \cap B)}{0.3}$$
$$\Rightarrow P(A \cap B) = 0.4 \times 0.3 = 0.12$$

<u>Question 3:</u>

Givens:

$$\boldsymbol{P(A) = 0.3,\quad P(B) = 0.4,\quad P(A \cap B \cap C) = 0.03,\quad P(\overline{A \cap B}) = 0.88}$$

(1) Are the event A and b independent?

$$P(A \cap B) = 1 - P(\overline{A \cap B}) = 1 - 0.88 = 0.12$$

$$P(A) \times P(B) = 0.3 \times 0.4 = 0.12$$
$$\Rightarrow P(A \cap B) = P(A) \times P(B)$$
$$\textit{Therefore, A and B are independent.}$$

(2) ***What is the probability of*** $\boldsymbol{P(C \mid A \cap B)}$:

$$P(C \mid A \cap B) = \frac{P(A \cap B \cap C)}{P(A \cap B)} = \frac{0.03}{0.12} = 0.25$$

<u>Question 4:</u>

Givens:

$$\boldsymbol{P(A_1) = 0.4,\ \ P(A_1 \cap A_2) = 0.2,\ \ P(A_3 \mid A_1 \cap A_2) = 0.75}$$

(1) ***Find the*** $\boldsymbol{P(A_2|A_1)}$:

$$P(A_2|A_1) = \frac{P(A_1 \cap A_2)}{P(A_1)} = \frac{0.2}{0.4} = 0.5$$

(2) ***Find the*** $\boldsymbol{P(A_1 \cap\ A_2 \cap A_3)}$:

$$P(A_3 \mid A_1 \cap A_2) = \frac{P(A_1 \cap\ A_2 \cap A_3)}{P(A_1 \cap A_2)}$$

$$0.75 = \frac{P(A_1 \cap\ A_2 \cap A_3)}{0.2}$$

$$P(A_1 \cap\ A_2 \cap A_3) = 0.75 \times 0.2 = 0.15$$