Calculus I (Math 101)
Sample Exam-Final
Second Semester
2016-2017

Question .1

In each of the followings find the limit, if it exists.

(a) $\lim_{x \to 3} \frac{\sqrt{x+1}}{x-2}$

(b) $\lim_{x \to \infty} \frac{3x-2}{x^2+7}$

(c) $\lim_{x \to 4^+} \frac{2}{x-4}$

(d) $\lim_{x \to 0} \frac{1-cos2x}{x^2}$

Question .2

Find $f'(x)$ in each of the followings:

(a) $f(x) = x^2 e^x$

(b) $f(x) = \sqrt{3x^2 + 5x}$

(c) $f(x) = \dfrac{x+2}{3x+5}$

(d) $f(x) = \ln(tan^{-1}x)$

Question .3

(a) Let

$$f(x) = \begin{cases} x^2 + 2 \ , & x < 1 \\ 3x \quad \ , & 1 \le x \le 4 \\ \dfrac{x}{x-1} \ , & x > 4 \end{cases}$$

(i) Determine whether $f(x)$ is continuous at $x = 1$.

(ii) Determine whether $f(x)$ is continuous at $x = 4$.

(b) Find an equation of the tangent line to the curve $y = x^3 + 2x$ at $x = 2$.

Question .4

(a) Find the absolute maximum and the absolute minimum of $f(x) = x^2 - 4x$ over $[1, 5]$.

(b) Suppose that x and y are differentiable functions of t and are related by the equation $y = x^3 - x$. Find dy/dt at $t = 2$ if $x = 1$ and $dx/dt = 3$ at $t = 2$.

Question .5

Consider the function $f(x) = \frac{1}{3}x^3 - x^2 - 3x$.

(a) Find the critical points of $f(x)$.

(b) Find the intervals on which $f(x)$ is increasing or decreasing.

(c) Find the relative minimums and maximums of $f(x)$, if any exist.

(d) Find the intervals on which $f(x)$ is concave up or down and find the points of inflection.

Calculus I (Math 101)

Sample Exam-Final

First Semester

2017-2018

Question .1

In each of the followings find the limit, if it exists.

(a) $\lim\limits_{x\to 0} \dfrac{3+xe^{2x}}{x^2+1}$

(b) $\lim\limits_{x\to 6} \dfrac{x^2-5x-6}{x^2-6x}$

(c) $\lim\limits_{t\to 0} \dfrac{e^{4t}-1}{\sin 2t}$

(d) $\lim\limits_{x\to\infty} \dfrac{4x^5-50x^3+15}{30x^2-7x^5}$

(e) $\lim\limits_{x\to 0^+} x\ln x$

Question .2

(a) Write the answer for each of the followings

(i) $\dfrac{d}{dx}(2x^8 - x^6) =$

(ii) $\dfrac{d}{dx}(\sin x) =$

(iii) $\dfrac{d}{dx}(\sec x) =$

(iv) $\dfrac{d}{dx}(\tan^{-1} x) =$

(v) $\dfrac{d}{dx}(\ln(x + 7)) =$

(vi) $\dfrac{d}{dx}(4^x) =$

(b) Find $\dfrac{dy}{dx}$ where $y = \dfrac{x^3 - x}{x^2 + 4}$

(c) Find $\dfrac{d^2y}{dx^2}$ where $y = x^3e^{-3x}$

Question .3

(a) Find $f'(x)$ where $f(x) = \left(x^2 + \tan x\right)^{10}$

(b) Find $\dfrac{dy}{dx}$ where $y = x^{\cos x}$

(c) Find the absolute maximum and minimum values of $f(x) = x^3 - 6x^2$ on the interval $[1, 5]$

Question .4

Consider the function $f(x) = x^4 - 4x^3$.

(a) Find the critical points of $f(x)$.

(b) Find the intervals on which $f(x)$ is increasing or decreasing.

(c) Find the relative minimums and maximums of $f(x)$, if any exist.

(d) Find the intervals on which $f(x)$ is concave up or down and find the points of inflection.

Question .5

The function $s(t) = t^3 - 6t^2$ describes the position of a particle moving along $s-$axis, where s is in meters and t is in seconds.

(a) Find the velocity and acceleration functions.

(b) At what times is the particle stopped?

(c) When is the particle speeding up? Slowing down?

Calculus I (Math 101)

Sample Final Exam

Second Semester

2017-2018

Question 1.

Answer the followings

(a) Given that $\lim_{x\to a} f(x) = 5$ and $\lim_{x\to a} g(x) = 4$, find $\lim_{x\to a} (2f(x) - 5g(x))^2$

(b) Find $\lim_{x\to 0} \dfrac{\sqrt{x^2+36}-6}{x}$

(c) Find $\lim_{x\to 1} f(x)$, if it exists, where $f(x) = \begin{cases} \frac{1}{x+7} & x \leq 1 \\ 1-7x & x > 1 \end{cases}$.

(d) Find $\lim_{x\to +\infty} \dfrac{e^{9x}}{x^4}$

(e) Find $\lim_{x \to +\infty} cos\left(\frac{8}{x}\right) sin\left(\frac{\pi x}{2x+1}\right)$

(f) Find $\frac{d^2y}{dx^2}|_{x=4}$ where $y = 3\sqrt{x} + 9x^2$

(g) Find $f'(x)$ where $f(x) = \ln(\sqrt{x}\tan x)$.

(h) Find $\frac{dy}{dx}$ where $y = 5x\sin(3x) + x^2\cos(3x)$.

Question 2.

(a) Find the equation of the tangent line to the curve $y^3 + yx = 3y^2 - x^2$ at the point $(0, 3)$

(b) A point P is moving along a curve whose equation is $y = \sqrt{x^2 + 64}$. When $P = (6, 10)$, y is increasing at a rate of 3 unites per second. How fast is x changing?

(c) Find the absolute maximum and minimum of $f(x) = 2x^3 - 9x^2 + 12x + 5$ on the interval $[-1, 4]$.

(d) Verify that $f(x) = x^3 - 5x + 4$ satisfies the hypothesis of the Mean-Value Theorem over the interval $[-2, 3]$ and find all values of c that satisfy the conclusion of the theorem.

Question 3.

Consider the function $f(x) = x^4 - 12x^2 + 7$.

(a) Find the critical points of $f(x)$ and the intervals on which f is increasing or decreasing, and determine the relative maximums and minimums of $f(x)$.

(b) Find the intervals on which $f(x)$ is concave up or down and find the points of inflection.

Question 4.

The function $s(t) = t^4 - 6t^3 + 4t^2$ describes the position of a particle moving along $s-$axis, where s is in meters and t is in seconds.

(a) Find the velocity and acceleration functions.

(b) Find the total distance traveled from $t = 0$ to $t = 5$?

(1) Find $\lim_{x\to 8} \dfrac{x^2-64}{x-8}$

(a) 8 (b) 16 (c) -16 (d) -8 (e) 0

(2) Find $\lim_{x\to -\infty} \dfrac{10x^4-1}{2x^4+5}$

(a) 0 (b) $-\infty$ (c) ∞ (d) 5 (e) $-\dfrac{1}{5}$

(3) If $f(x) = \sqrt{x^5+3}$, then $f'(x) =$

(a) $\dfrac{5x^4}{\sqrt{x^5+3}}$

(b) $\dfrac{5x^4}{2}$

(c) $\dfrac{5x^5}{2}$

(d) $\dfrac{5x^4+3}{\sqrt{x^5+3}}$

(e) $\dfrac{5x^4}{2\sqrt{x^5+3}}$

(4) Find y', where $y = x^3 \cos 2x$

(a) $3x^2 \sin 2x$

(b) $-6x^2 \sin 2x$

(c) $3x^2 \cos 2x + x^3 \sin 2x$

(d) $-3x^2 \sin 2x$

(e) $3x^2 \cos 2x - 2x^3 \sin 2x$

(5) If $g(x) = x^2 + \ln x$, then $\lim_{x\to 1} \left(\dfrac{g(x) - g(1)}{x-1} \right) =$

(a) 0 (b) 1 (c) 2 (d) 3 (e) -1

(6) Let f and g be differentiable functions. Suppose that $f(5) = -4$, $g(5) = 6$, $f'(6) = 3$, $g'(5) = 11$. Find $(f \circ g)'(5)$

(a) 33 (b) 3 (c) 11 (d) 12 (e) -44

(7) Find the slope of tangent line to the graph of $y = \dfrac{x^4 + 2x - 3}{7 - x^3}$ at $x = 0$.

(a) $-\dfrac{3}{7}$ (b) $-\dfrac{2}{7}$ (c) $\dfrac{2}{7}$ (d) 2 (e) -1

(8) Let $f(x) = \begin{cases} 2 - 2x & x \geq 2 \\ x - x^2 & x < 2 \end{cases}$.

Which one of the following statements is true about f? (Only one is true).

(a) f is continuous and differentiable at $x = 2$.

(b) f is continuous but not differentiable at $x = 2$.

(c) f is neither continuous nor differentiable at $x = 2$.

(d) f is not continuous but it is differentiable at $x = 2$.

(e) None of the above.

(9) Find $f''(x)$ where $f(x) = (1 - 2x)^7$.

(a) $f''(x) = 42(1 - 2x)^5$

(b) $f''(x) = -42(1 - 2x)^5$

(c) $f''(x) = 7(-2)^6$

(d) $f''(x) = -84(1 - 2x)^5$

(e) $f''(x) = 168(1 - 2x)^5$

(10) $\dfrac{d}{dx}\left(sin^{-1}(5x)\right) =$

(a) $5\cos^2(5x)$

(b) $\dfrac{5}{1 + 25x^2}$

(c) $\dfrac{5}{\sqrt{1 - 25x^2}}$

(d) $5\cos^{-2}(5x)$

(e) $\dfrac{1}{1 - 25x^2}$

(11) The local linear approximation of $f(x) = \sqrt{x}$ at $x_0 = 1$ is given by

(a) $f(x) \approx \frac{x}{2} + \frac{1}{2}$

(b) $f(x) \approx -\frac{x}{2} + \frac{1}{2}$

(c) $f(x) \approx x - \frac{1}{2}$

(d) $f(x) \approx \frac{x}{2} - \frac{1}{2}$

(e) $f(x) \approx -\frac{x}{2} - \frac{1}{2}$

(12) $\lim_{x \to \infty} \frac{e^{9x}}{x^4} =$

(a) 1 (b) 0 (c) $+\infty$ (d) $-\infty$ (e) $\frac{1}{24}$

(13) If $f'(x) > 0$ and $f''(x) < 0$ for all x in (a, b), then f(x) is

(a) increasing on $[a, b]$ and concave up on (a, b).

(b) increasing on $[a, b]$ and concave down on (a, b).

(c) decreasing on $[a, b]$ and concave up on (a, b).

(d) decreasing on $[a, b]$ and concave down on (a, b).

(e) None of the above.

(14) The function $f(x) = x^2 + 2$ satisfies the hypothesis of the Mean-Value Theorem on the interval $[2, 5]$. Find the value c that satisfies the conclusion of Mean-Value Theorem.

(a) 2.5 (b) 3.5 (c) 4.5 (d) 3 (e) 4

Question 2. [ILOs: 2.1 , 2.2] (6 + 3 =9 Marks)

(a) Use L'Hôpital's rule to find each of the following limits

(i) $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$

(ii) $\lim_{x \to +\infty} \left(7e^x + x^4\right)^{\frac{6}{x}}$

(b) Let $g(x) = \tan^{-1} x$. Find $f(x)$ so that $f'(x) = g'(x)$ and $f(1) = 2$.

Question 3. [ILOs: 2.3] (4 + 4 = 8 Marks)

(a) Find the equation of the tangent line to the curve $x^2y - y^3 = x - 7$ at the point $(1, 2)$.

(b) Find the absolute maximum and absolute minimum of $f(x) = \left(x^2 + x\right)^{\frac{2}{3}}$ on $[-2, 3]$.

Question 4. [ILOs: 2.3] (4 + 4 + 4 =12 Marks)

Let $f(x) = x^4 - 8x^3 + 5$.

(a) Find the critical points of $f(x)$ and

(b) Find the intervals on which f is increasing or decreasing, and determine the relative maximums and minimums of $f(x)$.

(c) Find the intervals on which $f(x)$ is concave up or down and find the points of inflection.

(1) Find $\lim_{x\to-\infty} \frac{10x^4-1}{2x^4+5}$

(a) $-\frac{1}{5}$ (b) 0 (c) 5 (d) $-\infty$ (e) ∞

(2) Find $\lim_{x\to 8} \frac{x^2-64}{x-8}$

(a) -8 (b) 8 (c) 0 (d) 16 (e) -16

(3) Find y', where $y = x^3 \cos 2x$

(a) $3x^2\cos 2x - 2x^3 \sin 2x$

(b) $3x^2 \sin 2x$

(c) $-6x^2 \sin 2x$

(d) $3x^2 \cos 2x + x^3 \sin 2x$

(e) $-3x^2 \sin 2x$

(4) If $f(x) = \sqrt{x^5+3}$, then $f'(x) =$

(a) $\frac{5x^5}{2}$

(b) $\frac{5x^4}{\sqrt{x^5+3}}$

(c) $\frac{5x^4+3}{\sqrt{x^5+3}}$

(d) $\frac{5x^4}{2}$

(e) $\frac{5x^4}{2\sqrt{x^5+3}}$

(5) Let f and g be differentiable functions. Suppose that $f(5) = -4$, $g(5) = 6$, $f'(6) = 3$, $g'(5) = 11$. Find $(f \circ g)'(5)$

(a) 12 (b) -44 (c) 33 (d) 3 (e) 11

(6) If $g(x) = x^2 + \ln x$, then $\lim_{x\to 1} \frac{g(x)-g(1)}{x-1} =$

(a) 2 (b) 3 (c) -1 (d) 0 (e) 1

(7) Let $f(x) = \begin{cases} 2-2x & x \geq 2 \\ x - x^2 & x < 2 \end{cases}$.

Which one of the following statements is true about f? (Only one is true).

(a) f is continuous and differentiable at $x = 2$.

(b) f is neither continuous nor differentiable at $x = 2$.

(c) f is continuous but not differentiable at $x = 2$.

(d) f is not continuous but it is differentiable at $x = 2$.

(e) None of the above.

(8) Find $f''(x)$ where $f(x) = (1-2x)^7$.

(a) $f''(x) = 168(1-2x)^5$

(b) $f''(x) = -84(1-2x)^5$

(c) $f''(x) = 42(1-2x)^5$

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(e) $f''(x) = 7(-2)^6$

(9) $\dfrac{d}{dx}\left(sin^{-1}(5x)\right) =$

(a) $\dfrac{1}{1-25x^2}$

(b) $\dfrac{5}{\sqrt{1-25x^2}}$

(c) $\dfrac{5}{1+25x^2}$

(d) $5\cos^2(5x)$

(e) $5\cos^{-2}(5x)$

(10) Find the slope of tangent line to the graph of $y = \dfrac{x^4+2x-3}{7-x^3}$ at $x = 0$.

(a) $\dfrac{2}{7}$ (b) 2 (c) -1 (d) $-\dfrac{3}{7}$ (e) $-\dfrac{2}{7}$

(11) If $f'(x) > 0$ and $f''(x) < 0$ for all x in (a, b), then f(x) is

(a) increasing on $[a, b]$ and concave down on (a, b).

(b) increasing on $[a, b]$ and concave up on (a, b).

(c) decreasing on $[a, b]$ and concave down on (a, b).

(d) decreasing on $[a, b]$ and concave up on (a, b).

(e) None of the above.

(12) The function $f(x) = x^2 + 2$ satisfies the hypothesis of the Mean-Value Theorem on the interval $[2, 5]$. Find the value c that satisfies the conclusion of Mean-Value Theorem.

(a) 3 (b) 4 (c) 2.5 (d) 3.5 (e) 4.5

(13) The local linear approximation of $f(x) = \sqrt{x}$ at $x_0 = 1$ is given by

(a) $f(x) \approx -\frac{x}{2} + \frac{1}{2}$

(b) $f(x) \approx x - \frac{1}{2}$

(c) $f(x) \approx \frac{x}{2} - \frac{1}{2}$

(d) $f(x) \approx -\frac{x}{2} - \frac{1}{2}$

(e) $f(x) \approx \frac{x}{2} + \frac{1}{2}$

(14) $\lim_{x \to \infty} \frac{e^{9x}}{x^4} =$

(a) $\frac{1}{24}$ (b) 1 (c) 0 (d) $-\infty$ (e) $+\infty$

Question 2. [ILOs: 2.1 , 2.2] (6 + 3 =9 Marks)

(a) Use L'Hôpital's rule to find each of the following limits

(i) $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$

(ii) $\lim_{x \to +\infty} \left(7e^x + x^4\right)^{\frac{6}{x}}$

(b) Let $g(x) = \tan^{-1} x$. Find $f(x)$ so that $f'(x) = g'(x)$ and $f(1) = 2$.

Question 3. [ILOs: 2.3] (4 + 4 = 8 Marks)

(a) Find the equation of the tangent line to the curve $x^2y - y^3 = x - 7$ at the point $(1, 2)$.

(b) Find the absolute maximum and absolute minimum of $f(x) = \left(x^2 + x\right)^{\frac{2}{3}}$ on $[-2, 3]$.

Question 4. [ILOs: 2.3] (4 + 4 + 4 =12 Marks)

Let $f(x) = x^4 - 8x^3 + 5$.

(a) Find the critical points of $f(x)$ and

(b) Find the intervals on which f is increasing or decreasing, and determine the relative maximums and minimums of $f(x)$.

(c) Find the intervals on which $f(x)$ is concave up or down and find the points of inflection.

Calculus I (Math 101)

Sample Final Exam

Second Semester

2018-2019

QUESTION 1: **(8×1=8 points, ILO's 1.1)**

Multiple-Choice: Select the correct answer for each of the following questions:

(1) $\frac{d}{dx}(\log_2(4x)) =$

(a) $\frac{1}{x \ln 2}$ (b) $\frac{1}{x}$ (c) $\frac{1}{4x}$ (d) $\frac{\ln 2}{4x}$

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(2) $\frac{d}{dx}(\tan^{-1} x^2) =$

(a) $\frac{1}{\sqrt{x^4+1}}$ (b) $\frac{2x}{x^4+1}$ (c) $\sec^{-1} x^2$ (d) $\frac{1}{x^4+1}$

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(3) If $f''(x) < 0$ for every $x \in (a, b)$, then $f(x)$ is **...........** on (a, b)

(a) increasing (b) decreasing (c) concave up (d) concave down

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(4) If $f(x) = \frac{x^2+2}{|x-3|}$, then f has infinite discontinuity at $x =$ **...........**

(a) ∞ **(b)** 3 **(c)** 0 **(d)** -3

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(5) If $\lim_{x\to\infty} f(x) = L$, then f has **...........**

(a) a vertical asymptote **(b)** a horizontal asymptote **(c)** a tangent line **(d)** a symmetric line

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(6) $\lim_{x\to-\infty} \frac{3+x^5}{1-x^5} =$

(a) 3 **(b)** 1 **(c)** $-\infty$ **(d)** -1

---

(7) If $f'(x) < 0$ for every $x \in (a, b)$, then $f(x)$ is **...........** on $[a, b]$

(a) increasing **(b)** decreasing **(c)** concave up **(d)** concave down

---

(8) If $f(-x) = f(x)$, then f is symmetric about the **...........**

(a) y −axis **(b)** x −axis **(c)** origin **(d)** $y = x$

QUESTION 2: **(2+2+3=7 points, ILO's 2.2)**

Find the derivative for each of the following functions:

(a) $f(x) = \frac{x^4+1}{x-6} + \tan^{-1} 2$

(b) $g(x) = \ln(\cos(3x + 1)) + (1 + 3x^2)^3$

(c) $f(x) = e^{x^2} \sin^{-1} x$

QUESTION 3: **(4+3=7 points, ILO's 2.3)**

(I) Use implicit differentiation to find the derivative of

$$x^2 + y^2 = xy$$

(II) Determine whether the following function is continuous everywhere

$$f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ 1 + 2x, & x > 1 \end{cases}$$

QUESTION 4: **(4+2=6 points, ILO's 2.2)**

(I) Use L'Hôpital's rule to find the following limit

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2}$$

(II) Suppose that $\boldsymbol{x}$ and $\boldsymbol{y}$ are differentiable functions of $\boldsymbol{t}$ and are related by the equation $\boldsymbol{y = x^4 + 5}$. Find $\frac{dy}{dt}$ at $\boldsymbol{t = 2}$ if $\boldsymbol{x = 1}$ and $\frac{dx}{dt} = 3$ at $\boldsymbol{t = 2}$.

QUESTION 5: (4+3=7 points, ILO's 1.2)

(I) Use logarithmic differentiation to find the derivative of $y = \frac{x^2\sqrt{x-14}}{(1+x^3)^4}$

(II) Find the absolute maximum and the absolute minimum of $f(x) = x^2 + 2x + 3$ over $[1, 5]$.

QUESTION 6: **(5×3=15 points, ILO's 2.1)**

Consider the function

$$\boldsymbol{f}(\boldsymbol{x}) = \frac{1}{3}\boldsymbol{x}^3 - \boldsymbol{x}^2$$

(a) Find the $\boldsymbol{x}-$ and $\boldsymbol{y}-$intercepts of $\boldsymbol{f}(\boldsymbol{x})$

(b) Determine the intervals on which $\boldsymbol{f}(\boldsymbol{x})$ is increasing and decreasing

(c) Determine the intervals on which $\boldsymbol{f}(\boldsymbol{x})$ is concave up and concave down

(d) Locate critical points, relative extrema and inflection points of $f(x)$

(e) Sketch the graph of $f(x)$ by using the above information

Good luck

QUESTION 1: **(8×1=8 points, ILO's 1.1)**

Multiple-Choice: Select the correct answer for each of the following questions:

(1) $\frac{d}{dx}(\cot^{-1} x^2) =$

(a) $\frac{-x}{x^4+1}$ (b) $\frac{1}{\sqrt{x^4+1}}$ (c) $\frac{-2x}{x^4+1}$ (d) $\csc^{-1} x^2$

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(2) $\frac{d}{dx}(\ln 2x) =$

(a) $\frac{1}{x \ln 2}$ (b) $\frac{1}{x}$ (c) $\frac{1}{2x}$ (d) $\frac{\ln 2}{2x}$

---

(3) If $\lim_{x\to\infty} f(x) = L$, then f has ....

(a) a vertical asymptote **(b)** a horizontal asymptote **(c)** a tangent line **(d)** a symmetric line

---

(4) $\lim_{x\to-\infty} \frac{3+4x^4}{1-2x^4} =$

(a) 3 (b) 2 (c) $-\infty$ (d) -2

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(5) If $f''(x) < 0$ for every $x \in (a, b)$, then $f(x)$ is .... on (a, b)

(a) increasing (b) decreasing (c) concave up (d) concave down

---

(6) If $f(x) = \frac{x^3+1}{|x-2|}$, then f has infinite discontinuity at $x =$ ....

(a) ∞ (b) 1 (c) 0 (d) 2

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(7) If $f'(x) < 0$ for every $x \in (a, b)$, then $f(x)$ is .... on $[a, b]$

(a) increasing **(b)** decreasing **(c)** concave up **(d)** concave down

---

(8) If $f(-x) = f(x)$, then f is symmetric about the ....

(a) y −axis **(b)** x −axis **(c)** origin **(d)** $y = x$

QUESTION 2: **(2+2+3=7 points, ILO's 2.2)**

Find the first derivative for each of the following functions:

(a) $f(x) = \frac{x^3+2}{x-4} + \sec(2)$

(b) $g(x) = (1 + x^2)^2 - \cos^2(x)$

(c) $f(x) = e^{2x} \cos^{-1} x$

QUESTION 3: **(4+3=7 points, ILO's 2.3)**

(I) Use implicit differentiation to find the derivative of

$$x^2 - y^2 = 2xy$$

(II) Determine whether the following function is continuous everywhere

$$f(x) = \begin{cases} x^3 - 2, & x \le 1 \\ 1 - x, & x > 1 \end{cases}$$

QUESTION 4: **(4+2=6 points, ILO's 2.2)**

(I) Use L'Hôpital's rule to find the following limit

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2}$$

(II) Suppose that x and y are differentiable functions of t and are related by the equation $y = x^3 - 6$. Find $\frac{dy}{dt}$ at $t = 2$ if $x = 1$ and $\frac{dx}{dt} = 3$ at $t = 2$.

QUESTION 5: (4+3=7 points, ILO's 1.2)

(I) Use logarithmic differentiation to find the derivative of $\boldsymbol{y} = \frac{\boldsymbol{x^3\sqrt{x+5}}}{\boldsymbol{(1+x^2)^4}}$

(II) Find the absolute maximum and the absolute minimum of $\boldsymbol{f(x)} = \boldsymbol{x^2} - \boldsymbol{2x} + 5$ over $[\mathbf{0}, \mathbf{2}]$.

QUESTION 6: **(5×3=15 points, ILO's 2.1)**

Consider the function

$$f(x) = \frac{1}{3}x^3 - x^2$$

(a) Find the $x-$ and $y-$intercepts of $f(x)$

(b) Determine the intervals on which $f(x)$ is increasing and decreasing

(c) Determine the intervals on which $f(x)$ is concave up and concave down

(d) Locate critical points, relative extrema and inflection points of $f(x)$

(e) Sketch the graph of $f(x)$ by using the above information

Good luck