3.3 Function Operations and Composition

العمليات على الدوال وتركيب الدوال.

مراجعة مجال الدوال

كثيرة حدود

Domain = $(-\infty, \infty)$

$f(x) = x^2 - 2x + 1$

$Df = (-\infty, \infty)$

مجال كثيرات الحدود دائماً يساوي جميع الأعداد الحقيقية.

دالة كسرية

المقام $\neq$ الصفر

$f(x) = \frac{1}{x+1}$

$x + 1 = 0$

$x \neq -1$

$Df = (-\infty, -1) \cup (-1, \infty)$

جميع الأعداد الحقيقية ماعدا صفر المقام.

دالة جذرية

البسط

ماتحت الجذر ≥ 0

$f(x) = \sqrt{2x+1}$

$2x + 1 \geq 0$

$x \geq -\frac{1}{2}$

$Df = [-\frac{1}{2}, \infty)$

المقام

ماتحت الجذر > 0

$f(x) = \frac{1}{\sqrt{2x+1}}$

$2x + 1 > 0$

$x > -\frac{1}{2}$

$Df = (-\frac{1}{2}, \infty)$

Operation on functions and Domains: مجال

العمليات على الدوال ومجالها.

Let $f(x)$, $g(x)$ defined functions.

$$(f+g)(x) = f(x) + g(x)$$

$$(f-g)(x) = f(x) - g(x)$$

$$(fg)(x) = f(x) \cdot g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad , \ g(x) \neq 0$$

Domains of $f+g$, $f-g$, fg is $D_f \cap D_g \Rightarrow$ تقاطع المجالات

Domain of $\frac{f}{g}$ is $D_f \cap D_g$ which $g \neq 0 \Rightarrow$ استبعاد اصفار المقام.

Example 1 p.(105): $f(x) = x^2+1$, $g(x) = 3x+5$

Find:

ⓐ $(f+g)(1) = f(1) + g(1) = \left((1)^2+1\right) + \left(3(1)+5\right) = 10$

ⓑ $(f-g)(-3) = f(-3) - g(-3) = \left((-3)^2+1\right) - \left(3(-3)+5\right) = 10-(-4) = 14$

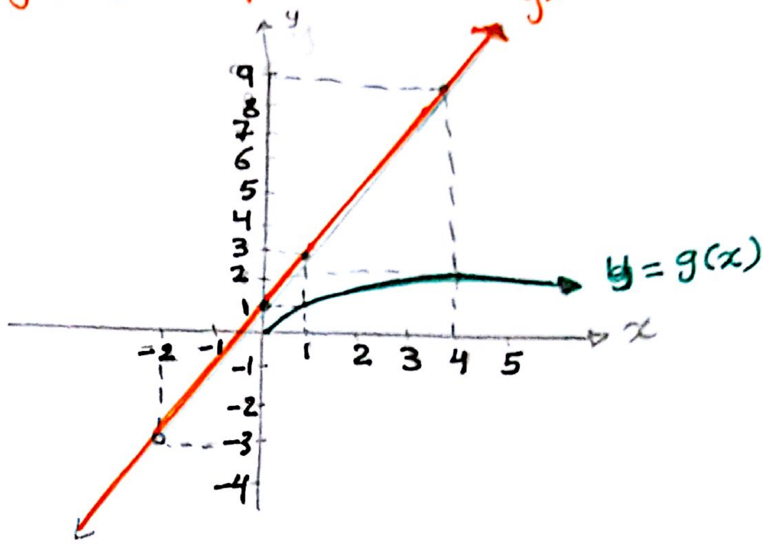
ⓒ $(fg)(5) = f(5) \cdot g(5) = (5^2+1)(3(5)+5) = (26)(20) = 520$

ⓓ $\left(\frac{f}{g}\right)(0) = \frac{f(0)}{g(0)} = \frac{0^2+1}{2(0)+5} = \frac{1}{5}$

Example 2 p.(106)

$f(x) = 2x+1$, $g(x) = \sqrt{x}$

x	$f(x)$	$g(x)$
-2	-3	undefined
0	1	0
1	3	1
4	9	2



احسبي:

Evaluate :~

$$(f+g)(4) = f(4) + g(4) = 9+2 = 11$$

$$(f-g)(-2) = f(-2) - g(-2) = \text{undefined.}$$ غير معرف

$$(fg)(1) = f(1) \cdot g(1) = 3(1) = 3$$

$$\left(\frac{f}{g}\right)(0) = \frac{f(0)}{g(0)} = \frac{1}{0} \quad \text{undefined.}$$ غير معرف

The Difference Quotient حاصل الفرق

$$\frac{f(x+h)-f(x)}{h}$$

<u>HW2</u> p. (108) Let $f(x) = 2x^2 - 3x$

Find $\dfrac{f(x+h)-f(x)}{h}$

$$\frac{f(x+h)-f(x)}{h} = \frac{2(x+h)^2 - 3(x+h) - (2x^2 - 3x)}{h}$$

$$= \frac{2(x^2 + 2hx + h^2) - 3x - 3h - 2x^2 + 3x}{h}$$

$$= \frac{\cancel{2x^2} + 4hx + 2h^2 - \cancel{3x} - 3h - \cancel{2x^2} + \cancel{3x}}{h}$$

$$= \frac{4hx + 2h^2 - 3h}{h} = \frac{h(4x + 2h - 3)}{h}$$

$$= 4x + 2h - 3$$

<u>ملاحظة</u> :

$$f(x+h) \neq f(x) + f(h)$$

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– Composition of Functions and Domain:

تركيب الدوال ومجالها.

$$(f \circ g)(x) = f(g(x))$$

$$(g \circ f)(x) = g(f(x))$$

Example 3 p.(109): $f(x) = 2x-1$, $g(x) = \frac{4}{x-1}$

Find: (a) $(f \circ g)(2)$, (b) $(g \circ f)(-3)$

(a) $(f \circ g)(2) = f(g(2))$

$$\boxed{g(2) = \frac{4}{2-1} = 4}$$

$\therefore (f \circ g)(2) = f(g(2)) = f(4) = 2(4)-1$

$= 8-1 = 7$

(b) $(g \circ f)(-3) = g(f(-3))$

$$\boxed{f(-3) = 2(-3)-1 = -7}$$

$\therefore (g \circ f)(-3) = g(f(-3)) = g(-7) = \frac{4}{-7-1}$

$= \frac{4}{-8} = -\frac{1}{2}$

Example 4 p. (110) : $f(x) = \frac{6}{x-3}$, $g(x) = \frac{1}{x}$

Find : (a) $(f \circ g)(x)$ and its domain

(b) $(g \circ f)(x)$ and its domain

ⓐ $(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x}\right)$

domain
$x \neq 0$
$(-\infty, 0) \cup (0, \infty)$

$$= \frac{6}{\frac{1}{x} - 3} = \frac{6}{\frac{1-3x}{x}} = 6 \cdot \frac{x}{1-3x}$$

$$= \frac{6x}{1-3x} \rightarrow$$

دالة كسرية
مجالها المقام ≠ صفر.

$$1 - 3x = 0 \Rightarrow -3x = -1 \Rightarrow x \neq \frac{1}{3}$$

$(-\infty, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$

$\therefore$ Domain of $(f \circ g)(x) = (-\infty, 0) \cup (0, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$

ⓑ $(g \circ f)(x) = g(f(x)) = g\left(\frac{6}{x-3}\right)$

domain
$x - 3 \neq 0$
$x \neq 3$
$(-\infty, 3) \cup (3, \infty)$

$$= \frac{1}{\frac{6}{x-3}} = 1 \cdot \frac{x-3}{6} = \frac{x-3}{6}$$

مجالها
جميع الأعداد الحقيقية.

$\therefore$ Domain of $(g \circ f)(x) = (-\infty, 3) \cup (3, \infty)$

ملاحظة هامة جداً :-

$(f\circ g)(x) \neq (g\circ f)(x)$

Example 5 p. (111): Find f and g

$(f\circ g)(x) = (x^2-5)^3 - 4(x^2-5) + 3$

$g(x) = x^2-5 \quad , \quad f(x) = x^3 - 4x + 3$

$(f\circ g)(x) = f(g(x)) = f(x^2-5)$

$= (x^2-5)^3 - 4(x^2-5) + 3$

Or

$g(x) = x^2 \quad , \quad f(x) = (x-5)^3 - 4(x-5) + 3$

$(f\circ g)(x) = f(g(x)) = f(x^2) = (x^2-5)^3 - 4(x^2-5) + 3$

Exercises 3.3 p.(111)

$f(x) = x^2 + 3$, $g(x) = -2x + 6$

Find:

① $(f+g)(3) = f(3) + g(3)$

$= (3^2+3) + (-2(3)+6)$

$= (9+3) + (-6+6)$

$= 12 + 0 = 12$

② $(f-g)(-1) = f(-1) - g(-1)$

$= ((-1)^2+3) - (-2(-1)+6)$

$= (1+3) - (2+6)$

$= 4 - 8 = -4$

③ $(fg)(4) = f(4)\,g(4)$

$= (4^2+3)(-2(4)+6)$

$= (16+3)(-8+6) = 19(-2) = -38$

④ $\left(\frac{f}{g}\right)(-1) = \frac{f(-1)}{g(-1)} = \frac{((-1)^2+3)}{-2(-1)+6} = \frac{4}{8} = \frac{1}{2}$

Find $(f+g)(x)$, $(f-g)(x)$, $(fg)(x)$ and $(\frac{f}{g})(x)$

Give the Domain. مجالاتها عبارة عن تقاطع مجال f و g.

(5) $f(x) = 3x+4$, $g(x) = 2x-5$

لأنها كثيرات حدود
← مجالها $(-\infty,\infty)$

Domain of $f(x)$: $(-\infty,\infty)$
$g(x)$: $(-\infty,\infty)$

$$* \quad (f+g)(x) = f(x)+g(x)$$
$$= (3x+4)+(2x-5)$$
$$= 3x+4+2x-5$$
$$= 5x-1$$

Domain $(f+g)(x)$: $(-\infty,\infty)$

تقاطع مجالي f, g

$$* \quad (f-g)(x) = f(x) - g(x)$$
$$= (3x+4)-(2x-5)$$
$$= 3x+4-2x+5$$
$$= x+9$$

Domain $(f-g)(x)$: $(-\infty,\infty)$

$$* \quad (fg)(x) = f(x)\,g(x) = (3x+4)(2x-5)$$
$$= 6x^2-15x+8x-20$$
$$= 6x^2-7x-20$$

Domain $(fg)(x)$: $(-\infty,\infty)$

$*\quad \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{3x+4}{2x-5} \longrightarrow$ المقام ≠ الصفر.

$2x-5 \neq 0 \Rightarrow x \neq \frac{5}{2}$

Domain $\left(\frac{f}{g}\right)(x)$: $\left(-\infty, \frac{5}{2}\right) \cup \left(\frac{5}{2}, \infty\right)$

(6) $f(x) = 2x^2 - 3x \quad , \quad g(x) = x^2 - x + 3$

Domain $f(x), g(x)$: $(-\infty, \infty)$ لأنها كثيرات حدود

$(f+g)(x) = f(x) + g(x)$

$= 2x^2 - 3x + x^2 - x + 3$

$= 3x^2 - 4x + 3$

Domain $(f+g)(x)$: $(-\infty, \infty)$

$(f-g)(x) = f(x) - g(x)$

$= (2x^2 - 3x) - (x^2 - x + 3)$

$= 2x^2 - 3x - x^2 + x - 3$

$= x^2 - 2x - 3$

Domain $(f-g)(x)$: $(-\infty, \infty)$

$$(fg)(x) = f(x)\,g(x)$$

$$= (2x^2 - 3x)(x^2 - x + 3)$$

$$= 2x^4 - 2x^3 + 6x^2 - 3x^3 + 3x^2 - 9x$$

$$= 2x^4 - 5x^3 + 9x^2 - 9x$$

Domain $(fg)(x)$: $(-\infty, \infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x^2 - 3x}{x^2 - x + 3}$$

$$x^2 - x + 3 \neq 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(3)}}{2}$$

$$= \frac{1 \pm \sqrt{1 - 12}}{2} = \frac{1}{2} \pm \frac{\sqrt{-11}}{2} = \frac{1}{2} \pm \frac{\sqrt{11}}{2}\, i$$

أصفار المقام عبارة عن أعداد مركبة ـــه جميع الأعداد الحقيقية مسموحة .

ـــه Domain $\left(\frac{f}{g}\right)(x)$: $(-\infty, \infty)$

⑦ $f(x) = \sqrt{4x-1}$

ما تحت الجذر أكبر من أو يساوي الصفر.

$$4x - 1 \geq 0$$

$$x \geq \frac{1}{4}$$

Domain $f(x)$: $[\frac{1}{4}, \infty)$

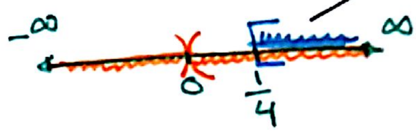
$g(x) = \frac{1}{x}$

المقام ≠ الصفر

$$x \neq 0$$

Domain $g(x)$: $(-\infty, 0) \cup (0, \infty)$

التقاطع هنا العناصر المشتركة ،،



$$(f+g)(x) = f(x) + g(x)$$
$$= \sqrt{4x-1} + \frac{1}{x}$$

Domain $(f+g)(x)$: $[\frac{1}{4}, \infty) \cap \left((-\infty, 0) \cup (0, \infty)\right)$

$$= [\frac{1}{4}, \infty)$$

التقاطع عبارة عن الفترة الأصغر.

$$(f-g)(x) = f(x) - g(x)$$
$$= \sqrt{4x-1} - \frac{1}{x}$$

Domain $(f-g)(x)$: $[\frac{1}{4}, \infty)$

تقاطع مجالي f, g

$$(fg)(x) = f(x)\,g(x) = \left(\sqrt{4x-1}\right)\left(\frac{1}{x}\right) = \frac{\sqrt{4x-1}}{x}$$

تقاطع المجالين $[\frac{1}{4}, \infty)$

Domain $(fg)(x)$: $[\frac{1}{4}, \infty)$

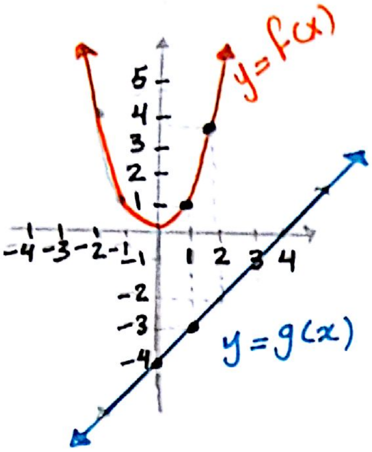
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{4x-1}}{\frac{1}{x}} = x\sqrt{4x-1}$$

Domain $\left(\frac{f}{g}\right)(x)$: $[\frac{1}{4}, \infty)$

Use the graph to evaluate :-

الرسم

(12)



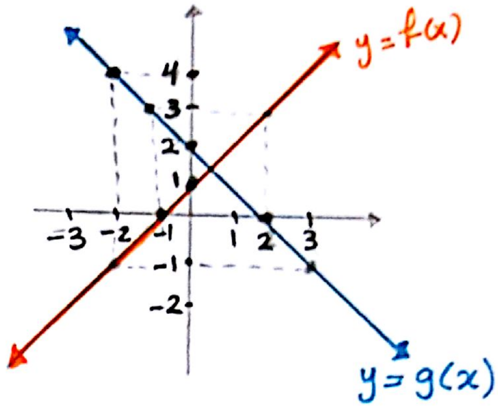
(a) $(f+g)(2) = f(2) + g(2)$
$= 4 + (-2) = 2$

(b) $(f-g)(1) = f(1) - g(1)$
$= 1 - (-3)$
$= 1 + 3 = 4$

(c) $(fg)(0) = f(0)\,g(0)$
$= 0(-4) = 0$

(d) $\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)} = \frac{1}{-3} = -\frac{1}{3}$

(13)



(a) $(f+g)(-1) = f(-1) + g(-1)$
$= 0 + 3 = 3$

(b) $(f-g)(-2) = f(-2) - g(-2)$
$= -1 - 4 = -5$

(c) $(fg)(0) = f(0)\,g(0)$
$= 1(2) = 2$

(d) $\left(\frac{f}{g}\right)(2) = \frac{f(2)}{g(2)} = \frac{3}{0}$ undefined
غير معرفة .

(14)

x	$f(x)$	$g(x)$
-2	0	6
0	5	0
2	7	-2
4	10	5

(a) $(f+g)(2) = f(2) + g(2) = 7 + (-2) = 5$

(b) $(f-g)(4) = f(4) - g(4) = 10 - 5 = 5$

(c) $(fg)(-2) = f(-2)\,g(-2) = 0(6) = 0$

(d) $\left(\frac{f}{g}\right)(0) = \frac{f(0)}{g(0)} = \frac{5}{0}$ undefined.

x	3	4	6
$f(x)$	1	3	9

x	2	7	1	9
$g(x)$	3	6	9	12

Find :-

(28) $(f \circ g)(2) = f(g(2)) = f(3) = 1$

(29) $(g \circ f)(3) = g(f(3)) = g(1) = 9$

(30) $(f \circ f)(4) = f(f(4)) = f(3) = 1$

– Find $(f \circ g)(x)$ and its domain.

$(g \circ f)(x)$ and its domain.

(36) $f(x) = \frac{2}{x}$, $g(x) = x + 1$

Domain $(-\infty, \infty)$

$(f \circ g)(x) = f(g(x)) = f(x+1)$

$= \frac{2}{x+1}$, $x + 1 \neq 0 \Rightarrow x \neq -1$

Domain $(f \circ g)(x)$: $(-\infty, -1) \cup (-1, \infty)$

domain $x \neq 0$

$(-\infty, 0) \cup (0, \infty)$

$(g \circ f)(x) = g(f(x)) = g\left(\frac{2}{x}\right)$

$= \frac{2}{x} + 1 = \frac{2+x}{x} \rightarrow x \neq 0$

Domain $(g \circ f)(x)$: $(-\infty, 0) \cup (0, \infty)$

(39) $f(x) = \dfrac{1}{x-2}$, $g(x) = \dfrac{1}{x}$

domain $x \neq 0$

$(-\infty, 0) \cup (0, \infty)$

$(f \circ g)(x) = f(g(x)) = f\left(\dfrac{1}{x}\right)$

$= \dfrac{1}{\frac{1}{x} - 2} = \dfrac{1}{\frac{1-2x}{x}} = \dfrac{x}{1-2x}$

مقام المقام بسط

$1 - 2x \neq 0 \Rightarrow x \neq \dfrac{1}{2}$

$(-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$

$\therefore$ Domain $(f \circ g)(x)$: $(-\infty, 0) \cup (0, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$

domain $x \neq 2$

$(-\infty, 2) \cup (2, \infty)$

$(g \circ f)(x) = g(f(x)) = g\left(\dfrac{1}{x-2}\right)$

$= \dfrac{1}{\frac{1}{x-2}} = x - 2$

$\therefore$ Domain $(g \circ f)(x)$: $(-\infty, 2) \cup (2, \infty)$

Find f, g such that $(f \circ g)(x) = h(x)$.

أوجدي الدوال f , g .

(44) $h(x) = (6x-2)^2$

$g(x) = 6x - 2$, $f(x) = x^2$

التأكد $(f \circ g)(x) = f(g(x)) = f(6x-2) = (6x-2)^2$

(45) $h(x) = \sqrt{x^2-1}$

$g(x) = x^2 - 1$, $f(x) = \sqrt{x}$

$(f \circ g)(x) = f(g(x)) = f(x^2-1) = \sqrt{x^2-1}$

(46) $h(x) = \sqrt{6x} + 12$

$g(x) = 6x$, $f(x) = \sqrt{x} + 12$

$(f \circ g)(x) = f(g(x)) = f(6x) = \sqrt{6x} + 12$