# Workshop Solutions to Sections 2.1 and 2.2

1) Find the domain of the function $f(x) = 9 - x^2$ .
Solution:
Since $f(x)$ is a polynomial, then
$$D_f = \mathbb{R} = (-\infty, \infty)$$

**Note:** The domain of any polynomial is $\mathbb{R}$ .

2) Find the range of the function $f(x) = 9 - x^2$ .
Solution:
$$R_f = (-\infty, 9]$$

3) Find the domain of the function $f(x) = 6 - 2x$ .
Solution:
Since $f(x)$ is a polynomial, then
$$D_f = \mathbb{R} = (-\infty, \infty)$$

4) Find the range of the function $f(x) = 6 - 2x$ .
Solution:
Since $f(x)$ is a polynomial of degree one ($i.e.$ is of an odd degree), then
$$R_f = \mathbb{R} = (-\infty, \infty)$$

5) Find the domain of the function $f(x) = x^2 - 2x - 3$ .
Solution:
Since $f(x)$ is a polynomial, then
$$D_f = \mathbb{R} = (-\infty, \infty)$$

6) Find the domain of the function $f(x) = 1 + 2x^3 - x^5$ .
Solution:
Since $f(x)$ is a polynomial, then
$$D_f = \mathbb{R} = (-\infty, \infty)$$

7) Find the domain of the function $f(x) = 5$ .
Solution:
Since $f(x)$ is a polynomial, then
$$D_f = \mathbb{R} = (-\infty, \infty)$$

8) Find the range of the function $f(x) = 5$ .
Solution:
$$R_f = \{5\}$$

9) Find the domain of the function $f(x) = |x - 1|$ .
Solution:
Since $f(x)$ is an absolute value of a polynomial, then
$$D_f = \mathbb{R} = (-\infty, \infty)$$

**Note:** The domain of an absolute value of any polynomial is $\mathbb{R}$ .

10) Find the domain of the function $f(x) = |x + 5|$ .
Solution:
Since $f(x)$ is an absolute value of a polynomial, then
$$D_f = \mathbb{R} = (-\infty, \infty)$$

11) Find the domain of the function $f(x) = |x|$ .
Solution:
Since $f(x)$ is an absolute value of a polynomial, then
$$D_f = \mathbb{R} = (-\infty, \infty)$$

12) Find the range of the function $f(x) = |x|$ .
Solution:
$$R_f = [0, \infty)$$

**Note:** The range of an absolute value of any polynomial is always $[0, \infty)$ .

13) Find the domain of the function $f(x) = |3x - 6|$ .
Solution:
Since $f(x)$ is an absolute value of a polynomial, then
$$D_f = \mathbb{R} = (-\infty, \infty)$$

14) Find the domain of the function $f(x) = |9 - 3x|$ .
Solution:
Since $f(x)$ is an absolute value of a polynomial, then
$$D_f = \mathbb{R} = (-\infty, \infty)$$

15) Find the domain of the function
$$f(x) = \frac{x + 2}{x - 3}$$
Solution:
$f(x)$ is defined when $x - 3 \neq 0 \implies x \neq 3$. So,
$$D_f = \mathbb{R} \setminus \{3\} = (-\infty, 3) \cup (3, \infty)$$

16) Find the domain of the function
$$f(x) = \frac{x - 2}{x + 3}$$
Solution:
$f(x)$ is defined when $x + 3 \neq 0 \implies x \neq -3$. So,
$$D_f = \mathbb{R} \setminus \{-3\} = (-\infty, -3) \cup (-3, \infty)$$

17) Find the domain of the function

$$f(x) = \frac{x+2}{x^2-9}$$

Solution:
$f(x)$ is defined when $x^2 - 9 \neq 0 \Longrightarrow x^2 \neq 9 \Longrightarrow x \neq \pm 3$. So,

$$D_f = \mathbb{R} \setminus \{-3,3\} = (-\infty,-3) \cup (-3,3) \cup (3,\infty)$$

18) Find the domain of the function

$$f(x) = \frac{x+2}{x^2-5x+6}$$

Solution:
$f(x)$ is defined when $x^2 - 5x + 6 \neq 0$
$\Longrightarrow (x-2)(x-3) \neq 0 \Longrightarrow x \neq 2$ or $x \neq 3$. So,

$$D_f = \mathbb{R} \setminus \{2,3\} = (-\infty,2) \cup (2,3) \cup (3,\infty)$$

19) Find the domain of the function

$$f(x) = \frac{x+2}{x^2-x-6}$$

Solution:
$f(x)$ is defined when $x^2 - x - 6 \neq 0$
$\Longrightarrow (x+2)(x-3) \neq 0 \Longrightarrow x \neq -2$ or $x \neq 3$. So,

$$D_f = \mathbb{R} \setminus \{-2,3\} = (-\infty,-2) \cup (-2,3) \cup (3,\infty)$$

20) Find the domain of the function

$$f(x) = \frac{x+2}{x^2+9}$$

Solution:
$f(x)$ is defined when $x^2 + 9 \neq 0$ but for any value $x$ the denominator $x^2 + 9$ cannot be 0. So,

$$D_f = \mathbb{R} = (-\infty,\infty)$$

21) Find the domain of the function

$$f(x) = \sqrt[3]{x-3}$$

Solution:

$$D_f = \mathbb{R} = (-\infty,\infty)$$

**Note:** The domain of an odd root of any polynomial is $\mathbb{R}$.

22) Find the domain of the function

$$f(x) = \sqrt{x-3}$$

Solution:
$f(x)$ is defined when $x - 3 \geq 0 \Longrightarrow x \geq 3$ because $f(x)$ is an even root. So,

$$D_f = [3,\infty)$$

23) Find the domain of the function

$$f(x) = \sqrt{3-x}$$

Solution:
$f(x)$ is defined when $3 - x \geq 0 \Longrightarrow -x \geq -3 \Longrightarrow x \leq 3$ because $f(x)$ is an even root. So,

$$D_f = (-\infty,3]$$

24) Find the domain of the function

$$f(x) = \sqrt{x+3}$$

Solution:
$f(x)$ is defined when $x + 3 \geq 0 \Longrightarrow x \geq -3$ because $f(x)$ is an even root. So,

$$D_f = [-3,\infty)$$

25) Find the domain of the function

$$f(x) = \sqrt{-x}$$

Solution:
$f(x)$ is defined when $-x \geq 0 \Longrightarrow x \leq 0$ because $f(x)$ is an even root. So,

$$D_f = (-\infty,0]$$

26) Find the range of the function

$$f(x) = \sqrt{-x}$$

Solution:

$$R_f = [0,\infty)$$

**Note:** The range of an even root is always $\geq 0$.

27) Find the domain of the function

$$f(x) = \sqrt{9-x^2}$$

Solution:
$f(x)$ is defined when $9 - x^2 \geq 0 \Longrightarrow -x^2 \geq -9 \Longrightarrow$
$x^2 \leq 9 \Longrightarrow \sqrt{x^2} \leq \sqrt{9} \Longrightarrow |x| \leq 3 \Longrightarrow -3 \leq x \leq 3$.
So,

$$D_f = [-3,3]$$

28) Find the domain of the function

$$f(x) = \frac{x+2}{\sqrt{x-3}}$$

Solution:
$f(x)$ is defined when $x - 3 > 0 \Longrightarrow x > 3$. So,

$$D_f = (3,\infty)$$

29) Find the domain of the function

$$f(x) = \frac{x+2}{\sqrt{9-x^2}}$$

Solution:
$f(x)$ is defined when $9 - x^2 > 0 \Longrightarrow -x^2 > -9$
$\Longrightarrow x^2 < 9 \Longrightarrow \sqrt{x^2} < \sqrt{9} \Longrightarrow |x| < 3 \Longrightarrow -3 < x < 3$.
So,

$$D_f = (-3,3)$$

30) Find the domain of the function

$$f(x) = \sqrt{x^2-9}$$

Solution:
$f(x)$ is defined when $x^2 - 9 \geq 0 \Longrightarrow x^2 \geq 9$
$\Longrightarrow \sqrt{x^2} \geq \sqrt{9} \Longrightarrow |x| \geq 3 \Longrightarrow x \geq 3$ or $x \leq -3$.
So,

$$D_f = (-\infty,-3] \cup [3,\infty)$$

31) Find the range of the function

$$f(x) = \sqrt{x^2 - 9}$$

Solution:

$$R_f = [0, \infty)$$

32) Find the domain of the function

$$f(x) = \frac{x+2}{\sqrt{x^2 - 9}}$$

Solution:

$f(x)$ is defined when $x^2 - 9 > 0 \implies x^2 > 9$

$\implies \sqrt{x^2} > \sqrt{9} \implies |x| > 3 \implies x > 3$ or $x < -3$ .

So,

$$D_f = (-\infty, -3) \cup (3, \infty)$$

33) Find the domain of the function

$$f(x) = \sqrt{9 + x^2}$$

Solution:

$f(x)$ is defined when $9 + x^2 \geq 0$ but it is always true for any value $x$ . So,

$$D_f = \mathbb{R}$$

34) Find the domain of the function

$$f(x) = \sqrt[4]{x^2 - 25}$$

Solution:

$f(x)$ is defined when $x^2 - 25 \geq 0 \implies x^2 \geq 25$

$\implies \sqrt{x^2} \geq \sqrt{25} \implies |x| \geq 5 \implies x \geq 5$ or $x \leq -5$ .

So,

$$D_f = (-\infty, -5] \cup [5, \infty)$$

35) Find the domain of the function

$$f(x) = \sqrt[6]{16 - x^2}$$

Solution:

$f(x)$ is defined when $16 - x^2 \geq 0 \implies -x^2 \geq -16 \implies$
$x^2 \leq 16 \implies \sqrt{x^2} \leq \sqrt{16} \implies |x| \leq 4 \implies -4 \leq x \leq 4$ .

So,

$$D_f = [-4, 4]$$

36) Find the range of the function

$$f(x) = \sqrt{16 - x^2}$$

Solution:

We know that $f(x)$ is defined when $16 - x^2 \geq 0$

$\implies -x^2 \geq -16 \implies x^2 \leq 16 \implies \sqrt{x^2} \leq \sqrt{16}$

$\implies |x| \leq 4 \implies -4 \leq x \leq 4$ . So,

$$D_f = [-4, 4]$$

Using $D_f$ we find the outputs vary from $0$ to $4$ . Hence,

$$R_f = [0, 4]$$

37) Find the domain of the function

$$f(x) = \frac{x + |x|}{x}$$

Solution:

$f(x)$ is defined when $x \neq 0$. So,

$$D_f = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$$

38) Find the domain of the function

$$f(x) = \begin{cases} -\dfrac{1}{x}, & x < 0 \\ x, & x \geq 0 \end{cases}$$

Solution:

It is clear from the definition of the function $f(x)$ that

$$D_f = \mathbb{R} = (-\infty, \infty)$$

39) Find the domain of the function

$$f(x) = \frac{2 - \sqrt{x}}{\sqrt{x^2 + 1}}$$

Solution:

$f(x)$ is defined when

1- $x \geq 0 \implies D_{\sqrt{x}} = [0, \infty)$

2- $x^2 + 1 > 0$ but this is always true for all $x$

$\implies D_{\sqrt{x^2+1}} = \mathbb{R}$ .

Hence,

$$D_f = D_{\sqrt{x}} \cap D_{\sqrt{x^2+1}} = [0, \infty) \cap \mathbb{R} = [0, \infty)$$

40) Find the domain of the function

$$f(x) = \sqrt{x - 1} + \sqrt{x + 3}$$

Solution:

$f(x)$ is defined when

1- $x - 1 \geq 0 \implies x \geq 1 \implies D_{\sqrt{x-1}} = [1, \infty)$

2- $x + 3 \geq 0 \implies x \geq -3 \implies D_{\sqrt{x+3}} = [-3, \infty)$

Hence,

$$D_f = D_{\sqrt{x-1}} \cap D_{\sqrt{x+3}} = [1, \infty) \cap [-3, \infty) = [1, \infty)$$

41) The function $f(x) = 3x^4 + x^2 + 1$ is a polynomial function.

42) The function $f(x) = 5x^3 + x^2 + 7$ is a cubic function.

43) The function $f(x) = -3x^2 + 7$ is a quadratic function.

44) The function $f(x) = 2x + 3$ is a linear function.

45) The function $f(x) = x^7$ is a power function.

46) The function $f(x) = \frac{2x+3}{x^2-1}$ is a rational function.

47) The function $f(x) = \frac{x-3}{x+2}$ is a rational function and we can say it is an algebraic function as well.

48) The function $f(x) = \sin x$ is a trigonometric function.

49) The function $f(x) = e^x$ is a natural exponential function.

50) The function $f(x) = 3^x$ is a general exponential function.

51) The function $f(x) = x^2 + \sqrt{x-2}$ is an algebraic function.

52) The function $f(x) = -3$ is a constant function.

53) The function $f(x) = \log_3 x$ is a general logarithmic function.

54) The function $f(x) = \ln x$ is a natural logarithmic function.

55) The function $f(x) = 3x^4 + x^2 + 1$ is

<u>Solution:</u>

$$f(-x) = 3(-x)^4 + (-x)^2 + 1 = 3x^4 + x^2 + 1 = f(x)$$

Hence,

$f(x)$ is an even function.

56) The function $f(x) = 9 - x^2$ is

<u>Solution:</u>

$$f(-x) = 9 - (-x)^2 = 9 - x^2 = f(x)$$

Hence,

$f(x)$ is an even function.

57) The function $f(x) = x^5 - x$ is

<u>Solution:</u>

$$f(-x) = (-x)^5 - (-x) = -x^5 + x$$
$$= -(x^5 - x) = -f(x)$$

Hence,

$f(x)$ is an odd function.

58) The function $f(x) = 2 - \sqrt[5]{x}$ is

<u>Solution:</u>

$$f(-x) = 2 - \sqrt[5]{(-x)} = 2 - \sqrt[5]{-x} = 2 + \sqrt[5]{x}$$
$$= -\left(-2 - \sqrt[5]{x}\right)$$

Hence,

$f(x)$ is neither even nor odd.

59) The function $f(x) = 3x + \frac{2}{\sqrt{x^2+9}}$ is

<u>Solution:</u>

$$f(-x) = 3(-x) + \frac{2}{\sqrt{(-x)^2+9}} = -3x + \frac{2}{\sqrt{x^2+9}}$$
$$= -\left(3x - \frac{2}{\sqrt{x^2+9}}\right)$$

Hence,

$f(x)$ is neither even nor odd.

60) The function $f(x) = \frac{3}{\sqrt{x^2+9}}$ is

<u>Solution:</u>

$$f(-x) = \frac{3}{\sqrt{(-x)^2+9}} = \frac{3}{\sqrt{x^2+9}} = f(x)$$

Hence,

$f(x)$ is an even function.

61) The function $f(x) = \sqrt{4 + x^2}$ is

<u>Solution:</u>

$$f(-x) = \sqrt{4 + (-x)^2} = \sqrt{4 + x^2} = f(x)$$

Hence,

$f(x)$ is an even function.

62) The function $f(x) = 3$ is

<u>Solution:</u>

Since the graph of the constant function $3$ is symmetric about the $y - axis$, then

$f(x)$ is an even function.

63) The function $f(x) = \frac{9-x^2}{x-2}$ is

<u>Solution:</u>

$$f(-x) = \frac{9 - (-x)^2}{(-x) - 2} = \frac{9 - x^2}{-x - 2}$$
$$= -\left(\frac{9 - x^2}{x + 2}\right)$$

Hence,

$f(x)$ is neither even nor odd.

64) The function $f(x) = \frac{x^2-4}{x^2+1}$ is

<u>Solution:</u>

$$f(-x) = \frac{(-x)^2 - 4}{(-x)^2 + 1} = \frac{x^2 - 4}{x^2 + 1} = f(x)$$

Hence,

$f(x)$ is an even function.

65) The function $f(x) = 3|x|$ is

<u>Solution:</u>

$$f(-x) = 3|(-x)| = 3|x| = f(x)$$

Hence,

$f(x)$ is an even function.

66) The function $f(x) = x^{-2}$ is

<u>Solution:</u>

$$f(x) = x^{-2} = \frac{1}{x^2}$$
$$f(-x) = \frac{1}{(-x)^2} = \frac{1}{x^2} = f(x)$$

Hence, $f(x)$ is an even function.

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| 67) The function $f(x) = x^3 - 2x + 5$ is<br><u>Solution:</u><br>$f(-x) = (-x)^3 - 2(-x) + 5 \; = -x^3 + 2x + 5$<br>$= -(x^3 - 2x - 5)$<br>Hence,<br>$f(x)$ is neither even nor odd. | 68) The function $f(x) = \sqrt[3]{x^5} - x^3 + x$ is<br><u>Solution:</u><br>$f(-x) = \sqrt[3]{(-x)^5} - (-x)^3 + (-x) \; = -\sqrt[3]{x^5} + x^3 - x$<br>$= -\left(\sqrt[3]{x^5} - x^3 + x\right) = -f(x)$<br>Hence,<br>$f(x)$ is an odd function. |
| 69) The function $f(x) = 7$ is<br><u>Solution:</u><br>Since the graph of the constant function $7$ is symmetric about the $y - axis$, then<br><br>$f(x)$ is an even function. | 70) The function $f(x) = \frac{x^3-4}{x^3+1}$ is<br><u>Solution:</u><br>$f(-x) = \frac{(-x)^3 - 4}{(-x)^3 + 1} = \frac{-x^3 - 4}{-x^3 + 1} = -\frac{x^3 + 4}{-x^3 + 1}$<br>Hence,<br>$f(x)$ is neither even nor odd. |
| 71) The function $f(x) = \frac{x^2-1}{x^3+3}$ is<br><u>Solution:</u><br>$f(-x) = \frac{(-x)^2 - 1}{(-x)^3 + 3} = \frac{x^2 - 1}{-x^3 + 3} = -\frac{x^2 - 1}{x^3 - 3}$<br>Hence,<br>$f(x)$ is neither even nor odd. | 72) The function $f(x) = x^6 - 4x^2 + 1$ is<br><u>Solution:</u><br>$f(-x) = (-x)^6 - 4(-x)^2 + 1 = x^6 - 4x^2 + 1 = f(x)$<br>Hence,<br>$f(x)$ is an even function. |
| 73) The function $f(x) = x^2$ is increasing on $(0, \infty)$. | 74) The function $f(x) = x^2$ is decreasing on $(-\infty, 0)$. |
| 75) The function $f(x) = x^3$ is increasing on $(-\infty, \infty)$. | 76) The function $f(x) = x^3$ is not decreasing at all. |
| 77) The function $f(x) = \sqrt{x}$ is increasing on $(0, \infty)$. | 78) The function $f(x) = \sqrt{x}$ is not decreasing at all. |
| 79) The function $f(x) = \frac{1}{x}$ is not increasing at all. | 80) The function $f(x) = \frac{1}{x}$ is decreasing on $(-\infty, \infty)$. |