المملكة العربية السعودية

وزارة التعليم

MINISTRY OF EDUCATION

وفوق كل ذي علم عليم

هام

لكل المهتمين و المهتمات
بدروس و مراجع الجامعية

Notes

* التركيز على المفاهيم الاساسية.
* شرح أبواب المنهج حسب الخطة.
* أمثلة توضيحية وتدريبات.
* نماذج اختبارات.

السعدي

CH. 1

Stat. 110
إحصاء ١١٠

إعداد

جمال السعدي

أستاذ الرياضيات والإحصاء للمرحلة الجامعية

Ch.1 Part.1

* Statistics:

Is the science of conducting studies to collect, organize, summarize, analyze, and draw conclusions from the data.

(العلم – يربط – الدراسات – جمع – تنظيم – تلخيص – تحليل – رسم – النتائج)

فروع علم الإحصاء
Branches of statistics

وصفي
Descriptiv

Consists of:

- **The collection,**
- **organization,**
- **summarization,**
- **Presentation of the data by the tables and graphs.**
- بعض الكلمات التي تدل على هذا النوع.
- **Average – mean – median – mode.**
- الحديث عن الماضي
مثل عام 1996- 2000
- تقديم خبر – أو تقرير.

استدلالي
Inferential

Consists of:

- **Generalizing from sampling to population.**
- **Performing estimation.**
- **Determining relations between variables and making prediction.**
- **Soon – Maybe – next – Can**
- الحديث عن المستقبل مثل عام 2020 (التوقع).
- استخدام عينة للتعميم على المجتمع.

In each of these statements, tell whether descriptive or inferential statistics have been used.

يلتحقوا

a In the year **2020**, 148 million Americans will be enrolled in an HMO.

(Inferential)

ضحايا العمل

b Nine out of ten on- the – job fatalities are men

(Descriptive)

الإنفاق

c Expenditures for the cable industry were $ 5.66 billion in **1996**. **(Descriptive)**

متوسط الدخل القومي لعمال المنازل

d The **median** household income for people aged 25- 34 is $ 35.888. **(Descriptive)**

علاج الحساسة

e Allergy therapy makes bees go away

(Inferential) تم معرفة النسبة عن طريق عينة ثم تعميها على المجتمع

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f Drinking decaffeinated coffee **can** raise cholesterol levels by 7% **(Inferential)**

متوسط الإنفاق على الأدوية للشخص الواحد

g The national **average** annual medicine expenditure per person is $ 1052 **(Descriptive)**

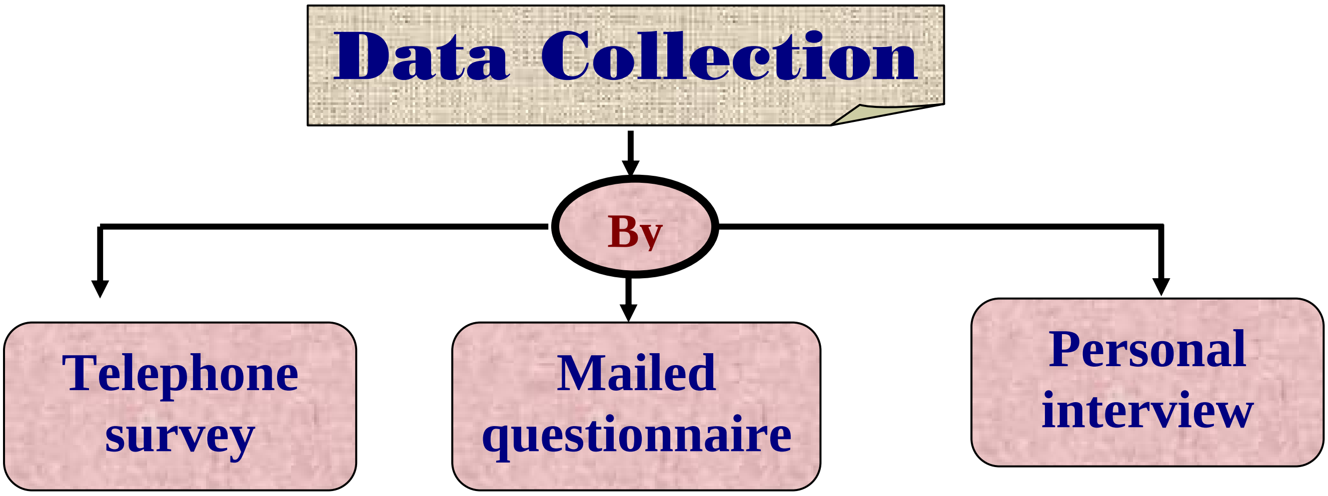
خبراء الرهن العقاري

h Experts say that mortgage rates may **soon** hit bottom.

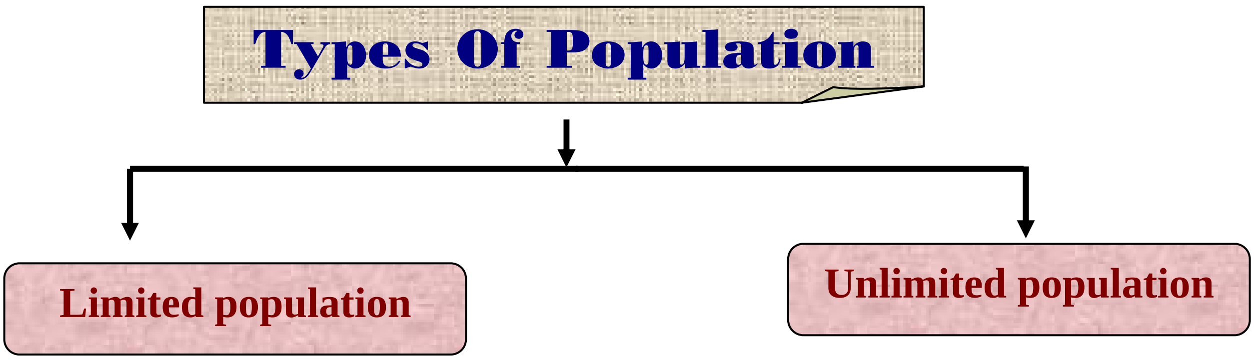
(Inferential)

Name and define the two areas of statistics

- Descriptive statistics: describe the data set.
- Inferential statistics: use the data to draw conclusions استنتاج about the population.



* Population: A set consists of all subjects.



* Example: Students grade (A , B , C , D , F)

* Example: integer number (…., -2, -1, 0, 1, 2, …….)

- **Sample: is a group of subjects selected from population.**
- **Sample $\subset$ population.**

Identify the sample and population in each of the following statements:

الطوارئ

1. In order to study the response times for emergency 988 calls in Jeddah 50 calls are selected randomly over a six month period and the response times are recorded.

** Population: all calls (988).

** Sample: 50 calls.

---

2. 1500 listeners to talk radio program of various types are selected.

** Population: all listeners to radio program.

** Sample: 1500 listeners.

Why we must use a sample Instead of population?

We must use a sample Instead of population because:

1. The size of population may be very large.

2. Study the whole population may be very expensive.

(Save Money)

3. Study the whole population may be need to a long time.

(Save Time)

4. Study the whole population may be destructive for the elements of population.

Type of Samples

Random sample عينة عشوائية	Systematic sample عينة منتظمة	Stratified sample عينة طبقية	Cluster sample عينة عنقودية

العينة العشوائية Random Sample:

* فيها تكون كل حالة من حالات المجتمع لها نفس فرصة الاختيار (اختيار عشوائي بدون شروط)

* All units of the population has the same chance of selecting.

العينة المنتظمة System sample:

فيها يقسم المجتمع إلى مجموعات عددها يساوي عدد أفراد العينة المختارة.
يتم اختيار حالة من المجموعة الأولى عشوائياً فإذا كان ترتيبها A مثلاً نختـار الحالـة التي ترتيبها A من كل المجموعات حتى نكون العينة المطاوبة.
بعض الكلمات = التي تدل عليه ← Seventh .... , ...100th

العينة الطبقية Stratified sample:

فيها يقسم المجتمع إلى مجموعات غير متقاطعة (مستقلة) تسمى طبقات (غير متداخلـة) ثم يتم اختيار عينة عشوائية بسيطة من كل طبقة.

العينة العنقودية Cluster sample:

فيها يكون المجتمع مقسم إلى مجموعات رئيسية نختار من هـذه المجموعـات **بعـض المجموعات** ثم نستخدم جميع عناصر المجموعات المختارة.

Classify each sample as random, systematic, stratified, or cluster.

مدرسة كبيرة مقسمة إلى مباني

a) In a large school district, all **teachers** from **two buildings** are interviewed to determine whether they believe the students have less homework to do now than in previous years. (السنوات السابقة)

(Cluster)

- اختيار مبنيين من مباني المدرسة.
- ثم اختيار كل المدرسين من المبنيين.

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b) Every **seventh** customer entering a shopping mall (مول تجاري) is asked to select her or his favorite store (المحل المفضل).

(Systematic)

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c) Nursing supervisors are selected using **random** numbers in order to determine annual salaries (الرواتب السنوية).

(Random)

الرقم ١٠٠ و مضاعفاته

d) Every **100th** hamburger manufactured is checked to determine its fat content. (مرضية)

(Systematic)

سعاة البريد

e) Mail carriers of a large city are divided into four groups according to gender **(male or female)** and according to whether they walk (يمشون) or ride (يركبون) on their routes. Then 10 are selected from each group and interviewed (مقابلتهم) to determine whether they have been bitten by a dog in the last year.

قسموا إلى ٤ مجموعات طبقاً للنوع

تم اختيار ١٠ من كل مجموعة

(Stratified)

Variables
متغيـرات

Qualitative
وصفية (ليست رقمية)

Nominal
أسمية

بيانات وصفية لا يمكن ترتيبها

مثال

- الجنسية.....
- الحالـة الاجتماعيـة (متزوج ، أعزب،...)
- فصائل الدم (A, B, O, AB)
- النوع (ذكر – أنثى)
- الرمز البريدي Zip Cod

Ranked (ordinal)
ترتيبية

بيانات وصفية يمكن ترتيبها

مثال

- المستوى الأكاديمي.. A, B, C, D, F
- تصنيف لاعبي التنس..
- تـصنيـف مـستويات الفنادق...
- تصنيف الدول من حيث عينة – متوسطة – فقيرة ....

Quantitative
كمية (رقمية)

Discrete
منفصلة

أعداد صحيحة تمثـل قـيم يمكن عدها

مثال

- عدد أفراد الأسرة...
- عدد طلاب الشعبة..
- عدد الموظفين......
- عدد السيارات...... أي جملة تبدأ بكلمة Number of…..

Continuous
متصلة

تأخذ قيم داخل مـدى معـين سـواء كانـت صحيحة أو كسور.

مثال

- أوزان........
- أطوال.....
- الدخل......
- الوقت......
- درجـــة الحرارة..... ....

*** Variables** : Is quantity can taken different values.

Example : length , weight, age, coller, time,……………

فئات أصناف

*** Qualitative** : The variables expressing by categories or classes.

Example: * Gender (Male, Female) → category.

* Classes (2-5, 6-9, 10-13) → classes.

*** Qualitative variables are two types.**

a **Nominal Variables:**

Gives names in which there is no order.

Example: * Types of blood.

* Name your country.

b **Ranked (ordinal)** Variables:

تصنيف أصناف ترتيبها

Classifies variables into categories that can be ranked.

Example: * Academic level. A, B, ……..

* Level of the Hotel. *, **, ……….,*****

Quantitative It is the variables which takes numerical values with measure scale and can be ordered or ranked.

Example: Age, height, weight,……….

*** Quantitative variables are two types.**

a **Discrete Variables** are can be count. **يمكن عدها**

Example: * Number of student in class.

* Number of cars in park.

b **Continuous variables:**

It is the variable which can take all possible numerical values in a given interval.

Example: * Height of a student.

- Weight of a student.

وصفى كمى

Classify each variable as qualitative or quantitative.

a. Number of bicycles sold in 1 year by a large sporting goods Store.

ألوان القبعات لكرة المضرب في متجر

b. Colors of baseball caps in a store.

الزمن المستغرق لقطع العشب

c. Times it takes to cut a lawn.

السعة بالقدم المكعب لـ ٦ أسـ ـرّة أطفال

d. Capacity in cubic feet of six truck beds.

رضيع

e. Classification of children in a day-care center (infant – toddler, preschool).

يحبو روضة

بحيرة جورج

f. Weights of fish caught in Lake George.

الحالة الاجتماعية لأعضاء كلية في جامعة كبيرة

g. Marital statues of faculty members in a large university.

Solution

a , c , d and f are Quantitative.

b , e and g are Qualitative.

Classify each variable as discrete or continuous.

a. Number of doughnuts sold each day by doughnut Heaven.

درجة حرارة ست حمامات سباحة

b. Water temperatures of six swimming pools in Pittsburgh on a given day.

ملجأ للحيوانات

c. Weights of cats in a pet shelter.

العمر الافتراضي بالساعات لعدد ١٢ حجر بطارية

d. Lifetime (in hours) of 12 flashlight batteries.

e. Number of cheeseburgers sold each day by a hamburger stand on a college campus.

المؤجرة

f. Number of DVDs rented each day by a video store.

السعة بالجالونات لعدد ٦ خزانات ماء في مقاطعة جيفر سون

g. Capacity (in gallons) of six reservoirs in Jefferson County.

Solution

a , e , and f are Discrete.

كل جملة تبدأ بكلمة
Number of

b, c , d and g are Continuous

Level Measurement Of The Data

- تقاس البيانات بأربعة أنواع من المقاييس

1 Nominal level data — المقياس الأسمي

هي بيانات أسمية غير رقمية (غير متقاطعة) لكل مجموعة خصائص تميزها عن غيرها.

Example: * blood types (A, B,O, AB).

* gender (male, female)

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2 Ordinal level data — المقياس الترتيبي

هي بيانات مقسمة إلى مستويات أو فئات يمكن ترتيبها تصاعدياً أو تنازلياً.

Example: * Grade of course (A, B, C, D. F)

* Rating scale (Poor, Fair, Good, Excellent).

* Ranking of tennis players.

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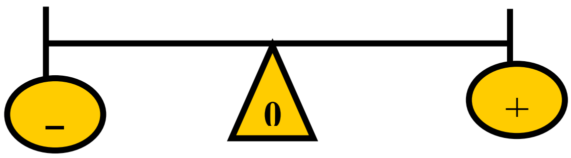
3 Interval level data — مقياس الفترة



هي بيانات رقمية تقاس بمقدار بعدها عن الصفر

Example: * Temperature

* I Q. test.

ممكن موجب أو سالب أو صفر

الصفر لا يعني انعدام الظاهرة

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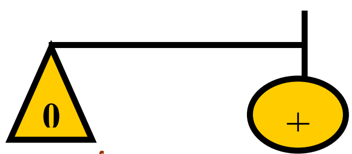
4 Ratio level data — مقياس النسبة



Example: * height , weight, time Salary, Age,……

بدايته صفر: أي الصفر له معنى وهو انعدام الظاهرة ويكون موجب ولا يكون سالب

Classify each as nominal – level, ordinal - level, interval level, or ratio – level measurement.

صفحات دليل التليفونات

a. Pages in the city of Cleveland telephone book. **(Ratio)**

صفحات دليل التليفون تبدأ من ١ إلى ....أي ليست سالبة ولا تبدأ من Zero

b. Rankings of tennis players. **(Ordinal)**

c. Weights of air conditioners. **(Ratio)**

ثلاجات

d. Temperatures inside 10 refrigerators. **(Interval)**

e. Salaries of the top five CEO in the United States. **(Ratio)**

ترتيب مسرحيات محلية رديئة مقبولة جيدة ممتازة

f. Ratings of eight local plays (Poor, Fair, Good, Excellent).

(Ordinal)

g. Times required for mechanics to do a turn-up. **(Ratio)**

h. Ages of students in a classroom. **(Ratio)**

الحالة الاجتماعية المرضى

i. Marital status of patients in a physician's office. **(Nominal)**

القدرة الجرار محرك

j. Horsepower of tractor engines. **(Ratio)**

Notes

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السعـــــــدي

CH. 5

Stat. 110
إحصـــاء ١١٠

إعداد

جمال السعدي

أستاذ الرياضيات والإحصاء للمرحلة الجامعية

Ch. 5

Discrete Probability Distributions

Probability Distributions

- **A random variable is a variable whose values are determined by chance.**
- Variables that can assume all values in the interval between any two given values are called **continuous variables**. For example, if the temperature goes from 60° to 70°.
- If a variable can assume only a specific number of values, such as the outcomes for the roll of a die or the outcomes for the toss of a coin, then the variable is called a **discrete variable**.
- **For these Exercises, state whether the variable is discrete or continuous.**

1. The speed of a jet airplane. **(Continuous)**

2. The number of cheeseburgers a fast-food restaurant serves each day. **(Discrete)**

3. The number of people who play the state lottery each day. **(Discrete)**

4. The weight of a Siberian tiger. **(continuous)**

5. The time it takes to complete a marathon. **(continuous)**

6. The number of mathematics majors in your school. **(Discrete)**

7. The blood pressures of all patients admitted to a hospital on a specific day. **(Discrete)**

Example:

Construct a probability for rolling a single die.

Solution

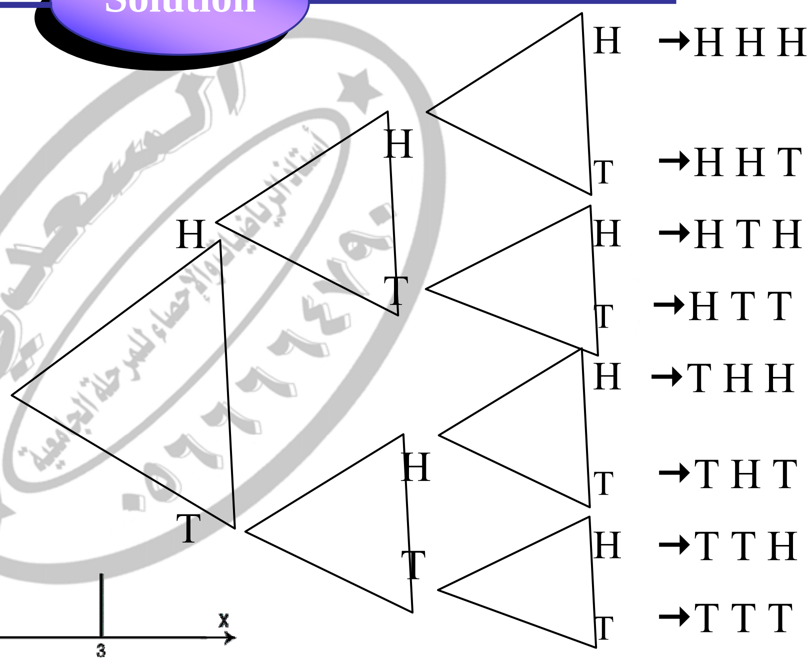
Outcome x	1	2	3	4	5	6
Probability: P(x)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

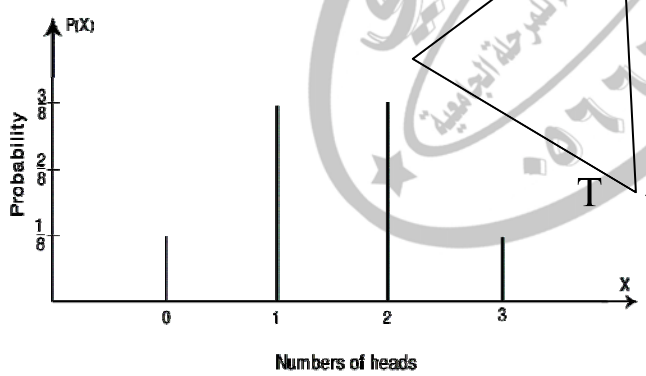
Example:

Represent graphically the probability distribution for the sample space for tossing three coins.

Number of heads x	0	1	2	3
Probability: P (x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Solution





شرطين أساسين
Two Requirements For a Probability Distribution

1. The Sum of the probabilities of all the events in the sample space must be equal 1 $\sum P(X) = 1$
2. The probability of each event in the sample space must be between or equal to 0 and 1. $0 \le P(X) \le 1$.

Example:

Determine whether each distribution is a probability distribution.

a

X	0	5	10	15	20
P (X)	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

Yes, it is a probability distribution.

b

X	0	2	4	6
P (X)	- 1.0	1.5	0.3	0.2

No. It is not a probability distribution, since P (x) cannot be 1.5 or – 1.0

c

X	1	2	3	4
P (X)	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{9}{16}$

Yes, it is a probability distribution.

d

X	2	3	7
P (X)	0.5	0.3	0.4

No, it is not, since $\sum p(X) = 1.2$

Mean, Variance, Standard Deviation, and Expectation

- **Formula for the mean of a probability distribution**

The mean of a random variable with a discrete probability distribution

$$\mu = X_1 \cdot P(X_1) + X_2 \cdot P(X_2) + X_3 \cdot P(X_3) + \ldots\ldots\ldots\ldots + X_n \cdot P(X_n)$$

$$\mu = \sum X \cdot P(X)$$

- **Formula for the variance of a probability distribution**

$$\sigma^2 = \sum [X^2 \cdot P(X)] - \mu^2$$

- **The standard deviation of a probability distribution is**

$$\sigma = \sqrt{\sigma^2} \quad \text{or} \quad \sqrt{\sum [X^2 \cdot P(X)] - \mu^2}$$

- **The expected value:**

$$\mu = E(X) = \sum X \cdot P(X)$$

- **Remember that variance and standard deviation cannot be negative.**

Example:

A pizza shop owner determines the number of pizza that are delivered each day. Find the mean variance, and standard deviation for the distribution shown. If the manager stated that 45 pizzas were delivered on one day. Do you think that this is a believable claim? (يستحق تصديقه)

Number of deliveries X	35	36	37	38	39
Probability: P (X)	0.1	0.2	0.3	0.3	0.1

Solution

X	P (x)	X. P (x)	X^2 . P (x)
35	0.1	3.5	122.5
36	0.2	7.2	259.2
37	0.3	11.1	410.7
38	0.3	11.4	433.2
39	0.1	3.9	152.1
		$\sum x.P(x) = 37.1$	$\sum x^2.p(x) = 1377.7$

- Mean: $\mu = \sum x \cdot p(x) = 37.1$
- Variance: $\sigma^2 = \sum x^2 \cdot p(x) - \mu^2$

  $= 1377.7 - (37.1)^2$

  $= 1.29$
- Standard deviation: $\sigma = \sqrt{\sigma} = \sqrt{1.29} = 1.1$

Example: متجر بيع بالتجزئة

The number of suits sold per day at a retail store is shown in the table, with the corresponding probabilities. Find the mean, variance, and standard deviation of the distribution.

Number of suits sold X	19	20	21	22	23
Probability P (X)	0.2	0.2	0.3	0.2	0.1

If the manager of the retail store wants to be sure that he has enough suits for the next 5 days, how many should the manager purchase ?

Solution

X	P (x)	X. P (x)	X^2 . P (x)
19	0.2	3.8	72.2
20	0.2	4	80
21	0.3	6.3	132.3
22	0.2	4.4	96.8
23	0.1	2.3	52.9
		$\sum$ x. P (x) = 20.8	$\sum$ x^2. P (x) = 434.2

- Mean. $\mu = \sum x \cdot p(x) = 20.8$
- Variance: $\sigma^2 = \sum x^2 \cdot p(x) - \mu^2$

  $= 434.2 - (20.8)^2$

  $= 1.56$
- Standard deviation: $\sigma = \sqrt{\sigma^2} = \sqrt{1.56} = 1.2$
- The number of suits = (20.8) × (5) = 104 suits

Example:

From past experience, a company has found that in cartons of transistors, 92 % contain no defective transistors, 3% contain one defective transistor, 3% contain two defective transistors, and 2% contain three defective transistors. Find the mean, variance, and standard deviation. For the defective transistors. About how many extra transistors per day would the company need to replace the defective ones if it used 10 cartons per day?

Solution

X	P (x)	X. P (x)	X^2 . P (x)
0	0.92	0	0
1	0.03	0.03	0.03
2	0.03	0.06	0.12
3	0.02	0.06	0.18
		$\sum x.P(x) = 0.15$	$\sum x^2.P(x) = 0.33$

- Mean. $\mu = \sum x.p(x) = 0.15$
- Variance: $\sigma^2 = \sum x^2 . p(x) - \mu^2$

$$= 0.33 - (0.15)^2$$
$$= 0.3075$$

- Standard deviation: $\sigma = \sqrt{\sigma^2} = \sqrt{0.3075} = 0.555$
- Number of extra transistors = (0.15) . (10) = 1.5 is $\cong$ 2.

Example:

A person decides to invest $ 50.000 in a gas well. Based on history, the Probabilities of the outcomes are as follows.

Outcome x	P (x)
$ 80.000 (Highly successful)	0.2
$ 40.000 (Moderately successful	0.7
- $ 50.000 (Dry well) خسارة كبيرة	0.1

استثمار

- Find the expected value of the investment.

  Would you consider this a good investment?

Solution

$E(x) = \sum x \cdot P(x)$

$= (80000)(0.2) + (40000)(0.7) + (-50000)(0.1)$

$= \$\ 39000$

This a good investment.

The Binomial Distribution

A binomial experiment is a probability experiment that satisfies the following four requirements:

1. There must be a fixed number of trials.
2. Each trial has only two outcomes: success or fail.
3. The outcomes of each trial must be independent of each other.
4. The probability of a success must remain the same for each trial.

Mean, Variance, and standard deviation for the binomial distribution

The mean, variance, and standard deviation of a variable that has the binomial distribution can be found by using the following formulas.

- **Mean:** $\mu = n \cdot p$
- **Variance:** $\sigma^2 = n \cdot p \cdot q$
- **Standard deviation:** $\sigma = \sqrt{n \cdot p \cdot q}$

Example:

A dice is rolled 480 times. Find the mean, variance, and standard deviation of the number of 2s that will be rolled.

Solution

Getting a 2 is a success and not getting a 2 is a failure:

- $n = 480,\ P = \frac{1}{6}$, and $q = \frac{5}{6}$
- $\mu = n.p = 480.\frac{1}{6} = 80$
- $\sigma^2 = n.p.q = 480.\left(\frac{1}{6}\right)\left(\frac{5}{6}\right) = 66.7$
- $\sigma = \sqrt{n.p.q} = \sqrt{66.7} = 8.2$

Example:

A coin is tossed 4 times. Find the mean, variance, and standard deviation of the number of heads that will be obtained.

Solution

The binomial distribution and

$n = 4$, $p = \frac{1}{2}$ and $q = \frac{1}{2}$

$$\mu = n\,.\,p = 4.\frac{1}{2}.\frac{1}{2} = 1$$

$$\sigma^2 = n\,.\,p\,.\,q = 4\,.\,\frac{1}{2}\,.\,\frac{1}{2} = 1$$

$$\sigma = \sqrt{1} = 1$$

Example:

If 3% of calculators are defective, find the mean, variance, and standard deviation of a lot of 300 calculators.

Solution

$n = 300$ $p = 0.03$ $q = 0.97$

- $\mu = n.p = (300)(0.03) = 9$
- $\sigma^2 = n.p.q = (300)(0.03)(0.97) = 8.7$
- $\sigma = \sqrt{\sigma^2} = \sqrt{8.73} = 2.9 \cong 3$

Example:

In a restaurant, a study found that 42% of all patrons smoked. If the seating capacity of the restaurant is 80 people, find the mean, variance, and standard deviation of the number of smokers. About how many seats should be available. For smoking customers?

Solution

$n = 80$ $p = 0.42$ $q = 0.58$

- $\mu = n.p = (80)(0.42) = 33.6$
- $\sigma^2 = n.p.q = (80)(0.42)(0.58) = 19.5$
- $\sigma = \sqrt{\sigma^2} = \sqrt{19.5} \cong 4.4$

Note

- **two outcomes**: yes or no → **(binomial)**
- **more than two outcomes** → **(not binomial)**

Which of the following are binomial experiments or can be reduced to binomial experiments?

صابون كثير الرغوة

a. Surveying 100 People to determine if they like sudsy soap.

(Binomial)

b. Tossing a coin 100 times to see how many heads occur

(Binomial)

c. Asking 1000 people which brand of cigarettes they smoke.

(Not binomial)

d. Testing one brand of aspirin by using 10 people to determine whether it is effective **(Binomial)**

f. Asking 100 people if they smoke **(Binomial)**

g. Checking 1000 applicants to see whether they were admitted to white Oak college. **(Binomial)**

h. Surveying 300 prisoners to see how many different crimes they were convicted of. **(Not binomial)**

أدينو بها

i. Surveying 300 prisoners to see whether this is their first offense.

(Binomial)

Binomial Probability Formula

In a binomial experiment, the probability of exactly X successes in n trials is

$$\frac{n!}{(n-x)!x!} \times p^{x} \times q^{n-x} = nC_{x} \times p^{x} \times q^{n-x} P(x) =$$

Example:

يخمن

A student takes a 20 – question, true/ false exam and guesses on each question. Find the probability of passing if the lowest passing grade is 15 correct out of 20. Would you consider this event likely to occur? Explain your answer.

اختبار

Solution

$n = 20 \qquad p = \frac{1}{2} \qquad q = \frac{1}{2}$

$$p\ (\text{passing}) = p\ (x \geq 15)$$

$= p\ (x = 15) + p\ (x = 16) + p\ (x = 17) + p\ (x = 18) + p\ (x = 19) + p\ (x = 20)$

$$= 20\,C_{15} \left(\frac{1}{2}\right)^{15} \left(\frac{1}{2}\right)^{5} + 20C_{16} \left(\frac{1}{2}\right)^{16} \left(\frac{1}{2}\right)^{4}$$

$$+ 20\,C_{17} \left(\frac{1}{2}\right)^{17} \left(\frac{1}{2}\right)^{3} + 20\,C_{18} \left(\frac{1}{2}\right)^{18} \left(\frac{1}{2}\right)^{2}$$

$$+ 20C_{19} \left(\frac{1}{2}\right)^{19} \left(\frac{1}{2}\right)^{1} + 20C_{20} \left(\frac{1}{2}\right)^{20} \left(\frac{1}{2}\right)^{0}$$

$= 0.015 + 0.005 + 0.001 + \ldots.. = 0.021 < 0.5$

There for P (passing) unlikely to occur.

Example:

مجتمع خدمة

If 80% of the people in a community have internet access from their homes, find these probabilities for a sample of 10 people.

على الأكثر

a. At most 6 have internet access.

b. Exactly 6 have internet access.

c. At least 6 have internet access.

d. Which event a, b, or c is most likely to occur? Explain why?

Solution

$n = 10$ $p = 0.8$ $q = 0.2$

(a) P (at most 6) = $p(x \le 6)$

= p (x = 6) + p (x = 5) + p (x = 4) + p (x = 3) + p (x = 2) + p (x = 1) + p (x = 0)

$= 10\,C_6\,(0.8)^6\,(0.2)^4 + 10\,C_5\,(0.8)^5\,(0.2)^5 + 10\,C_4\,(0.8)^4\,(0.2)^6$

$+ 10\,C_3\,(0.8)^3\,(0.2)^7 + 10\,C_2\,(0.8)^2\,(0.2)^8 + 10C_1\,(0.8)^1\,(0.2)^9$

$+ 10\,C_0\,(0.8)^0\,(0.2)^{10} = 0.121$

(b) $P(x = 6) = 10C_6\,(0.8)^6\,(0.2)^4 = 0.088$

(c) p (at least 6) = $p(x \ge 6)$ =..........= 0.967.

(d) Event c is most likely to occur because it's > 0.5

Example:

A survey found that 86% of Americans have never been a
ضحية جريمة العنف
victim of violent crime. If a sample of 12 Americans is selected at random, find the probability that 10 or more have never been
معقول يبدو
victims of violent crime. Does it seem reasonable that 10 or more have never been victims of violent crime?

Solution

$n = 12$ $p = 0.86$ $q = 0.14$

$p(x \geq 10) = p(x = 10) + p(x = 11) + p(x=12)$

$= 12C_{10}\,(0.86)^{10}\,(0.14)^{2} + 12C_{11}\,(0.86)^{11}\,(0.14)^{1} + 12C_{12}\,(0.86)^{12}\,(0.14)^{0}$

$= 0.77 > 0.5$

Yes: it seem reasonable.................

Chapter Quiz

Determine whether each statement is true or false. If the statement is false explain why.

1. The expected value of a random variable can be thought of as a long – run average. فترة طويلة (✓)

2. The number of courses a students is taking this semester is an example of a continuous random variable. (x)

3. when the multinomial distribution is used, the outcomes must be dependent. (x)

4. A binomial experiment has a fixed number of trials. (✓)

Complete these statements with the best answer:

5. Random variable values are determined by **chance**.

6. The mean for a binomial variable can be found be found by using the formula **$\mu = n \cdot p$**.

7. One requirement for a probability distribution is that the sum of all the events in the sample space must equal **1**.

Select the best answer:

8. What is the sum of the probabilities of all outcomes in a probability distribution?

a. 0 c. 1

b. 1/2 d. It cannot be determined.

9. How many outcomes are there in a binomial experiment?

a. 0 c. 2

b. 1 d. It varies

For questions 11 through 14, determine if the distribution represents a probability distribution. If not, state why.

11

X	1	2	3	4	5
P (X)	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{2}{7}$	$\frac{3}{7}$	$\frac{2}{7}$

→ No where $\sum P(x) > 1$

12

X	3	6	9	12	15
P (X)	0.3	0.5	0.1	0.08	0.02

→ yes

13

X	50	75	100
P (X)	0.5	0.2	0.3

→ yes

14

X	4	8	12	16
P (X)	$\frac{1}{6}$	$\frac{3}{12}$	$\frac{1}{2}$	$\frac{1}{12}$

→ yes

انتهى Ch 5

كل التمنيات بالنجاح والتوفيق

السعدي

.Ch.1 Part.2

Observational and

Observational study
دراسة مشاهدية

The researcher

Observe only and not effect

ملاحظة الحدث دون تدخل للتأثير فى النتائج

- مثال: ملاحظة عدد المرضى الذين تم شفاؤهم من مرض معين.
- كلمات تدل على هذا النوع …….Find – See …..

Experimental Study
دراسة تجريبيه

The researcher

Effect and observe.

يحدث تدخل من الباحث فى الظاهره محل الدراسه ثم ملاحظه مدى تـأثير هـذا التدخل على نتائج الظاهره.

- **مثال:** اعطاء المرضى عقار معين ثم ملاحظة تأثير هذا العقار على عدد مـن تـم شفاءهم.
- كلمات تدل على هذا النوع …….Placed on - given….

Identify each study as being either observational or experimental:

اشخاص عشوائياً قسموا

a. Subjects were randomly assigned to two groups, and one

اعشاب طبيه

group was given an herb and the other group a placebo. After

تنفس صناعي اصابات

6 months, the numbers of respiratory tract infections each group had were compared.

تم التدخل فى الدراسه وذلك بإعطاء نوع معين من العلاج لكل مجموعه

(Experimental)

واقف مزدحم تقاطع

b. A researcher stood at a busy intersection to see if the color of

السيارة التى يقودها الشخص مرتبط بـ السرعة

the automobile that a person drives is related to running red

الاضواء

lights.

يشاهد فقط بدون تدخل to see

(Observational)

---

c. A researcher find that people who are more hostile have higher total cholesterol levels than who are less hostile.

بدون تدخل * finds

(Observational)

أشخاص عشوائيا قسموا

e. Subjects are randomly assigned to four groups.

Each group is placed on one of four special diets – a low- fat diet,

اتحاد - ضخم منتظم

a high- fish diet, a combination of low – fat diet, and a regular diet. After 6 months, the blood pressures of the groups are compared to see if diet has any effect on blood pressure.

تم التدخل فى دراسة " تأثير الرجيم على ضغط الدم"

Each group is placed on diets

(Experimental)

$2x + 1 = y$

*** Independent**

متغير مستقل X : (مؤثر)

هو متغير محل اهتمام الباحث (موضوع الدراسه)

Another name: **Explanatory variable**

Example: * number of study hours

* Room temperature

*** Dependent variable**

متغير غير مستقل (تابع) y (متأثر)

هو ناتج العمليات التى أجريت على المتغير المستقل

Another name: **Outcome variable**

Example: * Student score.

* Bacteria growth.

*** Confounding**

متغير لا يمكن فصله عن المتغير المستقل

ويؤثر على النتائج يتداخل مع متغيرات أخرى

Interferes With other variables

مثال : ممارسه الرياضه (متغير مستقل)

\+

الامتناع عن التدخين

⇐ = تحسين الصحه (م. تابع)

Identify the independent variable and the dependent: variable for each of the studies in last Exercise:

a- Independent var. : " type of pill " مؤثر

دواء – أعشاب

b- Independent var. : " type of pill " مؤثر

Dependent var. : " number of infections " متأثر

---

c- Independent var. : " color of automobile " مؤثر

Dependent var. : " running red lights " متأثر

---

d- Independent var. : " level of hostility " مؤثر

Dependent var. : "cholesterol level" متأثر

---

e- Independent var. : " type of diet " مؤثر

Dependent var. : " blood pressure " متأثر

Summary

- The two major areas of statistics are **descriptive** and inferential.
- Descriptive statistics includes the collection. Organization, summarization and presentation of data.
- Inferential statistics includes making inferences from samples to populations, estimations, determining relationships and making predictions. Inferential statistics is based on probability theory.
- Since in most cases the populations under study are large, statisticians use subgroups called samples to get the necessary data for their studies. There are four basic methods used to obtain samples: **random**, **systematic**, **stratified** and **cluster.**

- Data can be classified as qualitative or quantitative. Quantitative data can be either discrete or continues, depending on the values they can assume. Data can also be measured by various scales the four basic levels of measurement are **nominal**, **ordinal**, **interval** and **ratio**.

- There are two basic types of statistical studies: observational studies and **experimental studies**.
- When conducting **observational studies**, researchers observe what is happening or what has happened and then draw conclusions based on these observations.

Chapter Quiz

Determine whether each statement is true or false if the statement is false explain why.

1- Probability is used as a basis for inferential statistics (✓)

2- The height of president Lincoln is an example of variable (×)

لان الارتفاع غير سالب Ratio

3- The highest level of measurement is the interval level (×)

4- When the population of college professors is divided into groups according to their rank (instructor, assistant professor. etc.) and then several are selected from each group to make up a sample, the sample is called cluster (×)

Stratified

تم التقسيم الى طبقات حسب الدرجة

5- The variable age is an example of a qualitative variable (×)

6- The weight of pumpkins is considered be a continuous variable (✓)

5.5 – 6.5

7- The boundary of a value such as 6 inches would be 5.9-6.1 inches (×)

Select the best answer.

غياب

8- The number of absences per year that a worker has is an example of what type of data?

a. nominal

b. Qualitative

c. Discrete

d. Continuous

9- What are the boundaries of 25.6 ounces?

a. 25-26 ounces

b. 25.55-25.65 ounces

c. 25.5 – 25.7 ounces

d. 20 – 39 ounces

10- A researcher divided subjects into two groups according to gender and then selected members from each group for her sample. What sampling method was the researcher using?

a. Cluster

b. Random

c. Systematic

d. Stratified

11- Data that can be classified according to color are measured on what scale?

a. Nominal

b. Ratio

c. Ordinal

d. Interval

يتضمن - يشمل يغير نتيجه

12- A study that involves <u>no</u> researcher intervention is called

a. An experimental study

b. A noninvolvement stud

c. An observational study

d. A quasi – experimental study

يتداخل

13- A variable that interferes with other variables in the study is called

a. An confounding variable.
b. An explanatory variable.
c. An outcome variable.
d. An interfering variable.

Use the best answer to complete these

14- Two major branches of statistics are **<u>descriptive</u>** and **<u>Inferential.</u>**

15- Two uses of probability are **<u>Gambling</u>** and Insurance.

16- The group of all subjects under study is called a(n) **<u>population.</u>**

17- A group of all subjects selected from the group of all subjects under study is called (n) **<u>sample.</u>**

18- Three reasons why samples are used in statistics are

<u>a. save time</u> **<u>b. save money</u>** **<u>c. when population is large.</u>**

19- The four basic sampling methods are….

a. Random **b. systematic** **c. stratified** **d. cluster**

20- A study that uses intact groups when it is not possible to randomly assign participants to the groups is called **Quasi** شبه **Experimental** تجريبى study.

21- In a research study, participants المشاركين should be assigned to groups **random** methods, if possible.

22- **For each statement decide whether descriptive or inferential statistics is used.**

a. **The average** المتوقع life expectancy in New Zealand is 78.49 years.

(descriptive).

b. A diet high in fruits and vegetables will lower blood pressure.

(Inferential)

c. the total amount كمية of estimated losses الخسارة المقدرة from hurricane اعصار Hougo was $ 4.2 billion.

(Descriptive)

d. Researchers stated يبين that the shape شكل، هيئه of a person's الشخص ear أذن is related مرتبط بـ to the person's aggression بمدى انحراف الشخص.

(Inferential) ... يستدل بشيء على شيء آخر

خريجين

e. In 2020, the number of high school graduates will be 3.2 million students.

(Inferential)

23- Classify each as nominal level, ordinal level, or ratio level measurement

Rating of movies as PG. and R **(nominal)**

بالماكينة المباعة اعمدة السكر(قوالب الحلوى)

b. Number of candy bars sold on a fund drive. **(ratio)**

سيارة صغيرة

c. Classification of automobiles as subcompact, compact,

عادية فخمة

standard, and luxury. **(ordinal)**

مجفف الشعر

d. Temperatures of hair dryers. **(Interval).**

تجاريه خطوط

e. Weights of suitcases on a commercial airline. **(ratio)**

24- Classify each variable as discrete or continues.

أعمار العمال

a. Ages of people working in a large factory. **(continuous)**

المقدمه مطعم

b. Number of cups of coffee served at a restaurant.

(discrete)

المحقونه

c. The amount of drug injected into a guinea pig

(continuous)

d. The time it takes a student to drive to school. **(continuous)**

e. The number of gallons of milk sold each day at a grocery store. **(discrete)**

25- Give the boundaries of each.

a. 48 seconds
b. 0.56 Centimeter
c. 9.1 quarts
d. 13.7 pounds
e. 7 feet

Solution

a 47.5 – 48.5
b 0.555 – 0.565
c 9.05 – 9.15
d 13.65 – 13.75
e 6.5 – 7.5

H.W

Find the boundaries:

a) 6.1 - 8.32
b) 6.2 - 8.42
c) 6 – 8.5

إنتهى Ch.1

كل التمنيات بالنجاح والتوفيق

السعدى

Notes

* التركيز على المفاهيم الاساسية.
* شرح أبواب المنهج حسب الخطة.
* أمثلة توضيحية وتدريبات.
* نماذج اختبارات.

السعـــــــــدي

CH. 4

Stat. 110
إحصـــاء ١١٠

إعداد

جمال السعدي

أستاذ الرياضيات والإحصاء للمرحلة الجامعية

Ch. 4 Part. 1

• sample space	فراغ العينه	• Product	حاصل ضرب
• Experiment	تجربه	• Certain occur	حادثة مؤكدة
• probability	احتمال	• Never occur	حادثة مستحيلة
• Toss	القاء	• Complement	حادثة مكملة
• Coin	قطعه نقود	• Outcomes	نواتج (عناصر)
• Roll	يتدحرج (القاء)	• Empirical	تجريبي (مبني على الملاحظة
• Dice	حجر نرد	• Subjective	وهمي (مبني على التخمين)
• Tree diagram	الشجرة البيانية	• Urn	صندوق
• Head	وجه القطعة النقدية	• Drawn	يسحب
• Tail	خلفيه القطعة النقدية	• Mutually exclusive	متنافية
• Event	حادثه	• Recent Study	دراسة حديثة
• Simple event	حادثه بسيطه	• Common	مشترك
• Odd number	عدد فردى	• Exactly	بالضبط
• Prime number	عدد أولى	• Contain	يحتوى على
• Even number	عدد زوجى	• Consists of	يتكون من
• Compound event	حدث مركب	• Select	يختار
• Random	عشوائى	• At least	على الأقل
• Gender	نوع	• At most	على الأكثر

Sample Spaces and Probability

A probability experiment

A chance leads to defined results called outcomes

An outcome

Is the result of a single trial of a probability experiment.

A sample space

Is the set of all possible outcomes.

An event

Consists of a set of outcomes of a probability experiment.

Equally likely events

Are events that have the same probability of occurring.

Classical Probability

The probability of any event E

Is $P(E) = \dfrac{\text{Number of outcomes in E}}{\text{Total number of outcomes in the sample space}}$ $P(E) = \dfrac{n(E)}{n(S)}$

Empirical probability

Based on observation

$$P(E) = \frac{\text{Frequency for the class}}{\text{Total frequency in the distribution}}$$

Subjective probability

Based on estimate and inexact information

Simple event

Is an event contain **one** outcome.

Compound event

Is an event contain **more than one** outcome.

Coin

- **The number of outcomes in the sample space:**

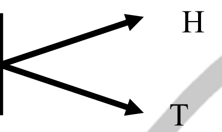
$n(S) = 2^N$ ⟶ عدد الرميات أو عدد القطع

Find the sample space for:

(1) Toss one coin → S = { H , T} ⟶ $N(s) = 2^1 = 2$

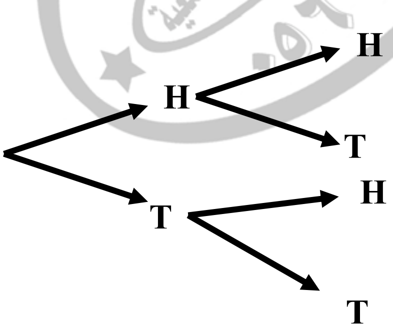
Head **Tail**

Tree diagram



(2) Toss two coins → S = {HH, HT, TH, TT} ⟶ $N(s) = 2^2 = 4$

Tree diagram



H.W

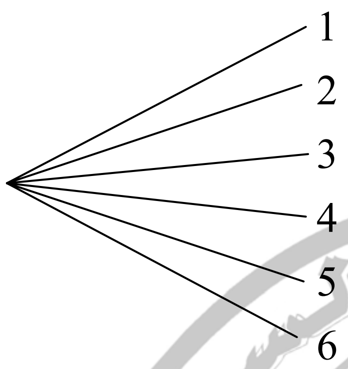
(3) Toss 3 Coins

Dice

The number of outcomes in the sample space.

$n(S) = 6^N$ ⟶ عدد الرميات أو عدد القطع

- Roll a dice ➔ S = {1, 2, 3, 4, 5, 6} ⟶ $N(s) = 6^1 = 6$

Tree diagram 

A is even number = {2, 4, 6}

B is odd number = {1, 3, 5}

C is prime number = {2, 3, 5}

D = {4} is simple event

A and B are mutually exclusive: where $A \cap B = \phi$

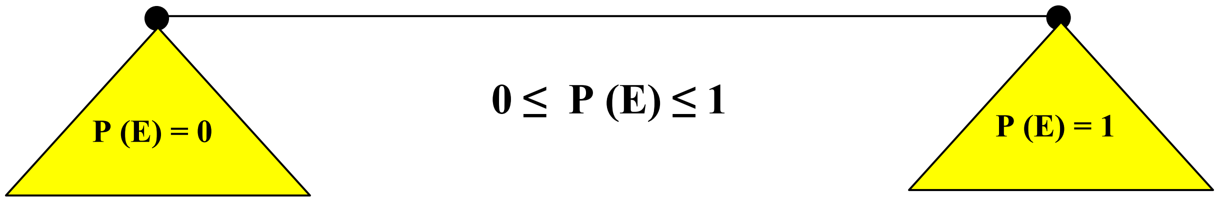
H.W

- Roll two dice ➔ S = ……………. , n (s) =………

  Tree diagram………………………..

Probability Rules

For any event E



- The Range of the values of the probability = [0 , 1]******
- P (E) = 0 where E can never occur.
- P (E) = 1 where E certain occur.
- $\sum_{i=1}^{n} P\ (a i) = 1$

The sum of the probabilities of all the outcomes in the sample space equal 1.

- $P\ (E') = 1 - P(E) \rightarrow P(E) + P\ (E') = 1$

Where E' is the complement of E

If s = {1, 2, 3 , 4, 5, 6}

A = {1, 2, 3, 4} ➜ A′ = {5 , 6 }

$P\ (A) = \frac{4}{6} \rightarrow P\ (A') = \frac{2}{6}$

11.Classify each statement as an example of classical probability, empirical probability, or subjective probability.

a) The probability that a person will watch the 6 o' clock evening news is 0.15 **→ (Empirical)**

b) The probability of winning at a Chuck-a-Luck game is $\frac{5}{36}$ **→ (Classical)**

c) The probability that a bus will be in an accident on a specific run is about 6% **→ (Empirical)**

d) The probability of getting a royal flush when five cards are selected at random is $\frac{1}{649.740}$ **→ (Classical)**

e) The probability that a student will get a C or better in a statistics course is about 70% **→ (Empirical)**

f) The probability that a new fast-food restaurant will be a success in Chicago is 35% **→ (Empirical)**

g) The probability that interest rates will rise in the next 6 months is 0.50 **→ (Subjective)**

Note

Empirical	Classical	Subjective
• نسبة مئوية • فاصلة عشرية	• كسور	• يبنى على التخمين ويشير إلى المستقبل Next………

10. A probability experiment is conducted. Which of these cannot be considered a probability of an outcome?

a. $\frac{1}{3}$	**b.** $-\frac{1}{5}$	**c. 0.80**
d. -0.59	**e. 0**	**f. 1.45**
g. 1	**h. 33%**	**i. 112%**

Solution

(b) $-\frac{1}{5}$

(d) – 0.59

(f) 1.45

(i) 112% $= 1.12$

Are can not be considered a probability of an outcome.

Where $0 \leq P(E) \leq 1$

12. If a die is rolled one time, find these probabilities.

a. **Of getting a 4.**

b. **Of getting an even number.**

c. **Of getting a number greater than 4.**

d. **Of getting a number less than 7.**

e. **Of getting a number greater than 0.**

f. **Of getting a number greater than 3 or an odd number.**

g. **Of getting a number greater than 3 and an odd number.**

Solution

$S = \{1, 2, 3, 4, 5, 6\}$

(a) $a = \{4\} \rightarrow P(a) = \frac{1}{6}$

(b) $b = \{2, 4, 6\} \rightarrow P(b) = \frac{3}{6} = \frac{1}{2}$

(c) $c = \{5, 6\} \rightarrow P(c) = \frac{2}{6} = \frac{1}{3}$

(d) $d = \{1, 2, 3, 4, 5, 6\} \rightarrow P(d) = \frac{6}{6} = 1 \longrightarrow$ certain occur

(e) $e = \{1, 2, 3, 4, 5, 6\} \rightarrow P(e) = \frac{6}{6} = 1 \longrightarrow$ certain occur

(f) $f = \{4, 5, 6, 1, 3\} \rightarrow P(f) = \frac{5}{6}$

(g) $g = \{5\} \rightarrow p(g) = \frac{1}{6}$

Example:

A couple has three children find each probability:

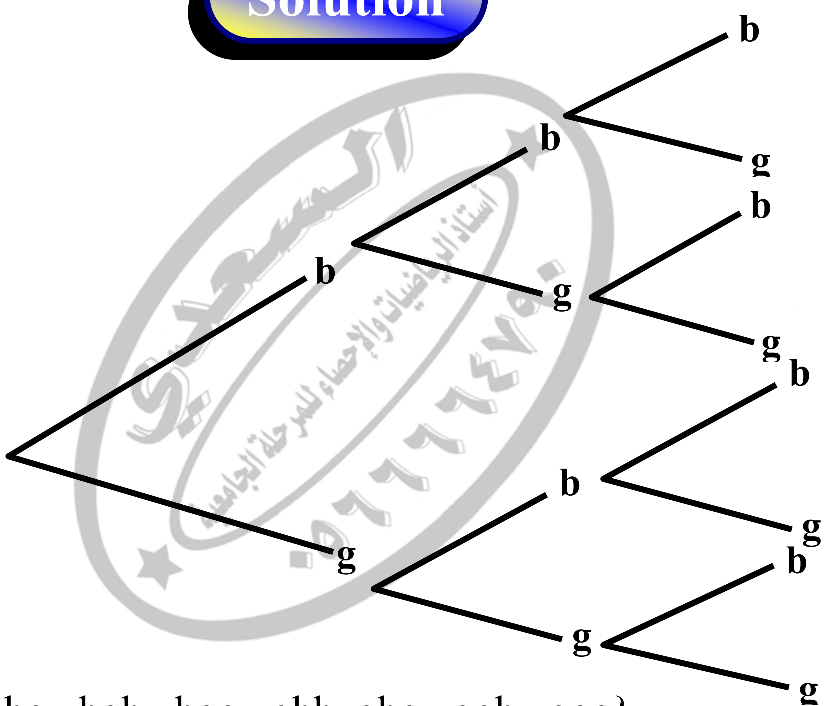
a) All boys

b) All girls or all boys

c) Exactly two boys or two girls.

D) At least one child of each gender.

Solution



S = { bbb , bbg , bgb , bgg , gbb, gbg , ggb , ggg}

(a) P (all boys) = $\frac{1}{8}$

(b) P (all girls or all boy) = $\frac{2}{8} = \frac{1}{4}$

(c) P (Exactly two boys or two girls) = $\frac{6}{8} = \frac{3}{4}$

(d) P (at least one child of each gender) = $\frac{6}{8} = \frac{3}{4}$

13. if two dice are rolled one time, find the probability of getting these results.

a. A sum of 6.

b. Doubles.

c. A sum of 7 or 11.

d. A sum greater than 9.

e. A sum less than or equal to 4.

Solution

S = {(1,1) , (1,2) ,………………., (1,6)
(2,1) , (2,2),………………...., (2,6)
(6,1) , (6,2),……………….., (6,6) }

(a) a = {(1,5) , (2,4) , (3,3) (4,2) , (5,1)}

$P(a) = \frac{5}{36}$

(b) b = { (1,1) , (2,2) , (3,3) , (4,4) , (5,5) , (6,6)}

$P(b) = \frac{6}{36} = \frac{1}{6}$

(c) c = {(1,6) , (2,5) , (3,4) , (4,3) , (5,2) , (6,1) , (5,6) , (6,5)}

$P(c) = \frac{8}{36} = \frac{2}{9}$

(d) d = {(4,6) , (5,5) , (5,6) , (6,4) , (6,5) , (6,6) }

$P(d) = \frac{6}{36} = \frac{1}{6}$

(c) c= {(1,1) , (1,2) , (1,3) , (2,1) , (2,2) , (3,1)}

$P(c) = \frac{6}{36} = \frac{1}{6}$

Roll two dice and multiply the number together.

a. **Write out the sample space.**

b. **What is the probability that the product is a multiple of 6 ?**

c. **What is the probability that the product is less than 10?**

Solution

a) S = {(1,1) , (1,2) , (1,3) , (1,4) , (1,5) , (1,6),

(2,1) , (2,2) , (2,3) , (2,4) , (2,5), (2,6),

(3,1) , (3,2) , (3,3) , (3,4) , (3,5), (3,6),

(4,1) , (4,2) , (4,3) , (4,4) , (4,5), (4,6),

(5,1) , (5,2) , (5,3) , (5,4) , (5,5), (5,6),

(6,1) , (6,2) , (6,3) , (6,4) , (6,5), (6,6)}

b) P (Product is multiple of 6) = $\frac{15}{36}$

c) P (Product is less than 10) = $\frac{17}{36}$

19. For a recent year, 51% of the families in the United Slates had no children under the age of 18; 20% had one children 19% had two children; 7% had three children; and 3% had four or more children. If a family is selected at random, find the probability that the family has:

a. Two or three children
b. More than one child
c. Less than three children
d. Based on the answers to parts a, b, and c, which is most likely to occur? Explain why.

Solution

Information's:

* P (0 children) = 0.51
* P (1 children) = 0.20
* P (2 children) = 0.19
* P (3 children) = 0.07
* P (4 children or more) = 0.03

(a) P (2 or 3 children)
= P (2) + P (3)
= 0.19 + 0.07
= **0.26**

(b) P (more than one children)
= P (2) + P (3) + P (4 children or more)
= 0.19 + 0.07 + 0.03
= **0.29**

(c) P (less than three children)
= P (0) + P (1) + P (2)
= 0.51 + 0.20 + 0.19
= 0.90

(d) In part c the event is most likely to occur.
Because the probability is greeter than any one.

Ch. 4 Part 2

The addition rules for

Two events are **mutually exclusive** events if they cannot occur at the same time they have no outcomes in common.

Addition Rule 1

When two events A and B are mutually exclusive, the probability that A or B will occur is

$P(A \text{ or } B) = P(A) + P(B)$

$\cup$

Addition Rule 2

If A and B are not mutually exclusive, then

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\cup$ $\cap$

ممرض طبيب

In a hospital unit there are 8 nurses and 5 physicians; 7 nurses and 3 physicians are females. If a staff person is selected, find the probability that the subject is a.

شخص

(1) Nurse or male.

(2) Physicians or females.

Solution

Staff	Females	Males	Total
Nurses	7	1	8
Physicians	3	2	5
Total	10	3	13

(1) P (Nurse or male)

= P (Nurse) + P (male) – P (nurse and male)

$$= \frac{8}{13} + \frac{3}{13} - \frac{1}{13} = \frac{10}{13}$$

(2) P (Physician or female)

= P (Physician) + P (female) – P (Physician and female)

$$= \frac{5}{13} + \frac{10}{13} - \frac{3}{13} = \frac{12}{13}$$

2. Determine whether these events are mutually exclusive:

a. Roll a die: Get an even number, and get a number less than 3.

b. Roll a die: Get a prime number, and get an odd number.

c. Roll a die: Get a number greater than 3, and get a number less than 3.

d. Select a student in your class: The student has blond hair (أشقر), and the student has blue eyes.

e. Select a student in your college: the student is a sophomore (طالب في السنة الثانية), and the student is a business (تجارة) major (تخصص).

f. Select any course: it is a calculus course, and it is an English course.

g. Select a registered voter (الناخب): the voter is a Republican (جمهوري), and the voter is a democrat.

Solution

(a) $A = \{2,4,6\}$ $B = \{1,2\}$

$A \cap B = \{2\}$ $\rightarrow$ not mutually exclusive.

(b) $A = \{2,3,5\}$ $B = \{1,3,5\}$

$A \cap B = \{3,5\}$ $\rightarrow$ not mutually exclusive.

(c) $A = \{4,5,6\}$ $B = \{1,2\}$

$A \cap B = \phi$ $\rightarrow$ A and B mutually exclusive.

(d) A: blond hair B = blue eyes

$A \cap B \neq \phi$ $\rightarrow$ not mutually exclusive.

(e) A: sophomore B: business major

$A \cap B \neq \phi$ $\rightarrow$ not mutually exclusive.

(f) A: calculus course B: English course

$A \cap B = \phi \rightarrow$ A and B mutually exclusive

(g) A: Republican B: Democrat

$A \cap B = \phi \rightarrow$ A and B mutually exclusive

Example:

At a convention there are 7 mathematics instructors, 5 computer science instructors, 3 statistics instructors, and 4 science instructors. If an instructor is selected, find the probability of getting a science instructor or a math instructor.

Solution

Total instructors = 7 + 5 + 3 + 4 = 19

P (science instructor or math instructor)

= P (science instructor) + P (math instructor)

$$= \frac{4}{19} + \frac{7}{19} = \frac{11}{19}$$

7. A recent study of 200 nurses found that of 125 female nurses, 56 had bachelor's degrees; and of 75 male nurses, 34 had bachelor's degrees. If a nurse is selected at random, find the probability that the nurse is

- **a. A female nurse with a bachelor's degree.**
- **b. A male nurse.**
- **c. A male nurse with a bachelor's degree.**
- **d. Based on your answers to parts a, b, and c, Explain which is most likely to occur. Explain why.**

Solution

	Male	Female
Bachelor's degree	34	56
Without bachelor degree	75-34 = 41	125-56 = 69
Total	75	125

(a) $P(A) = \frac{56}{200} = 0.28$

(b) $P(B) = \frac{75}{200} = 0.38$

(c) $P(C) = \frac{34}{200} = 0.17$

(d) Event B is most likely to occur

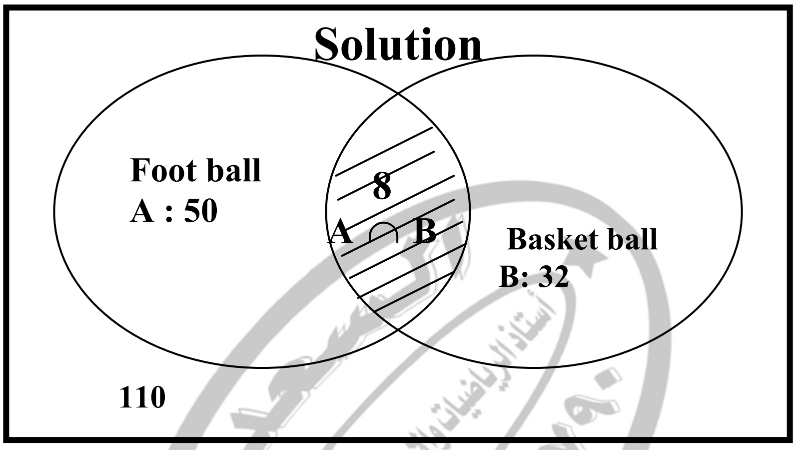
Because the probability is greeter than any one.

9. At a particular (خاصة) school (مدرسة) with <u>200</u> male students, <u>58</u> play football, <u>40</u> play basketball, and 8 play both.
What is the probability that a randomly selected male student?

a. Plays neither sport.

b. Plays (Football or Basketball).

c. Plays (Football and Basketball).



a. P (Play Neither sport) $= \dfrac{110}{200} = 0.55$

b. P (Play: Football or basketball)

= P (A or B)

= P (A) + P (B) – P (A ∩ B)

$= \dfrac{58}{200} + \dfrac{40}{200} - \dfrac{8}{200} = \dfrac{90}{200} = 0.45$

c. P (Play: Football and basketball)

$= \dfrac{8}{200} = 0.04$

مركز تجاري لمقايضة السيارات

13. The Bargain Auto Mall has these cars in stock.

	SUV	سيارة صغيرة Compact	متوسطة الحجم Mid – Sized
Foreign أجنبي	20	50	20
Domestic داخلي - وطني	65	100	45

If a car is selected at random, find the probability that it is:

a. Domestic

b. Foreign and mid – sized

c. Domestic or an SUV.

Solution

Total cars = 20 + 50 + 20 + 65 + 100 + 45 = 300

(a) $P\ (\text{Domestic}) = \dfrac{65 + 100\ 45}{300} = 0.7$

(b) $P\ (\text{Foreign and mid – sized}) = \dfrac{20}{300} = 0.07$

(c) P (Domestic or SUV)

$= P\ (\text{Domestic}) + P\ (\text{SUV}) - P\ (\text{Domestic and SUV})$

$$= \frac{65 + 100 + 45}{300} + \frac{20 + 65}{300} - \frac{65}{300} = 0.77$$

25. An urn contains 6 red balls, 2 green balls, 1 blue ball and 1 white ball. If a ball is drawn, find the probability of getting:

(a) Red or green.

(b) blue or white

(c) not green

(d) blue and white

Solution

Total balls = 6 + 2 + 1 + 1 = 10

(a) P (red or green) = P (red) + P (green) $= \frac{6}{10} + \frac{2}{10} = \underline{\underline{0.8}}$

(b) P (blue or white) = P (blue) + P (white) $= \frac{1}{10} + \frac{1}{10} = \underline{\underline{0.2}}$

(c) P (not green) = 1 – P (green) $= 1 - \frac{2}{10} = \underline{\underline{0.8}}$

(d) P (blue and white) = P (ϕ) = $\underline{\mathbf{0}}$

Ch. 4 Part. 3

The Multiplication Rules and Conditional Probability

Two events A and B are independent events if:

A occurs does not affect the probability of B occurring.

Multiplication Rule 1

When two events are independent, the probability of both occurring is

P(A and B) = P(A) × P(B)

Example:

An urn contains 3 red balls, 2 blue balls, and 5 white balls.

A ball is selected and its color noted. Then it is replaced. A second ball is selected and its color noted. Find the probability of each of these.

a. Selecting 2 blue balls.

b. Selecting 1 blue ball and then 1 white ball.

c. Selecting 1 red ball and then 1 blue ball.

Solution

a. P (blue and blue) = P(blue) × P(blue) = $\frac{2}{10} \times \frac{2}{10} = \frac{4}{100} = \frac{1}{25}$

b. P (blue and white) = P(blue) × P(white) = $\frac{2}{10} \times \frac{5}{10} = \frac{10}{100} = \frac{1}{10}$

c. P (red and blue) = P(red) × P(blue) = $\frac{3}{10} \times \frac{2}{10} = \frac{6}{100} = \frac{3}{50}$

Example:

Approximately 9% of men have a type of color blindness that prevents them from distinguishing between red and green. If 3 men are selected at random, find the probability that all of them will have this type of red-green color blindness.

Solution

Let C denote red-green color blindness. Then

$$P(\text{ C and C and C}) = P(C) \times P(C) \times P(C)$$
$$= (0.09)\,(0.09)\,(0.09)$$
$$= 0.000729$$

Example:

A coin is flipped and a die is rolled. Find the probability of getting a head on the coin and a 4 on the die.

Solution

$$P(\text{ head and 4}) = P(\text{head}) \times P(4) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

- **IF P(A) < 0.5**

A is unlikely to occur

- **If P(A) ≥ 0.5**

A is likely to occur

- **If P(A) = L**

➔ P(none A) = 1 - L

Example:

If 28% of U.S. medical degrees are conferred to women, find the probability that 3 randomly selected medical school graduates are men. Would you consider this event likely or unlikely to occur? Explain your answer.

Solution

- $P(W) = 0.28$
- $P(M) = 1 - P(W)$

$= 1 - 0.28 = 0.72$

$P(3M) = P(M) \, . \, P(M) \, , \, P(M)$

$= (0.72)\,(0.72)\,(0.72)$

$= 0.373$

The event is unlikely to occur because $P(3M) < 0.5$

Example:

Eighty-eight percent of U.S. children are covered by some type of health insurance. If 4 children are selected at random, what is the probability that none are covered?

Solution

P(covered) = 0.88

P(non covered) = 1 – 0.88 = 0.12

P(4 children are non covered) = (0.12) (0.12) (0.12) (0.12) = 0.0002

Multiplication Rule 2

When two events are dependent, the probability of both occurring is

P(A and B) = P(A) × P(B/A)

Example:

A person owns a collection of 30 CDs, of which 5 are country music. If 2 CDs are selected at random, find the probability that both are country music.

Solution

Since the events are dependent,

$$P(C_1 \text{ and } C_2) = P(C_1) \times P(C_2 \mid C_1) = \frac{5}{30} \times \frac{4}{29} = \frac{20}{870} = \frac{2}{87}$$

Example:

In a civic organization, there are 38 members; 15 are men and 23 are women. If 3 members are selected to plan the July 4th parade, find the probability that all 3 are women. Would you consider this event likely or unlikely to occur? Explain your answer.

Solution

The total members

= 15 men + 23 women = 38

$$P(3 \text{ women}) = \frac{23}{38} \times \frac{22}{37} \times \frac{21}{36} = 0.21 \quad < 0.5$$

There for: This event unlikely to occur.

Conditional probability

- P(A/B)

Probability that A occur
After B already occurred

$$P(A/B) = \frac{P(A \text{ and } B)}{P(B)} \rightarrow P(A \text{ and } B) = P(B) \times P(A/B)$$

$$P(B/A) = \frac{P(A \text{ and } B)}{P(A)} \rightarrow P(A \text{ and } B) = P(A) \times P(B/A)$$

Example:

A recent survey asked 100 people if they thought women in the armed forces should be permitted to participate in combat. The results of the survey are shown.

Gender	Yes	No	Total
Male	32	18	50
Female	8	42	50
Total	40	60	100

Find these probabilities.

a. The respondent answered yes, given that the respondent was a female.

b. The respondent was a male, given that the respondent answered no.

Solution

Let: M = Male F = Female
Y = yes N = No

$$\text{a. } P(Y/F) = \frac{P(Y \text{ and } F)}{P(F)} = \frac{\frac{8}{100}}{\frac{50}{100}} = \underline{\underline{0.16}}$$

$$\text{b. } P(M/N) = \frac{P(M \text{ and } N)}{P(N)} = \frac{\frac{18}{100}}{\frac{60}{100}} = \underline{\underline{0.3}}$$

Example:

An insurance company classifies drivers as low-risk, medium-risk, and high-risk. Of those insured, 60% are low-risk, 30% are medium-risk, and 10% are high-risk. After a study, the company finds that during a 1-year period, 1% of the low-risk drivers had an accident, 5% of the medium-risk drivers had an accident, and 9% of the high-risk drivers had an accident. If a driver is selected at random, find the probability that the driver will have an accident during the year.

Low
$P(A) \times P(B/A)$
$(0.60) \times (0.01)$ ⇒

Medium
$P(A) \times P(B/A)$
$(0.30) \times (0.05)$ ⇒

High
$P(A) \times P(B/A)$
$(0.10) \times (0.09)$ ⇒

Solution

P(have an accident)

= P(low – risk and have an accident) ➔ (0.6) (0.01)

+ P(medium – risk and have an accident) ➔ (0.3) (0.05)

+ P(high – risk and have an accident) ➔ (0.1) (0.09)

= 0.03

حالة الـ Coin

At least one ......

Find the probability of getting at least one .....

- (1) A coin is tossed 3 times:
  $N(s) = 2^3 = 8$

$\therefore$ P (at least one tail) $= \dfrac{N(s) - 1}{N(s)} = \dfrac{8-1}{8} = \dfrac{7}{8}$

- (2) A coin is tossed 5 times :
  $N(s) = 2^5 = 32$

$\therefore$ P (at least one head) $= \dfrac{N(s) - 1}{N(s)} = \dfrac{32-1}{32} = \dfrac{31}{32}$

حالة الـــنسب المئوية

Rule

- **A: at least one ....**
- **A' : no .... ≡ ....**
- **$P(A') = (\quad)(\quad)(\quad)$........**
- **$P(A) = 1 - P(A')$**

Example:

It has been found that 6% of all automobiles on the road have defective brakes. If 5 automobiles are stopped and checked by the state police, find the probability that at least one will have defective brakes.

Solution

A = at least one have defective brakes
A' = no have defective brakes
P(defective) = 0.06 P(undefective) = 1 – 0.06 = 0.94
$P(A') = (0.94)(0.94)(0.94)(0.94)(0.94) = $ **0.7339**
$P(A) = 1 - P(A') = 1 - 0.7339 = $ **0.266**

Ch. 4 Part. 4

Counting Rules

- Fundamental Counting rule:

In a sequence of n events in which the first one has k_1 possibilities and the second event has k_2 and the third has k_3, and so forth, the total number of possibilities of the sequence will be

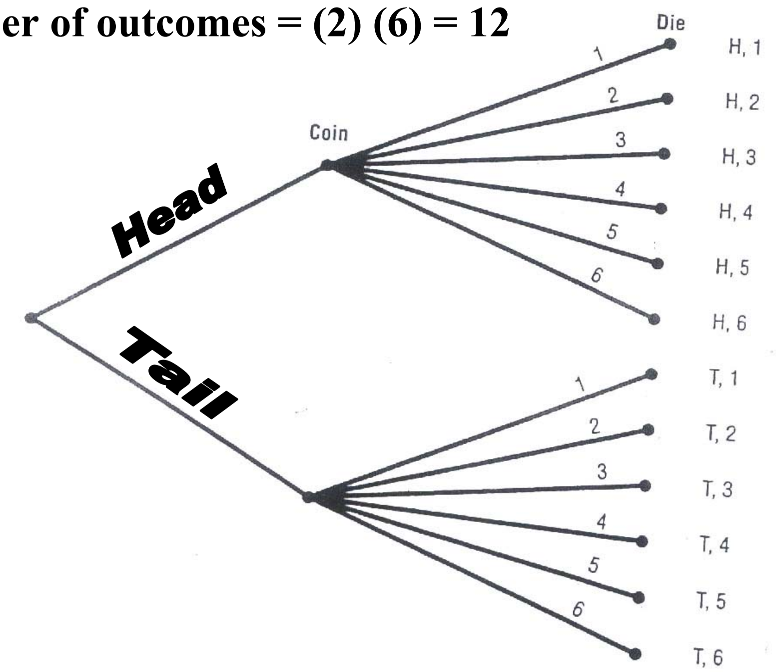
$K_1 \times K_2 \times K_3 \times \ldots\ldots\ldots \times K_n$

Example:

A coin is tossed and a die is rolled. Find the number of outcomes for the sequence of events.

Solution

The number of outcomes = (2) (6) = 12



Example:

The digits 0, 1,2, 3, and 4 are to be used in a **four-digit** ID card. How many different cards are possible if repetitions are permitted?

5	5	5	5

Solution

Since there are 4 spaces to fill and 5 choices for each space,

The number of cards = $5 \times 5 \times 5 \times 5 = 5^4 = 625$

Permutations

A permutation is an arrangement of n objects in a specific order.

Factorial Formulas

For any counting n

$n! = n \cdot (n-1) \cdot (n-2) \ldots\ldots 1$

$5! = (5)(4)(3)(2)(1) = 120$

$0! = 1$

Permutation Rule

The arrangement of n objects in a specific order using r objects at a time is called a permutation of n objects taking r objects at a time. It is written as ${}_nP_r$, and the formula is

$${}_nP_r = \frac{n!}{(n-r)!}$$

- order is important

$${}_5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{5 \times 4 \times 3 \times \cancel{2} \times \cancel{1}}{\cancel{2} \times \cancel{1}} = 60$$

ممكن بالآلة بدون الفك

$${}_5P_5 = \frac{5!}{(5-5)!} = \frac{5!}{0!} = \frac{5 \times 4 \times 3 \times \cancel{2} \times \cancel{1}}{\cancel{1}} = 120$$

${}_5P_5 = 5!$ **$0! = 1$** **${}_nP_n = n!$**

Example:

How many different ways can a chairperson and an assistant chairperson be selected for a research project if there are seven scientists available?

Solution

$$_7P_2 = \frac{7!}{(7-2)!} = \frac{7!}{5!} = 42$$

Example:

A store manager wishes to display 8 different brands of shampoo in a row. How many ways can this be done?

Solution

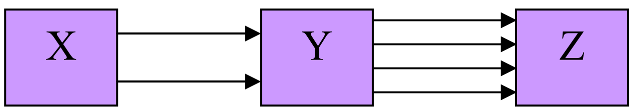
Numbers of ways

$= 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 8! = 40320$

Example:

There are 2 major roads from city X to city Y and 4 major roads from city Y to city Z. How many different trips can be made from city X to city Z passing through city Y?

Solution



Numbers of ways = $2 \times 4 = 8$

Example:

If 50 tickets are sold and 2 prizes are to be awarded.

Find the probability that one person will win 2 prizes if that person buys 2 tickets.

Solution

التذكرتان x , y

x ثم y أو y ثم y

$$P(2\ \text{prizes}) = \left(\frac{1}{50} \times \frac{1}{49}\right) + \left(\frac{1}{50} \times \frac{1}{49}\right) = \frac{1}{1225}$$

Combination Rule

The number of combinations of r objects selected from n objects is denoted by ${}_nC_r$ and is given by the formula: ${}_nC_r = \dfrac{n!}{(n-r)!r!}$

Example:

In a club there are 7'women and 5 men. A committee of 3 women and 2 men is to be chosen.
How many different possibilities are there?

Solution

$${}_7C_3 \times {}_5C_2 = \frac{7!}{(7-3)!3!} \times \frac{5!}{(5-2)!2!} = 350$$

How many different tests can be made from a test bank of 20 questions if the test consists of 5 questions?

Solution

Number of tests = $20\ C_5$ = 15504

Example:

There are 7 women and 5 men in a department.

1. How many ways can a committee of 4 people be selected?
2. How many ways can this committee be selected if there must be 2 men and 2 women on the committee?
3. How many ways can this committee be selected if there must be at least 2 women on the committee?

Solution

1- number of committee = $12\ C_4 = 495$

2- number of committee = $7\ C_2 \times 5C_2 = 210$

3- number of committee where at least 2 women

(2 w and 2 m) or (3 w and 1 m) or 4 w)

$= (7\ C_2 \times 5C_2) + (7\ C_3 \times 5C_1) + (7\ C_4)$

$= (21 \times 10) + (35 \times 5) + 35 = 420$

Example:

How many ways can a dinner patron select 3 appetizers and 2 vegetables if there are 6 appetizers and 5 vegetables on the menu?

Solution

appetizers — vegetables

$$\text{Number of ways} = 6\,C_3 \times 5\,C_2$$

$$= 20 \times 10 = 200$$

Example:

How many different ways can an instructor select 2 textbooks from a possible 17?

Solution

$$\text{Number of ways} = 17\,C_2 = 136$$

Example:

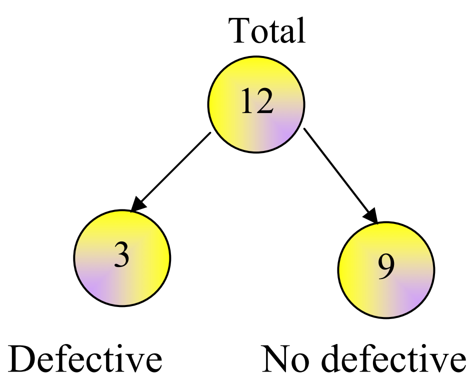
A package contains 12 resistors, 3 of which are defective.
If 4 are selected, find the probability of getting
a. No defective resistors
b. 1 defective resistor
c. 3 defective resistors

Solution

a. $P(\text{No defective}) = \dfrac{9\,C_4}{12\,C_4} = 0.255$

b. $P(1\text{ defective}) = \dfrac{3\,C_1 \times 9\,C_3}{12\,C_4} = 0.509$

c. $P(3\text{ defective}) = \dfrac{3\,C_3 \times 9\,C_1}{12\,C_4} = 0.018$



Example:

A box contains 24 transistors, 4 of which are defective. If 4 are sold at random, find the following probabilities.

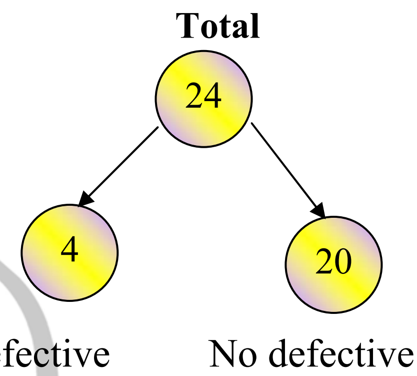
a. Exactly 2 are defective.

b. None is defective.

c. All are defective,

d. At least 1 is defective.

Solution



a.

$$P(\text{exactly 2 defectives}) = \frac{{}_{4}C_{2} \times {}_{20}C_{2}}{{}_{24}C_{4}} = \frac{1140}{10626} = \frac{190}{1771}$$

b.

$$P(\text{no defectives}) = \frac{{}_{20}C_{4}}{{}_{24}C_{4}} = \frac{4845}{10626} = \frac{1615}{3542}$$

c.

$$P(\text{all defective}) = \frac{{}_{4}C_{4}}{{}_{24}C_{4}} = \frac{1}{10626}$$

d.

P(at least 1 defective) = 1 —P(no defectives)

$$= 1 - \frac{{}_{20}C_{4}}{{}_{24}C_{4}} = 1 - \frac{1615}{3542} = \frac{1927}{3542}$$

Example:

A store has 6 TV Graphic magazines and 8 News time magazines on the counter. If two customers purchased a magazine, find the probability that one of each magazine was purchased.

Solution

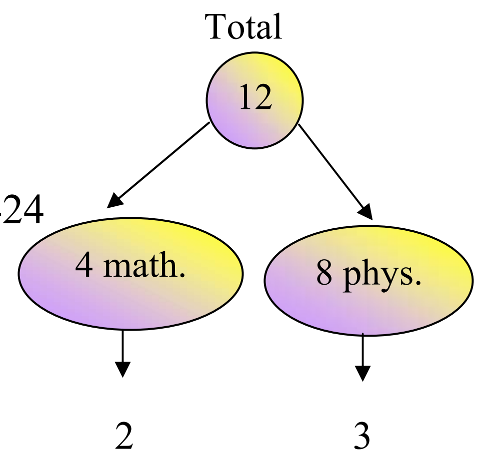
$$P(1 \text{ TV Graphic and } 1 \text{ News time}) = \frac{{}_6C_1 \times {}_8C_1}{{}_{14}C_2} = \frac{6 \times 8}{91} = \frac{48}{91}$$

Example:

Find the probability of randomly selecting 2 mathematics books and 3 physics books from a box containing 4 mathematics books and 8 physics books.

Solution

$$P(2 \text{ math and } 3 \text{ phys.}) = \frac{{}_4C_2 \times {}_8C_3}{{}_{12}C_5} = 0.424$$



Example:

An insurance sales representative select 3 policies to review. The group of policies she can select from contains 8 life policies, 5 automobile policies, and 2 homeowner policies.

Find the probability of selecting

a. All life policies

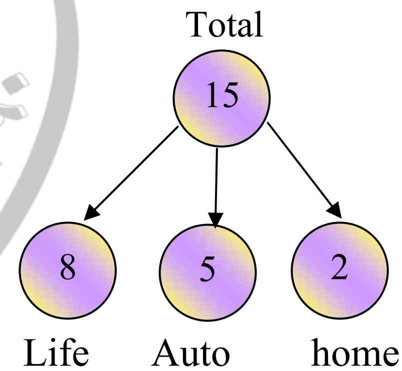
b. Both homeowner policies

c. All automobile policies

d. 1 of each policy

e. 2 life policies and 1 automobile policy

Solution



(a) P (All life) $= \dfrac{8C_3}{15C_3} = 0.123$

(b) P (Both homeowner) $= \dfrac{2C_2 \times 13C_1}{15\,C_3} = 0.029$

(c) P (3 Auto) $= \dfrac{5C_3}{15C_3} = 0.022$

(d) P (1 of each policy) $= \dfrac{8C_1 \times 5C_1 \times 2C_1}{15\,C_3} = 0.176$

(e) P (2 life and 1 Auto) $= \dfrac{8C_2 \times 5\,C_1}{15\,C_3} = 0.308$

Example:

There are **2 math** – students and **5 stat** – students in a class

How many ways can a group of **3 students be selected** if there must be **at least one math** – student on this group ?

Solution

2 math
5 stat
} Select 3 students

At least 1 math – student

= (1 math | and | 2 stat) | or | (2 math | and | 1 stat)

↓ ↓ ↓

$= (2\,c_1 \times 5\,c_2) + (2\,C_2 \times 5\,C_1)$

= 25

Chapter Quiz

Determine whether each statement is true or false. If the statement is false, explain why.

1. Subjective probability has little use in the real world. (×)
2. Classical probability uses a frequency distribution to compute probabilities. (×)
3. In classical probability, all outcomes in the sample space are equally likely. (√)
4. When two events are not mutually exclusive, P (A or B) = P (A) + P (B) (×)
5. If two events are dependent, they must have the same probability of occurring. (×)
6. An event and its complement can occur at the same time. (×)
7. The arrangement ABC is the same as BAC for combinations. (√)
8. When objects are arranged in a specific order, the arrangement is called a combination, (×)

Select the best answer.

9. The probability that an event happens is 0.42. What is the probability that the event won't happen?

a. -0.42 c. 0
b. 0.58 d. 1

10. When a meteorologist says that there is a 30% chance of showers, what type of probability is the person using?

a. Classical c. Subjective
b. Empirical d. b and c are correct

11. The sample space for tossing 3 coins consists of how many outcomes?

a. 2 c. 6
b. 4 d. 8

12. The complement of guessing 5 correct answers on a 5-question true/false exam is

a. Guessing 5 incorrect answers
b. Guessing at least 1 incorrect answer
S = {(5 inc.), (1c, 4 inc), (2c, 3 inc), (3c, 2 inc), (4c, inc), (5c)}
A: 5 correct answers
A' : at least 1 incorrect answers
c. Guessing at least 1 correct answer
d. Guessing no incorrect answers

13. When two dice are rolled, the sample space consists of how many events?

a. 6 c. 36
b. 12 d. 54

14. What is nP_0?

a. 0 c. n
b. 1 d. It cannot be determined.

15. What is the number of permutations of 6 different objects taken all together?

a. 0 c. 36
b. 1 d. 720

16. What is 0!?

a. 0 c. Undefined
b. 1 d. 10

17. What is $_nC_n$?

a. 0 c. n
b. 1 d. It cannot be determined.

Complete the following statements with the best answer.

18. The set of all-possible outcomes of a probability experiment is called the ***sample space***
19. The probability of an event can be any number between and including ***0*** and ***1***
20. If an event cannot occur, its probability is ***0***
21. The sum of the probabilities of the events in the sample space is ***1***
22. When two events cannot occur at the same time, they are said to be ***mutually exclusive***

انتهى Ch 4

كل التمنيات بالنجاح والتوفيق

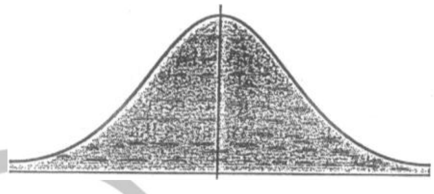
السعدى

Ch. 6 Part.1

التوزيع الطبيعي

The Normal Distribution

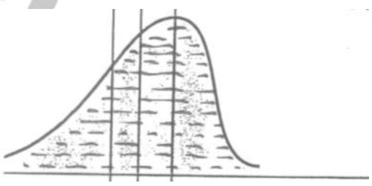
- When the data values are evenly distributed about the mean, a distribution is said to be a symmetric distribution. (A normal distribution is symmetric.).



Mean = Median = Mode

Normal

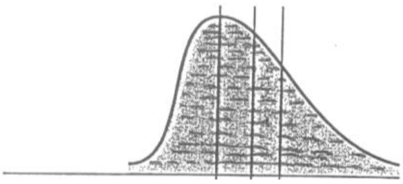
- When the majority of the data values fall to the right of the mean, the distribution is said to be a negatively or left-skewed distri-bution.



Mean Median Mode
Negatively skewed

Mean < Median < Mode

- When the majority of the data values fall to the left of the mean, a distribution is said to be a positively or right-skewed distribution.



Mode Median Mean
Positively skewed

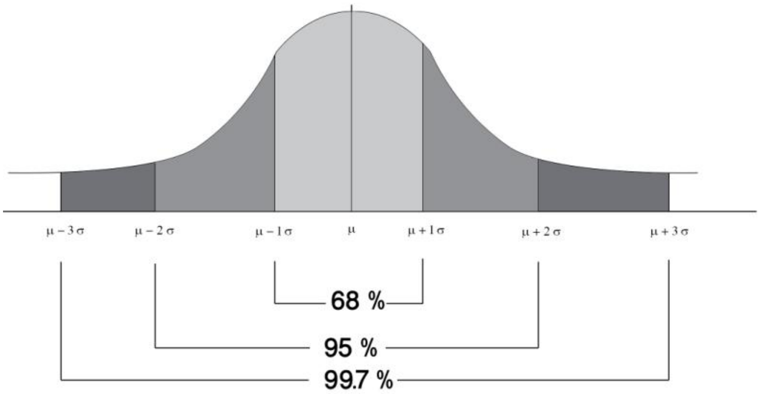
Mode < Median < Mean

Properties of a Normal Distribution

متصل متماثل شكل

A normal distribution is a continuous, symmetric, bell-shaped distribution of a variable.

Summary of the Properties of the Theoretical Normal Distribution
1. A normal distribution curve is bell-shaped. 2. The mean, median, and mode are equal and are located at the center of the distribution. 3. A normal distribution curve is unimodal (it has only one mode). 4. The curve is symmetric about the mean 5. The curve is continuous, that is, there are no gaps or holes. 6. The curve never touches the x axis. 7. The total area under a normal distribution curve is equal to 1.00, or 100%. 8. The area under the part of a normal curve that lies within 1 standard deviation of the mean is approximately 0.68, or 68%; within 2 standard deviations, about 0.95, or 95%; and within 3 standard deviations, about 0.997, or 99.7%.



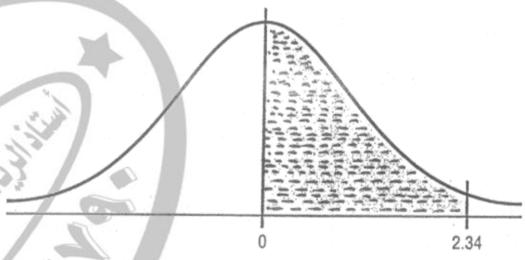
Finding the Area under the Standard Normal Distribution Curve

Example:

Find the area under the standard normal distribution curve between z = 0 and z = 2.34

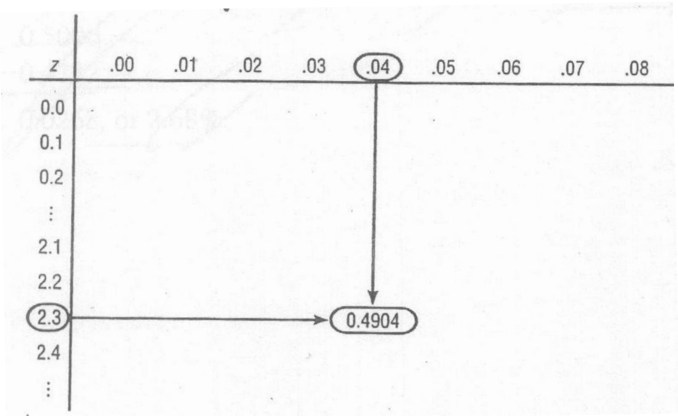
Solution

Draw the figure and represent the area as shown in Figure



Area = P (0 < z < 2.34) = 0.4904

طريقة الكشف موضحة كما يلي:



**** We use table E in last Page to find the area.**

0566664790 أستاذ الرياضيات والإحصاء للمرحلة الجامعية جمال السعدي

Example:

Find the area to the left of z = -1.93.

Solution

P (z < - 1.93)

= 0.5 – p (0< z < 1.93)

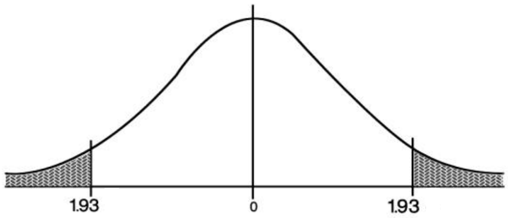
= 0.5 – 0.4732

= 0.0268

= 2.68 %

التظليل طرفي..

∴ المساحة = 0.5 – ناتج الكشف من

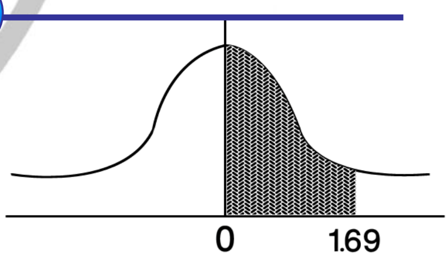


Example:

Find the area between:

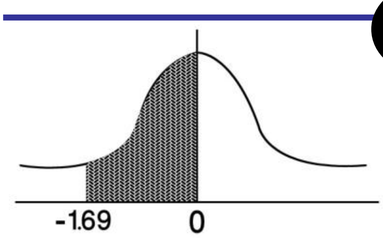
(1) [illegible] nd z = 1[illegible]

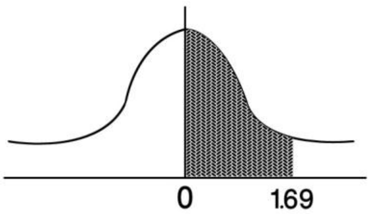
Solution



(2) z = 0 and z = - 1.69

Solution



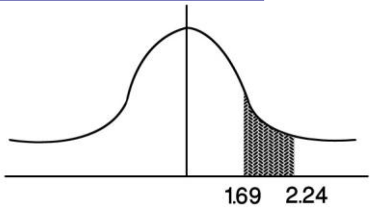


P (- 1.69 < z < 0) = p (0 < z < 1.69) = 0.4545

③ Find the area between

$Z = 1.69$ and $z = 2.24$

Solution



الحد ان فى جهه واحده: طرح

$P(1.69 < z < 2.24)$

الأكبر الأصغر

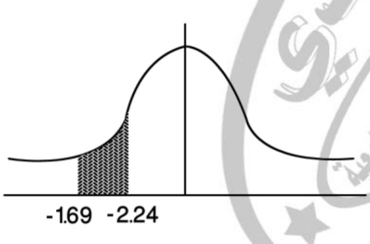
$= p(0 < z < 2.24) - p(0 < z < 1.69)$

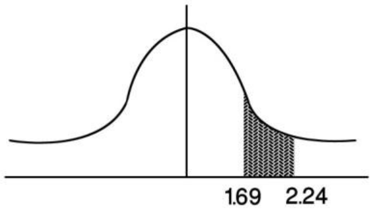
$= 0.4875 - 0.4545 = 0.033$

④ Find the area between

$Z = -1.69$ and $z = -2.24$

Solution





$P(-2.24 < z < -1.69) = P(1.69 < z < 2.24)$

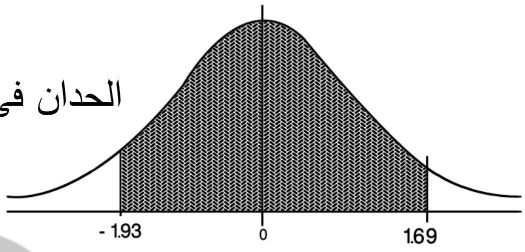
As shown in number (3)

= ………….. = 0.033

5) Find the area between

Z = - 1.56 and z = 1.69

Solution



الحدان فى جهتين مختلفتين نكشف عن الحدان ثم نجمع النتائج

P (- 1.56 < z < 1.69)

= P (0 < z < 1.56) + P (0 < z < 1.69)

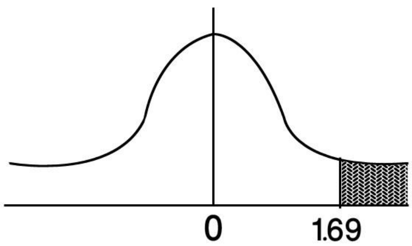
= 0.4406 + 0.4545 = 0.8951

6 Find the area to the right of Z = 1.69

Solution

∵ التظليل طرفى

∴ المساحة = 0.5 – الكشف من الجدول



P (Z > 1.69)

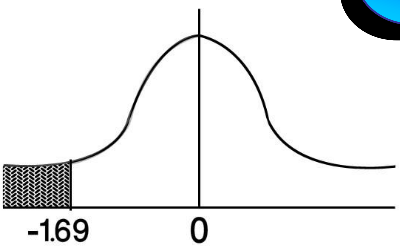
= 0.5 – p (0 < Z < 1.69)

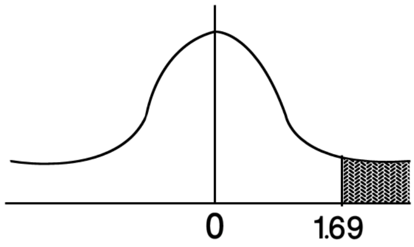
= 0.5 – 0.4545

= 0.0455

7. Find the area to the left of Z = - 1.69

Solution





P (Z < - 1.69)

= P (Z > 1.69)

= 0.5 – 0.4545 = 0.0455

---

8. Find the area to the left Z = 1.69

Solution

∴ التظليل زاد عن نصف المستوى

∴ المساحة = نصف المستوى + الجزء الباقي

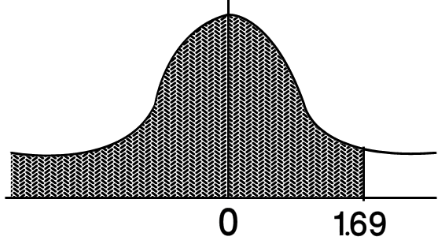
= 0.5 + الكشف عن الجزء الباقي من الجدول

P (Z < 1.69)

= 0.5 + p (0 < Z < 1.69)

= 0.5 + 0.4545

= 0.9545



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9. Find the area of the right z = - 1.69

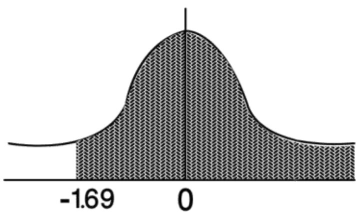
Solution

P (z > - 1.69)

= 0.5 + p (0 < z < 1.69)

= 0.5 + 0.4545

= 0.9545



Note:

إذا علمت المساحة وطلب قي

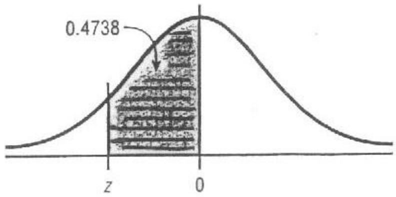
نكشف كشف عكسي في الجدول

For Exercises 40 through 45, find the z value that corresponds to the given area

4

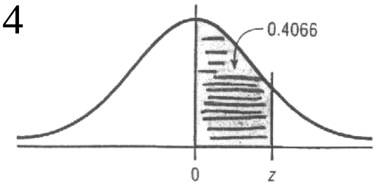
إذا كانت Z في اليمين تكون (+)

إذا كانت Z في اليسار تكون (−)



Z = - 1.94

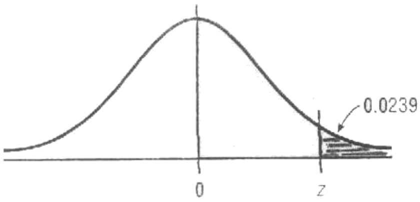
4



Z = 1.32

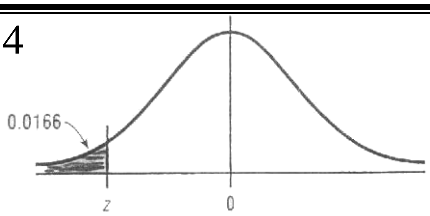
42

إذا كانت المساحة طرفية نطرح المساحة المعطاه من 0.5 ثم نكشف عن المساحة الناتجة في الجدول



Area = 0.5 – 0.0239
= 0.4761

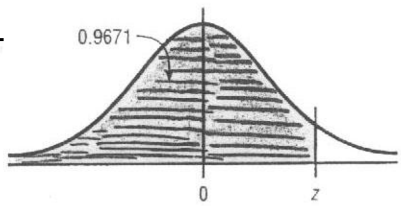
4



Area = 0.5 – 0.0166
= 0.4834

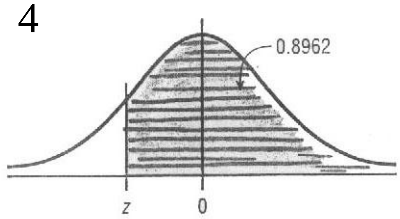
4

إذا زادت المساحة المعطاة عن 0.5 نطرح من المساحة المعطاة 0.5



Area = 0.9671 – 0.5
= 0.4761

4



Area = 0.8962 – 0.5
= 0.3962

Applications of the Normal Distribution

The standard normal distribution

Is normal distribution with $\mu = 0$ *and* $\sigma = 1$

التوزيع الطبيعي المعياري

To solve problems by using the standard normal distribution, transform the original variable to a standard normal distribution variable by using the formula

- $z = \dfrac{\text{value} - \text{mean}}{\text{standard deviation}}$ or $z = \dfrac{x - \mu}{\sigma}$
- $P(X > X_0)$

$$= P\left(Z > \frac{x_0 - \mu}{\sigma}\right)$$

ملحوظة

عند حساب الاحتمال حول المتغير x الذي يتبع توزيع طبيعي. يحول إلى توزيع طبيعي معياري Z

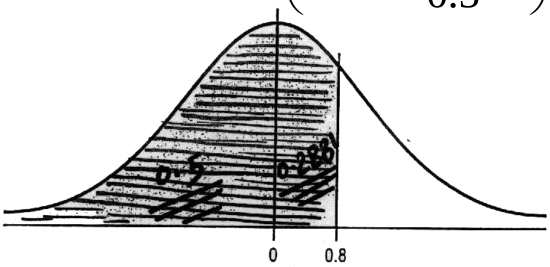
$$z = \frac{x - \mu}{\sigma}$$

Example:

The mean number of hours an American worker spends on the computer is 3.1 hours per workday. Assume the standard deviation is 0.5 hour. Find the percentage of workers who spend less than 3.5 hours on the computer. Assume the variable is normally distributed

Solution

$$P(x < 3.5) = p\left(z < \frac{3.5 - 3.1}{0.5}\right) = p(z < 0.8)$$



$= 0.5 + p(0 < z < 0.8)$

$= 0.5 + 0.2881$

$= 0.7881$

Therefore, 78.81 % of the workers spend less than 3.5 hours per workday on the computer

Example:

A survey found that people keep their microwave ovens (أفران الميكروويف) an average of 3.2 years. The standard deviation is 0.56 year. If a person decides to buy a new microwave oven.

Find the probability that he or she has owned (يملك) the old oven for the following amount of time. Assume the variable is normally distributed:

a. Less than 1.5 year's

b. Between 2 and 3 years

c. More than 3.2 years

d. What percent of microwave ovens would be replaced if a warranty (الضمان) of 18 months were given?

Note

$$Z = \frac{x - \mu}{\sigma}$$

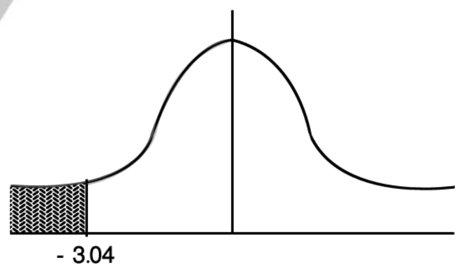
Solution

$\mu = 3.2$ $\sigma = 0.56$

(a) $p(x < 1.5) = p\left(z < \frac{1.5 - 3.2}{0.56}\right)$

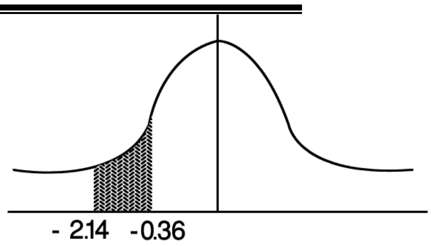
$= P(z < -3.04)$

$= 0.5 - 0.4988 = 0.0012$



(b) $P(2 < x < 3) = p\left(\frac{2-3.2}{0.56} < z < \frac{3-3.2}{0.56}\right)$

$= p(-2.14 < z < -0.36)$



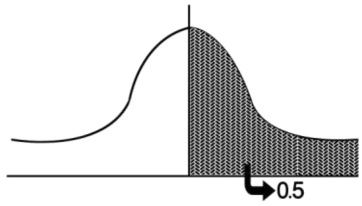
الحدان في جهه واحدة طرح بعد الكشف في الجدول الأكبر – الأصغر

$= 0.4838 - 0.1406 = 0.3432$

(c) $p\,(x > 3.2) = p\left(z > \dfrac{3.2 - 3.2}{0.56}\right)$

$= p\,(z > 0)$

$= 0.5$



(d) 18 months $= \dfrac{18}{12} = 1.5$ years

Were μ and σ by years.

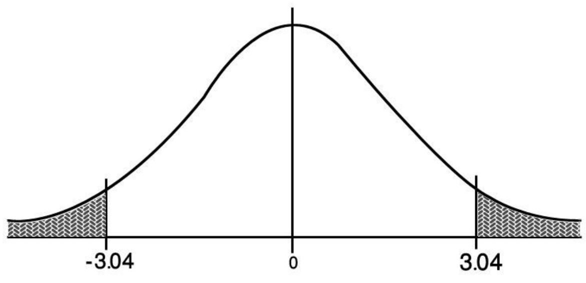
$p\,(x < 1.5) = p\left(z < \dfrac{1.5 - 3.2}{0.56}\right)$

$= p\,(z < -3.04)$

$= 0.5 - 0.4988$

$= 0.0012$

$0.0012 \times 100\% = 0.12\%$ of the ovens must be replaced.



Example:

The average time for a mail carrier to cover his route طريق is 380 minutes, and the standard deviation is 16 minutes. If one of these trips is selected at random, find the probability that the carrier will have the following route time. Assume the variable is normally distributed.

a. At least 350 minutes

b. At most 395 minutes

Solution

$\mu = 380$ $\sigma = 16$

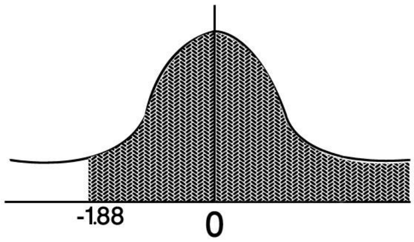
(a) At least على الأقل 350 minutes

$$P(x \geq 350) = p\left(Z \geq \frac{350-380}{16}\right)$$

$= p(z \geq -1.88)$

$= 0.5 + 0.4699$

$= 0.9699$



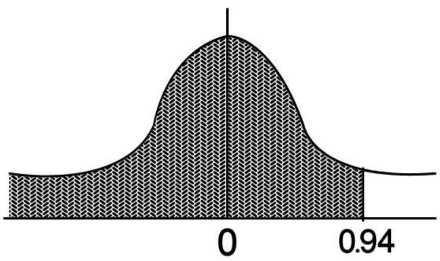
(b) At most على الأكثر 395 minutes

$$P(x \leq 395) = p\left(z \leq \frac{395-380}{16}\right)$$

$= p(z \leq 0.94)$

$= 0.5 + 0.3264$

$= 0.8264$



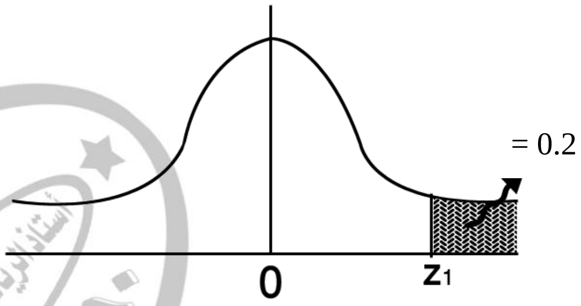
Example:

The average age of Amtrak passenger train cars is 19.4 years. If the distribution of ages is normal and 20% of the cars **are older** than 22.8 years, find the standard deviation.

Solution

The average $\mu = 19.4$

$P(0 < z < z_1) = 0.5 - 0.2 = 0.3$



There for

Z_1 corresponds to 0.3

$Z_1 = 0.84$

$$Z_1 = \frac{x - \mu}{\sigma}$$

$$0.84 = \frac{22.8 - 19.4}{\sigma}$$

$$0.84 \times \sigma = 3.4$$

$$\sigma = \frac{3.4}{0.84} = 4.048$$

Example:

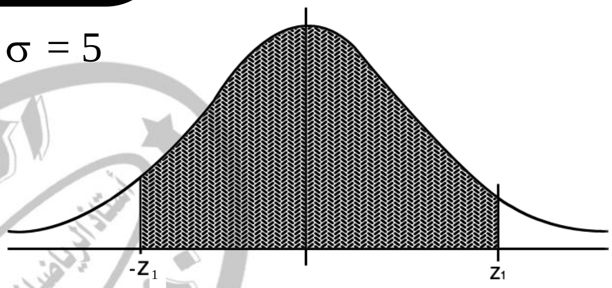
أحد أفراد الأسرة ينفق

If a one-person household spends an average of \$40 per week on groceries, find the maximum and minimum dollar amounts spent per week for the middle 50% of one-person households. Assume that the standard deviation is \$5 and the variable is normally distributed.

البقالة المستهلك

Solution

The average $\mu = 40$ and $\sigma = 5$



$$P(0 < z < z_1) = \frac{0.5}{2} = 0.25$$

Therefore $z_1 = \pm 0.67$

$$z_1 = \frac{x - \mu}{\sigma} \rightarrow \pm 0.67 = \frac{x - 40}{5}$$

- $0.67 = \dfrac{x - 40}{5}$

$x - 40 = (0.67)(5)$

$x = 3.35 + 40$

$x = 43.35$ max

- $-0.67 = \dfrac{x - 40}{5}$

$x - 40 = (-0.67)(5)$

$x = -3.35 + 40$

$x = 36.65$ min

Example:

ساعة يد

The mean lifetime of a wristwatch is 25 months, with a standard deviation of 5 months. If the distribution is normal.

ضمان

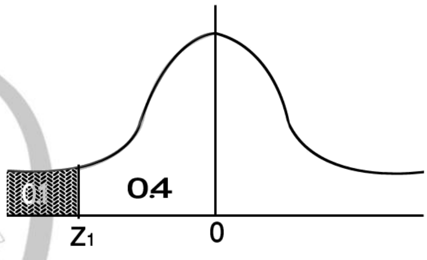
For how many months should a guarantee be made if the

الصانع (صاحب المصنع)

manufacturer **does not** want to exchange **more than 10%** of the watches? Assume the variable is normally distributed.

Solution

$\mu = 25$ $\sigma = 5$



$P(z < z_1) = 0.1$

$P(z_1 < z < 0) = 0.4$

Therefore $z_1 = -1.28$ & $z_1 = \dfrac{x - 25}{\sigma}$

$$\frac{x - 25}{5} = -1.28$$

$X - 25 = (-1.28)(5)$

$X = (-1.28)(5) + 25 = 18.6$ month

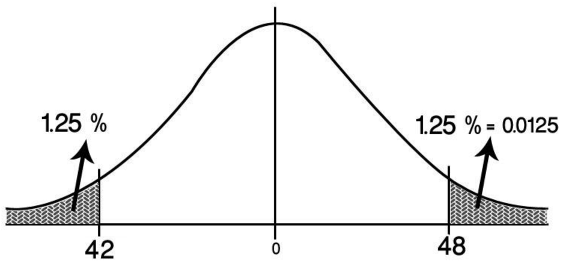
Example:

In a certain normal distribution, 1.25% of the area lies to the left of 42, and 1.25% of the area lies to the right of 48.

Find μ and σ.

Solution

$$\mu = \frac{42 + 48}{2} = \frac{90}{2} = 45$$



$0.5 - 0.0125 = 0.4875$

$P(0 < z < z_1) = 0.4875$

$Z_1 = 2.24$

There for $z = \frac{x - \mu}{\sigma}$

$$2.24 = \frac{48 - 45}{\sigma}$$

$$\sigma = \frac{48 - 45}{2.24} = 1.34$$

Ch. 6 Part. 2

نظرية النهاية المركزية
The Central Limit Theorem

n: sample size taken from population ●في حالة : مجتمع أخذت منه عنة حجمها n.

A sampling distribution of sample means is a distribution using the means computed from all possible random samples of a specific size taken from a population Properties of the Distribution of Sample Means

1. The mean of the sample means will be the same as the population mean.
2. The standard deviation of the sample means will be smaller than the standard deviation of the population, and it will be equal to the population standard deviation divided by the square root of the sample size.

1. $z = \frac{x - \mu}{\sigma}$ Used to gain information about an individual data value when the variable is normally distributed.

في حالة فرد

2. $z = \frac{\overline{x} - \mu}{\sigma / \sqrt{n}}$ Used to gain information when applying the central limit theorem about a sample mean when the variable is normally distributed

في حالة عينه حجمها n

Example:

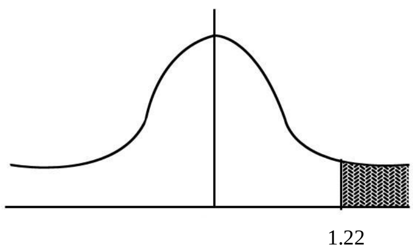
The mean weight of 15-year-old males is 142 pounds, and the standard deviation is 12.3 pounds. If a sample of thirty-six 15-year-old males is selected, find the probability that the mean of the sample will be greater than 144.5 pounds. Assume the variable is normally distributed. Based on your answer, would you consider the group overweight?

Solution

$\mu = 142$, $\sigma = 12.3$ and $n = 36$

$P\,(x' > 144.5\,)$

$$= P\left(z > \frac{144 - 142}{\frac{12.3}{\sqrt{36}}} \right)$$



$= P\,(z > 1.22)$

$= 0.5 - 0.3888$

$= 0.1112$

$= 11.12\ \%$

● No: since the average weight is within 2 standard deviation of the mean.

Example:

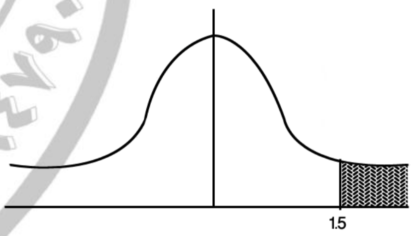
The average age of chemical engineers is 37 years a standard deviation of 4 years. If an engineering (شركة هندسية) firm employs 25 chemical engineers, find the probability that the average age of the group is greater than 38.2 years old. If this is the case, would it be safe to assume that the engineers in this group are generally much older than average?

Solution

$\mu = 37$, $\sigma = 4$ and $n = 25$

P (x' > 38.2)

$= p\left(z > \dfrac{38.2 - 37}{\frac{4}{\sqrt{25}}} \right)$



= p (z > 1.5)

= 0.5 – 0.4332

= 0.0668

= 6.68%

Example:

The average annual salary in Pennsylvania was \$24,393 in 1992. Assume that salaries were normally distributed for a certain group of wage earners, and the standard deviation of this group was \$4362.

a. Find the probability that a randomly selected individual مكسبه earned less than \$26,000. أشخاص

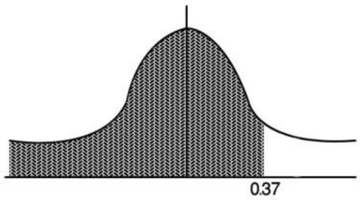
b. Find the probability that, for a randomly selected sample of 25 individuals, the mean salary was less than \$26,000.

c. Why is the probability for part b higher than the probability for part a.

Solution

$\mu = 24393$, $\sigma = 4362$

(a) $p(x < 26000) = p\left(z < \dfrac{26000 - 24393}{4362}\right)$

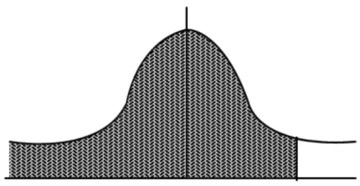


$= p(z < 0.37) = 0.5 + p(0 < z < 0.37)$

$= 0.5 + 0.1443 = 0.6443$

(b) $\mu = 24393$, $\sigma = 4362$ and $n = 25$

$P(x' < 26000) = p\left(z < \dfrac{26000 - 24393}{\frac{4362}{\sqrt{25}}}\right)$



$= p(z < 1.84) = 0.5 + p(0 < z < 1.84)$

$= 0.5 + 0.4671 = 0.9671$

(c) Sample means are less variable than individual data.

Example:

Assume that the mean systolic (الانقباض) blood pressure (ضغط الدم) of normal adults (بالغ سن الرشد) is 120 millimeters of mercury (mm Hg) and the standard deviation is 5.6. Assume the variable is normally distributed.

a. If an individual is selected, find the probability that the individual's pressure will be between 120 and 121.8 mm Hg.

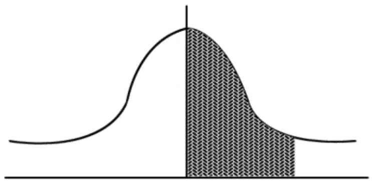
b. If a sample of 30 adults is randomly selected, find the probability that the sample mean will be between 120 and 121.8 mm Hg.

c. Why is the answer to part a so much smaller than the answer to part b?

Solution

(a) $p(120 < x < 121.8) = p\left(\frac{120-120}{5.6} < z < \frac{121.8-120}{5.6}\right)$

$= p(0 < z < 0.32) = 0.1255$

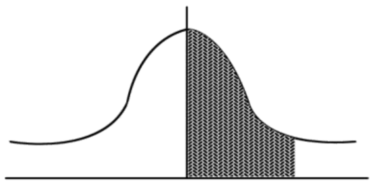


(b) $\mu = 120$, $\sigma = 5.6$ and $n = 30$

$$P(120 < \bar{x} < 121.8) = p\left(\frac{120-120}{\frac{5.6}{\sqrt{30}}} < z < \frac{121.8-120}{\frac{5.6}{\sqrt{30}}}\right)$$

$= p(0 < z < 1.76)$

$= 0.4608$



(c) Sample means are less variable than individual data .

Chapter Quiz

Determine whether each statement is true or false. If the statement is false, explain why.

1. The total area under a normal distribution is **infinite.** (x)
2. The standard normal distribution is a continuous distribution. (√)
3. All variables that are approximately normally distributed can be transformed to standard normal variables. (√)
4. The z value corresponding to a number **below** the mean is always negative. (√)
5. **The area** under the standard normal distribution to the left of z = 0 is negative. (x)
6. The central limit theorem applies to means of samples selected from **different** populations. (x)

Select the best answer.

7. The mean of the standard normal distribution is

a. 0 b. 1 c. 100 d. variable

8. Approximately what percentage of normally distributed data values will fall within 1 standard deviation above or below the mean?

a. 68% b. 95% c. 99.7% d. Variable

9. Which is not a property of the standard normal distribution?

a. It's symmetric about the mean. b. It's uniform.

c. It's bell-shaped. d. It's unimodal.

10. When a distribution is positively skewed, the relationship of the mean, median, and mode from left to right will be:

a. Mean, median, mode

b. Mode, median, mean

c. Median, mode, mean

d. Mean, mode, median

---

11. The standard deviation of all possible sample means equals:

a. The population standard deviation.

b. The population standard deviation divided by the population mean.

c. The population standard deviation divided by the square root of the sample size.

d. The square root of the population standard deviation.

Complete the following statements with the best answer.

12. When one is using the standard normal distribution, $P(z < 0) =$ **0.5**.

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13. The difference between a sample mean and a population mean is due to Sampling error.

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14. The mean of the sample means equals Population mean.

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15. The standard deviation of all possible sample means is called Standard error of the mean.

إنتهى Ch 6

كل التمنيات بالنجاح والتوفيق

السعدي