MAT1060 RULES 1

5.2Anti Derivatives

$\int f(x)\,dx = F(x) + c$

$\int f`(x)dx = f(x) + c$

$\frac{d}{dx}\int f(x)dx = f(x)$

Properties

$\int c\, f(x)\,dx = c\int f(x)dx$

$\int (f(x)\, g(x))\,dx = \int f(x)dx \pm \int g(x)dx$

$\int f(x)\,.\,g(X)\,dx \neq \int f(x)dx.\int g(x)dx$

$\int \frac{f(x)}{g(X)}\,dx \neq \frac{\int f(x)dx}{\int g(x)dx}$

Trigonometric Identities :

$secx = \frac{1}{cosx}$, $cscx = \frac{1}{sinx}$,

$\tan x = \frac{\sin x}{\cos x}$, $cotx = \frac{\cos x}{sinx} = \frac{1}{tanx}$

$Sin^2x + cos^2x = 1$, $Sin^2x = 1 - cos^2x$, $cos^2x = 1 - sin^2x$

$tan^2x + 1 = sec^2x$, $tan^2x = sec^2x - 1$

$1 + cot^2x = csc^2x$, $cot^2x = csc^2x - 1$

$sin\,2x = 2\,sin\,x\,cos\,x$

$cos\,2x = cos^2x - sin^2x$

$= 2\,cos^2x - 1$, $cos^2x = \frac{1}{2}(1 + cos\,2x)$

$= 1 - 2sin^2x$, $sin^2x = \frac{1}{2}(1 - cos\,2x)$

Basic Table :

$\int 1\,dx = x + c$

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$

$\int cosx\,dx = sinx + c$

$\int sinx\,dx = -cosx + c$

$\int sec^2x\,dx = tanx + c$

$\int csc^2x\,dx = -cotx + c$

$\int secx\,tanx\,dx = secx + c$

$\int cscx\,cotx\,dx = -cscx + c$

$\int \frac{1}{x}\,dx = ln|x| + c$

$\int e^x\,dx = e^x + c$

$\int b^x\,dx = \frac{b^x}{lnb} + c$

$\int \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x} + c$

$\int \frac{1}{\sqrt{1-x^2}}dx = sin^{-1}(x) + c$

$\int \frac{1}{1+x^2}dx = tan^{-1}(x) + c$

$\int \frac{1}{x\sqrt{x^2-1}}dx = sec^{-1}|x| + c$

Linear Function

$\int (ax+b)^n\,dx = \frac{1}{a}\frac{(ax+b)^{n+1}}{n+1} + c$

$\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + c$

$\int \frac{1}{(ax+b)}dx = \frac{1}{a}\,ln|(ax+b)| + c$

$\int e^{(ax+b)}\,dx = \frac{1}{a}e^{(ax+b)} + c$

$\int b^{(ax+b)}\,dx = \frac{1}{a}\frac{b^{(ax+b)}}{lnb} + c$

$\int \frac{1}{\sqrt{ax+b}}\,dx = \frac{2}{a}\sqrt{ax+b} + c$

5.3Integration by Substitution

$\int f(g(x))\,.\,g`(x)\,dx = F(g(x)) + c$

$\int [f(x)]^n\,.\,f`(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + c$

$\int \cos(f(x))\,.\,f`(x)\,dx = \sin(f(x)) + c$

$\int \frac{f`(X)}{f(x)}dx = \ln|f(x)| + c$

$\int e^{f(x)}\,.\,f`(x)\,dx = e^{f(x)} + c$

$\int b^{f(x)}\,.\,f`(x)\,dx = \frac{b^{f(x)}}{lnb} + c$

$\int \frac{f`(X)}{\sqrt{f(x)}}dx = 2\sqrt{f(x)} + c$

Inverse Trigo

$\int \frac{1}{\sqrt{a^2-x^2}}dx = sin^{-1}\left(\frac{x}{a}\right) + c$

$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}tan^{-1}\left(\frac{x}{a}\right) + c$

$\int \frac{1}{x\sqrt{x^2-a^2}}dx = \frac{1}{a}sec^{-1}\left|\frac{x}{a}\right| + c$

Additional Formulas

$\int tanx\,dx = -\ln|\cos x| + c$

$\int cotx\,dx = \ln|\sin x| + c$

$\int secx\,dx = ln|\sec x + tanx| + c$

$\int \csc x\,dx = ln|cscx - \cot x| + c$

5.5 Definite Integral

Name of Plane Figures	Areas (in sq. Units)	Perimeter (in Units)
Circle	π (Radius)^2	2π x Radius
Rectangle	Length x Width	2(Length + Width)
Square	Side x Side	4 x Side
Triangle	½ x Base x Height	Sum of all Sides
Rhombus	Length x Height	4 Side
Trapezoidal	$\frac{1}{2}(b_1+b_2)XHeight$	4 side

5.6 Fundamental Theorem

F.T1 $\int_a^b f(x)\,dx = F(b) - F(a)$

F.T2 $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$

The Mean Value Theorem

M.V.T $\int_a^b f(x)\,dx = (b-a)f(x^*)$

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Ch6

6.1 Area Between Two Curves

With respect to x

$$A = \int_a^b [f(x) - g(x)]dx$$

$$, f(x) \geq g(x)$$

With respect to y

$$A = \int_c^d [w(y) - v(y)]\, dy$$

$$, w(x) \geq v(x)$$

6.2 Volume of Revolution

Rotation About X-Axis

$$V = \pi \int_a^b ([f(x)]^2 - [g(x)]^2)dx$$

$$, f(x) \geq g(x)$$

Rotation About Y-Axis

$$V = \pi \int_c^d ([w(y)]^2 - [v(y)]^2)\, dy$$

$$, w(x) \geq v(x)$$

6.4 Arc Length

With Respect to X-Axis

$y = f(x), a \leq x \leq b$

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

With Respect to Y-Axis

$x = g(y), c \leq y \leq d$

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

6.5 Surface Area

Rotation About X-Axis

$y = f(x) \quad , a \leq x \leq b$

$$S = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Rotation About Y-Axis

$x = g(y) \quad , c \leq y \leq d$

$$S = \int_c^d 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

6.9 Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$\operatorname{sech} x = \frac{2}{e^x + e^{-x}}$$

$$\operatorname{csch} x = \frac{2}{e^x - e^{-x}}$$

Hyperbolic Derivatives

$$\frac{d}{dx}(\sinh x) = \cosh x$$

$$\frac{d}{dx}(\cosh x) = \sinh x$$

$$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$$

$$\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$$

$$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$$

$$\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$$

Inverse Hyperbolic Derivatives

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1 + x^2}}$$

$$\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2 - 1}}$$

$$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1 - x^2}$$

$$\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1 - x^2}$$

$$\frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\operatorname{csch}^{-1} x) = -\frac{1}{|x|\sqrt{1+x^2}}$$

Basic Identities

$\cosh^2 x - \sinh^2 x = 1$

$\cosh x + \sinh x = e^x$

$\cosh x - \sinh x = e^{-x}$

$\tanh^2 x = 1 - \operatorname{sech}^2 x$

$\coth^2 x = 1 + \operatorname{csch}^2 x$

$\sinh 2x = 2 \sinh x \cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

$= 2\cosh^2 x - 1$

$= 2\sinh^2 x + 1$

$\cosh^2 x = \frac{1}{2}(\cos 2x + 1)$

$\sinh^2 x = \frac{1}{2}(\cosh 2x - 1)$

Hyperbolic Integrals

$$\int \cosh x\, dx = \sinh x + c$$

$$\int \sinh x\, dx = \cosh x + c$$

$$\int \operatorname{sech}^2 x\, dx = \tanh x + c$$

$$\int \operatorname{csch}^2 x\, dx = -\coth x + c$$

$$\int \operatorname{sech} x \tanh x\, dx = -\operatorname{sech} x + c$$

$$\int \operatorname{csch} x \coth x\, dx = -\operatorname{cshc} x + c$$

Inverse Hyperbolic Integrals

$$\int \frac{du}{\sqrt{a^2 + u^2}} = \sinh^{-1}\left(\frac{u}{a}\right) + c, a > 0$$

$$\int \frac{du}{\sqrt{u^2 - a^2}} = \cosh^{-1}\left(\frac{u}{a}\right) + c, u > a > 0$$

$$\int \frac{du}{a^2 - u^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{u}{a}\right) + c, u^2 < a^2 \\ \frac{1}{a}\coth^{-1}\left(\frac{u}{a}\right) + c, u^2 > a^2 \end{cases}$$

$$\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\operatorname{sech}^{-1}\left(\frac{u}{a}\right) + c$$

$$, 0 < u < a$$

$$\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\operatorname{csch}^{-1}\left|\frac{u}{a}\right| + c,$$

$$u \neq 0 \text{ and } a > 0$$

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7.2 Integration by parts

$$\int u\,dv = uv - \int v\,du$$

How to choose u ? (LIATE)

Logarithmic, **I**nverse trigonometric, **A**lgebraic, **T**rigonometric, **E**xponential

7.3 Powers of Trigonometric

$$\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x\,dx$$

$$\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}\int \cos^{n-2}x\,dx$$

$$\int \tan^n x\,dx = \frac{1}{n-1}\tan^{n-1}x - \int \tan^{n-2}x\,dx$$

$$\int \sec^n x\,dx = \frac{1}{n-1}\sec^{n-2}x\tan x + \frac{n-2}{n-1}\int \sec^{n-2}x\,dx$$

Product of Power Sine and Cosine

$$\int \sin^m x\,\cos^n x\,dx$$

n odd — split $\cos x$ — use $\sin^2 x = 1 - \cos^2 x$
take $u = \sin x$

m odd — split $\sin x$ — use $\cos^2 x = 1 - \sin^2 x$
take $u = \cos x$

$\begin{cases} m \text{ even} \\ n \text{ even} \end{cases}$ — use $\begin{cases} \cos^2 x = \frac{1}{2}(1+\cos 2x) \\ \sin^2 x = \frac{1}{2}(1-\cos 2x) \end{cases}$

Product of Power tangent and Sec

$$\int \tan^m x\,\sec^n x\,dx$$

n even — split $\sec^2 x$ — use $\sec^2 x = \tan^2 x + 1$
take $u = \tan x$

m odd — split $\sec x\tan x$ — use $\tan^2 x = \sec^2 x - 1$
take $u = \sec x$

$\begin{cases} m \text{ even} \\ n \text{ odd} \end{cases}$ — $\sec x$ Alone — use $\tan^2 x = \sec^2 x - 1$

Identities of Sine and Cosine

$$\sin\alpha\,\cos\beta = \frac{1}{2}[\sin(\alpha-\beta) + \sin(\alpha+\beta)]$$

$$\sin\alpha\,\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

$$\cos\alpha\,\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$

7.5Rational Integration:

- Partial Fractions
- Completing the square
- Long Division (Or Factoring)

7.4Trigonometric Substitution

$\sqrt{a^2 - x^2}$ $\rightarrow x = a\sin\theta$
$dx = a\cos\theta\,d\theta$

$\sqrt{a^2 + x^2}$ $\rightarrow x = a\tan\theta$
$dx = a\sec^2\theta\,d\theta$

$\sqrt{x^2 - a^2}$ $\rightarrow x = a\sec\theta$
$dx = a\sec\theta\tan\theta\,d\theta$

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