King Abdul Aziz University
Faculty of Sciences
Mathematics Department

Second Exam 2016-17
Calculus I- Math 110
Allowed Time: 90 M

لا يُسمح باستخدام الالة الحاسبة

B

Name:	ID:

تعليمات هامة:

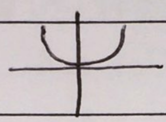
- ◻ يجب أن يكون نموذج الإجابة الذي أمامك هو B
- ◻ التأكد من أن عدد أسئلة الاختبار 33 سؤالاً.
- ◻ كتابة البيانات وتظليل الرقم الجامعي بطريقة صحيحة.
- ◻ التأكد من اجابتك قبل تظليلها.
- ◻ ركز على رقم السؤال الذي ستظلل اجابته و الحرف الذي يحمل الإجابة الصحيحة ، وتظليل اجابة واحدة فقط ولن يسمح بالتظليل بعد انتهاء الوقت المحدد.
- ◻ تظليل جميع الإجابات في نموذج الاجابة بشكل واضح وكامل.
- ◻ الرجاء اغلاق الجوال وعدم استخدامه نهائيا .

Q.1	If $a>0$ and $x,y\in\mathbb{R}$, then $a^{x-y}=$						
(A)	$a^x a^x$	(B)	$a^x a^y$	(C)	$\frac{a^x}{a^y}$	(D)	$(a^x)^y$

a^{x-y} $a^{\frac{x}{y}}$

Q.2	The domain of the function $y=e^{-x}+1$ is						
(A)	$\mathbb{R}$	(B)	$(0,\infty)$	(C)	$(-\infty,0)$	~~(D)~~	$(1,\infty)$

Q.3	The function $f(x)=x^2$ is one to one.		
(A)	True	(B)	False



Q.4	If f is a function, then the domain of f equals the range of f^{-1}.		
(A)	True	(B)	False

Q.5	If f is a one to one function and $f(4)=8$, then $f^{-1}(8)=$						
(A)	8	(B)	32	(C)	4	(D)	$\frac{1}{2}$

Q.6	The inverse function of $f(x)=x^3-2$ is						
(A)	$\sqrt[3]{x+2}$	(B)	$2-x^3$	(C)	$\sqrt{x+2}$	(D)	$\sqrt[3]{x-2}$

$Y = X^3 - 2$ $\quad \sqrt[3]{Y+2} = X$ $\quad f^{-1}(x) = \sqrt[3]{x+2}$

$y = x^3 - 2$

$y + 2 = x^3$

$\sqrt[3]{y+2} = x$ $\quad \sqrt[3]{x+2} = f^{-1}(x)$

Q.7	The graph of f^{-1} is obtained by reflecting the graph of the function f about the line $y=0$.		
(A)	True	(B)	False

Q.8	For every $x>0$, $5^{\log_5(x)} =$						
(A)	5^x	**(B)**	x^5	**(C)**	$25x$	**(D)**	x

Q.9	$\log_2 2 + \log_2 8 =$ $1 + \log_2 2^3$						
(A)	4	**(B)**	64	**(C)**	32	**(D)**	16

$1 + \log_2 2^3$ $1 + 3 = 4$

Q.10	The solution of the equation $e^{2+3x} = 10$ is ~~the~~ $2+3x = \frac{\ln 10 - 2}{3}$						
(A)	$x = \frac{5-\ln 10}{3}$	**(B)**	$x = \frac{\ln 10 - 2}{3}$	**(C)**	$x = \ln 10$	**(D)**	$x = 3$

$2+3x = \ln 10$

$\frac{\ln 10 - 2}{3}$

0	30	45	60	90
0	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	

Q.11	$\cos^{-1}\left(\frac{1}{2}\right) =$						
(A)	$\frac{\pi}{2}$	**(B)**	$\frac{\pi}{4}$	**(C)**	$\frac{\pi}{3}$	**(D)**	$\frac{\pi}{8}$

	0	30	45	60	90
sin $\sqrt{\ }$	0	1	2	3	4
cos	4	3	2	1	0

Q.12	$\lim_{t\to 0} \frac{\sqrt{t^2+4}-2}{t^2} =$ $\frac{2\sqrt{t^2+4} - 2}{0}$ $\frac{0}{2\sqrt{4}-2}$						
(A)	$\frac{1}{3}$	**(B)**	$\frac{1}{6}$	**(C)**	$\frac{1}{4}$	**(D)**	~~0~~

$\frac{\frac{2t}{2\sqrt{t^2+4}}}{2t} = \lim_{t\to 0} \frac{1}{2\sqrt{t^2+4}} = \frac{1}{2(2)} = \frac{1}{4}$

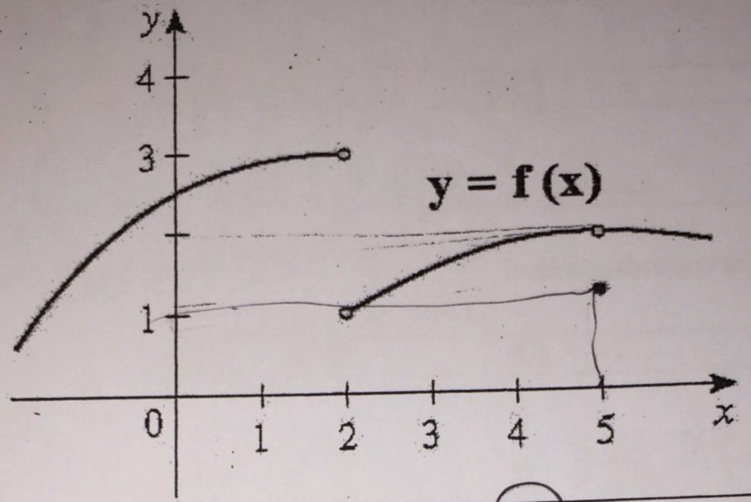
Q.13	$\lim_{x\to 2} \frac{x-2}{x^2-4} =$ $\frac{1}{2x}$						
(A)	$\frac{1}{4}$	**(B)**	∞	**(C)**	2	**(D)**	0

$\frac{1}{2x} = \frac{1}{4}$

Q.14	$\lim_{x\to 0} \frac{1}{x^2} = \infty$ $\frac{1}{0} = \infty$		
(A)	**True**	**(B)**	**False**

Q.15	$\lim\limits_{x\to a} f(x)=L$ **if and only if** $\lim\limits_{x\to a^-} f(x)=L$		
(A)	**True**	**(B)**	**False**

Q.16	**From the graph below** $f(5)=$



(A)	*does not exist*	**(B)**	3	**(C)**	1	**(D)**	2

8

Q.17	**The vertical asymptote of the graph of the function** $f(x)=\dfrac{2x}{x-4}$ **is**						
(A)	$y=3$	**(B)**	$x=3$	**(C)**	$y=4$	**(D)**	$x=4$

$x-4 \neq 0$
$x \neq 4$

$x=4$

Q.18	**If** $\lim\limits_{x\to a} f(x)=5$, **then** $\lim\limits_{x\to a}\left[f(x)\,f(x)\right]=$						
(A)	$\lim\limits_{x\to a} f(5x)$	**(B)**	25	**(C)**	*does not exist*	**(D)**	5

5×5

Q.19	$\lim\limits_{x\to 2}\dfrac{x^3+2x^2+1}{5+3x}=$ $= \dfrac{8+8+1}{5+6} = \dfrac{17}{11}$						
(A)	$\dfrac{17}{11}$	**(B)**	$\dfrac{11}{17}$	**(C)**	17	**(D)**	0

$$\frac{2^3+2\cdot 2^2+1}{5+3\cdot 2} = \frac{8+8+1}{11} = \frac{17}{11}$$

Q.20	If f is a rational function, then $\lim_{x \to a} f(x) = f(a)$ at any real number a.		
(A)	~~True~~	(B)	False

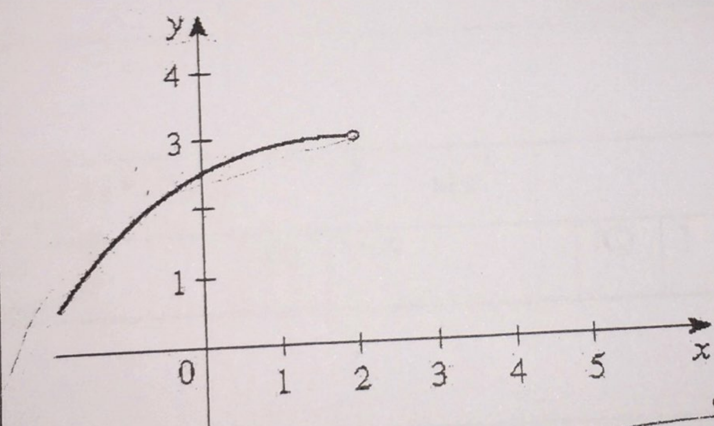
Q.21	$\lim_{x \to 0} x^2 \cos \frac{1}{x} =$						
(A)	0	(B)	∞	(C)	$-\infty$	(D)	1

$\lim \frac{2x-1}{1} = \frac{3}{}$

Q.22	The function $f(x) = \begin{cases} \frac{x^2 - x - 2}{x-2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$ is discontinuous at						
(A)	1	(B)	$\boxed{2}$	(C)	-1	(D)	0

$f(2) = 1$

$\lim_{x \to 2} = \frac{x^2 - x - 2}{x - 2} = \frac{0}{0}$ $\quad \lim_{x \to 2} \frac{2x-1}{1} = 3 \neq f(2)$

Q.23	From the graph below of the function $f(x)$, $\lim_{x \to 2^-} f(x) = 3$		
			
(A)	True	(B)	~~False~~

Q.24	The logarithmic functions are continuous at every number in their domain.		
(A)	True	(B)	~~False~~

4

Q.25	The function $f(x)=\cos(x^2)$ is continuous on						
(A)	$\mathbb{R}$	(B)	$(0,\infty)$	(C)	$(-\infty,0]$	(D)	$(2,\infty)$

Q.26	A function f is continuous on an interval $[a,b]$ if it is continuous at every number in the interval.		
(A)	True	(B)	False

Q.27	If $f(x)$ and $g(x)$ are continuous at a, then $f(x)-g(x)$ is discontinuous at a.		
(A)	True	(B)	False

Q.28	$\lim\limits_{x\to\infty}\dfrac{1}{x^{-3}}=$						
(A)	$\frac{1}{2}$	(B)	0	(C)	∞	(D)	$-\infty$

$\frac{1}{x^3}$ $\frac{1}{\frac{1}{x^3}}$ x^3

Q.29	$\lim\limits_{x\to\infty}\left(4x^2-2x\right)=$						
(A)	∞	(B)	$-\infty$	(C)	1	(D)	0

$4x^2$

Q.30	The horizontal asymptote of the graph of the function $f(x)=\dfrac{1}{x}$ is						
(A)	$y=0$	(B)	$y=2$	(C)	$x=0$	(D)	$x=2$

∞

$\lim\limits_{x\to\infty}\frac{1}{x}$

Q.31	$\lim\limits_{x\to a} x^5 = 5^a$		
(A)	True	(B)	False

Q.32	$\log_3 x=$						
(A)	$\dfrac{\ln x}{\ln 3}$	(B)	$-\dfrac{x}{3}$	(C)	$\log_3 \dfrac{x}{3}$	(D)	$\dfrac{\ln 3}{\ln x}$

Q.33	If $\ln x=0$, then $x=$						
(A)	1	(B)	$\ln 4$	(C)	$\ln 9$	(D)	e^5

$e^{\cancel{\ln} x} = e^0$

1. $a > 0$ $\quad a^{x-y} = \dfrac{a^x}{a^y}$ C

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2. $y = e^{-x} + 1$

$D = (-\infty, \infty) = R$ A

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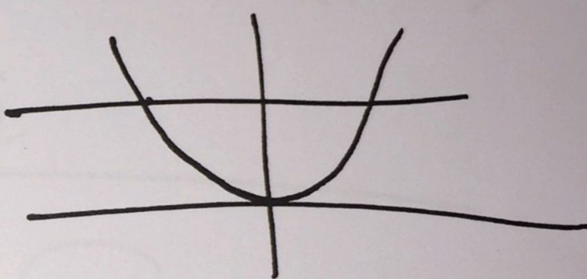
3. $f(x) = x^2$



not one-to-one

B

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4. $R_{f^{-1}} = D_f$ A

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5. $f(4) = 8 \Longrightarrow f^{-1}(8) = 4$ C

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6. $f(x) = x^3 - 2 \Longrightarrow y = x^3 - 2$

$y + 2 = x^3 \xrightarrow{\text{"}\sqrt[3]{\ }\text{"}} x = \sqrt[3]{y+2}$

$f^{-1}(x) = \sqrt[3]{x+2}$ A

7 The graph of f^{-1} is obtained by reflecting the graph of the function f about $\boxed{y = x}$

B

---

8 $5^{\log_5 x} = x \quad (x > 0)$

D

---

9 $\log_2 2 + \log_2 8 = 1 + \log_2 2^3$

$= 1 + 3 \log_2 2 = 1 + 3(1) = 4$ A

---

10 $e^{2+3x} = 10$ $\underline{\underline{\ln}}$

$2 + 3x = \ln 10$

$3x = \ln 10 - 2$

$\boxed{x = \frac{\ln 10 - 2}{3}}$

B

(3)

(11) $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$ (c)

(12) $\lim_{t \to 0} \frac{\sqrt{t^2+4} - 2}{t^2} = \frac{\sqrt{0+4} - 2}{0} = 0/0$

$\overset{H}{=} \lim_{t \to 0} \frac{\cancel{\frac{2t}{2\sqrt{t^2+4}}}}{\cancel{2t}} = \lim_{t \to 0} \frac{\cancel{2t}}{\cancel{2t}(2\sqrt{t^2+4})}$

$= \lim_{t \to 0} \frac{1}{2\sqrt{t^2+4}} = \frac{1}{2\sqrt{0+4}} = \frac{1}{4}$ (c)

(13) $\lim_{x \to 2} \frac{x-2}{x^2-4} = \frac{2-2}{4-4} = \frac{0}{0}$

$\overset{H}{=} \lim_{x \to 2} \frac{1}{2x} = \frac{1}{2(2)} = \frac{1}{4}$ (A)

(14) $\lim_{x \to 0} \frac{1}{x^2} = \frac{1}{(0)^2} = \infty$ (A)

(15) $\lim_{x \to a} f(x) = L$ if and only if

$$\lim_{x \to a^+} f(x) = \boxed{\lim_{x \to a^-} f(x)} = L$$

(B)

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(16) $f(5) = 1$ (C)

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(17) $$f(x) = \frac{2x}{x-4}$$

$x - 4 \neq 0 \Rightarrow x \neq 4$

$$f(4) = \frac{2(4)}{4-4} = \frac{8}{0}$$

$\therefore$ vertical asymptote is $\boxed{x = 4}$ (D)

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(18) $\lim_{x \to a} f(x) = 5$

$$\lim_{x \to a} [f(x) \cdot f(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} f(x)$$

$$= (5)(5) = 25$$ (B)

(5)

(19) $\lim_{x \to 2} \dfrac{x^3+2x^2+1}{5+3x}$

$$\Rightarrow \frac{(2)^3+2(2)^2+1}{5+3(2)} = \frac{8+8+1}{5+6}$$

$$= \frac{17}{11} \qquad (A)$$

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(20) If f is a rational function

then $\lim_{x \to a} f(x) = f(a)$ مع مجالها

(R - {أصفار المقام})

(B)

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(21) $\lim_{x \to 0} x^2 \cos \frac{1}{x} = 0$

$$-1 \leq \cos \frac{1}{x} \leq 1$$

$$-x^2 \leq x^2 \cos \frac{1}{x} \leq x^2$$

$$\lim_{x \to 0} -x^2 = \lim_{x \to 0} x^2 = 0 \Rightarrow \lim_{x \to 0} x^2 \cos \frac{1}{x} = \quad (A)$$

من نظرية الساندوتش

6

22 $f(x) = \begin{cases} \dfrac{x^2 - x - 2}{x - 2} & x \neq 2 \\ 1 & x = 2 \end{cases}$

$f(2) = 1$

$\lim\limits_{x \to 2} f(x) = \lim\limits_{x \to 2} \dfrac{x^2 - x - 2}{x - 2} = \dfrac{4 - 2 - 2}{2 - 2}$

$= \dfrac{0}{0}$

$\overset{H}{=} \lim\limits_{x \to 2} \dfrac{2x - 1}{1} = \dfrac{4 - 1}{1} = 3 \neq f(2)$

$\therefore$ $f(x)$ is discontinuous at $x = 2$ B

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23 $\lim\limits_{x \to 2^-} f(x) = 3$ A

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24 أي دالة متصلة ع مجالها

A

25 $f(x) = \cos(x^2)$ is continuous

on $\mathbb{R} = (-\infty, \infty)$ A

---

26 A

---

27 $f(x)$ and $g(x)$ are continuous

at a, then $f(x) - g(x)$ is

continuous at a B

---

28 $\lim_{x \to \infty} 4x^2 - 2x = \lim_{x \to \infty} 4x^2 = 4(\infty)^2$

$= \infty$

A

---

29 $f(x) = \frac{1}{x}$

$\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \frac{1}{x} = 0$

The horizontal asymptote is $\boxed{y = 0}$

A

8

31 $\lim_{x \to a} x^5 = a^5$ B

---

32 $\log_3 x = \frac{\ln x}{\ln 3}$ A

---

33 $\ln x = 0$ $\underline{\underline{e}}$

$\cancel{e}^{\cancel{\ln} x} = e^0$

$x = e^0 = 1$ A

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A

Name:	ID:

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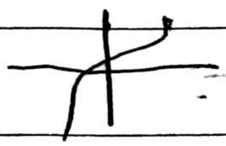
Q.1	If $a>0$ and $x, y \in \mathbb{R}$, then $a^{x+y} =$						
(A)	$a^x a^x$	(B)	$a^x a^y$	(C)	$\frac{a^x}{a^y}$	(D)	$(a^x)^y$

Q.2	The domain of the function $y = \frac{1}{2}e^{-x} - 1$ is						
(A)	$\mathbb{R}$	(B)	$(0,\infty)$	(C)	$(-\infty,0)$	(D)	$(1,\infty)$

Q.3	The function $f(x) = x^3$ is one to one.		
(A)	True	(B)	False

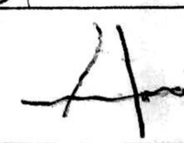


Q.4	If f is a function, then the domain of f^{-1} equals the range of f.		
(A)	True	(B)	False

Q.5	If f is a one to one function and $f(3)=7$, then $f^{-1}(7) =$						
(A)	3	(B)	21	(C)	4	(D)	$\frac{3}{7}$

Q.6	The inverse function of $f(x) = x^3 + 2$ is						
(A)	$x^3 - 2$	(B)	$2 - x^3$	(C)	$\sqrt{x-2}$	(D)	$\sqrt[3]{x-2}$

$y = x^3 + 2$
$\sqrt[3]{y-2} = x$



Q.7	The graph of $f^{-1}(x)$ is obtained by reflecting the graph of the function f a[bout the] line $y=x$.		
(A)	True	(B)	False

Q.8	For every $x \in \mathbb{R}$, $\log_7(7^x) =$						
(A)	7^x	(B)	x^7	(C)	$49x$	(D)	x

$x \log_7 7 = x$

Q.9	$\log_2 80 - \log_2 5 =$						
(A)	4	(B)	75	(C)	400	(D)	16

$\log_2 \frac{80}{5} = \log_2 16 = \log_2 2^4 = 4 (1)$

Q.10	The solution of the equation $e^{5-3x} = 10$ is						
(A)	$x = \frac{1}{3}$	(B)	$x = \frac{5 - \ln 10}{3}$	(C)	$x = \ln 10$	(D)	$x = 3$

$5 - 3x = \ln 10$

$\frac{-\ln 10 + 5}{3} = \frac{5 - \ln 10}{3}$

Q.11	$\sin^{-1}\left(\frac{1}{2}\right) =$						
(A)	$\frac{\pi}{2}$	(B)	$\frac{\pi}{4}$	(C)	$\frac{\pi}{6}$	(D)	$\frac{\pi}{8}$

	0	30	45	60	90
sin	0	1	2	3	4
cos	4	3	2	1	0

2

Q.12	$\lim_{t \to 0} \frac{\sqrt{t^2 + 9} - 3}{t^2} =$						
(A)	$\frac{1}{3}$	(B)	$\frac{1}{6}$	(C)	$\frac{1}{4}$	(D)	0

$\frac{\frac{\cancel{2t}}{2\sqrt{t^2+9}}}{\cancel{2t}} = \frac{1}{2\sqrt{t^2+9}} = \frac{1}{2\sqrt{9}} = \frac{1}{6}$

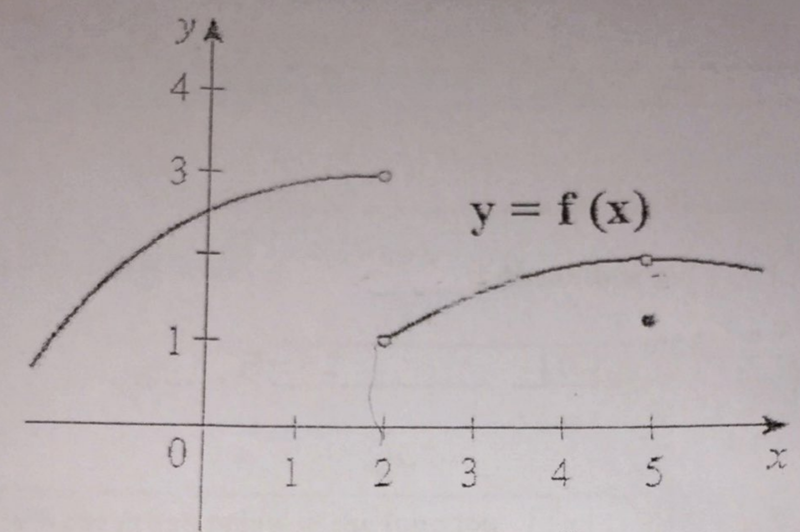
Q.13	$\lim_{x \to 1} \frac{x-1}{x^2 - 1} =$						
(A)	$\frac{1}{2}$	(B)	1	(C)	2	(D)	0

$\frac{1}{2x} = \frac{1}{2}$

Q.14	$\lim_{x \to 0} \frac{1}{x^2} = -\infty$		
(A)	True	(B)	False

Q.15	$\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^+} f(x) = L$		
(A)	True	(B)	False

Q.16	From the graph below $\lim_{x \to 2^+} f(x) =$						



(A)	*does not exist*	(B)	3	(C)	1	(D)	2

Q.17	The vertical asymptote of the graph of the function $f(x) = \dfrac{2x}{x-3}$ is						
(A)	$y = 3$	(B)	$x = 3$	(C)	$y = 2$	(D)	$x = 2$

$x = 3$

Q.18	If $\lim_{x \to a} f(x) = 5$, then $\lim_{x \to a} [5f(x)] =$						
(A)	$\frac{1}{5} \lim_{x \to a} f(x)$	(B)	$-5 \lim_{x \to a} f(x)$	(C)	*does not exist*	(D)	25

$5 \cdot 5$

Q.19	$\lim_{x \to -2} \dfrac{x^3 + 2x^2 - 1}{5 - 3x} =$						
(A)	$-\frac{1}{11}$	(B)	$\frac{5}{3}$	(C)	15	(D)	39

$-8 + 8 - 1$

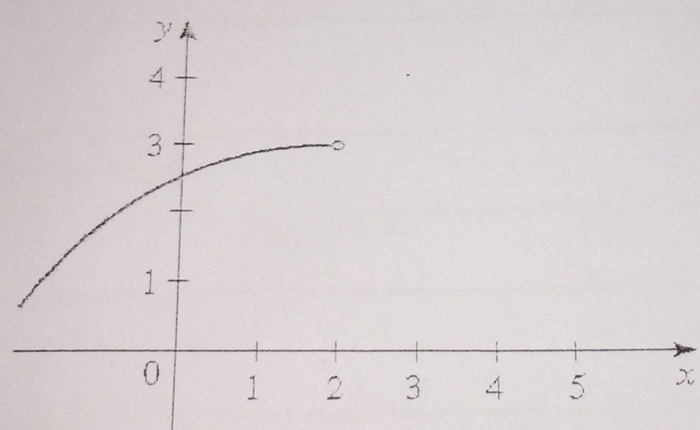
$\dfrac{-1}{5+6} = \dfrac{-1}{11}$

Q.20	If f is a polynomial function, then $\lim\limits_{x \to a} f(x) = f(a)$.		
(A)	True	(B)	False

Q.21	$\lim\limits_{x \to 0} x^2 \sin \frac{1}{x} =$						
(A)	0	(B)	∞	(C)	$-\infty$	(D)	1

Q.22	The function $f(x) = \begin{cases} \frac{1}{x^2} & if \quad x \neq 0 \\ 1 & if \quad x = 0 \end{cases}$ is discontinuous at						
(A)	1	(B)	2	(C)	-1	(D)	0

$\lim \frac{1}{x^2}$ $\frac{1}{0}$ ∞

Q.23	From the graph below of the function $f(x)$, $\lim\limits_{x \to 2^-} f(x) = 3$		
			
(A)	True	(B)	False

Q.24	The trigonometric functions are continuous at every number in their domain.		
(A)	True	(B)	False

Q.25	The function $f(x)=\sin(x^2)$ is continuous on						
(A)	$(-\infty,0)$	(B)	$(0,\infty)$	(C)	$(-\infty,0]$	(D)	$\mathbb{R}$

Q.26	A function f is continuous on an interval $[a,b]$ if it is continuous at a and b.		
(A)	True	(B)	False

Q.27	If $f(x)$ and $g(x)$ are continuous at a, then $f(x)\,g(x)$ is continuous at a.		
(A)	True	(B)	False

Q.28	$\lim\limits_{x\to\infty}\dfrac{1}{x^{\frac{1}{2}}}=$						
(A)	$\frac{1}{2}$	(B)	0	(C)	∞	(D)	$-\infty$

Q.29	$\lim\limits_{x\to\infty}(x^2-x)=$						
(A)	∞	(B)	$-\infty$	(C)	1	(D)	0

Q.30	The horizontal asymptote of the graph of the function $f(x)=\dfrac{1}{3x-1}$ is						
(A)	$y=0$	(B)	$y=-2$	(C)	$x=-2$	(D)	$x=0$

$\lim\limits_{x\to\pm\infty}\dfrac{1}{3x-1}$

$y=0$

Q.31	$\lim\limits_{x\to a}\sqrt[3]{x}=\sqrt[3]{a}$		
(A)	True	(B)	False

Q.32	For any $a > 0\ (a \neq 1)$, $\log_a x =$						
(A)	$\dfrac{\ln x}{\ln a}$	(B)	$\dfrac{x}{a}$	(C)	$\log_a \dfrac{x}{a}$	(D)	$\dfrac{\ln a}{\ln x}$

Q.33	If $\ln x = 5$, then $x =$						
(A)	5	(B)	$\ln 5$	(C)	$\ln \dfrac{1}{5}$	(D)	e^5

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