بسم الله الرحمن الرحيم

تدريبات (٥) للامتحان النهائي

Question 2: (4 points)

Perform and simplify the following:

1. $\frac{5x+3}{3}-\frac{2-7x}{2}+x+1$

Solution:

$$\frac{5x+3}{3}-\frac{2-7x}{2}+x+1=\frac{2(5x+3)}{6}-\frac{3(2-7x)}{6}+\frac{6(x+1)}{6}$$

$$=\frac{10x+6-6+21x+6x+6}{6}$$

$$=\frac{37x+6}{6}$$

2. $\dfrac{2x-6}{(x-2)^2} \div \dfrac{x-3}{x^2-4}$

Solution:

$$\frac{2x-6}{(x-2)^2} \div \frac{x-3}{x^2-4} = \frac{2x-6}{(x-2)^2} \times \frac{x^2-4}{x-3}$$

$$= \frac{2(x-3)}{(x-2)^2} \times \frac{(x-2)(x+2)}{x-3}$$

$$= \frac{2(x-3)(x-2)(x+2)}{(x-2)(x-2)(x-3)}$$

$$= \frac{2(x+2)}{(x-2)}$$

Question 3: (4 points)

Solve the following equations:

1. $\frac{2}{x} = \frac{3}{x+4}$

Solution:

$$\frac{2}{x} = \frac{3}{x+4} \Leftrightarrow 2(x+4) = 3x$$

$$\Leftrightarrow 2x + 8 = 3x$$

$$\Leftrightarrow 2x - 3x = -8$$

$$\Leftrightarrow -x = -8$$

$$\Leftrightarrow x = 8$$

The solution set is $\{8\}$.

2. $3x^2 - 5x + 2 = 0$

Solution:

$$\Delta = b^2 - 4ac$$
$$= (-5)^2 - 4 \times 3 \times 2$$
$$= 25 - 24$$
$$= 1$$

$\Delta > 0$, then the equation has 2 real solutions:

$$x_1 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{5+1}{6} = \frac{6}{6} = 1$$
$$x_2 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{5-1}{6} = \frac{4}{6} = \frac{2}{3}$$

The solution set is $\left\{\frac{2}{3}, 1\right\}$

Question 4: (4 points)
Solve the following inequalities:

1. $\dfrac{2x-7}{4} \leq \dfrac{3x+1}{3}$

Solution:

$$\frac{2x-7}{4} \leq \frac{3x+1}{3} \Leftrightarrow 3(2x-7) \leq 4(3x+1)$$
$$\Leftrightarrow 6x-21 \leq 12x+4$$
$$\Leftrightarrow 6x-12x \leq 21+4$$
$$\Leftrightarrow -6x \leq 25$$
$$\Leftrightarrow x \geq \frac{25}{-6}$$
$$\Leftrightarrow x \geq -\frac{25}{6}$$

The solution set is

$$\left[-\frac{25}{6}, +\infty\right).$$

2. $-2 \le 3x - 5 \le 7$

Solution:

$$-2 \le 3x - 5 \le 7 \Leftrightarrow -2 + 5 \le 3x \le 7 + 5$$

$$\Leftrightarrow 3 \le 3x \le 12$$

$$\Leftrightarrow \frac{3}{3} \le x \le \frac{12}{3}$$

$$\Leftrightarrow 1 \le x \le 4$$

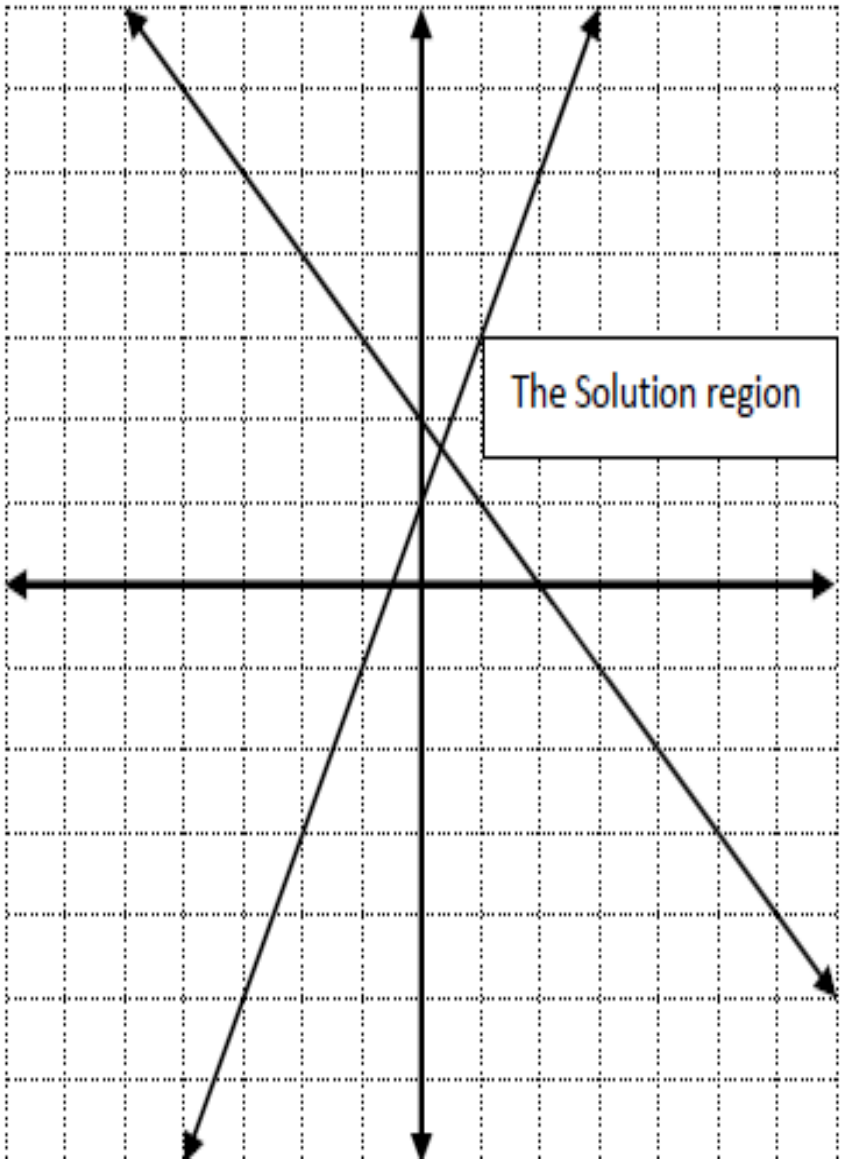
The solution set is

$$[1, 4].$$

Question 5:

1. Graph the lines $y = 2x + 1$ and $y = -x + 2$ and then shade the solutions of the system:

$$\begin{cases} y \le 2x + 1 \\ y \ge -x + 2 \end{cases}$$



Solution:

(4 points)

2. Solve the following system of equations:

$$\begin{cases} 3x - y = 1 \\ x + 2y = 5 \end{cases}$$

Solution:

$$\begin{cases} 3x - y = 1 & R1 \\ x + 2y = 5 & R2 \end{cases}$$

$$\begin{cases} 6x - 2y = 2 & R1 \leftarrow 2R1 \\ x + 2y = 5 & R2 \end{cases}$$

$$\begin{cases} 7x = 7 & R1 \leftarrow R1 + R2 \\ x + 2y = 5 & R2 \end{cases}$$

$$\begin{cases} x = 1 \\ 1 + 2y = 5 \end{cases}$$

$$\begin{cases} x = 1 \\ y = 2 \end{cases}$$

The solution set is

$$\{(1, 2)\}.$$