

مدة الاختبار: ساعتان السنة الدراسية :الفصل الدراسي الأول لعام ٤١٤٤/١٤٤ هـ

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Second Midterm Exam (Monday 22 /4/1442 H)

Answer the following Questions

Question 1:

Prove in full detail, with the standard operations in \mathbb{R}^2 , that the set $\{(x, 2x): x \text{ is real number}\}$ is a vector space (3 degrees)

Question 2:

In (a), and (b) W is not a subspace of the vector space. Verify this by giving a specific example that violates the test for a vector subspace:

- (a) W is the set of all vectors in R^3 Whose third component is -1.
- (b) W is the set of all matrices in $M_{n,n}$ such that $A^2 = A$. (2 degrees)

Question 3:

(a) Determine whether the set $S = \{ (1,2,-2), (2,-1,1) \}$ in \mathbb{R}^3 is a linear combination of :

(*i*)
$$u = (1, -5, -5)$$

(*ii*) $v = (-2, -6, 6)$
(*iii*) $w = (-1, -22, 22)$
(*v*) $z(-4, -3, 3)$

(b) Determine whether the following matrices from $M_{2,2}$ form a linearly independent set:

$$A = \begin{bmatrix} 1 & -1 \\ 4 & 5 \end{bmatrix}, B = \begin{bmatrix} 4 & 3 \\ -2 & 3 \end{bmatrix}, C = \begin{bmatrix} 1 & -8 \\ 22 & 23 \end{bmatrix}$$
(4 degrees)

Question 4:

Explain why $S = \{2, x, x + 3, 3x^2\}$ is not a basis for P_2 . (2 degrees)

Question 5:

Determine whether the function is a liner transformation or not:

(a)
$$T : R^2 \to R^2$$
, $T(x, y) = (x, 1)$.
(b) $T : M_{2,2} \to R.T(A) = a + b + c + d$, where $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ (3 degrees)

Q(6) Find the kernel of the fowling linear transformations:

(a)
$$T: P_3 \to P_2$$
, $T(a_0 + a_1x + a_2x^2 + a_3x^3) = a_1 + 2a_2x + 3a_3x^2$
(b) $T: R^2 \to R^2$, $T(y) = (x + 2y, y - x)$. (3 degrees).

 $\underline{\mathbf{Q7}}$ Find the eigenvalues and the corresponding eigenvectors of :

$$A = \begin{bmatrix} 1 & -4 \\ -2 & 8 \end{bmatrix}.$$
 (3 degrees)

With all the best