حل الجلسة الامتحانية تحليل :

عيّن عدد حقيقي A :

① $f(x) = \dfrac{2x+1}{x-1}$

* $\lim\limits_{x\to+\infty} f(x) = 2$

* $f(x) \in]1.95, 2.05[$

$1.95 < f(x) < 2.05$

$1.95-2 < f(x)-2 < 2.05-2$

$-0.05 < f(x)-2 < 0.05$

$|f(x)-2| < \dfrac{5}{100}$

$\left|\dfrac{2x+1}{x-1} - \dfrac{2}{1}\right| < \dfrac{1}{20}$

$\left|\dfrac{2x+1-2x+2}{x-1}\right| < \dfrac{1}{20}$

$\left|\dfrac{3}{x-1}\right| < \dfrac{1}{20}$

$\dfrac{3}{x-1} < \dfrac{1}{20}$

$\dfrac{x-1}{3} > 20$

$x-1 > 60$

$x > 61$

$\boxed{A = 61}$

* $\lim\limits_{x\to+\infty} f(f(x)) = f\left(\lim\limits_{x\to+\infty} f(x)\right)$

$= f(2) = \dfrac{4+1}{2-1} = 5$

② $f(x) = \dfrac{2+\ln x}{1+\ln x}$

* $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} \left(\dfrac{2+\ln x}{1+\ln x}\right)$

$= \lim\limits_{x\to+\infty} \dfrac{\ln x\left(\frac{2}{\ln x}+1\right)}{\ln x\left(1+\frac{1}{\ln x}\right)}$

$= \lim\limits_{x\to+\infty} \dfrac{\frac{2}{\ln x}+1}{1+\frac{1}{\ln x}}$

$= \dfrac{0+1}{1+0} = 1$

* $f(x) \in]0.9, 1.1[$

$0.9 < f(x) < 1.1$

$0.9-1 < f(x)-1 < 1.1-1$

$-0.1 < f(x)-1 < 0.1$

$|f(x)-1| < \dfrac{1}{10}$

$\left|\dfrac{2+\ln x}{1+\ln x} - 1\right| < \dfrac{1}{10}$

$\left|\dfrac{2+\ln x-1-\ln x}{1+\ln x}\right| < \dfrac{1}{10}$

$\dfrac{1}{1+\ln x} < \dfrac{1}{10}$

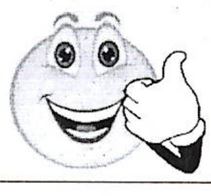
$1+\ln x > 10$

$\ln x > 9$

$x > e^9$

$\boxed{A = e^9}$

للتواصل: 0991070187



② $u_n = \frac{2n-1}{n+1}$

$\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{2n-1}{n+1} = 2$

$u_n \in]1.9 , 2.1[$

$1.9 < u_n < 2.1$

$1.9 - 2 < u_n - 2 < 2.1 - 2$

$-0.1 < u_n - 2 < 0.1$

$|u_n - 2| < \frac{1}{10}$

$\left|\frac{2n-1}{n+1} - 2\right| < \frac{1}{10}$

$\left|\frac{2n-1-2n-2}{n+1}\right| < \frac{1}{10}$

$\frac{3}{n+1} < \frac{1}{10}$

$\frac{n+1}{3} > 10$

$n+1 > 30$

$n > 29$

$\boxed{n_0 = 29}$

④ $f(x) = \frac{3e^x+2}{e^x-1}$

$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{3e^x+2}{e^x+1}$

$= \lim_{x\to+\infty} \frac{e^x\left(3+\frac{2}{e^x}\right)}{e^x\left(1+\frac{1}{e^x}\right)}$

$= \lim_{x\to+\infty} \frac{3+\frac{2}{e^x}}{1+\frac{1}{e^x}}$

$= \frac{3+0}{1+0} = 3$

$f(x) \in]2.95 , 3.05[$

$2.95 < f(x) < 3.05$

$2.95 - 3 < f(x) - 3 < 3.05 - 3$

$-0.05 < f(x) - 3 < 0.05$

$|f(x) - 3| < \frac{5}{100}$

$\left|\frac{3e^x+2}{e^x-1} - \frac{3}{1}\right| < \frac{1}{20}$

$\left|\frac{3e^x+2-3e^x+3}{e^x-1}\right| < \frac{1}{20}$

$\frac{5}{e^x-1} < \frac{1}{20}$

$\frac{e^x-1}{5} > 20$

$e^x - 1 > 100$

$e^x > 101$

$x > \ln(101)$

$\Rightarrow \boxed{A = \ln 101}$

اوراق مكثفة :

(*) للاستمرار :

①
$$f(x)=\begin{cases} A\,\dfrac{\sqrt{2x-1}-\sqrt{3x-2}}{x^2+3x-4} & ; x>1 \\ -\dfrac{1}{5} & ; x=1 \\ B\,\dfrac{\sin(x-1)}{(x^2+1)|x-1|} & ; x<1 \end{cases}$$

(*) f مستمر على IR فهو مستمر عند الواحد فهو يحقق الشرط :

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x) = f(1) \quad \boxed{*}$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} A\,\frac{\sqrt{2x-1}-\sqrt{3x-2}}{x^2+3x-4}$$

$$= \lim_{x\to 1^+} A\,\frac{(\sqrt{2x-1}-\sqrt{3x-2})(\sqrt{2x-1}+\sqrt{3x-2})}{(x^2+3x-4)(\sqrt{2x-1}+\sqrt{3x-2})}$$

$$= \lim_{x\to 1^+} A\,\frac{2x-1-3x+2}{(x^2+3x-4)(\sqrt{2x-1}+\sqrt{3x-2})}$$

$$= \lim_{x\to 1^+} A\,\frac{\cancel{(1-x)}}{(x+4)\cancel{(x-1)}(\sqrt{2x-1}+\sqrt{3x-2})}$$

$$= \lim_{x\to 1^+} A\,\frac{-1}{(x+4)(\sqrt{2x-1}+\sqrt{3x-2})}$$

$$= A\cdot\frac{-1}{5*2} = -\frac{A}{10}$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} B\,\frac{\sin(x-1)}{(x^2+1)\cdot|x-1|}$$

$$= \lim_{x\to 1^-} -B\,\frac{\sin(x-1)}{(x^2+1)\cdot(x-1)}$$

نفرض $x-1=t$

$$\Rightarrow x = t+1$$

عندما $x\to 1$ فإن $t\to 0$

$$\Rightarrow \lim_{x\to 1^-} f(x) = \lim_{t\to 0} -B\,\frac{\sin t}{((t+1)^2+1)\,t}$$

$$= -B\cdot\frac{1}{2} = -\frac{B}{2}$$

نعوض في (*) :

$$\underset{(1)}{-\frac{A}{10}} = \underset{(2)}{-\frac{B}{2}} = \underset{(3)}{-\frac{1}{5}}$$

من ① و ③ نجد :

$$A = \frac{-10\times(-1)}{5} = 2$$

من ② و ③ نجد :

$$B = \frac{-2\times -1}{5} = \frac{2}{5}$$

②
$$f(x)=\begin{cases} \dfrac{x.\sin x}{\sqrt{x^2+1}-1} & ; x\neq 0 \\ m & ; x=0 \end{cases}$$

بما أن f مستمر على IR فهو مستمر عند الصفر

$$\lim_{x\to 0} f(x) = f(0)$$

$$\lim_{x\to 0} \frac{x.\sin x}{\sqrt{x^2+1}-1} = m$$

$$= \lim_{x\to 0} \frac{x.\sin x(\sqrt{x^2+1}+1)}{(\sqrt{x^2+1}-1)(\sqrt{x^2+1}+1)} = m$$

$$\lim_{x\to 0} \frac{x \cdot \sin x\,(\sqrt{x^2+1}+1)}{x^2+1-1} = m$$

$$\lim_{x\to 0} \frac{\cancel{x} \cdot \sin x\,(\sqrt{x^2+1}+1)}{\cancel{x^2}} = m$$

$$\lim_{x\to 0} \frac{\sin x}{x} \cdot (\sqrt{x^2+1}+1) = m$$

$$1 \cdot (\sqrt{0+1}+1) = m$$

$$m = 2$$

(٢) شرط الاستمرار عند الصفر:

$$\lim_{x\to 0} f(x) \overset{?}{=} f(0)$$

$$\lim_{x\to 0} \frac{4(1-\cos x)}{x^2} = \lim_{x\to 0} \frac{4 \cdot 2\sin^2(\frac{x}{2})}{x^2}$$

$$= \lim_{x\to 0} 8 \cdot \frac{\sin^2(\frac{x}{2})}{4 \cdot \frac{x^2}{4}}$$

$$= \lim_{x\to 0} \frac{8}{4} \left(\frac{\sin(\frac{x}{2})}{\frac{x}{2}}\right)^2$$

$$= 2 \cdot (1)^2 = 2$$

$$f(0) = 2 \Rightarrow$$

$$\lim_{x\to 0} f(x) = f(0) = 2$$

f مستمر عند الصفر.

- الاشتقاقات.

1

$$f(x) = (x-3)\sqrt{x(3-x)}$$

$$\lim_{x\to 3} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x\to 3} \frac{(x-3)\sqrt{x(3-x)} - 0}{x-3}$$

$$= \lim_{x\to 3} \frac{\cancel{(x-3)}\sqrt{x(3-x)}}{\cancel{x-3}}$$

$$= \lim_{x\to 3} \sqrt{x(3-x)} = 0$$

إذاً: f اشتقاقي عند 3.

2

$$f(x) = \cos x$$

① $f(\frac{\pi}{3}) = \cos\frac{\pi}{3} = \frac{1}{2}$

$$f'(x) = -\sin x$$

$$f'(\frac{\pi}{3}) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

② حسب تعريف العدد المشتق:

$$\lim_{x\to x_0} \frac{f(x)-f(x_0)}{x-x_0} = f'(x_0)$$

$$\lim_{x\to \frac{\pi}{3}} \frac{\cos x - \frac{1}{2}}{x-\frac{\pi}{3}} = -\frac{\sqrt{3}}{2}$$

3

$$f(x) = \begin{cases} x^2(1-\ln x) & ; x>0 \\ 0 & ; x=0 \end{cases}$$

$$\lim_{x\to 0} \frac{f(x)-f(0)}{x-0}$$

$$= \lim_{x\to 0} \frac{x^2(1-\ln x)-0}{x}$$

$$= \lim_{x\to 0} \frac{x^2(1-\ln x)}{x}$$

$$= \lim_{x\to 0} x(1-\ln x)$$

$$= \lim_{x\to 0} x - x\ln x = 0-0=0$$

$\Leftarrow$ f اشتقاقي عند الصفر.

$$f(x) = \frac{x^2+|x|}{x^2+1}$$

① $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} \frac{x^2+x}{x^2+1}$

$$= \lim_{x\to+\infty} \frac{x^2}{x^2} = 1$$

$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} \frac{x^2-x}{x^2+1}$

$$= \lim_{x\to-\infty} \frac{x^2}{x^2} = 1$$

② $\lim\limits_{x\to 0^+} \frac{f(x)-f(0)}{x-0}$

$$= \lim_{x\to 0^+} \frac{\frac{x^2+x}{x^2+1}-0}{x}$$

$$= \lim_{x\to 0^+} \frac{x^2+x}{x(x^2+1)}$$

$$= \lim_{x\to 0^+} \frac{x(x+1)}{x(x^2+1)}$$

$$= \lim_{x\to 0^+} \frac{x+1}{x^2+1} = \frac{0+1}{0+1} = 1$$

إذاً: f اشتقاقي عند الصفر من اليمين.

ⓧ معادلة المماس من اليمين

$A(0,0)$ و $m = f'(0) = 1$

$T: y - y_0 = m(x - x_0)$

$T: y - 0 = 1.(x-0)$

$T: y = x$

$$f(x) = \sqrt{x+1}$$

① $f(1) = \sqrt{1+1} = \sqrt{2}$

$f'(x) = \frac{1}{2\sqrt{x+1}}$

$f'(1) = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{2\sqrt{2}.\sqrt{2}} = \frac{\sqrt{2}}{4}$

② حسب تعريف المشتق:

$$\lim_{x\to x_0} \frac{f(x)-f(x_0)}{x-x_0} = f'(x_0)$$

$$\lim_{x\to 1} \frac{\sqrt{x+1}-\sqrt{2}}{x-1} = \frac{\sqrt{2}}{4}$$

٦

$f(x) = \sqrt{x} \cdot \ln(x+1)$

a

$$\lim_{x \to 0} \frac{f(x) - f(0)}{x - 0}$$

$$= \lim_{x \to 0} \frac{\sqrt{x} \cdot \ln(x+1) - 0}{x}$$

$$= \lim_{x \to 0} \sqrt{x} \cdot \frac{\ln(x+1)}{x} \quad (1)$$

$$= \sqrt{0} \cdot 1 = 0$$

إذاً: f اشتقاقي عند الصفر.

b

$$f'(x) = \frac{1}{2\sqrt{x}} \cdot \ln(x+1) + \frac{1}{x+1} \cdot \sqrt{x}$$

$$= \frac{\ln(x+1)}{2\sqrt{x}} + \frac{\sqrt{x}}{x+1}$$

c

$$g'(x) = \left(\frac{\ln(\cos x + 1)}{2\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\cos x + 1} \right) \times \sin x$$

٧

$$f(x) = \frac{2x+3}{x-1}$$

(1)

$$f'(x) = \frac{2(x-1) - 1 \cdot (2x+3)}{(x-1)^2}$$

$$= \frac{2x - 2 - 2x - 3}{(x-1)^2}$$

$$= \frac{-5}{(x-1)^2}$$

f اشتقاقي على $\mathbb{R} \setminus \{1\}$

(2)

$$g(x) = \frac{2\sqrt{x} + 3}{\sqrt{x} - 1}$$

$$g'(x) = \frac{-5}{(\sqrt{x}-1)^2} \cdot (\sqrt{x})'$$

$$= \frac{-5}{(\sqrt{x}-1)^2} \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{-5}{2\sqrt{x}} \left(\frac{1}{(\sqrt{x}-1)^2} \right)$$

من $g(x) = \frac{2\sqrt{x}+3}{\sqrt{x}-1}$

البسط تابع اشتقاقي على J

والمقام اشتقاقي على J ولا ينعدم

$\Rightarrow$ g تابع اشتقاقي على J.

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* معادلات تفاضلية

① $y' = 2y + 1$

$a = 2 ,\ b = 1$

$f(x) = k \cdot e^{ax} - \frac{b}{a}$

$f(x) = k \cdot e^{2x} - \frac{1}{2}$

لتعيين k نستفيد من الشرط $f(0) = 1$:

$k \cdot e^{0} - \frac{1}{2} = 1$

$k \cdot 1 = 1 + \frac{1}{2}$

$k = \frac{3}{2}$

$\Rightarrow f(x) = \frac{3}{2} e^{2x} - \frac{1}{2}$

② $2y' + y = 1$

$2y' = -y + 1$

$y' = -\frac{1}{2} y + \frac{1}{2}$

$a = -\frac{1}{2} ,\ b = \frac{1}{2}$

$f(x) = k e^{ax} - \frac{b}{a}$

$f(x) = k \cdot e^{-\frac{1}{2}x} + 1$

لتعيين k نستفيد من الشرط :

$f(0) = -1$

$k \cdot e^{0} + 1 = -1$

$\boxed{k = -2}$

$f(x) = -2 e^{-\frac{1}{2}x} + 1$

③ $2y' = y$

$y' = \frac{1}{2} y$

$f(x) = k \cdot e^{ax}$

$f(x) = k \cdot e^{\frac{1}{2}x}$

لتعيين k :

$f'(x) = \frac{1}{2} k \cdot e^{\frac{1}{2}x}$

$f'(0) = 2$

$\frac{1}{2} k \cdot e^{0} = 2 \Rightarrow \boxed{k = 4}$

$f(x) = 4 \cdot e^{\frac{1}{2}x}$

④ $f(x) = x \cdot e^{x}$

$y = f(x) = x \cdot e^{x}$

$y' = 1 \cdot e^{x} + e^{x} \cdot x$

$= e^{x}(1 + x)$

نعوض في المعادلة :

$y' - y = e^{x}$

$e^{x} + x \cdot e^{x} - x \cdot e^{x} = e^{x}$

محققة $e^{x} = e^{x}$

اوراق مكثفة :

* قيمة مجهول :

1- $f(x) = ax^3 + bx + 1$

قيمة حدية $A(1,-1)$

$f(1) = -1$ $f'(1) = 0$

$f(1) = -1 \rightarrow a(1)^3 + b(1) + 1 = -1$

$a + b + 1 = -1$ — (1)

$f'(x) = 3ax^2 + b$

$f'(1) = 0$

$3a + b = 0$ — (2)

نطرح :

$-2a + 1 = -1$

$-2a = -2$

$\boxed{a = -1}$

نعوض في (2) :

$-3 + b = 0 \Rightarrow \boxed{b = 3}$

$f(x) = -x^3 + 3x + 1$

2- $f(x) = ax + b + \frac{\ln x}{x}$

قيمة حدية $A(1,0)$

$f(1) = 0$ $f'(1) = 0$

$f(1) = 0 \Rightarrow a(1) + b + \frac{\ln(1)}{1} = 0$

$a + b = 0$ — (1)

$f'(x) = a + \frac{\frac{1}{x}\cdot x - 1\cdot \ln x}{x^2}$

$f'(x) = a + \frac{1 - \ln x}{x^2}$

$f'(1) = 0 \Rightarrow a + \frac{1 - \ln(1)}{1^2} = 0$

$a + 1 = 0$

$\boxed{a = -1}$

نعوض في (1) :

$-1 + b = 0$

$\boxed{b = +1}$

3- $f(x) = \frac{2x}{x^2 - 4}$

$f(x) = \frac{a}{x-2} + \frac{b}{x+2}$

$(x+2)$ $(x-2)$

$= \frac{ax + 2a + bx - 2b}{(x-2)(x+2)}$

$f(x) = \frac{(a+b)x + 2a - 2b}{x^2 - 4}$

$\frac{2x}{x^2-4} = \frac{(a+b)x + 2a - 2b}{x^2 - 4}$

$2x = (a+b)x + 2a - 2b$

$\Rightarrow a + b = 2$ — (1)

$2a - 2b = 0$ — (2)

بقسمة المعادلة (2) على (2)

$a - b = 0$ — (3)

نجمع (1) و (3) : $2a = 2 \Rightarrow \boxed{a = 1}$

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نعوض في ① : $1+b=2$

$\boxed{b=1}$

$f(x)=\frac{1}{x-2}+\frac{1}{x+2}$

④ $f(x)=\frac{x^2+2x-2}{x+3}$

$$\begin{array}{r|l} x^2+2x-2 & x+3 \\ \mp x^2 \mp 3x & x-1 \\ \hline -x-2 & \\ \pm x \pm 3 & \\ \hline 1 & \end{array}$$

$f(x)=x-1+\frac{1}{x+3}$

$f(x)=ax+b+\frac{c}{x+3}$

$a=1,\ b=-1,\ c=1$

* مقارب مائل :

دورة ٢٠٢١ ثانية -

① $f(x)=x+\sqrt{x^2+1}$

① $\Delta: y=2x$

نشكل الفرق :

$f(x)-y_\Delta = x+\sqrt{x^2+1}-2x$

$=\sqrt{x^2+1}-x$

$=\frac{(\sqrt{x^2+1}-x)(\sqrt{x^2+1}+x)}{\sqrt{x^2+1}+x}$

$=\frac{x^2+1-x^2}{\sqrt{x^2+1}+x}$

$=\frac{1}{\sqrt{x^2+1}+x}$

نحسب الفرق عند $+\infty$:

$\lim_{x\to+\infty}[f(x)-y_\Delta]=\lim_{x\to+\infty}\left(\frac{1}{\sqrt{x^2+1}+x}\right)$

$=\frac{1}{+\infty}=0$

إذاً $\Delta: y=2x$ مقارب مائل للخط C عند $+\infty$

② الوضع النسبي : ندرس إشارة الفرق

$f(x)-y_\Delta=\frac{1}{\sqrt{x^2+1}+x}>0$

C فوق المقارب

③ $f(x)=x-\ln\left(\frac{3x+1}{x}\right)$

ⓐ $\Delta: y=x-\ln 3$

* نشكل الفرق :

$f(x)-y_\Delta=x-\ln\left(\frac{3x+1}{x}\right)-x+\ln 3$

$=\ln 3-\ln\left(\frac{3x+1}{x}\right)$

$=\ln\left(\frac{\frac{3}{1}}{\frac{3x+1}{x}}\right)=\ln\left(\frac{3x}{3x+1}\right)$

نحسب الفرق عند $+\infty$:

$\lim_{x\to+\infty}(f(x)-y_\Delta)=$

$$= \lim_{x \to +\infty} \ln\left(\frac{3x}{3x+1}\right)$$

$$= \ln\left(\lim_{x \to +\infty} \frac{3x}{3x+1}\right) = \ln(1) = 0$$

إذاً: $\Delta : y = x - \ln 3$ مقارب مائل.

(b) الوضع النسبي: ندرس إشارة الفرق:

$$f(x) - y_{\Delta} = \ln\left(\frac{3x}{3x+1}\right) < 0$$

C تحت المقارب Δ

(2) $f(x) = \ln(e^x + 3)$

(a) $f(x) = \ln\left(e^x \cdot \left(1 + \frac{3}{e^x}\right)\right)$

$$= \ln e^x + \ln(1 + 3e^{-x})$$

$$= x + \ln(1 + 3e^{-x})$$

(b) $\Delta : y = x$

• نشكل الفرق:

$$f(x) - y_{\Delta} = x + \ln(1 + 3e^{-x}) - x$$

$$= \ln(1 + 3e^{-x})$$

• ننهي الفرق في جوار $+\infty$:

$$\lim_{x \to +\infty} (f(x) - y_{\Delta}) = \lim_{x \to +\infty} \left(\ln(1 + 3e^{-x})\right)$$

$$= \ln(e^{-\infty} + 1) = \ln(1) = 0$$

إذاً: $\Delta : y = x$ مقارب مائل لـ C عند $+\infty$

دورة أولى 2019

(4) $f(x) = x + 3 - \frac{1}{x^2}$

$\Delta : y = x + 3$

• نشكل الفرق:

$$f(x) - y_{\Delta} = x + 3 - \frac{1}{x^2} - (x + 3)$$

$$= \frac{-1}{x^2}$$

• ننهي الفرق في جوار $+\infty$:

$$\lim_{x \to +\infty} (f(x) - y_{\Delta}) = \lim_{x \to +\infty} \left(\frac{-1}{x^2}\right) = 0$$

إذاً Δ مقارب مائل لـ C

• الوضع النسبي: ندرس إشارة الفرق:

$$f(x) - y_{\Delta} = \frac{-1}{x^2} < 0$$

C تحت المقارب

دورة 2019

(5) $f(x) = ax + b - \frac{\ln x}{x}$

$A(1, 0) \in f \quad d : y = 3x$

$f(1) = 0 \qquad f'(1) = 3$

$f(1) = 0$

$a(1) + b - \frac{\ln(1)}{1} = 0 \Rightarrow a + b = 0$ (1)

$$f'(x) = a - \frac{\frac{1}{x} \cdot x - \ln x \cdot 1}{x^2} = a - \frac{1 - \ln x}{x^2}$$

$$f'(1) = 3 \Rightarrow a - \frac{1 - 0}{1} = 3$$

القسم: الوحدة:

$a - 1 = 3 \Rightarrow \boxed{a = 4}$

نعوض في ①:

$4 + b = 0 \Rightarrow \boxed{b = -4}$

$f(x) = 4x - 4 - \frac{\ln x}{x}$

② $\Delta : y = 4x - 4$

لنشكل الفرق:

$f(x) - y_{\Delta} = 4x - 4 - \frac{\ln x}{x} - (4x - 4)$

$= -\frac{\ln x}{x}$

نتعين الفرق في جوار $+\infty$:

$\lim_{x \to +\infty} (f(x) - y_{\Delta}) = \lim_{x \to +\infty} \left(-\frac{\ln x}{x}\right) = 0$

* الوضع النسبي:

ندرس إشارة الفرق:

$f(x) - y_{\Delta} = -\frac{\ln x}{x}$

• نعدم الفرق:

$f(x) - y_{\Delta} = 0$

$-\frac{\ln x}{x} = 0 \Rightarrow \ln x = 0$

$\boxed{x = 1}$

x	0 1 $+\infty$	
$f(x) - y$	$+$ 0	$-$
الوضع النسبي	C فوق Δ	C تحت Δ

دورة 2017:

$f(x) = \sqrt{4x^2 + 5}$

• $\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \sqrt{4x^2 + 5} = +\infty$

• $\lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \frac{\sqrt{4x^2+5}}{x} = \frac{+\infty}{\infty}$

$= \lim_{x \to +\infty} \frac{\sqrt{x^2\left(4 + \frac{5}{x^2}\right)}}{x}$

$= \lim_{x \to +\infty} \frac{|x| \sqrt{4 + \frac{5}{x^2}}}{x}$

$= \lim_{x \to +\infty} \frac{x \sqrt{4 + \frac{5}{x^2}}}{x}$

$= \lim_{x \to +\infty} \sqrt{4 + \frac{5}{x^2}} = \sqrt{4 + 0} = \boxed{2 = a}$

• $\lim_{x \to +\infty} (f(x) - 2x) = +\infty - \infty$

$= \lim_{x \to +\infty} \frac{(\sqrt{4x^2+5} - 2x)(\sqrt{4x^2+5} + 2x)}{\sqrt{4x^2+5} + 2x}$

$= \lim_{x \to +\infty} \frac{4x^2 + 5 - 4x^2}{\sqrt{4x^2+5} + 2x}$

$= \lim_{x \to +\infty} \frac{5}{\sqrt{4x^2+5} + 2x} = \frac{5}{\infty} = 0$

$\Delta : y = 2x$

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الطريقة ١ لحلها

● $f(x) = \sqrt{x^2 - 2x + 2}$

ⓐ $x^2 - 2x + 2$

$= \underbrace{x^2 - 2x + 1} - 1 + 2$

$= (x-1)^2 + 1$

ⓑ $\lim\limits_{x \to +\infty} \left(f(x) - \sqrt{(x-1)^2} \right)$

$= \lim\limits_{x \to +\infty} \left(\sqrt{(x-1)^2 + 1} - \sqrt{(x-1)^2} \right)$

$= \lim\limits_{x \to +\infty} \dfrac{\left(\sqrt{(x-1)^2+1} + \sqrt{(x-1)^2}\right)\left(\sqrt{(x-1)^2+1} - \sqrt{(x-1)^2}\right)}{\sqrt{(x-1)^2+1} + \sqrt{(x-1)^2}}$

$= \lim\limits_{x \to +\infty} \dfrac{(x-1)^2 + 1 - (x-1)^2}{\sqrt{(x-1)^2+1} + \sqrt{(x-1)^2}}$

$= \lim\limits_{x \to +\infty} \dfrac{1}{\sqrt{(x-1)^2+1} + \sqrt{(x-1)^2}} = 0$

ⓒ الوضع النسبي: ندرس إشارة الفرق:

$f(x) - y_{\Delta} = \dfrac{1}{\sqrt{(x-1)^2+1} + \sqrt{(x-1)^2}} > 0$

C فوق Δ.

ⓔ $f(x) = 2x - 3 - \dfrac{\ln x}{x}$

① * $\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} \left(2x - 3 - \dfrac{\ln x}{x}\right)$ ⓐ

$= +\infty - 0 = +\infty$

② * $\lim\limits_{x \to +\infty} \dfrac{f(x)}{x} = \lim\limits_{x \to +\infty} \left(2 - \dfrac{3}{x} - \dfrac{\ln x}{x^2}\right)$

$= 2 - 0 - 0 = \boxed{2 = a}$

③ * $\lim\limits_{x \to +\infty} \left(2x - 3 - \dfrac{\ln x}{x} - 2x\right)$

$= \lim\limits_{x \to +\infty} \left(-3 - \dfrac{\ln x}{x}\right)$

$= -3 - 0 = \boxed{-3 = b}$

④ $\Rightarrow \Delta : y = 2x - 3$

مقارب مائل للخط C.

ⓑ * الوضع النسبي:

ندرس إشارة الفرق:

$f(x) - y_{\Delta} = 2x - 3 - \dfrac{\ln x}{x} - (2x - 3)$

$= \dfrac{-\ln x}{x}$

نعدم الفرق:

$f(x) - y_{\Delta} = 0 \Rightarrow -\ln x = 0$

$\boxed{x = 1} \Leftarrow \ln x = 0$

x	0		1		$+\infty$
$f(x) - y_{\Delta}$		$+$	0	$-$	
الوضع النسبي		C فوق Δ		C تحت Δ	

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القسم: الوحدة:

$\boxed{*}$ نهايات :

① $f(x) = \dfrac{e^{3x}-1}{\ln(2x+1)}$; $a=0$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{e^{3x}-1}{\ln(2x+1)} = \frac{0}{0}$$

$$= \lim_{x\to 0} \frac{\cancel{x}\cdot \frac{e^{3x}-1}{x}}{\cancel{x}\cdot \frac{\ln(2x+1)}{x}}$$

$$= \lim_{x\to 0} \frac{3\cdot \frac{e^{3x}-1}{3x}}{2\cdot \frac{\ln(2x+1)}{2x}}$$

$$= \frac{3(1)}{2(1)} = \frac{3}{2}$$

② $f(x) = \dfrac{\ln(x+2)}{x+1}$; $a=-1$

$$\lim_{x\to -1} f(x) = \lim_{x\to -1} \frac{\ln(x+2)}{x+1} = \frac{0}{0}$$

$= \cancel{\lim_{x\to}}$

نفرض $x+2 = t$

$x+1 = t-1$

عندما $x \to -1$ فإن $t \to 1$

$$\lim_{t\to 1} \frac{\ln(t)}{t-1} = 1$$

③ $f(x) = \dfrac{e^{x}+e^{-x}-2}{1-\cos 2x}$; $a=0$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{e^{x}+e^{-x}-2}{1-\cos 2x} = \frac{0}{0}$$

$$= \lim_{x\to 0} \frac{1}{e^x}\cdot \frac{e^x\cdot e^x + e^{-x}\cdot e^x - 2e^x}{2\sin^2 x}$$

$$= \lim_{x\to 0} \frac{1}{e^x}\cdot \frac{e^{2x} - 2e^x + 1}{2\sin^2 x}$$

$$= \lim_{x\to 0} e^{-x}\cdot \frac{(e^x-1)^2}{2\sin^2 x}$$

$$= \lim_{x\to 0} e^{-x}\cdot \frac{\cancel{x^2}\cdot \frac{(e^x-1)^2}{x^2}}{\cancel{x^2}\cdot \frac{2\sin^2 x}{x^2}}$$

$$= \lim_{x\to 0} e^{-x}\cdot \frac{\left(\frac{e^x-1}{x}\right)^2}{2\cdot\left(\frac{\sin x}{x}\right)^2}$$

$$= \frac{e^0}{2}\cdot \frac{1^2}{1^2} = \frac{1}{2}$$

④ $f(x) = \dfrac{e^{3x}-e^{x}}{\ln(2x+1)}$; $a=0$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{e^{3x}-e^{x}}{\ln(2x+1)} = \frac{0}{0}$$

$$= \lim_{x\to 0} \frac{e^x(e^{2x}-1)}{\ln(2x+1)}$$

$$= \lim_{x\to 0} e^x \frac{\cancel{x}\cdot\frac{e^{2x}-1}{x}}{\cancel{x}\cdot\frac{\ln(2x+1)}{x}} = \lim_{x\to 0} e^x \frac{2\cdot\frac{e^{2x}-1}{2x}}{2\cdot\frac{\ln(2x+1)}{2x}}$$

$$= \frac{2\cdot e^0}{2} = 1$$

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⑤ $f(x) = x \cdot \ln\left(1+\frac{1}{x}\right)$; $a=+\infty$

$\lim_{x\to+\infty} f(x) = 0 \cdot \infty$

نفرض $t = \frac{1}{x} \Leftarrow x = \frac{1}{t}$

عندما $x \to +\infty$ فإن $t \to 0$

$\lim_{t\to 0} \frac{1}{t} \ln(1+t)$

$= \lim_{t\to 0} \frac{\ln(1+t)}{t} = 1$

$\boxed{6}$ $f(x) = x \cdot \cos\left(\frac{1}{x}\right)$; $a=0$

$-1 \le \cos\left(\frac{1}{x}\right) \le 1$

$-x \le x \cdot \cos\left(\frac{1}{x}\right) \le x$

$-x \le f(x) \le x$

نتهي الأطراف عند الصفر

$\lim_{x\to 0} (-x) = 0$

$\lim_{x\to 0} (x) = 0$

حسب مبرهنة الإحاطة :

$\lim_{x\to 0} f(x) = 0$

$\boxed{7}$ $f(x) = \frac{2x+\sin x}{x-2}$; $a=+\infty$

$-1 \le \sin x \le 1$

$2x-1 \le 2x+\sin x \le 2x+1$

$\frac{2x-1}{x-2} \le \frac{2x+\sin x}{x-2} \le \frac{2x+1}{x-2}$

$\frac{2x-1}{x-2} \le f(x) \le \frac{2x+1}{x-2}$

$\lim_{x\to+\infty} \left(\frac{2x-1}{x-2}\right) = 2$

$\lim_{x\to+\infty} \left(\frac{2x+1}{x-2}\right) = 2$

$\lim_{x\to+\infty} f(x) = 2 \Leftarrow$

$\boxed{8}$ $|f(x) - 3| \le \frac{1}{x^2}$

نفرض $g(x) = \frac{1}{x^2}$

نتهي $g(x)$ عند $+\infty$:

$\lim_{x\to+\infty} g(x) = \lim_{x\to+\infty} \left(\frac{1}{x^2}\right) = 0$

$\lim_{x\to+\infty} f(x) = 3 \Leftarrow$

$\boxed{9}$ $|f(x) + 2| \le \frac{x \cdot \cos(e^x)}{x^2+1}$

نفرض $g(x) = \frac{x \cdot \cos(e^x)}{x^2+1}$

$-1 \le \cos(e^x) \le +1$

$-x \le x \cdot \cos(e^x) \le x$

$\frac{-x}{x^2+1} \le \frac{x \cdot \cos(e^x)}{x^2+1} \le \frac{x}{x^2+1}$

$$\frac{-x}{x^2+1} \le g(x) \le \frac{x}{x^2+1}$$

ننهي الأطراف عند $+\infty$:

$$\lim_{x\to+\infty} \frac{-x}{x^2+1} = 0$$

$$\lim_{x\to+\infty} \frac{x}{x^2+1} = 0$$

$$\Leftarrow \lim_{x\to+\infty} g(x) = 0$$

$$\Rightarrow \lim_{x\to+\infty} f(x) = -2$$

$\boxed{10}$ $$|f(x) - 5| \le \frac{E(x)}{x^2+1}$$

نفرض $g(x) = \frac{E(x)}{x^2+1}$

$$x-1 \le E(x) \le x$$

$$\frac{x-1}{x^2+1} \le \frac{E(x)}{x^2+1} \le \frac{x}{x^2+1}$$

$$\frac{x-1}{x^2+1} \le g(x) \le \frac{x}{x^2+1}$$

ننهي الأطراف في جوار $+\infty$:

$$\left.\begin{aligned} \lim_{x\to+\infty} \frac{x-1}{x^2+1} &= 0 \\ \lim_{x\to+\infty} \frac{x}{x^2+1} &= 0 \end{aligned}\right\}$$

$$\Leftarrow \lim_{x\to+\infty} g(x) = 0$$

$$\Leftarrow \lim_{x\to+\infty} f(x) = 5$$

١٨-٢-٢٠

$$f(x) = \frac{1}{3+\cos x}$$

① $$-1 \le \cos x \le 1$$

$$3-1 \le 3+\cos x \le 3+1$$

$$2 \le 3+\cos x \le 4$$

$$\frac{1}{2} \ge \frac{1}{3+\cos x} \ge \frac{1}{4}$$

$$\frac{1}{4} \le f(x) \le \frac{1}{2}$$

إذاً f محدودة.

② لدينا من الطلب السابق:

$$\frac{1}{4} \le \frac{1}{3+\cos x} \le \frac{1}{2}$$

$$\frac{x^2}{4} \le \frac{x^2}{3+\cos x} \le \frac{x^2}{2}$$

ننهي الأطراف عند $+\infty$:

$$\lim_{x\to+\infty} \left(\frac{x^2}{4}\right) = +\infty$$

$$\lim_{x\to+\infty} \left(\frac{x^2}{2}\right) = +\infty$$

فحسب مبرهنة الإحاطة:

$$\lim_{x\to+\infty} \frac{x^2}{3+\cos x} = +\infty$$

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القسم: الوحدة:

نهايات أساسية

① $f(x) = 2x \cdot e^{-x} \; ; \; a = +\infty$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} 2x \cdot e^{-x}$$

$$= \lim_{x \to +\infty} \frac{2x}{e^{x}} = 0$$

② $f(x) = \dfrac{\sqrt{x}}{\ln x} \; ; \; a = +\infty$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{\sqrt{x}}{\ln x}$$

$$= \lim_{x \to +\infty} \frac{\sqrt{x}}{\ln \sqrt{x}^{2}} = \lim_{x \to +\infty} \frac{\sqrt{x}}{2 \ln \sqrt{x}}$$

$$= +\infty$$

③ $f(x) = \dfrac{e^{x} - 1}{x - 1} \; ; \; a = +\infty$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{e^{x}\left(1 - \frac{1}{e^{x}}\right)}{x\left(1 - \frac{1}{x}\right)}$$

$$= +\infty \left(\frac{1 - 0}{1 - 0}\right) = +\infty$$

④ $f(x) = \dfrac{1}{x} + \ln x \; ; \; a = 0, +\infty$

- $\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \left(\frac{1}{x} + \ln x\right) = 0 + \infty = +\infty$
- $\lim_{x \to 0^{+}} f(x) = \lim_{x \to 0^{+}} \frac{1}{x}\left(1 + x \ln x\right) = +\infty (1 + 0) = +\infty$

⑤ $f(x) = x - e^{x} \; ; \; a = +\infty$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} (x - e^{x}) = +\infty - \infty$$

$$= \lim_{x \to +\infty} e^{x}\left(\frac{x}{e^{x}} - 1\right) = +\infty (0 - 1)$$

$$= -\infty$$

⑥ $f(x) = (x^{2} - x) \cdot \ln x \; ; \; a = 0$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} (x^{2} - x) \cdot \ln x$$

$$= \lim_{x \to 0} x^{2} \ln x - x \ln x$$

$$= 0 - 0 = 0$$

⑦ $f(x) = e^{x} - \ln x \; ; \; a = +\infty$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} (e^{x} - \ln x) = +\infty - \infty$$

$$= \lim_{x \to +\infty} e^{x}\left(1 - \frac{\ln x}{e^{x}}\right)$$

$$= +\infty (1 - 0) = +\infty$$

⑧ $f(x) = e^{x} - x^{2} \; ; \; a = +\infty, -\infty$

- $\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} (e^{x} - x^{2}) = 0 - \infty = -\infty$
- $\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} e^{x}\left(1 - \frac{x^{2}}{e^{x}}\right)$

$$= +\infty (1 - 0) = +\infty$$

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القسم: الوحدة:

اكتشافات:

① $f(x) = \dfrac{\sqrt{x+1}-2}{x-3}$; $a=3$

$$\lim_{x\to 3} f(x) = \lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} = \frac{0}{0}$$

$$\lim_{x\to 3} \frac{(\sqrt{x+1}-2)(\sqrt{x+1}+2)}{(x-3)(\sqrt{x+1}+2)}$$

$$= \lim_{x\to 3} \frac{(x+1-4)}{(x-3)(\sqrt{x+1}+2)}$$

$$= \lim_{x\to 3} \frac{x-3}{(x-3)(\sqrt{x+1}+2)}$$

$$= \lim_{x\to 3} \frac{1}{\sqrt{x+1}+2} = \frac{1}{4}$$

② $f(x) = \dfrac{1-\cos x}{x.\sin x}$; $a=0$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{1-\cos x}{x.\sin x} = \frac{0}{0}$$

$$= \lim_{x\to 0} \frac{2\sin^2(\frac{x}{2})}{x.2\sin(\frac{x}{2}).\cos(\frac{x}{2})}$$

$$= \lim_{x\to 0} \frac{\sin(\frac{x}{2})}{2.\frac{x}{2}\cos(\frac{x}{2})}$$

قاعدة

$$= \frac{1}{2.1} = \frac{1}{2}$$

③ $f(x) = \dfrac{x^2-4x-12}{x^2-4}$; $a=-2$

$$\lim_{x\to -2} f(x) = \lim_{x\to -2} \frac{x^2-4x-12}{x^2-4} = \frac{0}{0}$$

$$\lim_{x\to -2} \frac{(x-6)(x+2)}{(x-2)(x+2)}$$

$$= \lim_{x\to -2} \frac{x-6}{x-2} = \frac{-8}{-4} = 2$$

④ $f(x) = \dfrac{\sqrt{2x^3-1}-1}{x-1}$; $a=1$

$$\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{\sqrt{2x^3-1}-1}{x-1} = \frac{0}{0}$$

$$= \lim_{x\to 1} \frac{(\sqrt{2x^3-1}-1)(\sqrt{2x^3-1}+1)}{(x-1)(\sqrt{2x^3-1}+1)}$$

$$= \lim_{x\to 1} \frac{2x^3-1-1}{(x-1)(\sqrt{2x^3-1}+1)}$$

$$= \lim_{x\to 1} \frac{2(x^3-1)}{(x-1)(\sqrt{2x^3-1}+1)}$$

$$= \lim_{x\to 1} \frac{2(x-1)(x^2+x+1)}{(x-1)(\sqrt{2x^3-1}+1)}$$

$$= \lim_{x\to 1} \frac{2(x^2+x+1)}{\sqrt{2x^3-1}+1} = \frac{6}{2} = 3$$

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القسم: الوحدة:

$\circledast$ حل جمل معادلتين:

①

$2\ln x + \ln y = 7$ —①

$3\ln x - 5\ln y = 4$ —②

شرط الحل: $y>0,\ x>0$

نفرض $\ln x = X$ و $\ln y = Y$

$2X + Y = 7$ —①

$3X - 5Y = 4$ —②

نضرب المعادلة ① بـ (5) ثم نجمع مع الثانية:

$$10X + 5Y = 35$$
$$3X - 5Y = 4 \quad (+)$$

$$13X = 39 \Rightarrow \boxed{X = 3}$$

نعوض في ①:

$$6 + Y = 7 \Rightarrow \boxed{Y = 1}$$

نعود إلى المتحولات الأصلية:

$\ln x = 3 \Rightarrow x = e^3$

$\ln y = 1 \Rightarrow y = e^1$

②

$\ln(x+y) = 2\ln 2$ —①

$\ln x + \ln y = \ln 3$ —②

شرط الحل: $x+y>0,\ y>0,\ x>0$

نطبق خواص اللوغاريتم.

$\ln(x+y) = \ln 2^2$

$\ln(x+y) = \ln 4$

$x + y = 4$ —①

$\ln x + \ln y = \ln 3$

$\ln(x \cdot y) = \ln 3$

$x \cdot y = 3$ —②

من ① نعزل x:

$x = 4 - y$ —$\circledast$

نعوض في ②:

$(4-y) \cdot y = 3$

$4y - y^2 = 3 \Rightarrow y^2 - 4y + 3 = 0$

$(y-3)(y-1) = 0$

إما $y = 3$

$\Rightarrow x = 1$

أو $y = 1$

$\Rightarrow x = 3$

③

$\ln(x \cdot y) = 2$ —①

$2\ln x - 3\ln y = 1$ —②

شرط الحل: $y>0,\ x>0,\ x \cdot y>0$

$\ln x + \ln y = 2$ —①

$2\ln x - 3\ln y = 1$ —②

نفرض $\ln x = X$ و $\ln y = Y$

$X + Y = 2$ — ①

$2X - 3Y = -1$ — ②

نضرب المعادلة الأولى بـ (3) ونجمع مع الثانية :

$3X + 3Y = 6$ — ①

$2X - 3Y = -1$ — ② (+)

$5X = 5 \Rightarrow \boxed{X = 1}$

نعوض في ① :

$1 + Y = 2 \Rightarrow \boxed{Y = 1}$

فنعود إلى المجهول الأصلي :

$Lnx = 1 \Rightarrow x = e^1$

$Lny = 1 \Rightarrow y = e^1$

4

$(Lnx).(Lny) = -12$ — ①

$Ln(x \cdot y) = 1$ — ②

شرط الحل : $x.y>0, y>0, x>0$

من ② :

$Lnx + Lny = 1$ — *

نفرض $Lny = Y$, $Lnx = X$

$X \cdot Y = -12$ — ①

$X + Y = 1$ — ②

من ② نعزل X :

$X = 1 - Y$ — **

نعوض في ① :

$(1 - Y)\,Y = -12$

$Y - Y^2 = -12$

$Y^2 - Y - 12 = 0$

$(Y-4)(Y+3) = 0$

إما $Y = 4$

$Lny = 4 \Rightarrow y = e^4$

أو $Y = -3$

$Lny = -3 \Rightarrow y = e^{-3}$

5

$e^x - \frac{1}{e} e^y = 1$ — ①

$2e^x + e^y = 4 + e$ — ②

نفرض $e^y = Y$, $e^x = X$

$X - \frac{1}{e} Y = 1$ — ①

$2X + Y = 4 + e$ — ②

نضرب المعادلة ① بـ (−2) ونجمع ① و ② :

$-2X + \frac{2}{e} Y = -2$

$2X + Y = 4 + e$ (+)

$(\frac{2}{e} + 1) Y = 2 + e$

$(\frac{2+e}{e}) Y = 2 + e$

$Y = e$

نعوض في ①

$X - \frac{1}{e} \cdot e = 1 \Rightarrow X = 2$

نعود إلى المجهول الأصلي :

$e^y = e^1 \Rightarrow \boxed{y = 1}$

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$e^x = 2 \Rightarrow \boxed{x = \ln 2}$

٦ $e^{4x} \cdot e^{y} = \frac{1}{e^{2}}$ — ①

$x \cdot y = -2$ — ②

من ① :

$e^{4x+y} = e^{-2} \Rightarrow 4x+y=-2$ — *

نعزل y : $y = -2-4x$ — ③

نعوض ③ في ② :

$x \cdot (-2-4x) = -2$

$-2x - 4x^{2} = -2$

$+4x^{2} + 2x - 2 = 0$

$2x^{2} + x - 1 = 0$

$a=2,\ b=1,\ c=-1$

$\Delta = b^{2} - 4ac$

$= 1 - 4(2)(-1)$

$= 1 + 8 = 9 > 0$

$\sqrt{\Delta} = 3$

$x_1 = \frac{-b+\sqrt{\Delta}}{2a} = \frac{-1+3}{2(2)} = \frac{2}{4} = \frac{1}{2}$

$y_1 = -2-4\left(\frac{1}{2}\right) = -2-2 = -4$

$x_2 = \frac{-b-\sqrt{\Delta}}{2a} = \frac{-1-3}{2(2)} = \frac{-4}{4} = -1$

$y_2 = -2-4(-1) = -2+4$

حل المعادلة:

① $3^{x+1} = 7^{x}$

$\ln 3^{x+1} = \ln 7^{x}$

$(x+1) \cdot \ln 3 = x \cdot \ln 7$

$x \cdot \ln 3 + \ln 3 = x \cdot \ln 7$

$x \cdot \ln 7 - x \cdot \ln 3 = \ln 3$

$x(\ln 7 - \ln 3) = \ln 3$

$x = \frac{\ln 3}{\ln 7 - \ln 3}$

٥ $9^{x} + 3^{x+1} - 4 = 0$

$(3^{2})^{x} + 3^{x} \cdot 3^{1} - 4 = 0$

$3^{2x} + 3 \cdot 3^{x} - 4 = 0$

نفرض $3^{x} = X$

$X^{2} + 3X - 4 = 0$

$(X+4)(X-1) = 0$

إما $X + 4 = 0 \Rightarrow X = -4$

$3^{x} = -4$ مرفوض

أو $X - 1 = 0 \Rightarrow X = 1$

$3^{x} = 1$

$\ln 3^{x} = \ln 1$

$x \cdot \ln 3 = 0$

$x = 0$

٤ $e^{2x} - 5e^{x} + 6 = 0$

نفرض: $e^{x} = X$

$X^{2} - 5X + 6 = 0$

$(X-3)(X-2) = 0$

إما $X = 3$

$e^{x} = 3 \Rightarrow x = \ln 3$

أو $X = 2$

$e^{x} = 2 \Rightarrow x = \ln 2$

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④ $e^{3x-1} = e^{2x}$

$3x - 1 = 2x$

$3x - 2x = 1$

$x = 1$

⑤ $(\ln x)^2 - 4\ln x - 5 = 0$

$x > 0$

نفرض : $\ln x = X$

$X^2 - 4X - 5 = 0$

$(X-5)(X+1) = 0$

إما $X = 5$

$\ln x = 5 \Rightarrow x = e^5$

أو $X = -1$

$\ln x = -1 \Rightarrow x = e^{-1}$

⑥ $\frac{1}{2}\ln(2x) = \ln(3-x) - \ln\sqrt{x+1}$

$]0,+\infty[$ $]-\infty,3[$ $]-1,+\infty[$

شرط الحل: $D =]0,3[$

$\frac{1}{2}\ln(2x) = \ln(3-x) - \ln\sqrt{x+1}$

$\ln(2x)^{\frac{1}{2}} = \ln\left(\frac{3-x}{\sqrt{x+1}}\right)$

$\ln\sqrt{2x} = \ln\left(\frac{3-x}{\sqrt{x+1}}\right)$

$\sqrt{2x} = \frac{3-x}{\sqrt{x+1}}$

$\sqrt{2x}\cdot\sqrt{x+1} = 3-x$

$\sqrt{2x^2+2x} = 3-x$

$2x^2 + 2x = (3-x)^2$

$2x^2 + 2x = 9 - 6x + x^2$

$2x^2 - x^2 + 2x + 6x - 9 = 0$

$x^2 + 8x - 9 = 0$

$(x+9)(x-1) = 0$

إما $x = -9$ مرفوض

أو $x = 1$ مقبول

حل متراجحة:

① $e^{2x} - e^x - 6 = 0$

نفرض : $e^x = X$

$X^2 - X - 6 = 0$

$(X-3)(X+2) = 0$

إما $X = 3$

$e^x = 3 \Rightarrow \cancel{\ln}\ x = \ln 3$

أو $X = -2$

$e^x = -2$ مرفوض

(*) $e^{2x} - e^x - 6 \leq 0$

حلولها:

$D =]-\infty, \ln 3]$

③ $(\ln x)^2 - 2\ln x - 3 = 0$

شرط الحل: $x > 0$

نفرض $\ln x = X$

$X^2 - 2X - 3 = 0$

$(X-3)(X+1) = 0$

إما $X = 3$

$\ln x = 3 \Rightarrow x = e^3$

أو $X = -1$

$\ln x = -1 \Rightarrow x = e^{-1}$

المتراجحة:

$(\ln x)^2 - 2\ln x - 3 \geq 0$

$\Rightarrow D' =]-\infty, e^{-1}] \cup [e^3, +\infty[$

$\Rightarrow E = D' \cap D = [e^3, +\infty[$

② $\ln(3x^2 - x) \leq \ln x + \ln 2$

$]-\infty, 0[\cup]\frac{1}{3}, +\infty[$ $\qquad]0, +\infty[$

شرط الحل:

$D =]\frac{1}{3}, +\infty[$

$\ln(3x^2 - x) \leq \ln x + \ln 2$

$\ln(3x^2 - x) \leq \ln(x \cdot 2)$

$3x^2 - x \leq 2x$

$3x^2 - x - 2x \leq 0$

$3x^2 - 3x \leq 0$

$3x^2 - 3x = 0$

$3x(x-1) = 0$

إما $x = 0$

أو $x = 1$

$\Rightarrow D' = [0, 1]$

$\Rightarrow E = D' \cap D = [\frac{1}{3}, 1]$

④ $(\ln x + 2)(\ln x - 3) \leq 0$

$D =]0, +\infty[\Leftarrow \boxed{x > 0}$

$(\ln x + 2) \cdot (\ln x - 3) = 0$

إما $\ln x = -2$

$x = e^{-2}$

أو $\ln x = 3$

$x = e^3$

$\Rightarrow D' = [e^{-2}, e^3]$

* ~~~~ * ~~~~ * ~~~~ *

مسألة مميزة

$P(x) = 2x^3 + 5x^2 + x - 2$

1. a) $p(-1) = 0$

$p(-1) = 2(-1)^3 + 5(-1)^2 + (-1) - 2$

$= 2(-1) + 5(1) - 1 - 2$

$= -2 + 5 - 3 = 0$

$\Leftarrow P(-1) = 0$

$E' =]-\infty, -2] \cup [-1, \frac{1}{2}]$

$2\ln x + \ln(2x+5) \leq \ln(2-x)$ ج

$x > 0$ $\quad$ $x > -\frac{5}{2}$ $\quad$ $x < 2$

$D =]0, 2[$

$2\ln x + \ln(2x+5) \leq \ln(2-x)$

$\ln x^2 + \ln(2x+5) \leq \ln(2-x)$

$\ln(x^2 \cdot (2x+5)) \leq \ln(2-x)$

$2x^3 + 5x^2 \leq 2 - x$

$\underbrace{2x^3 + 5x^2 + x - 2}_{P(x)} \leq 0$

$P(x) \leq 0$

$\Rightarrow E' =]-\infty, -2] \cup [-1, \frac{1}{2}]$

$\Rightarrow E = D \cap E' =]0, \frac{1}{2}]$

————*——*

(b) $p(x) = (x+1) \cdot Q(x)$

$$\begin{array}{r|l} 2x^3 + 5x^2 + x - 2 & x+1 \\ \cline{2-2} \mp 2x^3 \mp 2x^2 & 2x^2 + 3x - 2 \\ \hline 3x^2 + x - 2 & \\ \mp 3x^2 \mp 3x & \\ \hline -2x - 2 & \\ \pm 2x \pm 2 & \\ \hline 0 & \end{array}$$

$p(x) = (x+1) \cdot Q(x)$

$= (x+1) \cdot (2x^2 + 3x - 2)$

(c) $P(x) = (x+1)(2x^2 + 3x - 2)$

$P(x) = 0 \Rightarrow$ إما $x = -1$

أو $2x^2 + 3x - 2 = 0$

$a = 2,\ b = 3,\ c = -2$

$\Delta = b^2 - 4ac$

$= 9 - 4(2)(-2)$

$= 9 + 16 = 25 > 0$

$\sqrt{\Delta} = 5$

$x_1 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{5 - 3}{2(2)} = \frac{1}{2}$

$x_2 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{-3 - 5}{2(2)} = -2$

x	$-\infty$		-2		-1		$\frac{1}{2}$		$+\infty$
$x+1$		$-$		$-$	0	$+$		$+$	
$2x^2+3x-2$		$+$	0	$-$		$-$	0	$+$	
$P(x)$		$-$	0	$+$	0	$-$	0	$+$	
$P(x) \leq 0$		محققة		////		محققة		////	

2020 أول $\quad$ حل متراجحة:

$\ln(x+1) < \sqrt{x+1}$

$\ln(x+1) - \sqrt{x+1} < 0$

لنعرف $g(x) = \ln(x+1) - \sqrt{x+1}$

ندرس إشارة $g(x)$: على المجال $]-1, +\infty[$

$g'(x) = \frac{1}{x+1} - \frac{1}{2\sqrt{x+1}}$

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$g'(x) = \frac{1}{x+1} - \frac{1}{2\sqrt{x+1}}$

$2 \qquad \sqrt{x+1}$

$= \frac{2-\sqrt{x+1}}{2(x+1)}$

$g'(x) = 0 \Rightarrow 2 - \sqrt{x+1} = 0$

$2 = \sqrt{x+1}$

$4 = x+1$

$x = 3$

$g(3) = \ln 4 - 2$

x	-1	3	$+\infty$
$g'(x)$		0	
$g(x)$	$\nearrow$	$\ln 4 - 2$	$\searrow$

من الجدول $g(3) = \ln 4 - 2$ قيمة كبرى $\Leftarrow$

$g(x) < g(3)$

$g(x) < 0$

$\Rightarrow \ln(x+1) - \sqrt{x+1} < 0$

$\ln(x+1) < \sqrt{x+1}$

$x < e^x$ ✱

$x - e^x < 0$

نفرض $g(x) = x - e^x$

ندرس اطراد التابع $g(x)$ على المجال $]-\infty, +\infty[$

$g'(x) = 1 - e^x$

$g'(x) = 0 \Rightarrow 1 - e^x = 0$

$e^x = 1$

$x = 0$

$g(0) = 0 - e^0 = -1$

x	$-\infty$	0	$+\infty$
$g'(x)$		0	
$g(x)$	$\nearrow$	-1	$\searrow$

$g(0) = -1$ قيمة كبرى

$\Leftarrow \; g(x) < -1$

$g(x) < 0$

$x - e^x < 0$

$x < e^x$

تكامل:

(A)

$$I=\int_0^{\ln 16}\frac{e^x+3}{e^x+4}\,dx$$

$$J=\int_0^{\ln 16}\frac{1}{e^x+4}\,dx$$

• $$I+J=\int_0^{\ln 16}\frac{e^x+3}{e^x+4}\,dx+\int_0^{\ln 16}\frac{1}{e^x+4}\,dx$$

$$=\int_0^{\ln 16}\frac{e^x+4}{e^x+4}\,dx$$

$$=\int_0^{\ln 16}1\,dx=[x]_0^{\ln 16}$$

$$=\ln 16-0=\ln 16$$

• $$I-3J=\int_0^{\ln 16}\frac{e^x+3}{e^x+4}\,dx+\int\frac{-3}{e^x+4}\,dx$$

$$=\int_0^{\ln 16}\frac{\overbrace{e^x}^{g'}}{\underbrace{e^x+4}_{g}}\,dx$$

$$=\left[\ln|e^x+4|\right]_0^{\ln 16}$$

$$=\ln(e^{\ln 16}+4)-\ln(e^0+4)$$

$$=\ln 20-\ln 5=\ln 4$$

$$\ominus\;\begin{cases} I+J=\ln 16\\ I-3J=\ln 4\end{cases}$$

$$4J=\ln 16-\ln 4$$

$$4J=\ln 4$$

$$J=\frac{\ln 4}{4}$$

$$I+\frac{\ln 4}{4}=\ln 16$$

$$I=2\ln 4-\frac{\ln 4}{4}$$

$$I=\frac{7\ln 4}{4}$$

(B)

$$I=\int_0^{\ln 2}\frac{2}{e^x+2}\,dx$$

$$J=\int_0^{\ln 2}\frac{e^x}{e^x+2}\,dx$$

(1) $$J=\int_0^{\ln 2}\frac{\overbrace{e^x}^{g'}}{\underbrace{e^x+2}_{g}}\,dx$$

$$=\left[\ln|e^x+2|\right]_0^{\ln 2}$$

$$=\ln(e^{\ln 2}+2)-\ln(e^0+2)$$

$$=\ln 4-\ln 3=\ln\left(\frac{4}{3}\right)$$

(2) $$I+J=\int_0^{\ln 2}\frac{e^x+2}{e^x+2}\,dx$$

$$=\int_0^{\ln 2}1\,dx=[x]_0^{\ln 2}=\ln 2$$

$$I + J = \ln 2$$

$$\ln\left(\frac{4}{3}\right) + J = \ln 2$$

$$J = \ln 2 - \ln\left(\frac{4}{3}\right)$$

$$* \quad J = \frac{\ln 2}{\ln 4 - \ln 3}$$

$$I = \int_0^1 \frac{x^3}{x^2+1}\,dx \quad - \boxed{C}$$

$$J = \int_0^1 \frac{x}{x^2+1}\,dx$$

$$J = \int_0^1 \frac{\frac{1}{2}\cdot 2x \rightarrow g'}{x^2+1 \rightarrow g}\,dx \quad ①$$

$$= \frac{1}{2}\left[\ln|x^2+1|\right]_0^1$$

$$= \frac{1}{2}\left[\ln 2 - \ln(1)\right)$$

$$= \frac{1}{2}\ln 2$$

$$I + J = \int_0^1 \frac{x^3+x}{x^2+1}\,dx \quad ②$$

$$= \int_0^1 \frac{x(x^2+1)}{x^2+1}\,dx$$

$$= \int_0^1 x\,dx = \left[\frac{x^2}{2}\right]_0^1$$

$$= \frac{1}{2} - 0 = \frac{1}{2}$$

$$\Rightarrow I + J = \frac{1}{2}$$

$$I + \frac{1}{2}\ln 2 = \frac{1}{2}$$

$$I = \frac{1}{2} - \frac{1}{2}\ln 2$$

كيوت :

Ⓐ

$$f(x) = e^x - 1$$

$$f(x) \leq 0 \quad \boxed{*}$$

$$e^x - 1 \leq 0$$

$$e^x \leq 1$$

$$x \leq \ln 1$$

$$x \leq 0$$

$$D' =]-\infty, 0]$$

$$\int_0^{\ln 2} f(x)\,dx \quad - \boxed{*}$$

$$= \int_0^{\ln 2} (e^x - 1)\,dx = \left[e^x - x\right]_0^{\ln 2}$$

$$= e^{\ln 2} - \ln 2 - (e^0 - 0)$$

$$= 2 - \ln 2 - 1$$

$$= 1 - \ln 2$$

$$f(x) = \frac{2x^2 + \ln x}{x} \quad Ⓑ$$

$$f(x) = 2x + \frac{\ln x}{x}$$

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① نشكل الفرق:

$$f(x) - y_\Delta = 2x + \frac{Lnx}{x} - 2x = \frac{Lnx}{x}$$

② نهاية الفرق في جوار $+\infty$:

$$\lim_{x\to+\infty}(f(x) - y_\Delta) = \lim_{x\to+\infty}\left(\frac{Lnx}{x}\right) = 0$$

إذاً: $\Delta : y = 2x$ مقارب مائل للخط C في جوار $+\infty$

③ الوضع النسبي: ندرس إشارة الفرق:

$$f(x) - y_\Delta = \frac{Lnx}{x}$$

نعدم الفرق:

$$f(x) - y_\Delta = 0 \Rightarrow Lnx = 0$$

$$\boxed{x = 1}$$

x	$-\infty$	1	$+\infty$
$f(x)-y_\Delta$	$-$	0	$+$
الوضع النسبي	C تحت Δ		C فوق Δ

④
$$S = \int_1^e (f(x) - y_\Delta)\,dx$$

$$= \int_1^e \left(\frac{Lnx}{x}\right) dx$$

$$= \int_1^e \underbrace{\frac{1}{x}}_{H'} \cdot \underbrace{Lnx}_{H}\, dx$$

$$= \left[\frac{(Lnx)^2}{2}\right]_1^e$$

$$= \frac{(Lne)^2}{2} - \frac{(Ln1)^2}{2}$$

$$= \frac{1}{2} - 0 = \frac{1}{2}$$

تكاملات على خفيف

$$I_1 = \int \frac{1}{1+e^{3x}}\,dx$$

$$= \int \frac{1+e^{3x}-e^{3x}}{1+e^{3x}}\,dx$$

$$= \int\left(\frac{1+e^{3x}}{1+e^{3x}} - \frac{e^{3x}}{1+e^{3x}}\right)dx$$

$$= \int 1 - \frac{1}{3}\cdot\frac{\overset{g'}{3e^{3x}}}{\underset{g}{1+e^{3x}}}\,dx$$

$$= \left[x - \frac{1}{3}Ln(1+e^{3x})\right]$$

$$I_2 = \int_1^e Lnx\,dx$$

نفرض $u = Lnx \Leftarrow u' = \frac{1}{x}$

$v' = 1 \Leftarrow v = x$

$$I = [u.v]_1^e - \int_1^e u'.v\,dx$$

$$= [x.Lnx]_1^e - \int_1^e \frac{1}{x}.x\,dx$$

$$I_2 = [x.Lnx]_1^e - [x]_1^e$$

$$= [x.Lnx - x]_1^e$$

$$= e\ln e - e - (1.Ln1 - 1)$$

$$= e - e - 0 + 1 = 1$$

$$I_3 = \int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$$

$$= \int \frac{1}{\sqrt{x}} . e^{\sqrt{x}}\,dx$$

$$= 2\int \underbrace{\frac{1}{2\sqrt{x}}}_{g'} . e^{\overbrace{\sqrt{x}}^{g}}\,dx$$

$$= 2e^{\sqrt{x}}$$

$$I_4 = \int \frac{1}{x.Lnx}\,dx$$

$$= \int \frac{\overbrace{\frac{1}{x}}^{g'}}{\underbrace{Lnx}_{g}}\,dx$$

$$= Ln|Lnx|$$

$$I_5 = \int x.\sqrt{x^2+1}\,dx$$

$$= \int x.(x^2+1)^{\frac{1}{2}}\,dx$$

$$= \int \frac{1}{2}.\underbrace{2x}_{H'}.\underbrace{(x^2+1)}_{H}^{\frac{1}{2}}\,dx$$

$$= \frac{1}{2}.\frac{(x^2+1)^{\frac{3}{2}}}{\frac{3}{2}}$$

$$= \frac{1}{2}.\frac{2}{3}.\sqrt{(x^2+1)^3}$$

$$= \frac{1}{3}\sqrt{(x^2+1)^3}$$

$$I_6 = \int \frac{x}{\sqrt{x^2+1}}\,dx$$

$$= \int \frac{x}{(x^2+1)^{\frac{1}{2}}}\,dx$$

$$= \int x.(x^2+1)^{-\frac{1}{2}}\,dx$$

$$= \int \frac{1}{2}.\underbrace{2x}_{H'}.\underbrace{(x^2+1)}_{H}^{-\frac{1}{2}}\,dx$$

$$= \frac{1}{2}\frac{(x^2+1)^{\frac{1}{2}}}{\frac{1}{2}}$$

$$= (x^2+1)^{\frac{1}{2}}$$

$$= \sqrt{x^2+1}$$

$$I_7 = \int \frac{Lnx}{x}\,dx$$

$$= \int \underbrace{\frac{1}{x}}_{H'}.\underbrace{Lnx}_{H}\,dx$$

$$= \frac{(Lnx)^2}{2}$$

$I_8 = \int x . e^x \, dx$

نفرض $u = x \Leftarrow u' = 1$

$v' = e^x \Leftarrow v = e^x$

$I_8 = u.v - \int u'.v \, dx$

$= x e^x - \int 1.e^x \, dx$

$= x.e^x - e^x$

$I_9 = \int x^2 . \cos x \, dx$

نفرض $u = x^2 \Leftarrow u' = 2x$

$v' = \cos x \Leftarrow v = \sin x$

$I_9 = u.v - \int u'.v \, dx$

$= x^2 . \sin x - \underbrace{\int 2x . \sin x \, dx}_{I'}$

$I' = \int 2x . \sin x \, dx$

نفرض $u = 2x \Leftarrow u' = 2$

$v' = \sin x \Leftarrow v = -\cos x$

$I' = u.v - \int u'.v \, dx$

$= -2x . \cos x + \int 2 . \cos x \, dx$

$= -2x . \cos x + \int 2 \cos x \, dx$

$= -2x \cos x + 2 \sin x$

$\Rightarrow I_9 = x^2 . \sin x - (-2x \cos x + 2 \sin x)$

$= x^2 . \sin x + 2x \cos x - 2 \sin x$

$I_{10} = \int x . \sin^2 x \, dx$

نفرض $u = x \Leftarrow u' = 1$

$v = \frac{1}{2}x - \frac{1}{4}\sin 2x \Leftarrow v' = \sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$

$I_{10} = u.v - \int u'.v \, dx$

$= x\left(\frac{1}{2}x - \frac{1}{4}\sin 2x\right) - \int \left(\frac{1}{2}x - \frac{1}{4}\sin x\right) dx$

$= \left(\frac{1}{2}x^2 - \frac{1}{4}x . \sin 2x - \frac{1}{2}\frac{x^2}{4} - \frac{1}{8}\cos 2x\right)$

$I_{11} = \int \frac{2x+1}{x^2+3x+2} \, dx$

$x^2+3x+2 = (x+2)(x+1)$ نحلل:

$\frac{2x+1}{(x+2)(x+1)} = \frac{A}{x+2} + \frac{B}{x+1}$

$(x+1) \quad (x+2)$

$\frac{2x+1}{(x+2)(x+1)} = \frac{Ax + A + Bx + 2B}{(x+2)(x+1)}$

$2x + 1 = (A+B).x + A + 2B$

$\Rightarrow A + B = 2 \quad —(1)$

$(-) \quad A + 2B = 1 \quad —(2)$

$-B = 1 \Rightarrow \boxed{B = -1}$

نعوض في ①:

$A - 1 = 2 \rightarrow A = 3$

$$\Rightarrow I_{11} = \int \left(\frac{A}{x+2} + \frac{B}{x+1} \right) dx$$

$$= \int \left(\frac{3}{x+2} - \frac{1}{x+1} \right) dx$$

$$= \int 3 \cdot \left(\frac{1}{x+2} - \frac{1}{x+1} \right) dx$$

$$= 3 \ln|x+2| - \ln|x+1|$$

$$I_{13} = \int \frac{e^x - 1}{e^x + 1} dx$$

$$= \int \frac{e^x - e^x + e^x - 1}{e^x + 1} dx$$

$$= \int \left(\frac{2e^x}{e^x+1} - \frac{(e^x+1)}{e^x+1} \right) dx$$

$$= \int \left(2 \frac{e^x}{e^x+1} - 1 \right) dx$$

$$= 2 \ln|e^x + 1| - x$$

$$I_{14} = \int x \cdot e^{x^2} dx$$

$$= \frac{1}{2} \int 2x \cdot e^{x^2} dx$$

$$= \frac{1}{2} e^{x^2}$$

$$I_{15} = \int \tan^2 3x \, dx$$

$$= \int (1 + \tan^2 3x - 1) dx$$

$$= \frac{1}{3} \tan 3x - x$$

$$I_{12} = \int \frac{x^2 - x + 1}{x - 1} dx$$

$$\begin{array}{r|l} & x \\ \hline x-1 & x^2 - x + 1 \\ & \mp x^2 \pm x \\ \hline & 1 \end{array}$$

$$I_{12} = \int \left(x + \frac{1}{x-1} \right) dx$$

$$= \frac{x^2}{2} + \ln|x-1|$$

انتهى العمل

Sam

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