Math 100

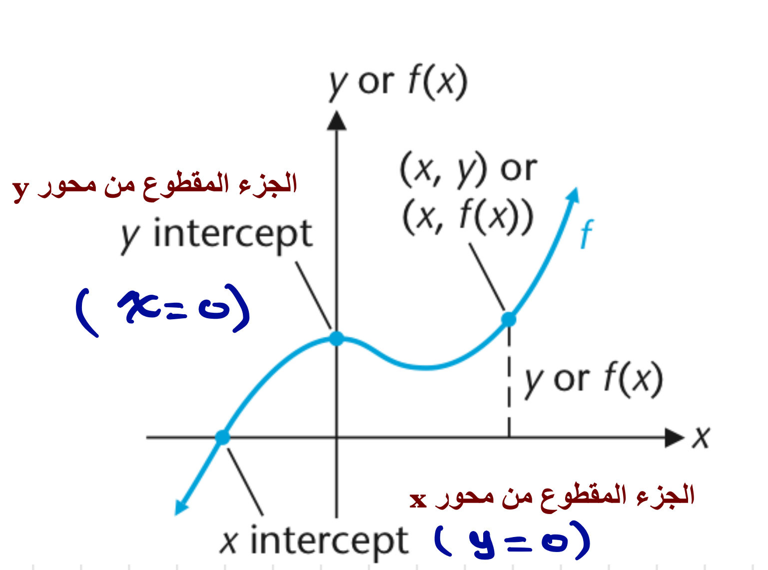
Mada Altiary

Graphing Functions

1. Intercepts of a function
2. Finding the domain &Range from a graph
3. Identifying increasing & decreasing function
4. Linear Function
5. Piecewise Functions

Graphing Function

1️⃣ Intercepts of a Function



Example: **find the domain, x intercept, y intercept of** $f(x) = \frac{4-3x}{2x+5}$

Solution:

$2x+5 = 0 \Rightarrow 2x = -5$

$\Rightarrow x = -\frac{5}{2}$

$\therefore$ Domain $= R - \{-\frac{5}{2}\}$

$= (-\infty, -\frac{5}{2}) \cup (-\frac{5}{2}, \infty)$

- x-intercept $(y=0)$

$0 = 4 - 3x$

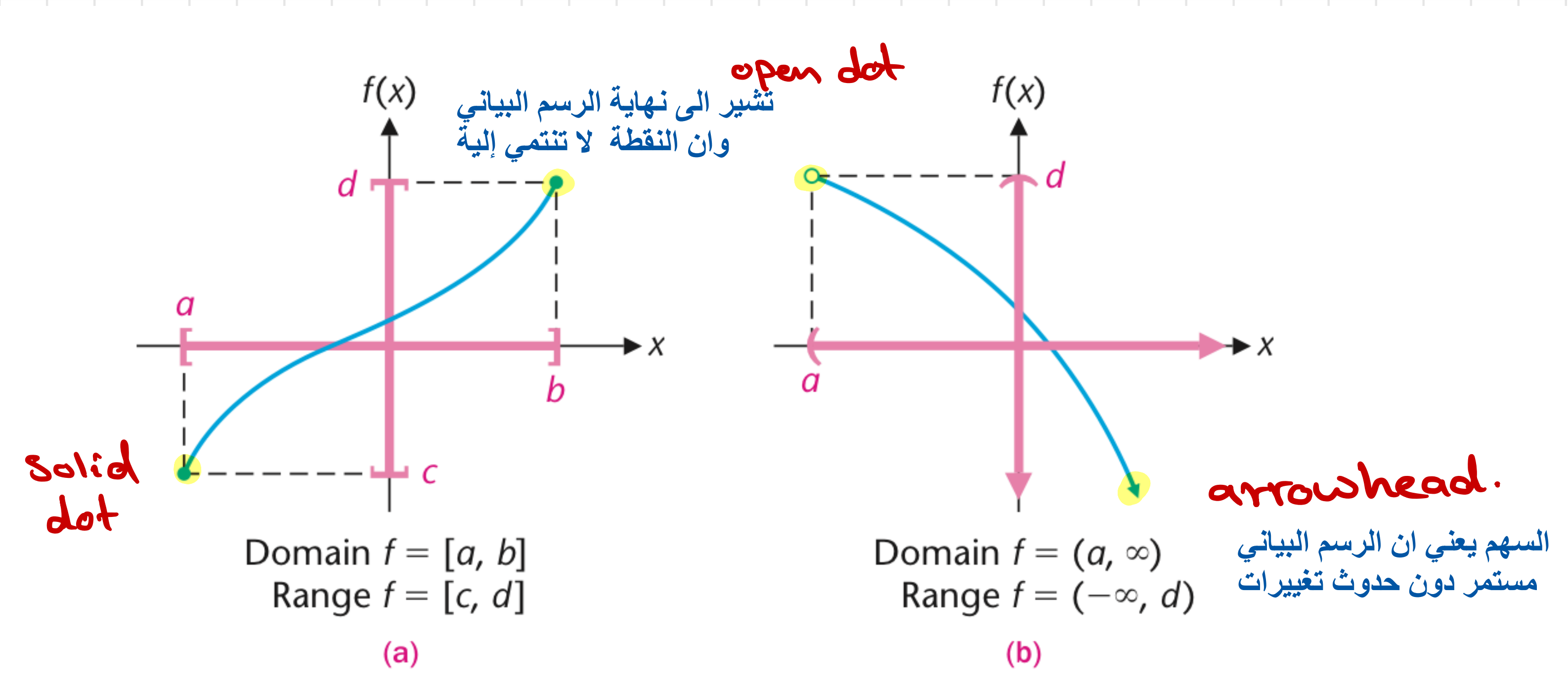
$3x = 4$

$\Rightarrow x = \frac{4}{3}$

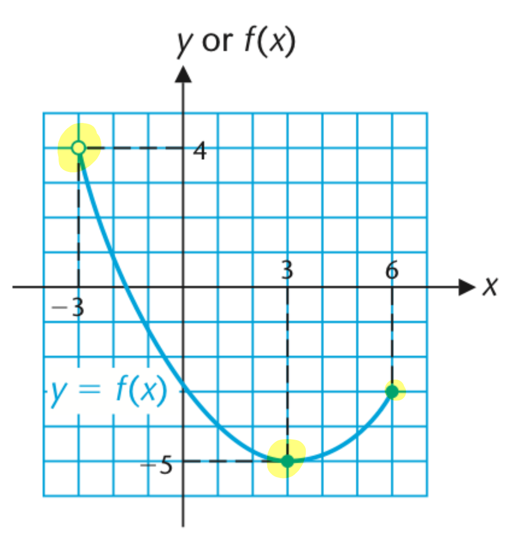
- y-intercept $(x=0)$

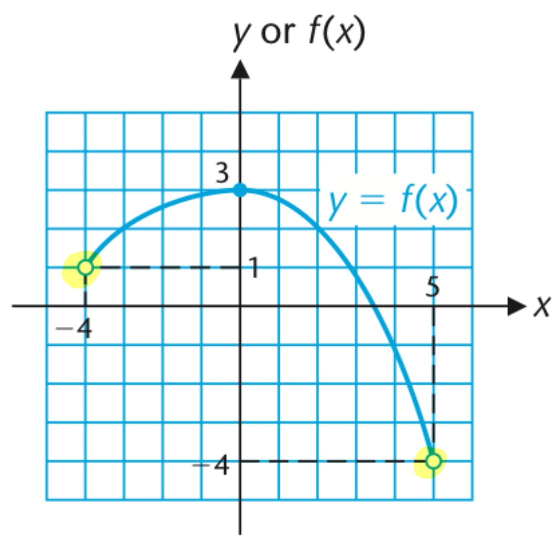
$f(0) = \frac{4-3(0)}{2(0)+5} = \frac{4}{5}$

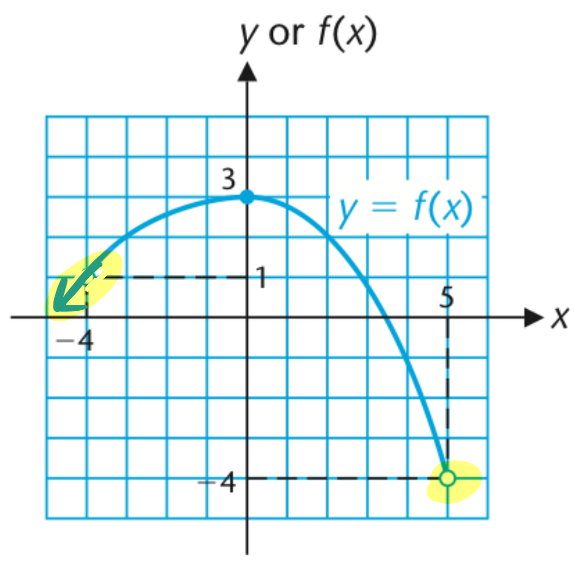
2 Finding the Domain and Range from the Graph



Example: **Find the domain and range for each graph**







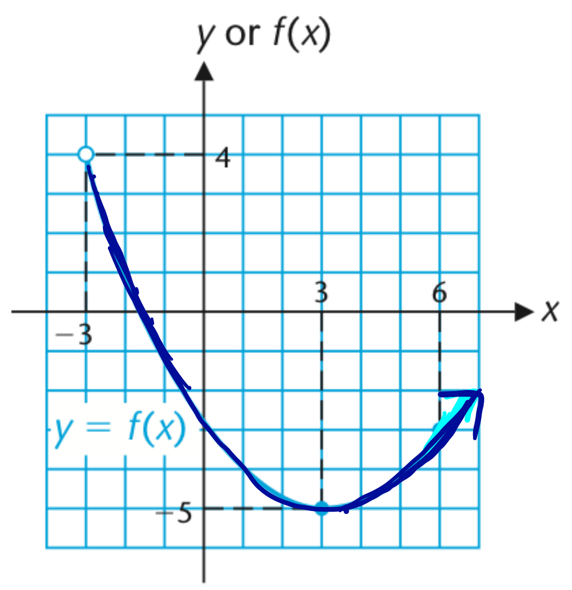
Domain = (-3, 6]
Range = [-5, 4)
f(3) = -5

Domain = (-4, 5)
Range = (-4, 3]
F(0) = 3

Domain = (-∞, 5)
Range = (-4, 3]
f(3) =

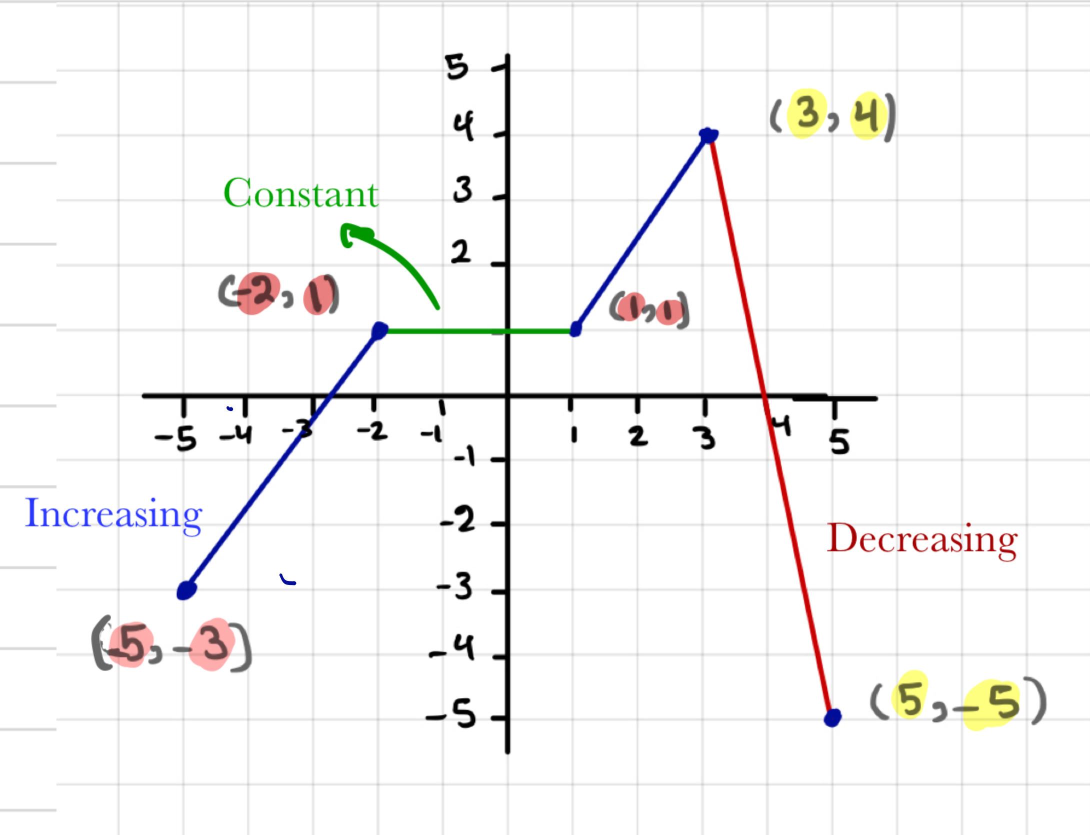
HW: **Find the domain and range for the following graph**

Find f(1), f(3), f(5).



تمارين : إيجاد المدى والمجال من التمثيل البياني

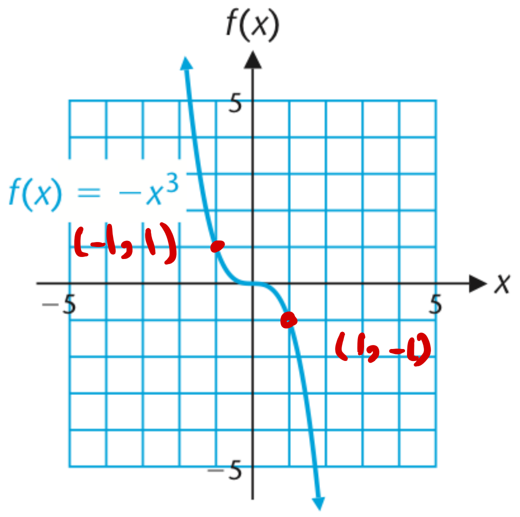
3 Identifying increasing and decreasing function.



Increasing: $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$

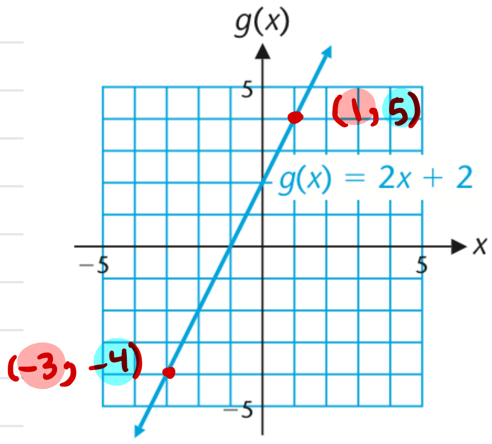
Decreasing: $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$

Constant: $x_1 < x_2 \Rightarrow f(x_1) = f(x_2)$.

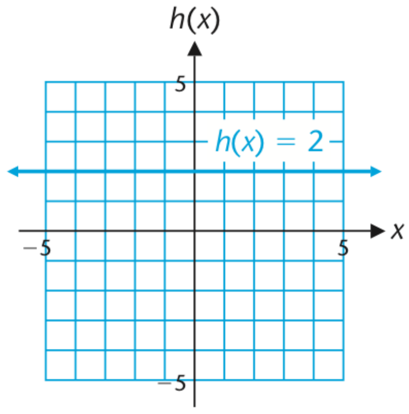


Decreasing on $(-\infty, \infty)$

(a)

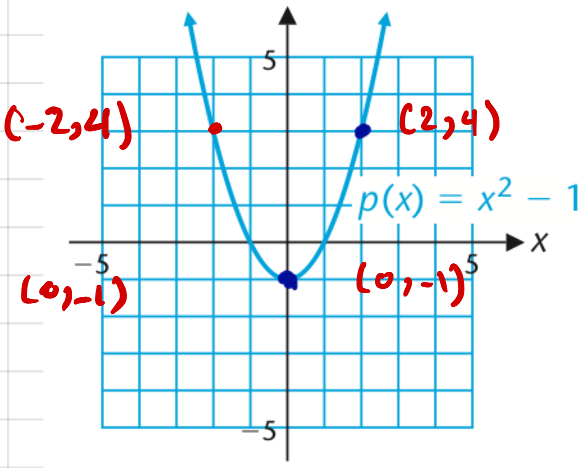


Increasing on $(-\infty, \infty)$



Constant on $(-\infty, \infty)$

(c)



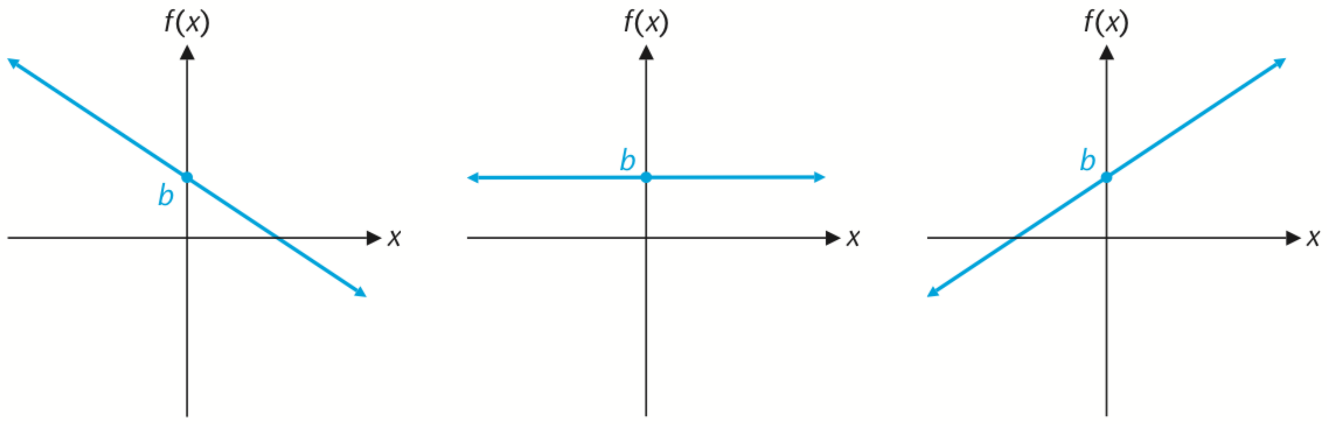
Decreasing on $(-\infty, 0]$
Increasing on $[0, \infty)$

(d)

4 Linear Function

GRAPH PROPERTIES OF $f(x) = mx + b$

The graph of a linear function is a line with slope m and y intercept b.



$m < 0$
Decreasing on $(-\infty, \infty)$
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

$m = 0$
Constant on $(-\infty, \infty)$
Domain: $(-\infty, \infty)$
Range: $\{b\}$

$m > 0$
Increasing on $(-\infty, \infty)$
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

5 Piecewise-Defined Function

Functions whose definitions involve more than one expression are called Piecwise- defined functions

Example:

The function f is defined by

$$f(x) = \begin{cases} 4x + 11 & \text{if } x < -2 \\ 3 & \text{if } -2 \le x \le 1 \\ -\frac{1}{2}x + \frac{7}{2} & \text{if } x > 1 \end{cases}$$

(A) Find $f(-3)$, $f(-2)$, $f(1)$, and $f(3)$.

(B) Graph f.

(C) Find the domain, range, and intervals where f is increasing, decreasing, or constant.

Piecewise-Defined Function

SOLUTIONS

(A) For $x < -2$, $f(x) = 4x + 11$, so

$$f(-3) = 4(-3) + 11 = -1$$

For $-2 \le x \le 1$, $f(x) = 3$, so

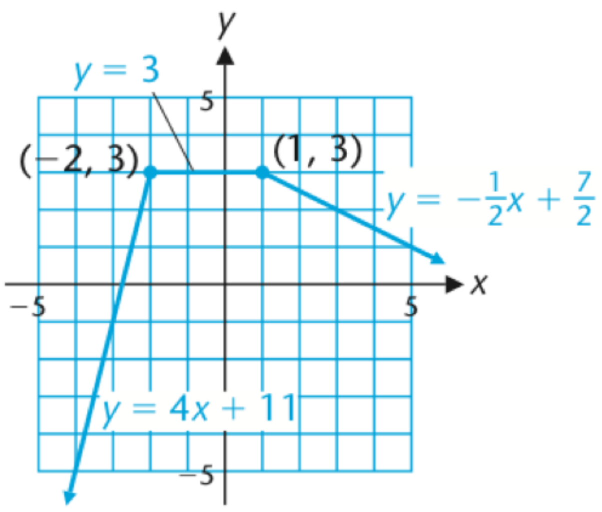
$$f(-2) = 3 \qquad \text{and} \qquad f(1) = 3$$

For $x > 1$, $f(x) = -\frac{1}{2}x + \frac{7}{2}$, so

$$f(3) = -\tfrac{1}{2}(3) + \tfrac{7}{2} = 2$$

(B) To graph f, we graph each expression in the definition of f over the appropriate interval. That is, we graph

$$\begin{aligned} y &= 4x + 11 && \text{for } x < -2 \\ y &= 3 && \text{for } -2 \le x \le 1 \\ y &= -\tfrac{1}{2}x + \tfrac{7}{2} && \text{for } x > 1 \end{aligned}$$



(C) Domain of f: $(-\infty, -2) \cup [-2, 1] \cup (1, \infty) = (-\infty, \infty)$

Range : $(-\infty, 3]$

F increasing on $(-\infty, -2)$

decreasing on $(1, \infty)$

Constant on $[-2, 1]$

Even and odd Function

Algebraically: A function is

Even : if $f(-x) = f(x)$

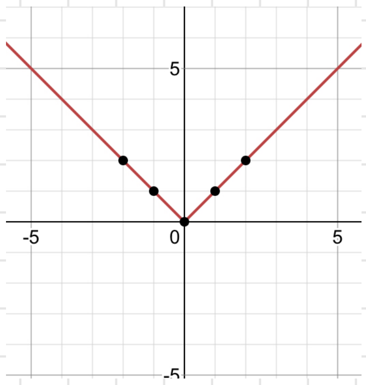
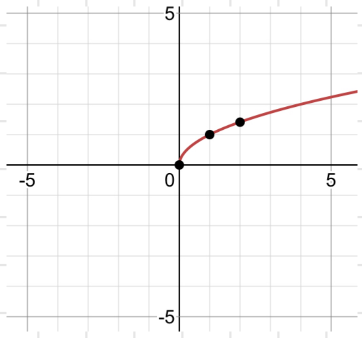
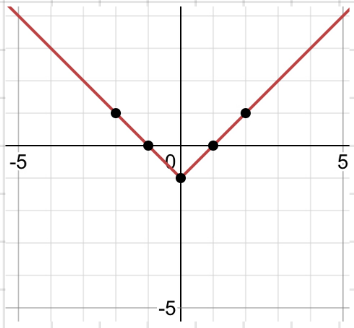
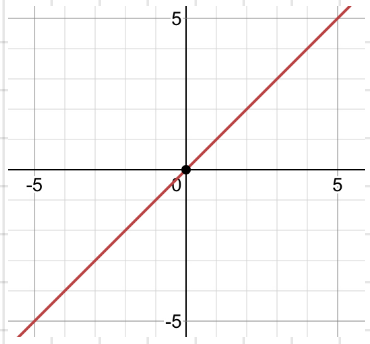
odd : if $f(-x) = -f(x)$.

Graphically:

Even function: symmetric with respect to y axis.

Odd function: symmetric with respect to origin.

Example	Solution	Comments.
$f(x) = x^2 + 1$	$f(-x) = (-x)^2 + 1$ $= x^2 + 1$ $= f(x)$ $\therefore f(x)$ is even	إذا كانت جميع أسس المتغير x زوجية فإن الدالة المعطاة زوجية ملاحظة: الحد الثابت يعتبر زوجي لأنه عبارة عن $1.x^0$ والصفر زوجي .
$f(x) = x^3 + x$	$f(-x) = (-x)^3 + (-x)$ $= -x^3 - x$ $= -(x^3 + x)$ $= -f(x)$ $\therefore f(x)$ is odd	إذا كانت جميع أسس المتغير x فردية ولا تحتوي على عدد ثابت فإن الدالة المعطاة فرديه.
$f(x) = x^4 + 3x.$	$f(-x) = (-x)^4 + 3(-x)$ $= x^4 - 3x$ $\neq f(x)$ $-f(x) = -x^4 - 3x$ $\neq f(x)$ Niether	إذا كانت أسس المتغير في الدالة المعطاة زوجي وفردي فإن الدالة لا زوجية ولا فردية.

Examples	Solutions	Comments.
$f(x) = \lvert x \rvert$	$f(-x) = \lvert -x \rvert = \lvert x \rvert = f(x)$ Even	
$f(x) = \sqrt{x}$	$f(-x) = \sqrt{-x} \neq f(x)$ $-f(x) = -\sqrt{x} \neq f(x)$ Neither	
$f(x) = \lvert x \rvert - 1$	$f(-x) = \lvert -x \rvert - 1$ $= \lvert x \rvert - 1$ $= f(x)$ Even.	
$f(x) = -x$	$f(-x) = -(-x) = -f(x)$ odd	

Remark:

$E \pm E = E$

$O \pm O = O$

$E \pm O =$ Nither

$E \times E = E$

$O \times O = E$

$E \times O = O$

$E/E = E$

$O/O = E$

$E/O = O$

Even, Odd, or Neither Worksheet

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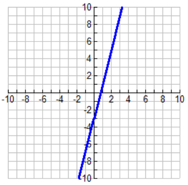
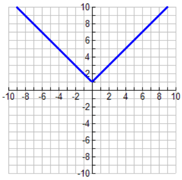
Determine whether the following functions are even, odd, or neither.

1. f(x)= 4x – 3



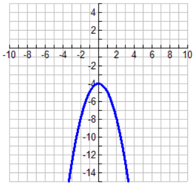
Nither

2. f(x)= $|x| + 1$



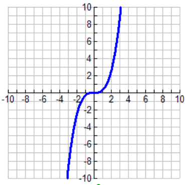
Even

3. f(x)= $-x^2 - 4$



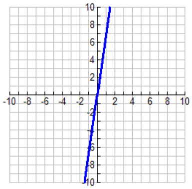
Even

4. f(x)= $\frac{1}{3}x^3$



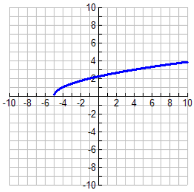
odd

5. f(x)= 7x



odd

6. f(x)= $\sqrt{x+5}$



Nither

7. f(x)= $3x^2$

Even

$f(-x) = 3(-x)^2$

$= 3x^2$

$= f(x)$

Even

8. f(x) = $x^3 - 2$

odd Even

$f(-x) = (-x)^3 - 2$

$= -x^3 - 2$

$\neq f(x)$

$-f(x) = -(x^3 - 2)$

$= -x^3 + 2$

$\neq f(x)$

Neither

9. f(x)= 3x + 4

odd Even

$f(-x) = 3(-x) + 4$

$= -3x + 4$

$\neq f(x)$

$-f(x) = -(3x + 4)$

$= -3x - 4$

$\neq f(x).$

Neither.

Even Even

10. $f(x)= x^2 - 5$

$$f(-x) = (-x)^2 - 5$$
$$= x^2 - 5$$
$$= f(x)$$

Even

odd even

11. $f(x)= 10x + 5$

$$f(-x) = 10(-x) + 5$$
$$= -10x - 5$$
$$\neq f(x)$$

$$-f(x) = -(10x+5)$$
$$= -10x - 5$$
$$\neq f(x)$$

Niether

12. $f(x)= 2(x+1)^2$

$$f(x) = 2(x^2 + 2x + 2)$$

even odd

$$= 2x^2 + 4x + 4$$

$$f(-x) = 2(-x^2) + 4(-x) + 4$$
$$= 2x^2 - 4x + 4$$
$$\neq f(x)$$

$$-f(x) = -(2x^2 + 4x + 4)$$
$$= -2x^2 - 4x - 4$$
$$\neq f(x)$$

Nither

Multiple Choice Questions

1)- whic of the following function is neither even nor odd.

a) $f(x) = 3$ b) $f(x) = x$ c) $x-1$ d) $f(x) = |x|$

2)- which of the following function is an odd function.

a) $f(x) = 3x^5$ b) $f(x) = x^2$ c) $f(x) = x^4$ d) $f(x) = 2x^8$

3)- The function $f(x) = 5$ is an even function.

a)- True b)- False.

4)- The function $f(x) = \frac{x}{x^2-1}$ is

a) even b)- odd c)- Nither