QUESTION 2: (4 points, ILO's 1.3)

Find the area of the region enclosed by the rose curve,

$$r = \cos(2\theta).$$

Solution:-

$$A = \int_{\alpha}^{\beta} \frac{1}{2} r^2 \, d\theta$$ (1points)

$$= 8 \int_{0}^{\pi/4} \frac{1}{2} \left(\cos(2\theta)\right)^2 \, d\theta$$ (1points)

$$= 4 \int_{0}^{\pi/4} \cos^2(2\theta) \, d\theta = 2 \int_{0}^{\pi/4} \left[1 + \cos(4\theta)\right] d\theta$$ (1points)

$$= 2 \left[\theta + \frac{1}{4} \sin(4\theta)\right]_{0}^{\pi/4}$$ ($\frac{1}{2}$ points)

$$= 2 \left[\frac{\pi}{4} + 0 - 0 - 0\right] = \frac{\pi}{2}$$ ($\frac{1}{2}$ points)

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N 1.
$r\cos\theta + r'$
$\sin\theta$

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QUESTION 1: (4 points, ILO's 1.3)

Find the slop of the tangent line to the curve $r = 4 - 3\sin\theta$ at $\theta = \pi$.

Solution:-

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{r\cos\theta + \frac{dr}{d\theta}\sin\theta}{-r\sin\theta + \frac{dr}{d\theta}\cos\theta}$$ (1points)

$$= \frac{(4-3\sin\theta)\cos\theta + (-3\cos\theta)\sin\theta}{-(4-3\sin\theta)\sin\theta + (-3\cos\theta)\cos\theta}$$ (1points)

At $\theta = \pi$

$$\frac{dy}{dx} = \frac{(4-3\sin\pi)\cos\pi + (-3\cos\pi)\sin\pi}{-(4-3\sin\pi)\sin\pi + (-3\cos\pi)\cos\pi}$$ (1points)

$$= \frac{(4-3(0))(-1) + (-3(-1))(0)}{-(4-3(0))(0) + (-3(-1))(-1)}$$ (1points)

$$= \frac{-4}{-3} = \frac{4}{3}$$

QUESTION 3:

(3+3 points, ILO's 1.5)

a) Find the center and the radius of the sphere whose equation is given by.

$$x^2+y^2+z^2-8x+2y-6z+1=0.$$

Solution:-

$(x^2-8x+16)+(y^2+2y+1)+(z^2-6z+9)=16+1+9-1$ (1points)

$(x-4)^2+(y+1)^2+(z-3)^2=25$ (1points)

The center is $C(4,-1,3)$ ($\frac{1}{2}$ points)

The radius $r=\sqrt{25}=5$ ($\frac{1}{2}$ points)

b) Find the direction cosines of the vector $\mathbf{v}=\mathbf{i}-2\mathbf{j}+2\mathbf{k}$

Solution:-

$\cos\alpha=\frac{v_1}{\|\bar{v}\|}=\frac{1}{\sqrt{(1)^2+(-2)^2+(2)^2}}=\frac{1}{3}\Rightarrow\alpha=1.231^R$ (1points)

$\cos\beta=\frac{v_2}{\|\bar{v}\|}=\frac{-2}{\sqrt{(1)^2+(-2)^2+(2)^2}}=\frac{-2}{3}\Rightarrow\beta=2.3005^R$ (1points)

$\cos\gamma=\frac{v_3}{\|\bar{v}\|}=\frac{2}{\sqrt{(1)^2+(-2)^2+(2)^2}}=\frac{2}{3}\Rightarrow\gamma=0.8411^R$ (1points)

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$$= \frac{(4-3\sin\theta)\cos\theta + (-3\cos\theta)\sin\theta}{-(4-3\sin\theta)\sin\theta + (-3\cos\theta)\cos\theta}$$ (1points)

At $\theta = \pi$

$$\frac{dy}{dx} = \frac{(4-3\sin\pi)\cos\pi + (-3\cos\pi)\sin\pi}{-(4-3\sin\pi)\sin\pi + (-3\cos\pi)\cos\pi}$$ (1points)

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