Page: 253 / /

A random variable is a variable whose values are determined by Chance.

If a variable can assum only a specific number of values, Such as the outcomes for the roll of a die or the outcomes for the toss of coin, then the variable is called a discrete variable.

Variable that can assum all values in the interval between any two given values are called Continuous variable.

Page: 254

A discrete probability distribution consists of the values a random variable can assume and the corresponding probabilities are determined theoretically or by observation.

Random variable (x)			
Probability P(x)			

Exp 5-1

Exp 5-2

Page: 255

Exp 5-3

Page: 257

Two Requirements for a probability Distribution:

1. The sum of the probabilities of all the events in the sample space must equal 1; that is, $\Sigma P(x) = 1$

2. The Probability of each event in the sample space must be between or equal to 0 and 1. That is, $0 \leqslant P(x) \leqslant 1$

Page: 257 / /

Exp 5-4

Page: 260

Formula for the mean of a probability distribution:

The mean of a random variable with a discrete probability distribution is:

$$\mu = X_1 \cdot P(X_1) + X_2 \cdot P(X_2) + X_3 \cdot P(X_3) + \dots + X_n \cdot P(X_n)$$
$$= \Sigma X \cdot P(X)$$

where $X_1, X_2, X_3, \dots, X_n$ are the outcomes and $P(X_1), P(X_2), P(X_3), \dots, P(X_n)$ are the corresponding probabilities.

note: $\Sigma X \cdot P(X)$ means to sum the products.

Exp 5-5

Solution:

	X	1	2	3	4	5	6	Σ	← ①
	P(X)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	1	← ②
③	X·P(X)	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	$\frac{6}{6}$	$\frac{21}{6}$	← ① x ② = ③

$$\therefore \mu = \frac{21}{6} = \frac{7}{2} = 3.5$$

Page: 261

Exp 5-6

Exp 5-7

Exp 5-8

Page: 262

Formula for the variance of a probability Distribution:

Find the variance of a probability distribution by multiplying the square of each outcome by its corresponding probability, summing those products, and subtracting the square of the mean.

The variance $\Rightarrow \sigma^2 = \Sigma[X^2 \cdot P(X)] - \mu^2$

The standard deviation $\Rightarrow \sigma = \sqrt{\sigma^2}$ or $\sqrt{\Sigma[X^2 \cdot P(X)] - \mu^2}$

Exp 5-9.

Page: 263 / /

Exp 5-10
Exp 5-11

Page: 264
Expectation: The expected value of a discrete random variable of a probability distribution is the theoretical average of the variable. The formula is:

$$\mu = E(X) = \sum X \cdot P(X)$$

note: The symbol $\underline{\underline{E(x)}}$ is used for the expected value.
imp: The expected value = The mean

Exp 5-12
Exp 5-13

Page: 271
A binomial experiment is a probability experiment that satisfies the following four requirements:

1. There must be a fixed number of trials.
2. Each trial can have only two outcomes or outcomes that can be reduced to two outcomes. These outcomes can be considered as either success or failure.
3. The outcomes of each trial must be independent of one another.
4. The probability of a success must remain the same for each trial.

The outcomes of a binomial experiment and the corresponding probabilities of these outcomes are called binomial distribution.

Page: 271

Notation for the Binomial Distribution:

$P(S)$ The symbol for the probability of success

$P(F)$ The symbol for the Probability of failure

p The numerical probability of a Success

q The numerical probability of a failure

$P(S) = p$

$P(F) = 1 - p = q$

$P(x) = C^n_x \, p^x \, q^{n-x}$

n = The number of trials

X = The number of successes in $\underline{n}$ trials

note that $0 \leqslant X \leqslant n$

$X = 0, 1, 2, 3, \dots, n.$

Page: 272

Exp 5-15

Solution: $S = \{HHH, \underline{HHT}, \underline{HTH}, HTT, ~~TTT~~, TTH, \underline{THH}\ THT\}$

$P(2H) = \frac{3}{8}$

$n = 3$

$x = 0, 1, \underline{\underline{2}}, 3$

$p = \frac{1}{2}$

$q = 1 - p$

$= 1 - \frac{1}{2}$

$= \frac{1}{2}$

$P(x=2) = C^3_2 \cdot \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^1$: بالآلة

$= 3 \cdot \frac{1}{4} \cdot \frac{1}{2} = \frac{3}{8}$

Exp 5-16

Solution:

$n = 10$

$x = 0, 1, 2, \underline{\underline{3}}, 4, 5, 6, 7, 8, 9, 10$

$P = \frac{1}{5}$

$q = 1 - \frac{1}{5} = \frac{4}{5}$

$(x=3)$

$P(\ ) = C^{10}_3 \cdot \left(\frac{1}{5}\right)^3 \cdot \left(\frac{4}{5}\right)^7 =$

$= 0.201$

Page: 273 / /

Exp 5-17

Solution:

$n = 5$

$X = 0, 1, 2, \underline{\underline{3}}, \underline{\underline{4}}, \underline{\underline{5}}$

$P = 0.30$

$q = 1 - 0.30 = 0.70$

$P(X \geq 3) = P(X=3) + P(X=4) + P(X=5)$

$= C^5_3 \cdot (0.30)^3 (0.70)^2 + C^5_4 (0.30)^4 \cdot (0.70)^1 + C^5_5 \cdot (0.30)^5 (0.70)^0$

$= 0.132 + 0.028 + 0.002$

$= 0.162$

ملاحظة: أي عدد توافيق له نفس الحدين يساوي واحد ($C^5_5 = 1$)

Page: 274

Mean, Variance, Standard Deviation for the Binomial Distribution:

Mean: $\mu = n \cdot p$

Variance: $\sigma^2 = n \cdot p \cdot q$

Standard deviation: $\sigma = \sqrt{\sigma^2} = \sqrt{n \cdot p \cdot q}$

Page: 275

Exp 5-21

Exp 5-22

Page: 276

Exp 5-23

Page: 300

A continuous variable can assum all values between any two given values of the variables.

Many continuous variables, such as the examples just mentioned, have distribution that are bell-shaped, and these are called approximately normally distributed variables.

Page: 300 - 301 / /

This distribution is also Known as a bell curve or a Gaussian distribution.

Page: 302

The shape and position of a normal distribution curve depend on two parameters, The mean and the standard deviation.

Page: 303

Normal distribution: A normal distribution is a ① Continuous, ② Symmetric, ③ bell-Shaped distribution of variable.

Summary of the Properties of the Theoretical Normal Distribution

1. A normal distribution curve is bell-shaped.
2. The mean, median, and mode are equal and are located at the center of the distribution.
3. A normal distribution curve is unimodal (i.e., it has only one mode).
4. The curve is symmetric about the mean, which is equivalent to saying that its shape is the same on both sides of a vertical line passing through the center.
5. The curve is continuous; that is, there are no gaps or holes. For each value of *X*, there is a corresponding value of *Y*.
6. The curve never touches the *x* axis. Theoretically, no matter how far in either direction the curve extends, it never meets the *x* axis—but it gets increasingly closer.
7. The total area under a normal distribution curve is equal to 1.00, or 100%. This fact may seem unusual, since the curve never touches the *x* axis, but one can prove it mathematically by using calculus. (The proof is beyond the scope of this textbook.)
8. The area under the part of a normal curve that lies within 1 standard deviation of the mean is approximately 0.68, or 68%; within 2 standard deviations, about 0.95, or 95%; and within 3 standard deviations, about 0.997, or 99.7%. See Figure 6–5, which also shows the area in each region.

Page: 304

Figure 6-5

The standard normal distribution is a normal distribution with a mean of zero = 0 and a standard deviation of one = 1

تسمى تحويلة z $\Rightarrow z = \frac{X - \mu}{\sigma}$

Page: 306

الجدول السالب و الموجب

مهم : الجداول صفحة ٧٨٤ و ٧٨٥ لحل الأمثلة.

~~Solution:~~

Page: 306 / /

$P(z<a) \cdot 100\%$

$P(z<a) \cdot N$

خطوات الحل:

١. التحويل من كلام إلى رموز.

٢. تحويل x إلى z . $z = \frac{x-\mu}{\sigma}$.

٣. التأكد بأنها على الصيغة $P(z<a)$.

a. $P(z<a)$ $\Leftarrow$ مباشرة "من الجدول.

b. $P(z>a)$ $\Leftarrow$ $1 - P(z<a)$.

c. $P(a<z<b)$ $\Leftarrow$ $P(z<b) - P(z<a)$.

٤. إيجاد الاحتمال من الجدول السالب أو الموجب.

784 785

Exp 6-1

Solution: $z = 2.06$

$P(z<2.06) = 0.9803$

Exp 6-2

Solution: $z = -1.19$

$P(z>-1.19)$

$= 1 - P(z<-1.19)$

$= 1 - 0.1170$

$= 0.8830$

Page: 307

Exp 6-3 $z = +1.68$ and $z = -1.37$

Solution:

$P(-1.37 < z < 1.68)$

$P(z<1.68) - P(z<-1.37)$

$0.9535 - 0.0853 = 0.8682$

$\therefore 0.8682 \times 100 = 86.82\%$

Page: 317

Exp 6-6

Solution:

$\mu = 146.21$	$P(X < 160) \cdot 100\%$	$P(X < 160) = P\left(z < \frac{160-146.21}{29.44}\right)$
$\sigma = 29.44$	$= (0.6808)(100)$	$= P(z < 0.47)$
$X \sim N$	$= 68.08\%$	$= 0.6808$
X = spending		

Exp 6-7

Page: 319

Exp 6-8

Page: 331

A Sampling distribution of Sample means is a distribution using the means computed from all possible random samples of a specific size taken from a population.

Sampling error is the difference between the sample measure and the corresponding population measure due to the fact that the sample is not a perfect representation of the population.

Properties of the Distribution of Sample means:

1. The mean of the sample means will be the same as the population mean. $\mu_{\bar{x}} = \mu$

2. The standard deviation of the sample means will be smaller than the standard deviation of the population, and it will be equal to the population standard deviation divided by the square root of the sample size.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Page: 333 / /

The Central limit Theorem:

As the sample size $\underline{n}$ increases without limit, the shape of the distribution of the sample means taken with replacment from a population with mean μ and standard deviation σ will approach a normal distribution. As previously shown, this distribution will have a mean $\underline{\mu}$ and a standard deviation $\underline{\sigma / \sqrt{n}}$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

Page: 334

Exp 6-13

Solution:

$\mu = 25$	$P(\bar{x} > 26.3)$	$= 1 - P(z < 1.94)$
$\sigma = 3$	$P\left(z > \frac{26.3 - 25}{3 / \sqrt{20}}\right)$	$= 1 - 0.9738$
$n = 20$	$P(z > 1.94)$	$= 0.0262$

Page: 335

Exp 6-14

Solution:

$\mu = 96$	$P(90 < \bar{x} < 100)$	$P(z > 1.5) - P(z < -2.25)$
$\sigma = 16$	$P\left(\frac{90 - 96}{16 / \sqrt{36}} < z < \frac{100 - 96}{16 / \sqrt{36}}\right)$	$= 0.9332 - 0.0122$
$n = 36$		$= 0.9210$
	$P(-2.25 < z < 1.5)$	

Page: 336

Exp 6-15

Solution:

$\mu = 218.4$

$\sigma = 25$

$n = 40$

a. $P(x < 224) = P\left(z < \frac{224 - 218.4}{25}\right) = P(z < 0.22) = 0.5871$

b. $P(\bar{x} < 224) = P\left(z < \frac{224 - 218.4}{25 / \sqrt{40}}\right) = P(z < 1.42) = 0.9222$