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موضوع الدرس

Sec 4.3 Zeros of Polynomial Functions

Factor theorem: For any polynomial f,
$x-k$ is a factor of $f \Leftrightarrow f(k)=0$

HW 1 130) $f(x)=6x^3+19x^2+2x-3$ if $x=-3$ is a zero of f

$$\begin{array}{r|rrrr} & x^3 & x^2 & x & \text{cons} \\ -3 & 6 & 19 & 2 & -3 \\ & & -18 & -3 & 3 \\ \hline & 6 & 1 & -1 & 0 \end{array}$$

$$\frac{f(x)}{x+3}=6x^2+x-1$$

$$f(x)=(x+3)(6x^2+x-1)$$

$$f(x)=(x+3)(2x+1)(3x-1)$$

Page 136: (3) $x-1$ is a factor of x^3-5x^2+3x+1
because the remainder is zero

$$\begin{array}{r|rrrr} & x^3 & x^2 & x & \text{Cons} \\ 1 & 1 & -5 & 3 & 1 \\ & & 1 & -4 & -1 \\ \hline & 1 & -4 & -1 & \boxed{0} \end{array}$$

(11) $f(x)=6x^3+25x^2+3x-4$, $k=-4$

Solution = $f(-4)=0$

$$\begin{array}{r|rrrr} & x^3 & x^2 & x & \text{cons} \\ -4 & 6 & 25 & 3 & -4 \\ & & -24 & -4 & 4 \\ \hline & 6 & 1 & -1 & 0 \end{array}$$

$$\frac{f(x)}{x+4}=6x^2+x-1$$

$$f(x)=(x+4)(2x+1)(3x-1)$$

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Sec 7.3 Continuity

(6/220) $\frac{x^2-4}{x-2}$ is not cont at 2

$\frac{x^2-4}{x-2} \to \frac{(x+2)(x-2)}{x-2}$ let $g(x) = x+2$ is an extension cont to f

P20 (8/220) $f(x) = \begin{cases} x^2, & x \le 2 \\ k-x^2, & x > 2 \end{cases}$

Find k such that f is cont

$\lim_{x \to 2^-} f(x) = 2^2 = 4$

$\lim_{x \to 2^+} f(x) = k - 2^2 = k-4$

$\boxed{k-4=4 \Rightarrow k=8}$

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Sec 7.3 Continuity

Hw3 215

(a) $3x^2-2x$ is cont on R (polynomial)

(b) $\frac{x-2}{x^2-4}$ is cont on $R\setminus\{-2,2\}$

(c) $|x^2-1|$ is cont on R

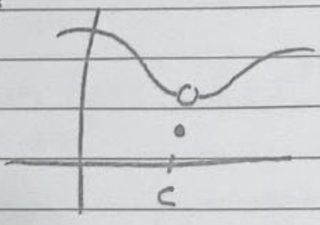
(d) $\sqrt{x}$ cont on $[0,\infty)$

(e) $\sqrt{x^2-2x-5} \Rightarrow x = \frac{2\pm\sqrt{4-4\cdot\cdot 5}}{2} = 1\pm\sqrt{6}$ $+\quad 1-\sqrt{6}\quad -\quad 1+\sqrt{6}\quad +$

is cont on $(-\infty, 1-\sqrt{6}]\cup[1+\sqrt{6},\infty)$

(f) $\frac{|x|}{\sqrt{|x+2|}}$ is cont on $R\setminus\{-2\}$

Ex



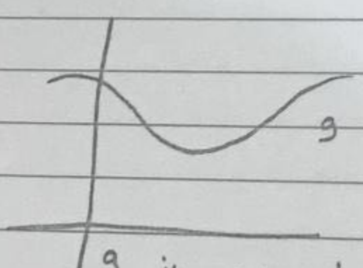
f is discontinuous at c

Hw4 216 $g(x) = \begin{cases} x, & x \neq 2 \\ 1, & x = 2 \end{cases}$

$\lim_{x\to 2} g(x) = \lim_{x\to 2} (x) = 2$

but $g(2) = 1$

$\lim_{x\to 2} g(x) \neq g(2)$



g is an extension continuity to f

Let $f(x) = \begin{cases} x, & x \neq 2 \\ 2, & x = 2 \end{cases} = x$ ⟵ discont at 2

is an extension cont to g

Sec 2.2 Limits at infinity and infinite limits موضوع الدرس / / التاريخ

Ex $\lim_{x\to\infty} \sqrt{x^2+x} - x \overset{\infty-\infty}{\Longrightarrow} \lim_{x\to\infty} \frac{(\sqrt{x^2+x}-x)(\sqrt{x^2+x}+x)}{\sqrt{x^2+x}+x}$

$= \lim_{x\to\infty} \frac{(x^2+x)-x^2}{\sqrt{x^2+x}+x} = \lim_{x\to\infty} \frac{x}{\sqrt{\frac{x^2}{x^2}+\frac{x}{x^2}}+\frac{x}{x}} = \lim_{x\to\infty} \frac{1}{\sqrt{1+\frac{1}{x}}+1} = \frac{1}{2}$

Ex $\lim_{x\to-\infty} \frac{-x^3+2x+1}{3x^2+5x-8} = \lim_{x\to-\infty} \frac{-x+\frac{2}{x}+\frac{1}{x^2}}{3+\frac{5}{x}-\frac{8}{x^2}} = \frac{\infty+0+0}{3+0-0} = \infty$

نقسم على x^2

Sec V.2 Limits at infinity and infinite Limits — موضوع الدرس — التاريخ / /

page 211 ① $\lim_{x \to \infty} \frac{x}{2x-3} = \frac{1}{2}$

② $\lim_{x \to \infty} \frac{3x^3 - 5x^2 + 7}{8 + 2x - 5x^3} = \frac{3}{-5} = \frac{-3}{5}$ ③ $= 0$ ④ $\frac{3}{-1} = -3$

⑤ $\lim \frac{2x-1}{\sqrt{3x^2+x+1}} = \lim_{x \to -\infty} \frac{2x}{\sqrt{3x^2}}$

مهم

$= \lim_{x \to -\infty} \frac{2x}{\sqrt{3}\sqrt{x^2}} = \lim_{x \to -\infty} \frac{2x}{\sqrt{3}\,|x|} = \lim_{x \to -\infty} \frac{2x}{\sqrt{3}(-x)} = \frac{-2}{\sqrt{3}}$

ووضعنا القيمة المطلقة بالسالب لأنه $-\infty$ ، لو كانت ∞ نضعها موجب

⑥ $\lim_{x \to 3} \frac{1}{3-x}$ DNE ⑦ $\lim_{x \to 3^-} \frac{1}{3-x} = \infty$

⑩ $\lim_{x \to 2^+} \frac{x}{(2-x)^3} = -\infty$ ⑪ $\lim_{x \to 1^+} \frac{1}{|x-1|} = \infty$

⑫ $\lim_{x \to 2} \frac{x-3}{x^2-4x+4} = \lim_{x \to 2} \frac{x-3}{(x-2)^2} = -\infty$

$x=2$ is vertical asymptote

⑬ $\infty \Leftarrow (5 > 3)$ ⑯ $\lim_{x \to \infty} \frac{1}{\sqrt{x^2-2x} - x} = \frac{\sqrt{x^2-2x} + x}{(x^2-2x) - x^2}$

$= \lim_{x \to \infty} \frac{\sqrt{x^2(1-\frac{2}{x})} + x}{-2x} = \lim_{x \to \infty} \frac{x\left(\sqrt{1-\frac{2}{x}} + 1\right)}{-2x}$

$= \frac{\sqrt{1-0} + 1}{-2} = -1$

التاريخ / /

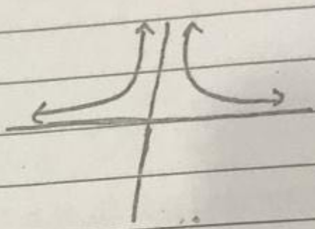
موضوع الدرس Sec 2.2 Limits at infinity and infinite limits

Hw 1 / 208 $\lim_{x \to \pm\infty} \frac{2x^2 - x + 3}{3x^2 + 5} = \frac{2}{3}$

Hw 2 / 208 $\lim_{x \to \pm\infty} \frac{5x + 2}{2x^3 - 1} = 0$

Hw 3 / 209 $\lim_{x \to 0^+} \frac{1}{x^2} = \infty$

$\lim_{x \to 0^-} \frac{1}{x^2} = \infty$



$\rightarrow \lim_{x \to 0} \frac{1}{x^2} = \infty$

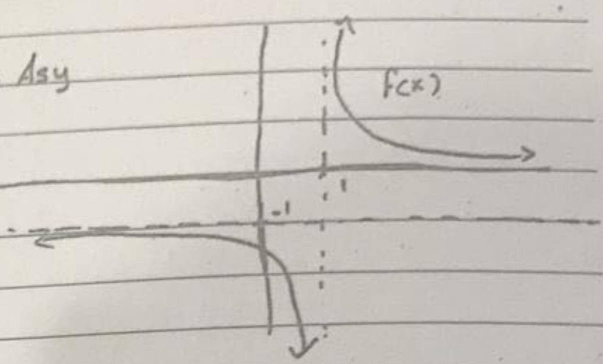
Hw 4 / 210 (a) $\lim_{x \to \infty} 3x^3 - x^2 + 2 \overset{\infty - \infty}{\Longrightarrow} \lim_{x \to \infty} x^3 \left(3 - \frac{1}{x} + \frac{2}{x^3}\right) = \infty(3 - 0 + 0) = \infty$

(b) $\lim_{x \to -\infty} 3x^3 - x^2 + 2 = -\infty - \infty + 2 = -\infty$

(c) ~~x~~
(d) $\rightarrow$ Homework

① $\lim_{x \to -\infty} f(x) = -1 \to y = -1$ Hor. Asy

② $\lim_{x \to \infty} f(x) = 0 \to y = 0$ Hor. Asy



③ $\lim_{x \to 1^+} f(x) = \infty$

④ $\lim_{x \to 1^-} f(x) = -\infty$

$\to x = 1$ Ver. Asy

Ex $\lim_{x \to 2^+} \frac{1}{x-2} = +\infty$

Ex $\lim_{x \to 2^-} \frac{1}{x-2} = -\infty$

Ex $\lim_{x \to \infty} \frac{1}{x-2} = 0$

Arithmatics	Undeterminate forms
① $\infty + \infty = \infty$	① $\frac{0}{0}$
② $-\infty - \infty = -\infty$	② $\frac{\pm\infty}{\pm\infty}$
③ $\infty \cdot \infty = \infty$	③ $\infty - \infty$
④ $-\infty \cdot \infty = -\infty$	④ $0^0, \infty^0, 1^\infty$
⑤ $-\infty \cdot -\infty = \infty$	⑤ $0 \cdot \infty$
⑥ $\frac{c}{0} = \pm\infty, c \neq 0$	
⑦ $\frac{c}{\pm\infty} = 0$	

Fact $p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots a_0$
and $q(x) = b_m x^m + b_{m-1} x^{m-1} + \cdots + b_0$
with $a_n \cdot b_m > 0$, Then $\lim_{x \to \infty} \frac{p(x)}{q(x)} = \begin{cases} \infty, & n > m \\ 0, & n < m \\ \frac{a_n}{b_m}, & n = m \end{cases}$

and $\lim_{x \to -\infty} \frac{p(x)}{q(x)} = \begin{cases} \infty, & n-m \text{ is even} \\ -\infty, & n-m \text{ is odd} \\ 0, & n < m \\ \frac{a_n}{b_m}, & n = m \end{cases}$

Page 206

(13) $\lim_{t\to 0} \frac{t}{\sqrt{4+t}-\sqrt{4-t}} = \lim_{t\to 0} \frac{t(\sqrt{4+t}+\sqrt{4-t})}{(\sqrt{4+t}-\sqrt{4-t})(\sqrt{4+t}+\sqrt{4-t})}$

$= \lim_{t\to 0} \frac{t(\sqrt{4+t}+\sqrt{4-t})}{(4+t)-(4-t)} = \lim_{t\to 0} \frac{\sqrt{4+t}+\sqrt{4-t}}{2} = 2$

(15) $\lim_{y\to 1} \frac{y-4\sqrt{y}+3}{y^2-1} = \lim_{y\to 1} \frac{(\sqrt{y}-1)(\sqrt{y}-3)}{(y-1)(y+1)} = \lim_{y\to 1} \frac{(\sqrt{y}-1)(\sqrt{y}-3)}{(\sqrt{y}-1)(\sqrt{y}+1)(y+1)}$

$= \frac{-2}{4} = -\frac{1}{2}$

(17) $\lim_{x\to 2} \frac{1}{x-2} - \frac{4}{x^2-4} = \lim_{x\to 2} \frac{x+2}{x^2-4} - \frac{4}{x^2-4} = \lim_{x\to 2} \frac{x-2}{(x-2)(x+2)}$

$= \lim_{x\to 2} \frac{1}{x+2} = \frac{1}{4}$

(22) $\lim_{x\to 2^-} \sqrt{2-x} = 0$

(26) $\lim_{x\to a^-} \frac{|x-a|}{x^2-a^2} = \lim_{x\to a^-} \frac{-(x-a)}{(x-a)(x+a)} = \frac{-1}{2a}$

(28) $f(x) = \begin{cases} x-1, & x \le -1 \\ x^2+1, & -1 < x \le 0 \\ (x+\pi)^2, & x > 0 \end{cases}$

$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} x-1 = -2$

(29) $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (x+\pi)^2 = \pi^2$

Ex $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x^2+1 = 1 \Rightarrow \lim_{x\to 0} f(x)$ DNE

(30) $\lim_{x\to 4} f(x) = 2$, $\lim_{x\to 4} g(x) = -3$

(a) $\lim_{x\to 4} (g(x)+3) = -3+3 = 0$

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(b) $\lim_{x\to 4} x\, f(x) = (4)(2) = 8$

Hw 4 / 203 $g(x) = \sqrt{1-x^2} \Rightarrow 1-x^2 \geq 0 \Rightarrow (1-x)(1+x) \geq 0$

$- - \underset{-1}{|} + + \underset{+1}{|} - - \Rightarrow Dom\, g = [-1, 1]$

$\lim_{x \to 1^+} g(x)$ DNE , $\lim_{x \to -1^-} g(x)$ DNE

$\lim_{x \to 1^-} g(x) = \sqrt{1-1^2} = 0$, $\lim_{x \to -1^+} g(x) = \sqrt{1-(-1)^2} = 0$

Read theorem 2, Page 203

Squeeze theorem

If $f(x) \leq g(x) \leq h(x)$ on interval containing c and $\lim_{x \to c} f(x) = \lim_{x \to c} h(x) = L$, then $\lim_{x \to c} g(x) = L$.

Hw 5 / 205 $3 - x^2 \leq u(x) \leq 3 + x^2$

$\lim_{x \to 0} 3 - x^2 = 3$, $\lim_{x \to 0} 3 + x^2 = 3$

$\Rightarrow \lim_{x \to 0} u(x) = 3$

Page 205 (7) $\Rightarrow \lim_{x \to 3} \frac{(x-3)^2}{(x-3)(x+3)} = \frac{x-3}{x+3} = \frac{0}{6} = 0$

(8) $\Rightarrow$ DNE (9) $\Rightarrow \lim_{x \to 9} \frac{\sqrt{x} - 3}{(\sqrt{x}-3)(\sqrt{x}+3)} = \lim_{x \to 9} \frac{1}{\sqrt{x}+3} = \frac{1}{6}$

(11) $\Rightarrow \frac{2}{-2} = -1$ Ex $\frac{|x-2|}{x-2} = \begin{cases} 1, & x > 2 \\ -1, & x < 2 \end{cases}$

$\lim_{x \to 2^+} f(x) = 1$ $\lim_{x \to 2^-} f(x) = -1$ $\Rightarrow \lim_{x \to 2} f(x)$ DNE

Sec 7.1 Limits of Functions موضوع الدرس / / التاريخ

HW2 201 (a) $\lim_{x\to -2} \frac{x^2+x-2}{x^2+5x+6} \overset{0}{\underset{0}{=}} \lim_{x\to -2} \frac{(x+2)(x-1)}{(x+2)(x+3)}$

$= \lim_{x\to -2} \frac{x-1}{x+3} = \frac{-3}{1} = -3$

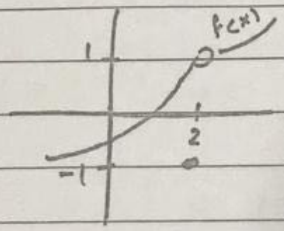
(b) $\lim_{x\to a} \frac{\frac{1}{x}-\frac{1}{a}}{x-a} \overset{0}{\underset{0}{=}} \lim_{x\to a} \frac{\frac{a-x}{xa}}{x-a} = \lim_{x\to a} \frac{-(x-a)}{xa(x-a)} = \lim_{x\to a} \frac{-1}{xa} = \frac{-1}{a^2}$

(c) $\lim_{x\to 4} \frac{\sqrt{x}-2}{x^2-16} \overset{0}{\underset{0}{=}} \lim_{x\to 4} \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{(x-4)(x+4)(\sqrt{x}+2)}$

$= \lim_{x\to 4} \frac{x-4}{(x-4)(x+4)(\sqrt{x}+2)} = \lim_{x\to 4} \frac{1}{(x+4)(\sqrt{x}+2)} = \frac{1}{32}$

ضرب بالمرافق

Ex



$\lim_{x\to 2} (f(x)) = 1$

$f(2) = -1$

دالة منفصلة

الصورة لاتساوي الغاية

HW3 202 $\text{sgn}(x) = \frac{x}{|x|} = \begin{cases} 1, & x>0 \\ -1, & x<0 \\ \text{Und}, & x=0 \end{cases}$

$\lim_{x\to 0^+} \text{sgn}(x) = 1, \quad \lim_{x\to 0^-} \text{sgn}(x) = -1$

$\to \lim_{x\to 0} \text{sgn}(x)$ DNE

x نطلع من اليسار -1 ومن اليمين 1

Sec 7.1 Limits of Functions موضوع الدرس / / التاريخ

① $\lim\limits_{x \to c^+} f(x) = L_1$ means as x approaches to c from the right side, we have $f(x)$ approaches to L_1

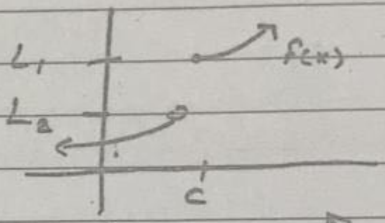
② $\lim\limits_{x \to c^-} f(x) = L_2$ means as x approaches to c from the left, we have $f(x)$ approaches to L_2

③ $\lim\limits_{x \to c} f(x) = L$ if

$$\boxed{\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x) = L}$$

Otherwise, $\lim\limits_{x \to c} f(x)$ does not exist (DNE)

Ex



$\lim\limits_{x \to c} f(x)$ DNE

Facts ① $\lim\limits_{x \to c} k = k$

② $\lim\limits_{x \to c} P(x) = P(c)$

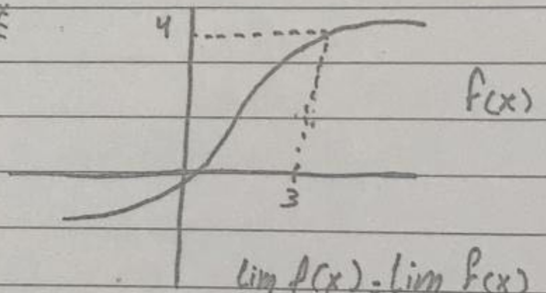
Ex ① $\lim\limits_{x \to -1} 3 = 3$

② $\lim\limits_{x \to 1} -5x^2 + 2x - 1 = -4$

③ $\lim\limits_{x \to 5} \dfrac{x+5}{x^2-25} = DNE$

④ $\lim\limits_{x \to -3} \dfrac{3+x}{x-2} = 0$

Ex



$\lim\limits_{x \to 3^+} f(x) = \lim\limits_{x \to 3^-} f(x) = 4$

$\Rightarrow \lim\limits_{x \to 3} f(x) = 4$

Sec 6.3 Evaluating Trigonometric Functions موضوع الدرس / /

Identities: (1) $\sin A = \cos(90-A)$

(2) $\cos A = \sin(90-A)$ (3) $\tan A = \cot(90-A)$

(4) $\cot A = \tan(90-A)$ (5) $\sec A = \csc(90-A)$

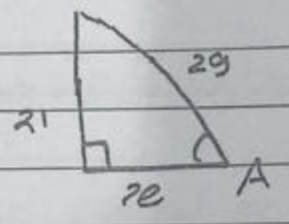
(6) $\csc A = \sec(90-A)$

HW 192 (a) $\cos 52 = \sin(90-52) = \sin 38$

(b) $\tan 71 = \cot 19$ (c) $\sec 24 = \csc 66$

~~195~~ Page 195

(1)

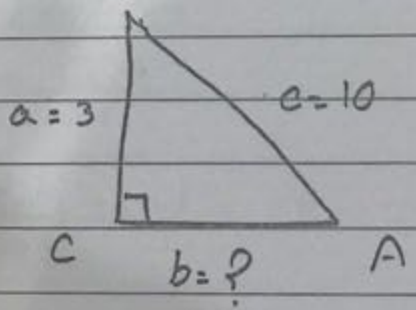


$\sin A =$ [illegible]

$\cos A = \frac{20}{29}$

$\tan A = \frac{21}{20}$

(15) $b = \sqrt{c^2 - a^2} = \sqrt{10^2 - 3^2} = \sqrt{91}$



التاريخ / /

Sec 6.2 Trigonometric Functions موضوع الدرس

Ex6 / 188 $\tan\theta = \frac{4}{3}$, $\theta \in Q\ III$

$y = -4$, $x = -3 \Rightarrow r = \sqrt{(3)^2+(4)^2} = \sqrt{25} = 5$

$\sin\theta = \frac{-4}{5}$, $\cos\theta = \frac{-3}{5}$

62 $\csc\theta = -2$, $\theta \in Q\ III$

$\cot\theta = \sqrt{\csc^2\theta - 1} = \sqrt{4-1} = \sqrt{3}$

التاريخ / / موضوع الدرس

Sec 6.2 Trignometric Functions

Ex 5 / 186 $\sin\theta = \frac{2}{3}$, $\theta \in Q\ II$

① $\cos\theta = -\sqrt{1-\sin\theta} = -\sqrt{1-\frac{4}{9}} = -\sqrt{\frac{5}{9}} = \frac{-\sqrt{5}}{3}$

② $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{2}{-\sqrt{5}} = \frac{-2}{\sqrt{5}}$ ③ $\csc\theta = \frac{3}{2}$ ④ $\sec\theta = \frac{-3}{\sqrt{5}}$

⑤ $\cot\theta = \frac{-\sqrt{5}}{2}$

طريقة اخرى

$\sin\theta = \frac{y}{r} = \frac{2}{3}$

$y = 2$, $r = 3 \Rightarrow x = -\sqrt{r^2 - y^2} = -\sqrt{3^2-2^2} = -\sqrt{5}$

Hw 4 / 186

ⓐ $\sin\theta = 2.5$ impossible

ⓑ $\tan\theta = 110.47$ possible

ⓒ $\sec\theta = 0.6$ impossible

Identities:

① $\sin^2\theta + \cos^2\theta = 1$

② $1 + \tan^2\theta = \cancel{\csc^2\theta}\ \sec^2\theta$

③ $1 + \cot^2\theta = \csc^2\theta$

Ex Find the range of $f(x) = 1 + \sin\theta$

$-1 \leq \sin\theta \leq 1$

$0 \leq f(x) \leq 2$

Range $f = [0,2]$

Hw 5 / 188 $\cos\theta = \frac{-\sqrt{3}}{4}$, $\sin\theta > 0$

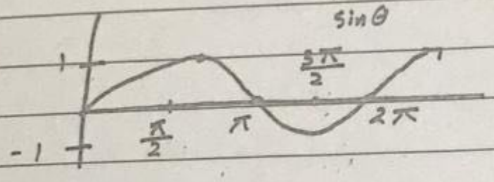
$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{3}{16}} = \sqrt{\frac{13}{16}} = \frac{\sqrt{13}}{4}$

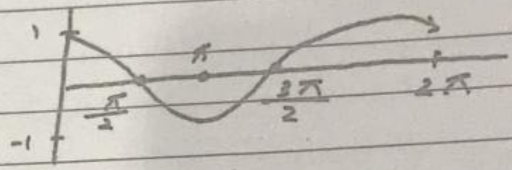
$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\sqrt{13}}{\sqrt{3}}$

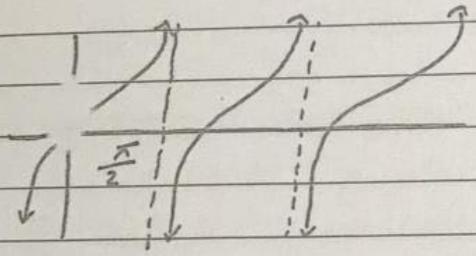
التاريخ / /

Sec 6.2 Trigonometric Functions موضوع الدرس

Function	Range
$\sin\theta, \cos\theta$	$[-1, 1]$
$\sec\theta, \csc\theta$	$[-\infty, -1] \cup [1, \infty)$
$\tan\theta, \cot\theta$	R





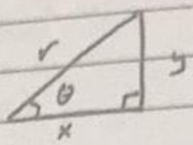


tan θ

موضوع الدرس Sec 6.2 Trigonometric Functions   التاريخ / /

phythagorian thereom:

$x^2 + y^2 = r^2$



① $\sin\theta = \frac{y}{r}$ ④ $\csc\theta = \frac{r}{y}$

② $\cos\theta = \frac{x}{r}$ ⑤ $\sec\theta = \frac{r}{x}$

③ $\tan\theta = \frac{y}{x}$ ⑥ $\cot\theta = \frac{x}{y}$

x = adjacent المجاور

y = opposite المقابل

r = hypotenuse الوتر

$\csc = \frac{1}{\sin\theta}$ $\tan = \frac{\sin\theta}{\cos\theta}$

$\sec = \frac{1}{\cos\theta}$

$\cot = \frac{1}{\tan} = \frac{\cos\theta}{\sin\theta}$

Hw1 / 181 $\sin\theta = \frac{-4}{5}$, $\cos\theta = \frac{-3}{5}$, $\tan\theta = \frac{4}{3}$

$\csc\theta = \frac{-5}{4}$, $\sec\theta = \frac{5}{-3}$, $\cot\theta = \frac{3}{4}$

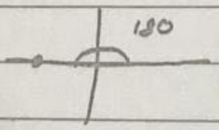
Ex2 / 181 $x + 2y = 0$, $x \geq 0$ choose $x = 2$, then $y = 1$

$r = \sqrt{2^2 + (-1)^2} = \sqrt{5} \Rightarrow \sin\theta = \frac{-1}{\sqrt{5}}$, $\cos\theta = \frac{2}{\sqrt{5}}$

$\tan\theta = \frac{-1}{2}$, $\sec\theta = \frac{\sqrt{5}}{2}$

$\csc\theta = -\sqrt{5}$, $\cot\theta = -2$

Hw2 / 182 (b) $(-3, 0)$



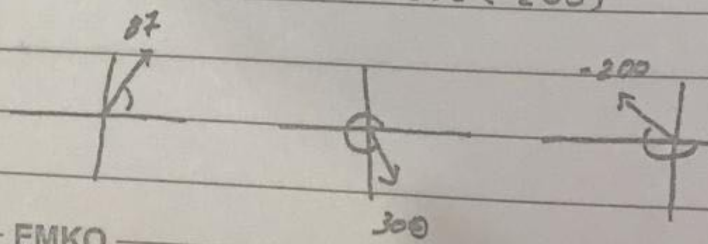
$\theta = 180° \Rightarrow$ Use the calculator

QII	QI
$\sin\theta$, $\csc\theta$ (+)	All (+)
QIII: $\tan\theta$, $\cot\theta$ (+)	QIV: $\cos\theta$, $\sec\theta$ (+)

Hw3 / 184 a- 87° → All (+)

b- 300° → $\cos 300$, $\sec 300$ (+)

c- -200 → $\sin(-200)$, $\csc(-200)$ (+)



Sec 2.1 Angles موضوع الدرس / / التاريخ

(29) $34^\circ\ 51'\ 35'' = 34^\circ + 0.85^\circ + 0.0097^\circ = 34.8597^\circ$

$51' = \left(\frac{51}{60}\right)^\circ = 0.85^\circ$

$35'' = \left(\frac{35}{3600}\right)^\circ = 0.0097^\circ$

(33) $-25.485^\circ = -(25^\circ\ 29'\ 6'')$

$0.485^\circ = (0.485 * 60)' = 29.1'$

$0.1' = (0.1 * 60)'' = 6''$

(44) $1000^\circ \equiv 1000^\circ - 2(360) = 280^\circ$

(46) $-8440^\circ \equiv -8440 + 24(360) = 200^\circ$

تاريخ / /

موضوع الدرس Sec 6.4 Exponential and logarithmic equations

(28) $\log_4 (x^3+37) = 3$

$\Rightarrow x^3+37 = 4^3 = 64 \Rightarrow x^3-27=0$

$\Rightarrow (x-3)(x^2+3x+9)=0 \Rightarrow x=3$

(23) $\ln x + \ln x^2 = 3 \Rightarrow \ln x^3 = 3 \Rightarrow e^{\ln x^3} = e^3 \Rightarrow x^3 = e^3 \Rightarrow x = e$

(28) $\log(x-10) - \log(x-6) = \log 2$

$\Rightarrow \log \frac{x-10}{x-6} = \log 2 \Rightarrow \frac{x-10}{x-6} = 2 \Rightarrow 2x-12 = x-10$

$\Rightarrow -x = -2 \Rightarrow x = 2$ غير معرفة بعد التعويض

$\Rightarrow$ The solution set is ϕ

(38) $\log_2(\log_2 x) = 1 \Rightarrow \log_2 x = 2^1 = 2 \Rightarrow x = 2^2 = 4$

(39) $\Rightarrow 2\log x = (\log x)^2$

$\Rightarrow (\log x)^2 - 2\log x = 0 \Rightarrow \log x(\log x - 2) = 0$

$\log x = 0$ or $\log x = 2$

$\Rightarrow x = 10^0$ or $x = 10^2$

$\Rightarrow x = 1$ or $x = 100$

التاريخ / /

موضوع الدرس Sec 5.4 Exponential and Logarithmic equations

Hw 167 $\log(3x+2)+\log(x-1)=1$

$\log[(3x+2)(x-1)]=1$

$\log[3x^2-x-2]=1$

$\Rightarrow 3x^2-x-2=10^1 \Rightarrow 3x^2-x-12=0$

$\Rightarrow x=\dfrac{1+\sqrt{1-4(3)(-12)}}{6} \Rightarrow x=\dfrac{1+\sqrt{145}}{6}$ ✓

اذاً عوضنا بالقيمة السالبة لتنتهي الحد تكون غير معرفة اذاً الجواب هو القيمة الموجبة فقط

page 169: (3) $3^x=7 \Rightarrow \ln 3^x=\ln 7 \Rightarrow x\ln 3=\ln 7$

$x=\dfrac{\ln 7}{\ln 3}=\log_3 7$

(8) $e^{x^2}=100 \Rightarrow \ln e^{x^2}=\ln 100 \Rightarrow x^2=\ln 100 \Rightarrow x=\pm\sqrt{\ln 100}$

(9) $e^{3x-7}\cdot e^{-2x}=4e \Rightarrow e^{x-7}=4e \Rightarrow \dfrac{e^{x-7}}{e}=4 \Rightarrow e^{x-8}=4 \Rightarrow \ln e^{x-8}=\ln 4$

$\Rightarrow x-8=\ln 4 \Rightarrow x=8+\ln 4$

(10) $=-3$ has no solution because $(\frac{1}{3})^x>0$

(12) $3(2)^{x+2}+1=100 \Rightarrow 2^{x-2}=33 \Rightarrow \log_2 2^{x-2}=\log_2 33$

$\Rightarrow x-2=\log_2 33 \Rightarrow x=2+\log_2 33$

(17) $5^{2x}+3(5^x)=28$. Let $y=5^x \Rightarrow y^2+3y-28=0 \Rightarrow (y+7)(y-4)=0$

$\Rightarrow y=-7$ or $y=4$

$\Rightarrow 5^x=-7$ or $5^x=4$

$\Rightarrow \log_5 5^x=\log_5 4 \Rightarrow x=\log_5 4$

(18) $5\ln x=10 \Rightarrow \ln x=2$

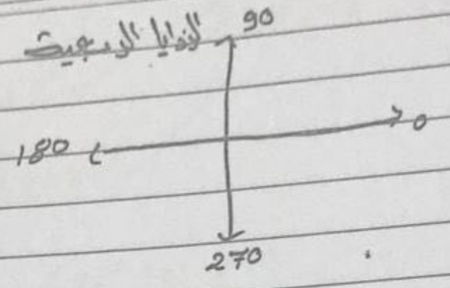
$\Rightarrow e^{\ln x}=e^2 \Rightarrow x=e^2$

التاريخ / /

موضوع الدرس Angles

Sec 6.1

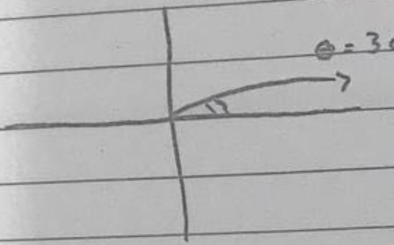
Quadrantal angles الزوايا الربعية



Coterminal angles: لها نفس ضلع النهاية

The difference between them is a multiple of 360

Ex 30, 390, 750, 1110, -330 are coterminal



page 178: (5)

The complement of $39^\circ 50'$ is $90^\circ - 39^\circ 50'$

$= 89^\circ 60' - 39^\circ 50' = 50^\circ 10'$

The supplement of $39^\circ 50'$

is $180^\circ - 39^\circ 50' = 179^\circ 60' - 39^\circ 50' = 140^\circ 10'$

(7) $20x + 10 + 3x + 9 = 180 \Rightarrow 23x + 19 = 180 \Rightarrow 23x = 161$

$\Rightarrow x = \frac{161}{23} = 7$

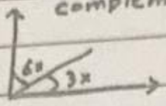
(19) $110^\circ 25' + 32^\circ 55' = 142^\circ 80' = 143^\circ 20'$

التاريخ / /

Sec 6.1 — Angles موضوع الدرس

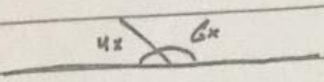
Hw1 176

(a) complementary



$3x+6x=90 \Rightarrow 9x=90 \Rightarrow x=10$

(b)



$6x+4x=180 \Rightarrow 10x=180 \Rightarrow x=18$

supplementary

Hw2 176

(a) convert $74^\circ\, 08'\, 14''$ into degrees

$8' = \left(\frac{8}{60}\right)^\circ = 0,1333^\circ$

$14'' = \frac{14}{3600} = 3,8\times10^{-3\circ}$

$74^\circ\, 08'\, 14'' = 74,1371^\circ$

(b) convert $34,817^\circ$ to, minutes, seconds and degrees

$0,817^\circ = (0,817 * 60)' = 49,02'$

$0,02' = (0,02 * 60)'' = 1,2'' \approx 1''$

$34.817^\circ = 34^\circ\, 49'\, 1''$

التاريخ / /

Sec 5.3 Logarithmic Functions موضوع الدرس

Ex solve

$\log_2 (x+1) - 2\log_2 x = \log_2 3$

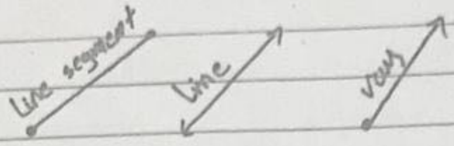
$\log_2 \left(\frac{x+1}{x^2}\right) = \log_2 3 \Rightarrow \frac{x+1}{x^2} = 3 \Rightarrow 3x^2 = x+1$

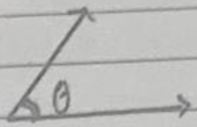
$\Rightarrow 3x^2 - x - 1 = 0 \Rightarrow x = \frac{1 \pm \sqrt{1-4(3)(-1)}}{6} = \frac{1+\sqrt{13}}{6}$ ✓

$x = \frac{1-\sqrt{13}}{6}$ ✗

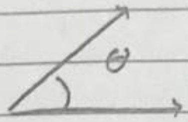
مرفوضة

Sec 6.1 Angles موضوع الدرس / / التاريخ





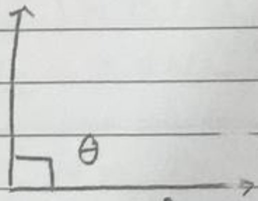
①



Acute حادة

$0 < \theta < 90^\circ$

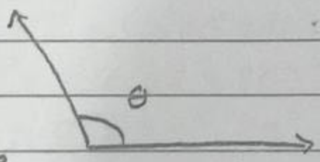
②



ght قائمة

$\theta = 90^\circ$

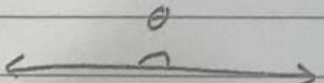
③



Obtuse منفرجة

$90^\circ < \theta < 180^\circ$

④



straight مستقيمة

$\theta = 180^\circ$

Note: ① The complement of θ is $90 - \theta$

② The supplement of θ is $180 - \theta$

③ 1° = one degree

$1'$ = one minute

$1''$ = one second

$\boxed{1^\circ = 60' = 3600''}$

$\boxed{1' = 60''}$

التاريخ / /

Sec 5.3 Logarithmic Functions موضوع الدرس

(7) $x = \log_5 \frac{1}{625} = \log_5 (\frac{1}{5})^4 = \log_5 5^{-4} = -4$

(8) $\log_x \frac{1}{32} = 5 \Rightarrow \frac{1}{32} = x^5 \Rightarrow (\frac{1}{2})^5 = x^5 \Rightarrow x = \frac{1}{2}$

(9) $\frac{1}{4}$ (10) 8 (11) 9

(12) $\log_x 25 = -2 \Rightarrow 25 = x^{-2} \Rightarrow x^2 = \frac{1}{25} \Rightarrow x = (\frac{1}{5})^2 \Rightarrow x = \frac{1}{5}$ ✓

مرفوض $\Leftarrow x = -\frac{1}{5}$ ✗

(15) $\log_9 x = \frac{5}{2} \Rightarrow x = 9^{\frac{5}{2}} = 3^5 = 243$

(16) $\log_{\frac{1}{2}} (x+3) = -4 \Rightarrow x+3 = (\frac{1}{2})^{-4} = 2^4 = 16 \Rightarrow x = 13$

(17) 3 (18) $3x - 15 = \log_x 1 = 0 \Rightarrow x = 5$

Ex $3x - 1 = 2\log_2 (y+1) + 7$

Write the equation in exponential form

$\frac{3x-8}{2} = \log_2 (y+1) \Rightarrow 2^{\frac{3x-8}{2}} = y+1 \Rightarrow y = 2^{\frac{3}{2}x-4} - 1$

Ex Let $f(x) = 1 - 3(2)^{4x-5}$. Find $f^{-1}(x)$

$x = 1 - 3(2)^{4y-5} \Rightarrow \frac{x-1}{-3} = 2^{4y-5} \Rightarrow \frac{1-x}{3} = 2^{4y-5}$

$\Rightarrow \log_2 (\frac{1-x}{3}) = 4y - 5 \Rightarrow y = \frac{1}{4}\log_2 (\frac{1-x}{3}) + \frac{5}{4} = f^{-1}(x)$

Ex $f(x) = 5 + 3\log_{\frac{1}{2}} (1-x)$

Dom $f = (-\infty, 1)$ $\quad 1 - x > 0$, $-x > -1 \Rightarrow x < 1$

Range $f = \mathbb{R}$ $\quad x = 1$ is the vertical asymptote

التاريخ / /

Sec 5.3 Logarithmic Functions موضوع الدرس

Properties of

for $x > 0$, $y > 0$ and $a > 0$ or $0 < a < 1$

we have ① $\log_a (xy) = \log_a (x) + \log_a (y)$

② $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

③ $\log_a x^r = r \log_a x$ ④ $\log_a 1 = 0$ ⑤ $\log_a a = 1$

⑥ $\log_a \frac{1}{a} = -1$

Hw 3 / 162 ⓐ $\log_3 (x+2) + \log_3 x - \log_3 2$

$= \log_3 (x+2)x - \log_3 2 = \log_3 \left(\frac{x(x+2)}{2}\right)$

ⓑ $2\log_a m - 3\log_a n = \log_a m^2 - \log_a n^3 = \log_a \left(\frac{m^2}{n^3}\right)$

Fact If $f(x) = \log_a x$, then $f^{-1}(x) = a^x$

Therefore ① $(f^{-1} \circ f)(x) = a^{\log_a x} = x$, $x > 0$

② $(f \circ f^{-1})(x) = \log_a a^x = x$, $x \in \mathbb{R}$

Page 163 ①

a- $\log_2 16 = \log_2 2^4 = 4$

b- $\log_3 1 = 0$

c- $\log_{10} 0.1 = -1$

d- $\log_2 \sqrt{2} = \log_2 2^{\frac{1}{2}} = \frac{1}{2}$

e- $\log_e \frac{1}{e^2} = \log_e e^{-2} = -2$

f- $\log_{\frac{1}{2}} 8 = \log_{\frac{1}{2}} 2^3$

$\Rightarrow \log_{\frac{1}{2}} \left(\frac{1}{2}\right)^{-3} = -3$

② $3^4 = 81 \Longrightarrow \log_3 3^4 = \log_3 81 \Leftrightarrow 4 = \log_3 81$

④ $\log_6 36 = 2 \Leftrightarrow 36 = 6^2$

Sec 5.3 Logarithmic Functions موضوع الدرس التاريخ / /

Hw 1 / 157 ⓐ $\log_x \frac{8}{27} = 3 \Rightarrow \frac{8}{27} = x^3 \Rightarrow x = \sqrt[3]{\frac{8}{27}} = \frac{2}{3}$

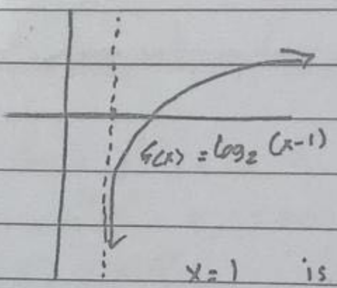
ⓑ $\log_4 x = \frac{5}{2} \Rightarrow x = 4^{\frac{5}{2}} = 2^5 = 32$

ⓒ $\log_{49} \sqrt[3]{7} = x \Rightarrow \sqrt[3]{7} = 49^x \Rightarrow 7^{\frac{1}{3}} = 7^{2x}$

$\Rightarrow \frac{1}{3} = 2x \Rightarrow x = \frac{1}{6}$

Hw 2 / 160 ⓐ $f(x) = \log_2 (x-1) \Rightarrow x-1 > 0 \Rightarrow x > 1$

Dom f = $(1, \infty)$, Range f = $\mathbb{R}$



$x = 1$ is the vertical asymptote

ⓑ $f(x) = \log_4 (x+2) + 1$

$x + 2 > 0 \Rightarrow x > -2$

Dom f = $(-2, \infty)$

Range f = $\mathbb{R}$

ⓒ Homework

التاريخ / /

موضوع الدرس

Sec 5.3 Logarithmic Functions

<u>Fact</u> $y = \log_a x \longleftrightarrow x = a^y$

where $0 < a < 1$ or $a > 1$ and $x > 0$

The function $f(x) = \log_a x$ is called

The logarithmic function to the base a

للتخلص من اللوغاريتم نضع الأساس تحت الطرفين

<u>properties of $f(x) = \log_a x$:</u>

① Dom $f = (0, \infty)$ ② Range $f = \mathbb{R}$

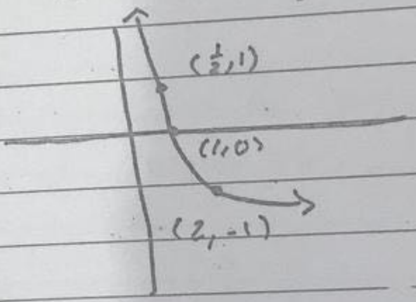
③ The y-axis is vertical asymptote

④ The points $(\frac{1}{a}, -1)$, $(1, 0)$, $(a, 1)$ are on the graph of f

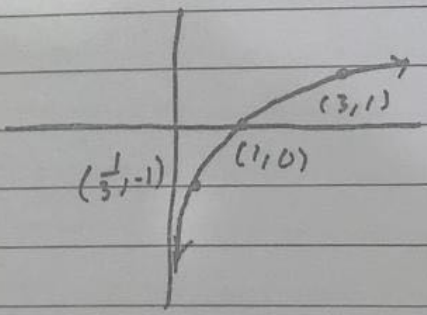
⑤ If $a > 1$, then f is increasing

If $0 < a < 1$ then f is decreasing

<u>Ex</u> graph $f(x) = \log_{\frac{1}{2}} x$

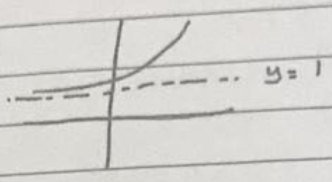


<u>Ex</u> $f(x) = \log_3 x$



Sec 5.2 Exponential Functions موضوع الدرس / / التاريخ

Page 155:

(15) 

is horizontal asymptote

Dom $(2^x + 1) = R$

Range $(2^x + 1) = (1, \infty)$

(20) $f(x) = 2^{x+2} - 4$

Range $f = (-4, \infty)$

The horizontal asymptote $\Rightarrow y = -4$

(31) $^{x} = 2 \Rightarrow 4^x = 4^{\frac{1}{2}} \Rightarrow x = \frac{1}{2}$

(32) $\left(\frac{5}{2}\right)^x = \frac{4}{25} \Rightarrow \frac{4}{25} = \left(\frac{2}{25}\right)^2 = \left(\frac{5}{2}\right)^{-2} = \left(\frac{5}{2}\right)^x \Rightarrow x = -2$

(33) $2^{3-2x} = 8 \Rightarrow 8 = 2^3 = 2^{3-2x} \Rightarrow x = 0$

(34) $e^{4x-1} = (e^2)^x \Rightarrow e^{4x-1} = e^{2x} \Rightarrow 2x = 4x - 1 \Rightarrow -2x = -1 \Rightarrow x = \frac{1}{2}$

(35) $27^{4x} = 9^{x+1} \Rightarrow (3^3)^{4x} = (3^2)^{x+1} \Rightarrow 3^{12x} = 3^{2x+2} \Rightarrow 12x = 2x + 2$

$\Rightarrow 10x = 2 \Rightarrow x = \frac{1}{2}$

(37) $\left(\frac{1}{e}\right)^{-x} = \left(\frac{1}{e^2}\right)^{x+1} \Rightarrow e^x = e^{-2x-2} \Rightarrow x = -2x - 2 \Rightarrow 3x - 2 \Rightarrow x = -\frac{2}{3}$

(41) $x^{5/2} = 32 \Rightarrow x = 32^{2/5} = 2^2 = 4$

(38) $(\sqrt{2})^{x+4} = 4^x \Rightarrow (2)^{\frac{x+4}{2}} = 2^{2x} \Rightarrow \frac{x+4}{2} = 2x \Rightarrow 4x = 4 + x$

$= 3x = 4 \Rightarrow x = \frac{4}{3}$

EMKO

موضوع الدرس Exponential Functions / / التاريخ

Sec 5.2

Hw2 / 154 solve $\left(\frac{1}{3}\right)^x = 81$

$81 = 3^4 = \left(\frac{1}{3}\right)^{-4} = \left(\frac{1}{3}\right)^x$

$\Rightarrow x = -4$

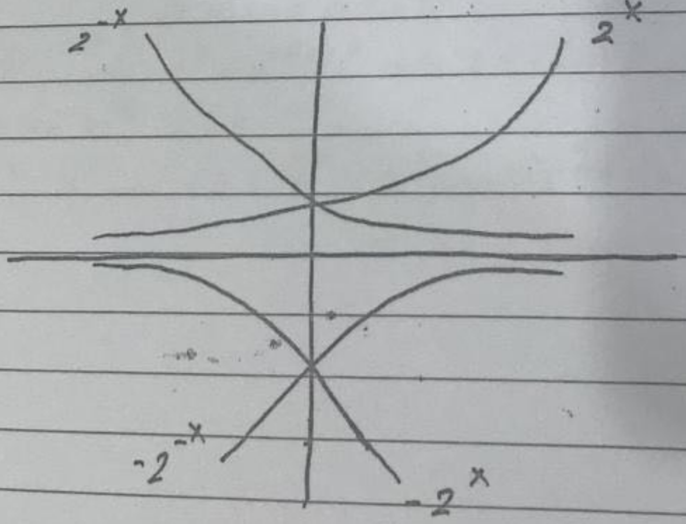
Hw3 / 154 $x^{4/3} = 81 \Rightarrow x = \pm 81^{3/4} = \pm \left(\sqrt[4]{81}\right)^3$

$= \pm 3^3 = \pm 27$

page 155

$f(x) = 3^x, \ g(x) = \left(\frac{1}{4}\right)^x$

1. $f(2) = 3^2 = 9$
2. $f(-2) = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$
3. $g(2) = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$
4. $g(-2) = \left(\frac{1}{4}\right)^{-2} = 4^2 = 16$
5. $f\left(\frac{3}{2}\right) = 3^{3/2} = \left(\sqrt{3}\right)^3 = 3\sqrt{3}$



Sec 5.2 Exponential Functions

Facts ① a^x is a unique number for every $x \in R$ where $0 < a < 1$ or $a > 1$

Base ← a ; x → Exponent

② $a^x = a^y \rightarrow x = y$

③ If $0 < a < 1$ and $n < m$, then $a^n > a^m$

④ If $a > 1$ and $n < m$, then $a^n < a^m$

The function $f(x) = a^x$ where $0 < a < 1$ or $a > 1$ is called the exponential function to the base a

base ← a ; x → exponent

Properties of $f(x) = a^x$:

① Dom $f = R$

② Range $f = (0, \infty)$

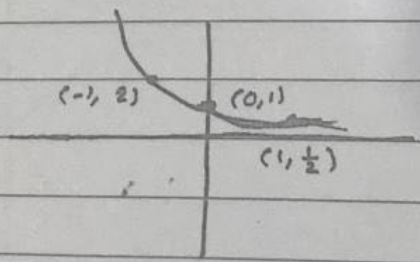
③ The x-axis is horizontal asymptote

④ The points $(-1, \frac{1}{a})$ $(0,1)$ $(1, a)$ are on the graph

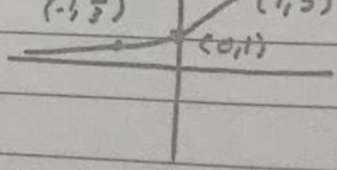
⑤ If $a > 1$ then f is increasing

⑥ If $0 < a < 1$ then f is decreasing

Ex graph $f(x) = (\frac{1}{2})^x$



Ex graph $f(x) = 3^x$



التاريخ / /

موضوع الدرس

Sec 5.1 Inverse Functions

Page 147:

① 1-1 ② Not 1-1 ⑤ Not 1-1

Page 149 | 30, 31, 35, 37, 32

EMKO

موضوع الدرس / / التاريخ

Sec 5.1 Inverse Functions

Hw 4 / 148

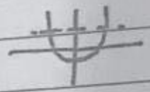
$f(x) = \sqrt{x+5}$, $x \geq 5$

$x = \sqrt{y+5} \Rightarrow x^2 = y+5 \Rightarrow x^2 - 5 = y = f^{-1}(x)$

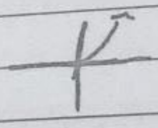
Dom f = $[5, \infty)$ = Range f^{-1}

Range f = $[0, \infty)$ = Dom f^{-1}

Ex $f(x) = x^2 \Rightarrow f$ is not one-to-one
so f^{-1} does not exist



$f(x) = x^2$, $x \geq 0$
is one-one

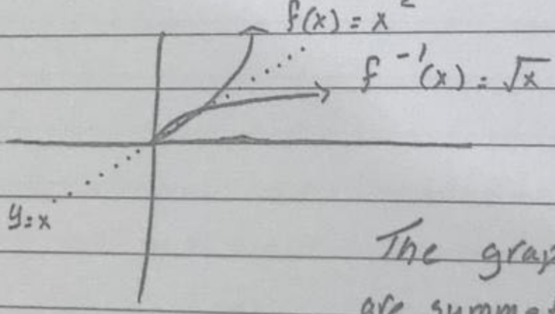


$f^{-1}(x) = \sqrt{x} = \sqrt{x^2} \rightarrow (f \circ g)(x)$
$= |x| = x$

$(g \circ f)(x) = (\sqrt{x})^2 = x$

→ Two inverse functions

Note



The graphs f and f^{-1} are symmetric about the line $y = x$

$y = x$ cuts across between any inverse functions forming two symmetric sides

التاريخ / /

Sec 7.3 Continuity موضوع الدرس

Defn ① f is right continuous at $x=c$ if
$$\boxed{\lim_{x\to c^+} f(x) = f(c)}$$

② f is left continuous at $x=c$ if
$$\boxed{\lim_{x\to c^-} f(x) = f(c)}$$

③ f is continuous at $x=c$ if
$$\boxed{\lim_{x\to c} f(x) = f(c)} \text{ or } \lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = f(c)$$

④ f is continuous on an interval $[a,b]$ if
ⓐ f is cont at c for every $c \in (a,b)$
ⓑ f is right cont at a
ⓒ f is left cont at b

Facts ① All polynomials are cont on $\mathbb{R}$
② $\sqrt{P(x)}$ is cont where $P(x) \geq 0$
③ $\frac{P(x)}{q(x)}$ is cont where $q(x) \neq 0$
④ $\sin x, \cos x, \tan x, \sec x, \csc x, \cot x$ are cont on their domains

Ex $\sin x, \cos x$ are cont on $\mathbb{R}$

Ex $\tan x$ is cont on $\mathbb{R}\setminus\{\frac{\pi}{2}+n\pi\}$ $n\in\mathbb{Z}$

HW 1 / 214 $f(x) = \sqrt{4-x^2} \Rightarrow 4-x^2 \geq 0 \rightarrow (2-x)(2+x) \geq 0$

Dom $f = [-2,2]$

$-$ (−2) $+$ (2) $-$

f is cont on $[-2,2]$

Ex $f(x) = \frac{5}{x^2-9}$ is cont on $\mathbb{R}\setminus\{-3,3\}$

موضوع الدرس

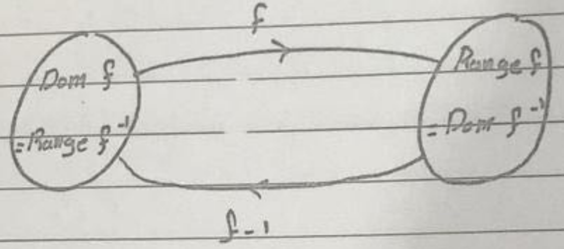
Sec 5.1 Inverse Functions

Defn A one-to-one function is a set of order pairs such that neither first component nor second component is repeated

Defn A function g is said to be the inverse of f if $(f \circ g)(x) = x$ for every $x \in Dom\, g$ and $(g \circ f)(x) = x$ for every $x \in Dom\, f$

We write $g = f^{-1}$

$$(f \circ f^{-1})(x) = (f^{-1} \circ f) = x$$



Hw2 / 144

(a) $F = \{(-2,1), (-1,0), (0,1), (1,2), (2,2)\}$

is not one-to-one $\Rightarrow F^{-1}$ does not exist

(b) $G = \{(3,1), (0,2), (2,3), (4,0)\}$

$G^{-1} = \{(1,3), (2,0), (3,2), (0,4)\}$

Ex If $f(2) = 3$, then $f^{-1}(3) = 2$

Ex find f^{-1} where $f(x) = 2x - 3$

How to find f^{-1} ?? (1) set $y = f(x)$

(2) interchange x with y

(3) find y which is $f^{-1}(x)$

التاريخ / / موضوع الدرس

Sec 5.1 Inverse Functions

Ex ① $f(x) = \sqrt{2}$ is not one-to-one
because $f(1) = f(2) = \sqrt{2}$

② $f(x) = 2x-1$ is one-to-one
because $f(a) = f(b) \Rightarrow 2a-1 = 2b-1 \Rightarrow 2a = 2b \Rightarrow a = b$

③ $f(x) = 5x^2+3$ is not one-to-one
because $f(2) = f(-2) = 23$

④ $f(x) = x^3$ is one-to-one
but $f(x) = x^3 - 7x^2$ is not one-to-one
because $f(0) = f(7) = 0$

⑤ $f(x) = 5\sqrt{x} - 1$ is one-to-one
because $f(a) = f(b) \Rightarrow 5\sqrt{a} - 1 = 5\sqrt{b} - 1$
$\Rightarrow 5\sqrt{a} = 5\sqrt{b} \Rightarrow \sqrt{a} = \sqrt{b} \Rightarrow a = b$

⑥ $f(x) = 2|x+1| - 9$ is not one-to-one
because $f(1) = f(-3) = -5$

⑦ $f(x) = 2\sqrt[3]{x-4} + 1$ is not one-to-one

⑧ $f(x) = 6(x-3)^2 + 5$ is not one-to-one
because $f(4) = f(2) = 11$

Zeros of Polynomials Functions الموضوع / / التاريخ

(17) $f(x) = x^4 + 5x^2 + 4$, $-i$ is a zero of f, find the other zeros

$$\begin{array}{r|ccccc} & x^4 & x^3 & x^2 & x & \text{cons} \\ -i & 1 & 0 & 5 & 0 & 4 \\ & & -i & -1 & -4i & -4 \\ \hline & 1 & -i & 4 & -4i & 0 \end{array}$$

$$\frac{f(x)}{x+i} = x^3 - ix^2 + 4x - 4i = x^2(x-i) + 4(x-i)$$

$= (x-i)(x^2+4)$

$f(x) = (x+i)(x-i)(x^2+4)$

$(x+i)(x-i)(x+2i)(x-2i)$

The zeros of f are $-i, i, -2i, 2i$

Recall that $x^2+y^2 = (x+iy)(x-iy)$

التاريخ / /

موضوع الدرس

Sec 5.1 Inverse Functions

Ex find f^{-1} where $f(x)=2x-3$

① $y=2x-3$

② $x=2y-3$

③ $x+3=2y \Rightarrow y=\frac{x+3}{2} \Rightarrow f^{-1}(x)=\frac{x+3}{2}$

Ex $f(x)=2\sqrt[3]{x-1}+4$ find $f^{-1}(x)$

① $y=2\sqrt[3]{x-1}+4$

② $x=2\sqrt[3]{y-1}+4$

③ $x-4=2\sqrt[3]{y-1}$

$\Rightarrow \frac{x-4}{2}=\sqrt[3]{y-1} \Rightarrow \left(\frac{x-4}{2}\right)^3=y-1$

$\Rightarrow 1+\left(\frac{x-4}{2}\right)=y$

$\Rightarrow y=1+\left(\frac{x-4}{2}\right)^3=f^{-1}(x)$

Note f^{-1} exists only if f is one-to-one

Hw3 146 $f(x)=\frac{2x+3}{x-4},\ x\neq 4$

$x=\frac{2y+3}{y-4} \Rightarrow xy-4x=2y+3 \Rightarrow xy-2y=4x+3$

$y(x-2)=4x+3 \Rightarrow y=\frac{4x+3}{x-2}=f^{-1}(x)$

Dom $f=\mathbb{R}\setminus\{4\}=$ Range f^{-1}

Range $f=\mathbb{R}\setminus\{2\}=$ Dom f^{-1}

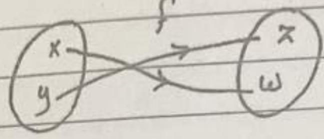
التاريخ / /

Sec 5.1 — Inverse Functions — موضوع الدرس

Defn A function f is one-to-one if for every $a, b \in \text{Dom } f$ with $a \neq b$, we have $f(a) \neq f(b)$

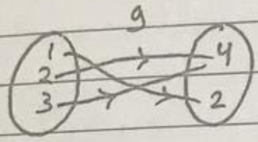
Or $f(a) = f(b) \Rightarrow a = b$

Ex



f is one-to-one

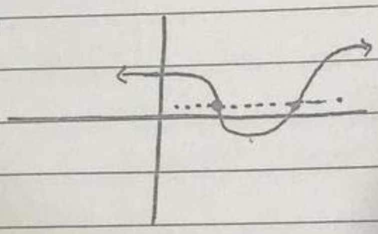
Ex



g is not one-to-one because $g(2) = g(3) = 4$

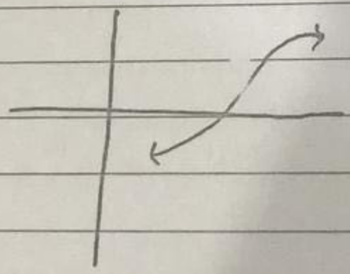
Horizontal Line test

Ex



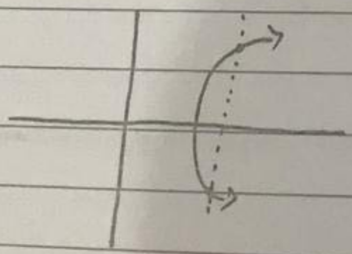
Not one-to-one

Ex



one-to-one

Ex



Is not even a function So, not one one-to-one

Sec 4.2 Synthetic Division موضوع الدرس التاريخ / /

10/128) $f(x) = 2x^3 + 3x^2 - 16x + 10$, $k = -4$

$$\begin{array}{r|rrrr} & x^3 & x^2 & x & \text{cons} \\ -4 & 2 & 3 & -16 & 10 \\ & & -8 & 20 & -16 \\ \hline & 2 & -5 & 4 & \underline{-6} \end{array}$$

$= \frac{f(x)}{x+4} = 2x^2 - 5x + 4 - \frac{6}{x+4}$

نضرب $(x+4)$ في جميع الأطراف

$$f(x) = \underbrace{(x+4)}_{x-k}\underbrace{(2x^2 - 5x + 4)}_{q(x)}\underbrace{-6}_{r}$$

19/129) $f(x) = x^2 - x + 3$, $k = 3 - 2i$

$$\begin{array}{r|rrr} & x^2 & x & \text{cons} \\ 3-2i & 1 & -1 & 3 \\ & & 3-2i & 2-10i \\ \hline & 1 & 2-2i & \underline{5-10i} \end{array}$$

$= f(3-2i) = 5 - 10i$

25/129) $f(x) = 3x^4 + 13x^3 - 10x + 8$, $k = \frac{-4}{3}$

$$\begin{array}{r|rrrrr} & x^4 & x^3 & x^2 & x & \text{cons} \\ -\frac{4}{3} & 3 & 13 & 0 & -10 & 8 \\ & & -4 & -12 & 16 & -8 \\ \hline & 3 & 9 & -12 & 6 & \boxed{0} \end{array}$$

Then $\frac{-4}{3}$ is a zero of f or $f\left(\frac{-4}{3}\right) = 0$

Sec 4.2 Synthetic Division موضوع الدرس / / التاريخ

Division Algorithm:

Let f and g be two polynomials with $\deg(f) \geq \deg(g)$.
Then there exists functions q and r such that

$$f(x) = g(x)\,q(x) + r(x)$$

Where $r(x)$ is called the remainder

Special case If $g(x) = x-k$, then

$$f(x) = (x-k)\,q(x) + r$$

The remainder theorem: The remainder of $\frac{f(x)}{x-k}$ is $f(k)$

Hw1
127

$f(x) = -x^4 + 3x^2 - 4x - 5$ Find $f(-3)$

$$\frac{-x^4 + 3x^2 - 4x - 5}{x+3}$$

$$\begin{array}{r|rrrrr} & x^4 & x^3 & x^2 & x & \text{cons} \\ -3 & -1 & 0 & 3 & -4 & -5 \\ & & 3 & -9 & 18 & -42 \\ \hline & -1 & 3 & -6 & 14 & -47 \end{array}$$

$$= -x^3 + 3x^2 - 6x + 14 - \frac{47}{x+3}$$

8
128

$$\begin{array}{r|rrrrr} & x^4 & x^3 & x^2 & x & \text{cons} \\ 1 & 1 & 0 & 0 & 0 & -1 \\ & & 1 & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 1 & 0 \end{array}$$

$$\frac{x^4 - 1}{x-1} = x^3 + x^2 + x + 1$$

موضوع الدرس   /   / تاريخ

Ex $f(x) = 2(3x-1)^2 + 4$ Find the vertex

sol: $f(x) = 2(9x^2 - 6x + 1) + 4 = 18x^2 - 12x + 6$

$h = \frac{12}{36} = \frac{1}{3}$ $k = f(\frac{1}{3}) = 4$

$(\frac{1}{3}, 4)$ is the vertex

Ex $f(x) = ax^2 + bx - 3$
and the parabola is open up, then f has

a- no x-intercept
b- one x-intercept
c- no y-intercept
d- two x-intercept

solution: $D = b^2 - 4a(-3)$
$D = b^2 + 12a > 0$
because $a > 0$ then it has two x-intercepts

EMKO

موضوع الدرس Quadratic Functions   التاريخ / /

Sec 4.1

Hw 1 / 118 $f(x) = x^2 - 6x + 7$

① $h = \frac{-b}{2a} = \frac{6}{2} = 3$ $k = f(3) = -2$

$f(x) = (x-3)^2 - 2$

② $x = h = 3$ is the axis

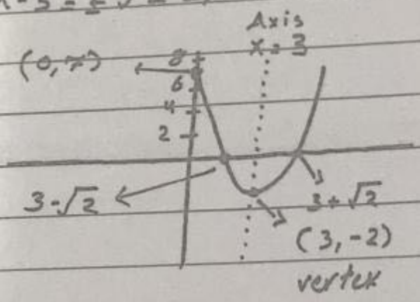
③ (h, k) = the vertex = $(3, -2)$

④ open up, Range = $[k, \infty) = [-2, \infty)$

⑤ $f(0) = 7 \Rightarrow$ is the y-intercept

⑥ $f(x) = (x-3)^2 - 2 = 0 \Rightarrow (x-3)^2 = 2 \Rightarrow x - 3 = \pm\sqrt{2} \Rightarrow x = 3 \pm \sqrt{2}$

are the x-intercepts



⑦ f decreases on $(-\infty, 3]$

and increases on $[3, \infty)$

Note: $D = b^2 - 4ac > 0 \Rightarrow$ then we have two x-intercept

$D = 0 \Rightarrow$ one x-intercept

$D < 0 \Rightarrow$ No x-intercept

Page 122:

② $f(x) = -2(x+3)^2 + 2$

$\Rightarrow h = -3, k = 2$

$a = -2 < 0 \Rightarrow$ open down

① Dom f = $\mathbb{R}$, Range f = $(-\infty, 2]$

② vertex = $(-3, 2)$ ③ the axis $\Rightarrow x = -3$

the y-intercept $\Rightarrow y = f(0) = -16$

$f(x) = -2(x+3)^2 + 2 = 0 \Rightarrow -2(x+3)^2 = -2 \Rightarrow (x+3)^2 = 1 \Rightarrow x + 3 = \pm 1$

$\Rightarrow x = -3 \pm 1 \Rightarrow x = -4$ or $x = -2$ are the x-intercept

Sec 4.1 Quadratic Functions   التاريخ / /

The standard form of a polynomial with degree n is

$P_n(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a$

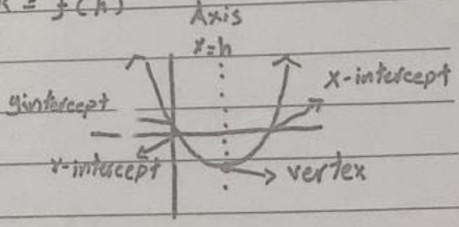
n	$P_n(x)$	Example
0	constant	$-\sqrt{2}$
1	Linear	$3x+4$
2	Quadratic	$-5x^2+4x-1$
3	Cubic	x^3+x-2
4	Quartic	$2x^4+x^3+7$

The standard form of a quadratic function is

$f(x) = ax^2+bx+c$ where $a \neq 0$

Properties of $f(x) = ax^2+bx+c$

1. $f(x) = a(x-h)^2+k$ where $h = \frac{-b}{2a}$, $k = f(h)$
2. $x = h$ is the axis
3. The point (h, k) is the vertex
4. The graph of f is called parabola
5. Domain $f = \mathbb{R}$



6. If $a>0$, then Range $f = [k, \infty)$ and the parabola is open up
7. If $a<0$, then Range $f = (-\infty, k]$ and the parabola is open down
8. $y = f(0)$ is the y-intercept
9. The solutions of $f(x)=0$ are the x-intercept
10. If $a>0$, then f decreases on $(-\infty, h]$ and increases on $[h, \infty)$
11. If $a<0$, then f increases on $(-\infty, h]$ and decreases on $[h, \infty)$

Note: If $0<|a|<1$, then then the parabola shrinks and if $|a|>1$ then the parabola $\rightarrow$ stretches (narrow) (wide)

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التاريخ / / موضوع الدرس

(20) $f(x) = \frac{1}{x} \Rightarrow \frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{\frac{x - x - h}{(x+h)x}}{h} = \frac{-h}{(x+h)x\,h}$

$= \frac{-1}{(x+h)x}$

التاريخ / /

موضوع الدرس

Sec 3.3 Function Operations and Composition

HW 4 / 110 $f(x) = 4x+1$, $g(x) = 2x^2+5x$

① $(f\circ g)(x) = f(2x^2+5x) = 4(2x^2+5x)+1 = 8x^2+20x+1$

② $(g\circ f)(x) = g(4x+1) = 2(4x+1)^2 + 5(4x+1)$
$= 2(16x^2+8x+1)+20x+5 = 32x^2+16x+2+20x+5$
$= 32x^2+36x+7 \Rightarrow (f\circ g)(x) \neq (g\circ f)(x)$

page 111 $f(x) = x^2+3$, $g(x) = -2x+6$
$(f+g)(3) = f(3)+g(3) = 12+0 = 12$

⑦ $f(x) = \sqrt{4x-1}$, $g(x) = \frac{1}{x}$
$Dom\ f = [\frac{1}{4}, \infty)$, $Dom\ g = R\setminus\{0\}$
$Dom\ (f+g) = [\frac{1}{4}, \infty) \cap R\setminus\{0\}$
$= [\frac{1}{4}, \infty)$ لا توجد قيمة
$Dom\ \frac{f}{g} = [\frac{1}{4}, \infty)\setminus\emptyset = [\frac{1}{4}, \infty)$ تجعل "g" المقام صفراً

⑫ ⓐ $(f+g)(2) = f(2)+g(2) = 4+{}^-2 = +2$

㉔ $f(x) = 2x-3$, $g(x) = -x+3$
$(f\circ g)(4) = f(g(4)) = f(-1) = -5$

㉗ $(f\circ f)(2) = f(f(2)) = f(1) = -1$

㊴ $f(x) = \frac{1}{x-2}$, $g(x) = \frac{1}{x}$
$(f\circ g)(x) = f(\frac{1}{x}) = \frac{1}{\frac{1}{x}-2} \Rightarrow \frac{1}{x}-2 \neq 0 \Rightarrow \frac{1}{x} \neq 2 \Rightarrow x \neq \frac{1}{2}$ and $x \neq 0$
$Dom\ (f\circ g) = R\setminus\{0, \frac{1}{2}\} \cap R\setminus\{0\} = R\setminus\{0, \frac{1}{2}\}$

ⓑ $(g\circ f)(x) = g(\frac{1}{x-2}) = \frac{1}{\frac{1}{x-2}} = x-2 \Rightarrow Dom(g\circ f) = R\setminus\{2\} \cap R\setminus\{2\}$
$= R\setminus\{2\}$

التاريخ / / موضوع الدرس

Sec 3.3 Function Operations and composition

① $Dom(f \pm g) = Dom(f \cdot g) = Dom(f g)$

$= Dom\ f \cap Dom\ g$

② $Dom \frac{f}{g} = (Dom\ f \cap Dom\ g) \setminus \{x \mid g(x) = 0\}$

Hw 1 / 106 $f(x) = 8x - 9$, $g(x) = \sqrt{2x-1}$

$(f + g)(x) = 8x - 9 + \sqrt{2x-1}$

$2x - 1 \geq 0$

$x \geq \frac{1}{2}$

$Dom\ f = \mathbb{R}$, $Dom\ g = [\frac{1}{2}, \infty)$

① $Dom(f+g) = \mathbb{R} \cap [\frac{1}{2}, \infty) = [\frac{1}{2}, \infty)$

② $Dom \frac{f}{g} = [\frac{1}{2}, \infty) \setminus \{\frac{1}{2}\} = (\frac{1}{2}, \infty)$

Hw / 108 $f(x) = 2x^3 - 3x$ The difference quotient

$$\frac{f(x+h) - f(x)}{h} = \frac{[2(x+h)^2 - 3(x+h)] - [2x^2 - 3x]}{h}$$

$$= \frac{2(x^2 + 2xh + h^2) - 3x - 3h - 2x^2 + 3x}{h} = \frac{4xh + 2h - 3h}{h}$$

$= 4x + 2h - 3$

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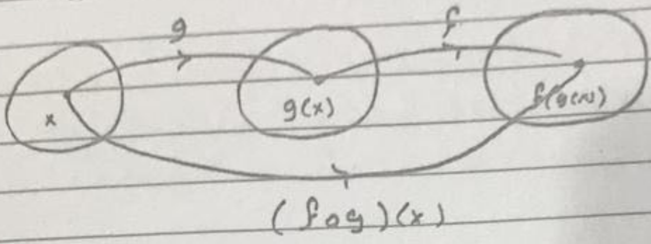
التاريخ / /

Sec 3.3 Function Operation and Composition موضوع الدرس

The composition of f and g is $(f \circ g)(x) = f(g(x))$
where $x \in$ Dom g and $g(x) \in$ Dom f



Dom $f \circ g$ = $D_{om}(f(g(x))) \cap$ Dom g

Ex: $f(x) = \sqrt{x}$, $g(x) = 4x+2$

(a) $(f \circ g)(x) = f(4x+2) = \sqrt{4x+2} \Rightarrow x \geq -\frac{1}{2}$

Dom $f \circ g = [-\frac{1}{2}, \infty) \cap \mathbb{R} = [-\frac{1}{2}, \infty) \Rightarrow$ مجال الناتج تقاطع مجال الدالة الثانية

(b) $(g \circ f)(x) = g(\sqrt{x}) = 4\sqrt{x} + 2$

Dom $g \circ f = [0, \infty) \cap [0, \infty) = [0, \infty) \Rightarrow$ مجال الناتج تقاطع مجال الثاني

Sec 3.3 Equations of lines موضوع الدرس / / التاريخ

(9) vertical, $(-6, 4) \Rightarrow x = -6$

(10) horizontal, $(-7, 4) \Rightarrow y = 4$

(11) $m = 5,\ b = 15 \rightarrow y = mx + b = 5x + 15$

(13) slope 0, y-intercept $\frac{3}{2} \Rightarrow m = 0,\ (0, \frac{3}{2}) \Rightarrow y = \frac{3}{2}$

(14) $x = -2$, does not, undefined, $y = \frac{1}{2}$, doesnot, 0

(15) Let x be $0 =$ $y = 3(0) - 1 = -1$

(18) $x + 2y = -4 \Rightarrow 2y = -4 - x \Rightarrow 2y = -4 - 0 \Rightarrow y = \frac{-4}{2} = -2$

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التاريخ / /

Sec 3.1 Functions موضوع الدرس

Ex $y = 3 + \frac{2}{x+5}$ Domain $= R / \{-5\}$
Range $= R / \{+3\}$

Ex $f(x) = 10 - \sqrt{x}$ Dom $= [0, \infty)$
Range $= (-\infty, 10]$

$\frac{16}{97}$ $y = -\sqrt{x}$ Dom $y = [0, \infty)$
Range $= (-\infty, 0]$

$\frac{17}{97}$ $xy = -6 \Rightarrow y = \frac{-6}{x}$ Dom $= R \setminus \{0\}$
Range $= R \setminus \{0\}$

(18) $y = \sqrt{7-2x} \Rightarrow 7 - 2x \geq 0 \rightarrow 7 \geq 2x$
$x \leq \frac{7}{2} \Rightarrow$ Dom $y = (-\infty, \frac{7}{2}]$
Range $y = [0, \infty)$

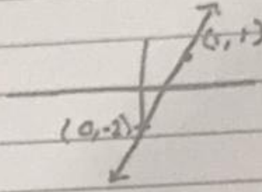
(19) $y = \frac{-7}{x-5} \Rightarrow$ Dom $y = R \setminus \{5\}$
Range $y = R \setminus \{0\}$

Ex $f(x) = 3x^2 - 5$
Dom $f = R$
Range $f = [-5, \infty)$

$x^2 \geq 0$
$\rightarrow 3x^2 \geq 0 \Rightarrow 3x^2 - 5 \geq -5$
$f(x) \geq -5$

EMKO

Sec 3.3 — Equations of lines — الموضوع — التاريخ / /



Hw2 / 100 $(1,1)\ (2,4) \Rightarrow m = \frac{4-1}{2-1} = 3$

$x_1\ y_1 \quad x_2\ y_2$

$y = 3(x-1)+1 \Rightarrow y = 3x-2$

Hw3 / 102 $(3,5)$

ⓐ Parallel to $2x+5y=4$

$m = \frac{-A}{B} = \frac{-2}{5}$

$y = \frac{-2}{5}(x-3)+5 \Rightarrow y = \frac{-2}{5}x + \frac{31}{5}$ (Point-slope)

$5y = -2x+31 \Rightarrow 2x+5y=31$ (standard)

ⓑ perpendicular to $2x+5y=4$

$m_1 = \frac{-1}{m_2} = \frac{-1}{\frac{-2}{5}} = \frac{5}{2}$

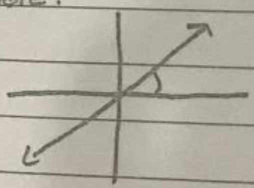
$y = \frac{5}{2}(x-3)+5 = \frac{5}{2}x - \frac{5}{2}$ (Point-slope)

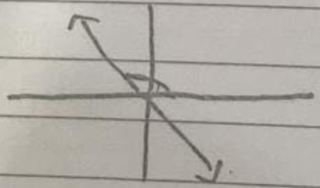
$-5x+2y=-5$ (standard)

Page 103: ① $y = \frac{1}{4}x+2$ Ⓓ

② $y-(-1) = \frac{3}{2}(x-1)$ Ⓒ

Note:





③ $m=-2,\ (1,3) \Rightarrow -2(x-1)+3 \Rightarrow y = -2x+5$

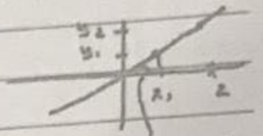
⑤ $\Rightarrow x = -8$

⑥ $\Rightarrow y = -8$

Sec 3.2 Equations of lines

التاريخ / /

① The slope of the line that passes through the points (x_1, y_1) (x_2, y_2) is given by $\Rightarrow m = \frac{y_2 - y_1}{x_2 - x_1}$



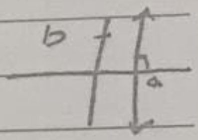
or $m = \tan\theta$

② the equation of this line is $\Rightarrow y = m(x - x_1) + y_1$
(Point-slope form)

③ The standard form of a line is $\Rightarrow Ax + By = C$
The slope here is $m = \frac{-A}{B}$

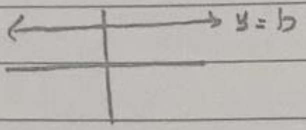
④ If the y-intercept is b then $y = mx + b$

⑤ The vertical line through the point (a, b) is $\boxed{x = a}$
and its slope is undefined



$x = a$

⑥ The horizontal line through (a, b) is $\boxed{y = b}$
the slope is zero



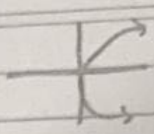
⑦ Two lines are parallel if they have the same slope
$m_1 = m_2$

⑧ Two lines are perpendicular
if $m_1 \cdot m_2 = -1$

HW 99: $(-3, 2)$ $(2, -4)$ $m = \frac{-4 - 2}{2 - -3} = \frac{-6}{5} \Rightarrow y = \frac{-6}{5}(x + 3) + 2$

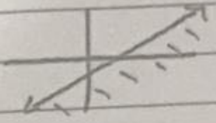
$\Rightarrow y = \frac{-6}{5}x - \frac{8}{5}$ (Point-slope) $\Rightarrow 5y = -6x - 8 \Rightarrow 6x + 5y = -8$
(standard)

Sec 3.1 Function الموضوع التاريخ / /



Ex3 92 (c) $y^2 = x$ is not a function of x
because $(1,1)(1,-1)$ are on the graph
Domain $= [0, \infty)$ Range $= \mathbb{R}$

(d) $y \le x-1$ is not a function because $(1,-1)(1,-2)$ are on the graph Domain $= \mathbb{R}$ Range $= \mathbb{R}$



(e) $y = \frac{5}{x-1}$ is a function Dom $= \mathbb{R} / \{1\}$
Range $= \mathbb{R} / \{0\}$

Hw3) $f(x) = -x^2 + 5x - 3$
$g(x) = 2x+3$

(a) $f(2) = -2^2 + 5(2) - 3 = 3$

(b) $f(q) = -q^2 + 5q - 3$

(c) $g(a+1) = 2(a+1) + 3 = 2a+5$

Defn A function is said to be (1) increasing on Interval I if for every $x_1 < x_2$ where $x_1, x_2 \in I$ we have $f(x_1) < f(x_2)$

(2) decreasing if $x_1 < x_2$ implies $f(x_1) > f(x_2)$

(3) constant if $f(x_1) = f(x_2)$ for every $x_1, x_2 \in I$

Ex The function $f(x) = 2x+1$ is increasing

Ex The function $f(x) = -2x+1$ is decreasing

Ex $f(x) = 10$ is constant

التاريخ / /

موضوع الدرس

Sec 1.1 Functions

Linear function $y = x + 4$

Dom $y = R$

Range $y = R$

Root function $y = \sqrt{2x-1} \Rightarrow 2x - 1 \geq 0 \Rightarrow x \geq \frac{1}{2}$

Dom $y = [\frac{1}{2}, \infty)$

Range $y = [0, \infty)$

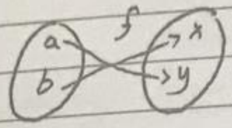
Quadratic functions

Domain: R

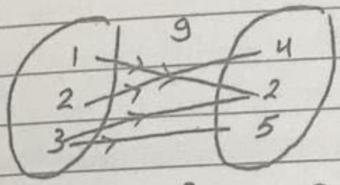
Range: $[-3, \infty)$

Sec 1.3 Function موضوع الدرس

Defn A function f is a relation from a set A into a set B such that each element in A assigns with one element from B. Here, A is called the domain of f and B is the range of f

Ex 

f is a function
Dom $f = \{a, b\}$, Range $f = \{x, y\}$

Ex 

g is not a function because 3 has two images

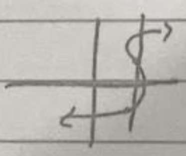
Dom $g = \{1, 2, 3\}$, Range $g = \{4, 2, 5\}$

Defn A function is a set of order pairs such that no first component is repeated

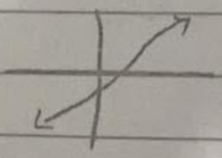
HW! 90 ⓐ $\{(3,-1), (4,2), (4,5), (6,8)\}$
is not a function because
4 has two images
Dom $= \{3, 4, 6\}$, Range $= \{-1, 2, 5, 8\}$

ⓑ function, Dom $= \{4, 6, 7, -3\}$
Range $= \{100, 200, 300\}$

Vertical line test



Not function



function

موضوع الدرس Absolute Value

Sec 2.5

⑨ $|3x-1| = 2$

$3x-1=2$ or $3x-1=-2$

$3x=3$ or $3x=-1$

$x=1$ or $x=\frac{-1}{3}$

⑫ $\left|\frac{5}{x-3}\right| = 10$ or $\frac{5}{x-3} = -10$

$5 = 10x-30$ or $5 = -10x+30$

$35 = 10x$ or $-25 = -10x$

$x = 3.5$ or $x = 2.5$

⑭ $|2x-3| = |5x+4|$

$2x-3 = 5x+4$ or $2x-3 = -5x-4$

$-7 = 3x$ or $7x = -1$

$x = \frac{-7}{3}$ or $\frac{-1}{7}$

㉖ $|4x+3|-2 = -1$

$|4x+3| = 1$

$4x+3 = 1$ or $4x+3 = -1$

$4x = -2$ or $4x = -4$

$x = \frac{-1}{2}$ or $x = -1$

㉟ $|4.3x+9.8| < 0 \rightarrow \emptyset$

㊱ $|2x+1| \leq 0 \rightarrow 2x+1=0 \rightarrow x = \frac{-1}{2}$

㊲ $|3x+2| > 0 \rightarrow \mathbb{R} \setminus \{\frac{-2}{3}\}$

Sec 2.5 Absolute Value موضوع الدرس / /

$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

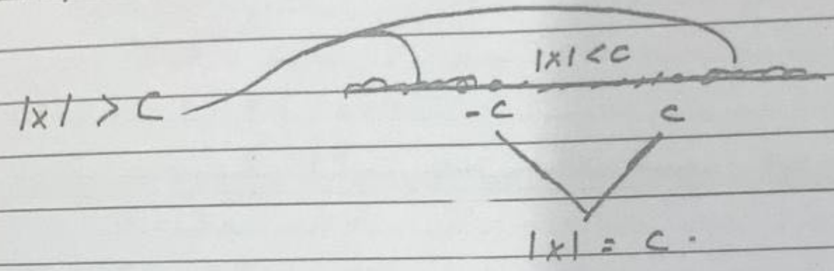
Ex $|-3| = 3$, $|0| = 0$, $|\frac{2}{5}| = \frac{2}{5}$

① $|x| = c \Leftrightarrow x = c$ or $x = -c$ where $c > 0$

② $|x| = |y| \Leftrightarrow x = y$ or $x = -y$

③ $|x| < c \Leftrightarrow -c < x < c$, where $c > 0$

④ $|x| > c \Leftrightarrow x > c$ or $x < -c$ where $c > 0$



Hw1 81 ⓐ $|2x+1| < 7$

$-7 < 2x+1 < 7$

$-8 < 2x < 6 \rightarrow -4 < x < 3$

Solution set is $(-4, 3)$

ⓑ $|2x+1| > 7$

$2x+1 > 7$ or $2x+1 < -7 \rightarrow 2x > 6$ or $2x < -8$

$x > 3$ or $x < -4 \rightarrow (3, \infty) \cup (-\infty, -4)$

Hw2 82 ⓐ $|2-5x| \geq -4 \rightarrow$ solution set is $\mathbb{R}$

ⓑ $|4x-7| < -3 \rightarrow \emptyset$

ⓒ $|5x+15| = 0 \rightarrow 5x+15 = 0$

$\rightarrow 5x = -15 \rightarrow x = -3$

Sec 2.4 Inqualities موضوع الدرس / / التاريخ

Page 78 :

(11) $\frac{4x+7}{-3} \le 2x+5 \to 4x+7 \ge -6x-15 \to 10x \ge -22$
$\to x \ge \frac{-22}{10} \to x \ge \frac{-11}{5}$

(19) $-3 \le \frac{x-4}{-5} < 4 \to 15 \ge x-4 > -20$
$\to 19 \ge x > -16$ Solution set is $(-16, 19]$

Ex $2x+1 \le 4+3x \le 2x+3 \to 1 \le 4+x \le 3$
$\to -3 \le x \le -1$ solution set is $[-3, -1]$

(24) $x^2 \le 9 \to x^2 - 9 \le 0$ solution set is $[-3, 3]$
(25) $x^2+5x+7 < 0$
$x = \frac{-5 \pm \sqrt{25-4(1)(7)}}{2} = \frac{-5 \pm \sqrt{3}\, i}{2}$ بخصوص أي جذر

Solution set is $\emptyset$ دائماً حاضنة

(38) $\frac{-4}{1-x} < 5 \to \frac{-4}{1-x} - 5 < 0 \to \frac{5x-9}{1-x} < 0$

$- - - \underset{1}{|} + + + + \underset{\frac{9}{5}}{|} - - -$

Solution set is $(-\infty, 1) \cup (\frac{9}{5}, \infty)$

Ex (1) $\frac{3}{2+x} \le 0 \to 2+x < 0 \to x < -2$ Solution set $(-\infty, -2)$
(2) $\frac{-2}{4+5x} > 0 \to 4+5x < 0 \to (-\infty, \frac{-4}{5})$

(3) $x^2+4x+4 \le 0 \to (x+2)^2 \le 0 \to (x+2)^2 = 0 \to x = -2$
(4) $x^2+4x+4 \ge 0 \to$ solution set is $\mathbb{R}$
(5) $x^2+4x+4 > 0 \to$ solution set is $\mathbb{R} \setminus \{-2\}$

Sec 2.4

Fact ① If $a<b$ then $a+c<b+c$

② If $c>0$ and $a<b$ then $ac<bc$

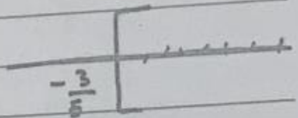
* ③ If $c<0$ and $a<b$ then $ac>bc$

Linear inequalities $ax>0$

HW1 / 73 $4-3x \le 7+2x$

$\to -3x-2x \le 7-4 \to -5x \le 3 \to x \ge \frac{3}{-5} \to x \ge \frac{-3}{5}$

The solution set is the interval $[-\frac{3}{5}, \infty)$



See the table Page 73

HW2 / 74 $-2<5+3x<20$

$-2-5<3x<20-5$

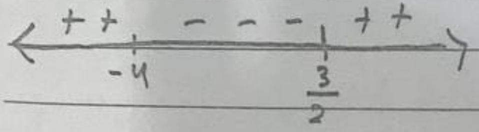
$-7<3x<15 \to \frac{-7}{3}<x<5 \to$ Solution set is $(-\frac{7}{3}, 5)$

Quadratic inequality

$ax^2+bx+c=0, \quad a\neq 0$

HW3 / 75 $2x^2+5x-12 \ge 0$

$x = \frac{-5 \pm \sqrt{25-4(2)(-12)}}{4} = \frac{-5 \pm \sqrt{121}}{4} = \frac{-5 \pm 11}{4} = \frac{3}{2}$ or -4



the solution set

$(-\infty, -4] \cup [\frac{3}{2}, \infty)$

موضوع الدرس Quadratic Equations التاريخ / /

Sec 2.3

Page 69. ① $x^2 = 25 \to x = \sqrt{25} \to x = \pm\sqrt{25} \to x = \pm 5$

② $x^2 + 5 = 0 \to x^2 = -5 \to x = \pm\sqrt{5}\,i$

③ $x^2 = -20 \to x = \pm\sqrt{20}\,i \to x = \pm 2\sqrt{5}\,i$

⑧ $5x^2 - 3x - 2 = 0 \to (5x+2)(x-1) = 0 \to 5x+2 = 0$ or $x - 1 = 0$

$\to x = \frac{-2}{5}$ or 1

⑩ $x^2 - 100 = 0 \to (x-10)(x+10) = 0 \to x = 10$ or $x = -10$

⑰ $(x+5)^2 = -3 \to x + 5 = \pm\sqrt{3}\,i \to x = -5 \pm \sqrt{3}\,i$

⑲ $x^2 - 4x + 3 = 0$, $h = \frac{4}{2} = 2$, $k = 2^2 - 4(2) + 3 = -1$

$(x-2) - 1 = 0 \to (x-2)^2 = 1 \to x - 2 = \pm\sqrt{1} = \pm 1$

$\to x = 2 \pm 1 \to x = 3$ or $x = 1$

㉖ $x^2 - x - 1 = 0 \to x = \dfrac{1 \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2} \to x = \dfrac{1 \pm \sqrt{5}}{2}$

㉝ $(x-9)(x-1) = -16 \to x^2 - x - 9x + 9 + 16 = 0$

$\to x^2 - 10x + 25 = 0 \to x = \dfrac{10 \pm \sqrt{100 - 4(1)(25)}}{2} \to x = 5$

㊺ $9x^2 + 11x + 4 = 0 \to D = b^2 - 4ac = 11^2 - 4(9)(4)$

$\to = 121 - 144 < 0 \to$ Two nonreal complex solutions

Sec 2.3 Quadratic Equations / / التاريخ

The quadratic formula

If $ax^2+bx+c=0$ then $\quad a\neq 0$

$$\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$ where $D=b^2-4ac$ is the discriminant

HW3 68 $\quad 2x^2=x-4$

$\Rightarrow 2x^2-x+4=0$

$a=2,\ b=-1,\ c=4$

$$x=\frac{1\pm\sqrt{(1)^2-4(2)(4)}}{4}=\frac{1\pm\sqrt{-31}}{4}=\frac{1}{4}\pm\frac{\sqrt{31}\,i}{4}$$

$$D=b^2-4ac$$

- $D>0$, D is perfect square $\downarrow$ Two rational Solution
- $D>0$, D is not perfect square $\downarrow$ Two irrational Solution
- $D=0$ $\downarrow$ one rational solution
- $D<0$ $\downarrow$ Two nonreal complex Solutions

HW4 68 $\quad A=\frac{\pi d^2}{4}$, for d

$\Rightarrow d^2=\frac{4A}{\pi}\Rightarrow d=\pm 2\sqrt{\frac{A}{\pi}}$

Zero factor property

7 P.70 $\quad x^2-5x+6=0\Rightarrow(x-2)(x-3)=0$

$\Rightarrow x-2=0$ or $x-3=0$

$\Rightarrow x=2,\ x=3$

EMKO

التاريخ / /

Sec 2.3 Quadratic Equations الموضوع

The standard form of a quadratic equation is
$ax^2+bx+c=0$ where a,b,c are constants

Square root property
If $x^2=c$ then $x=\pm\sqrt{c}$

Hw1
65
a- $x^2=17 \to x=\pm\sqrt{17}$
b- $x^2=-25 \to x=\pm\sqrt{-25} \to x=\pm\sqrt{25}\,i=\pm 5i$
c- $(x-4)^2=12 \to x-4=\pm\sqrt{12}$
$\to x=4\pm\sqrt{12}=4\pm\sqrt{4}\sqrt{3}=4\pm 2\sqrt{3}$

completing the square
If $ax^2+bx+c=0$
and $f(x)=a(x^2+bx+c$

$\boxed{h=\frac{-b}{2a}}$ $\boxed{k=f(h)}$ then $\boxed{f(x)=a(x-h)^2+k}$

Hw2
66
$9x^2-12x+9=0$
$h=\frac{-b}{2a}=\frac{12}{18}=\frac{2}{3}$

$k=f\left(\frac{2}{3}\right)=9\left(\frac{2}{3}\right)^2-12\left(\frac{2}{3}\right)+9$
$=4-8+9=5 \to 9\left(x-\frac{2}{3}\right)^2+5=0$
$\to \left(x-\frac{2}{3}\right)^2+5=0 \to \left(x-\frac{2}{3}\right)^2=\frac{-5}{9}$
$\to x-\frac{2}{3}=\pm\sqrt{\frac{-5}{9}}=\pm\frac{\sqrt{5}\,i}{3} \to x=\frac{2}{3}\pm\frac{\sqrt{5}\,i}{3}$

Sec 2.2 Complex Numbers موضوع الدرس / /

Page 12:

(7) -4 is real

(8) $13i$ is pure imaginary

(9) $5+i$ is nonreal complex

(10) π is real

(11) $\sqrt{-25} = \sqrt{25}\,i = 5i$ is pure imaginary

(15) $-\sqrt{-18} = -\sqrt{18}\,i = -\sqrt{9}\sqrt{2}\,i = -3\sqrt{2}\,i$

(19) $\frac{\sqrt{-24}}{\sqrt{8}} = \sqrt{\frac{24}{8}}\,i = \sqrt{3}\,i$

(18) $\frac{\sqrt{-30}}{\sqrt{-10}} \quad \frac{\sqrt{30}\,i}{\sqrt{10}\,i} = \sqrt{\frac{30}{10}} = \sqrt{3}$

(22) $\frac{-6-\sqrt{-24}}{2} = \frac{-6-\sqrt{6}\sqrt{4}\,i}{2} = -3-\sqrt{6}\,i$

(31) $(3-2i)^2 = 9-12i+4i^2 = 5-12i$

(34) $(\sqrt{6}+i)(\sqrt{6}-i) = (\sqrt{6})^2+1^2 = 6+1 = 7$

(50) $i^{-13} = (i^4)^{2}\cdot i^{-} = 1\cdot i^{-1} = i^{-1} = \frac{1}{i} = \frac{-i}{1} = -i$

(29) $(2+i)(3-2i) = 6-4i+3i-2i^2 = 8-i$

Sec 2.2 complex Numbers موضوع الدرس / / التاريخ

$x^2 = -1 \Longrightarrow$ has no real solution

suppose that $i = \sqrt{-1}$ then $i^2 = -1$

The standard form of a complex number is $a + ib$

Where a, b are $\in \mathbb{R}$, Here a is called the real part and b is called imaginary part. Ex $\underbrace{5}_{\text{Real Part}} - \underbrace{2i}_{\text{Imaginary Part}}$

$a + ib$

$a \neq 0$	$a = 0$	$a \neq 0$
$b = 0$	$b \neq 0$	$b \neq 0$
$a + ib = a$	$a + ib = ib$	$a + ib$ is nonreal
is real number	is pure imaginary	complex number
Ex $2, -3, \frac{5}{7}, \sqrt{3} \in \mathbb{R}$	Ex $3i, -7i, -\frac{1}{2}i$	Ex $-2+i$, $4+5i$

Fact If $x > 0$ then $\sqrt{-x} = \sqrt{x}\, i$

Hw1 10
a- $\sqrt{-7}\cdot\sqrt{-7} = \sqrt{7}i \cdot \sqrt{7}i = (\sqrt{7})^2 i^2 = -7$

c- $\sqrt{-6}\cdot\sqrt{-10} = \sqrt{6}i \cdot \sqrt{10}i = \sqrt{60}\, i^2$

$= -\sqrt{60} = -\sqrt{4}\sqrt{15} = -2\sqrt{15}$

EMKO

Sec 2.2 Complex Numbers

Addition $(a+ib)+(c+id)=(a+c)+i(b+d)$

Subtraction $(a+ib)-(c+id)=(a-c)+i(b-d)$

Multiplication $(a+ib)(c+id)=ac+iad+ibc+i^2bd$

$$=(ac-bd)+i(ad+bc)$$

Note: $(c+id)(c-id)=c^2+d^2$

Defn: The conjugate of $c+id$ is $c-id$

Division $\frac{a+ib}{c+id}=\frac{(a+ib)(c-id)}{(c+id)(c-id)}=\frac{ac-iad+ibc-i^2bd}{c^2+d^2}$

$$=\left(\frac{ac+bd}{c^2+d^2}\right)+i\left(\frac{bc-ad}{c^2+d^2}\right)$$

Hw2 / 60
(a) $(3-4i)+(-2+6i)=1+2i$
(b) $(-4+3i)-(6-7i)=-10+10i$

Hw3 / 62
(a) $\frac{3+2i}{5-i}=\frac{(3+2i)(5+i)}{(5-i)(5+i)}=\frac{15+3i+10i+2i^2}{5^2+1^2}$

$$=\frac{13}{26}+\frac{13}{26}i=\frac{1}{2}+\frac{1}{2}i$$

(b) $\frac{3}{i}=\frac{3(-i)}{i(-i)}=\frac{-3i}{1}=-3i$

Example
1. $i^3=i^2i=-i$
2. $i^4=i^2i^2=1$
3. $i^5=i^4i=i$
4. $i^{27}=(i^4)^6i^3=(1)(-i)=-i$
5. $i^{1013}=(i^4)^{253}i=1(i)=i$

Linear Equations

identity
infinity many solutions
when we solve it we get $0=0$
solution set is $\mathbb{R}$
Ex $2(3-5x)=-10x+6$
$6-10x=-10x+6 \Rightarrow 0=0$
$\Rightarrow$ Solution set is $\mathbb{R}$.

conditional
one solution
$x=\frac{-b}{a}$
Ex $2x-1=0$
$\Rightarrow x=\frac{1}{2}$

contradiction
No Solution
when we solve it we get two different numbers are equal
solution set is ϕ
Ex $2x+1=3+2x$
$\Rightarrow 1=3$ impossible
solution set is ϕ

Hw 2 56

(a) $I=Prt$, for t
$\Rightarrow t=\frac{I}{Pr}$

(b) $A-P=Prt$, for P
$\Rightarrow A=P+Prt$
$\Rightarrow A=P(rt+1)$
$\Rightarrow P=\frac{A}{1+rt}$

Defn Two equations are equivalent if they have the same solution set

Page 57: (1) $2x+5=x-3 \Rightarrow$ True
(2) $5(x-8)=5x-40 \Rightarrow$ True
(3) $x^2=4$, $x+2=4 \Rightarrow$ True
(4) False
(19) $\frac{1}{14}(3x-2)=\frac{x+10}{10} \Rightarrow 10(3x-2)=14(x+10)$
$\Rightarrow 30x-20=14x+140 \Rightarrow 16x=160$
$\Rightarrow x=10$

Sec 2.1 Linear Equations موضوع الدرس / / التاريخ

Recall that

① if $a = b$ then $a + c = b + c$

② if $a = b$ then $ac = bc$

The standard form of a linear Equation is $ax + b = 0$
Where a and b are constants

Ex Solve

$$2(3x - 1) + 7 = 2 - 5x$$

Sol $6x - 2 + 7 = 2 - 5x$

$\Rightarrow 6x + 5x = 2 - 5$

$\Rightarrow 11x = -3 \Rightarrow x = \frac{-3}{11}$

Hw1
65

$$\frac{2x + 4}{3} + \frac{1}{2}x = \frac{1}{4}x - \frac{7}{3}$$

$\Rightarrow \frac{2x}{3} + \frac{1}{2}x - \frac{1}{4}x = \frac{-4}{3} - \frac{7}{3}$

$\Rightarrow \frac{8 + 6 - 3}{12}x = -\frac{11}{3} \Rightarrow \frac{11}{12}x = \frac{-11}{3} \Rightarrow \left(\frac{-11}{3}\right)\left(\frac{12}{11}\right)$

$\Rightarrow x = \frac{-12}{3} \Rightarrow x = -4$

Sec 1.6 Rational Exponents موضوع الدرس / / التاريخ

Hw 4 / 41 $\dfrac{(x+y)^{-1}}{x^{-1}+y^{-1}} = \dfrac{\frac{1}{x+y}}{\frac{1}{x}+\frac{1}{y}} = \dfrac{\frac{1}{x+y}}{\frac{x+y}{xy}} = \dfrac{1}{x+y} \cdot \dfrac{xy}{x+y} = \dfrac{xy}{(x+y)^2}$

Hw 3 / 40 (c) $6y^{2/3} \cdot 2y^{1/2} = 12y^{2/3+1/2} = 12y^{7/6}$

(d) $\left(\dfrac{3m^{5/6}}{y^{3/4}}\right)^2 \left(\dfrac{8y^3}{m^4}\right)^{2/3} = \dfrac{9m^{5/3}}{y^{3/2}} \cdot \dfrac{4y^2}{m^4} = 36m^{-7/3}y^{1/2}$

(e) $m^{2/3}(m^{7/3} + 2m^{1/3}) = m^3 + 2m$

Fact: (1) if n is even then $\sqrt[n]{a} = |a|$

(2) if n is odd then $\sqrt[n]{a^n} = a$

Hw 6 / 43 (a) $\sqrt{p^4} = (p^4)^{\frac{1}{2}} = p^2$

(b) $\sqrt[4]{p^4} = |p|$

(d) $\sqrt[6]{(-2)^6} = |-2| = 2$

(e) $\sqrt[5]{m^5} = m$

(f) $\sqrt{(2k+3)^2} = |2k+3|$

(g) $\sqrt{x^2 - 4x + 4} = \sqrt{(x-2)^2} = |x-2|$

Ex $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y}) = x - y$

41 / P. 48 $y^{-5} - 3y^{-3} = y^{-5}(1 - 3y^2)$

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التاريخ / / موضوع الدرس

Sec 1.6 Rational Exponents

Recall that: ① $a^{-n} = \frac{1}{a^n}$, $a \neq 0$ ② $a^{\frac{1}{n}} = \sqrt[n]{a}$, $a > 0$

③ $a^{\frac{m}{n}} = (a^{\frac{1}{n}}) = (\sqrt[n]{a})^m$ or $a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} = \sqrt[n]{a^m}$ Ex $a^{\frac{1}{2}} = \sqrt{a}$

$a^{\frac{1}{3}} = \sqrt[3]{a}$

$a^{\frac{1}{4}} = \sqrt[4]{a}$

④ If $a < 0$ and n is even then $a^{\frac{1}{n}}$ is not a real number

Ex $(-4)^{\frac{1}{2}}$ is not a real number because $a = -4$ and $n = 2$

Ex $-4^{\frac{1}{2}} = -\sqrt{4} = -2$

Hw 1 / 37

ⓐ $\frac{12^5}{12^2} = 12^3$ ⓑ $\frac{a^5}{a^{-8}} = a^{5+8} = a^{13}$

ⓒ $\frac{16m^{-9}}{12m^{11}} = \frac{4}{3m^{20}}$ ⓓ $\frac{25r^{7}z^{5}}{10r^{9}z} = \frac{5z^4}{2r^2}$

Hw 2 / 39

ⓐ $36^{\frac{1}{2}} = \sqrt{36} = 6$

ⓑ $-100^{\frac{1}{2}} = -\sqrt{100} = -10$

ⓓ $625^{\frac{1}{4}} = \sqrt[4]{625} = 5$

ⓔ $(-1296)^{\frac{1}{4}}$ is not a real number

ⓕ $-1296^{\frac{1}{4}} = -\sqrt[4]{1296} = -6$

ⓖ $(-27)^{\frac{1}{3}} = \sqrt[3]{-27} = -3$

ⓗ $-32^{\frac{1}{5}} = -\sqrt[5]{32} = -2$

Hw 3 / 40

ⓐ $\frac{27^{\frac{1}{3}} \cdot 27^{\frac{5}{3}}}{27^3} = \frac{27^2}{27^3} = \frac{1}{27}$

ⓑ $81^{\frac{5}{4}} \cdot 4^{\frac{-3}{2}} = (\sqrt[4]{81})^5 \cdot (\sqrt{4})^{-3}$

$= 3^5 \cdot 2^{-3} = \frac{3^5}{2^3} = \frac{243}{8}$

Sec 1.3 Rational expressions موضوع الدرس / / التاريخ

Hw 2 / 34

ⓐ $\frac{5}{9x^2}+\frac{1}{6x}=\frac{30x+9x^2}{54x^3}=\frac{3x(10+3x)}{3x(18x^2)}=\frac{10+3x}{18x^2}, x\neq 0$

ⓑ $\frac{y}{y-2}+\frac{8}{2-y}=\frac{y}{y-2}-\frac{8}{y-2}=\frac{y-8}{y-2}, y\neq 2$

Page 35: ② $\frac{9x+12}{(2x+3)(x-5)}$

$2x+3=0 \Rightarrow x=\frac{-3}{2}$

$x-5=0 \Rightarrow x=+5$

Domain $= \mathbb{R}\setminus\{\frac{-3}{2}, 5\}$

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Sec 1.5 Rational expressions موضوع الدرس / / التاريخ

The function $R(x) = \dfrac{P(x)}{Q(x)}$

Where P and Q are two polynomials is called rational function

The domain of $R(x)$ is $\{x \mid Q(x) \neq 0\}$

Ex The domain of $\dfrac{x^2-5x+1}{x+3}$ is $\mathbb{R} \setminus \{-3\}$ or $(-\infty,-3) \cup (-3,\infty)$

-3

Zero factor property

If $ab=0$, then $a=0$ or $b=0$

Fact $\dfrac{ac}{bc} = \dfrac{a}{b}$, $b \neq 0$, $c \neq 0$

HW! 32 (a) $\dfrac{2x^2+7x-4}{5x^2+20x} = \dfrac{(2x-1)(x+4)}{5x(x+4)} = \dfrac{(2x+1)}{5x}$, $x \neq -4$, $x \neq 0$

(b) $\dfrac{6-3x}{x^2-4} = \dfrac{3(2-x)}{(x-2)(x+2)} = \dfrac{-3(x-2)}{(x-2)(x+2)} = \dfrac{-3}{(x+2)}$, $x \neq 2$, $x \neq -2$

Addition and Subtraction

$\dfrac{a}{b} \mp \dfrac{c}{d} = \dfrac{ad \mp bc}{bd}$ توحيد المقامات

Multiplication $\dfrac{a}{b} \cdot \dfrac{c}{d} = \dfrac{ac}{bd}$

Division $\dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{a}{b} \cdot \dfrac{d}{c} = \dfrac{ad}{bc}$

Sec 1.4 Factoring Polynomials موضوع الدرس تحليل كثيرات الحدود التاريخ / /

More Rules:

(4) $x^3+y^3 = (x+y)(x^2-xy+y^2)$

(5) $x^3-y^3 = (x-y)(x^2+xy+y^2)$

$\frac{Hw^2}{26}$ (a) $16p^2-40pq+5q^2$

$= (4p)^2 - 2(4p)(5q) + (5q)^5$

$= (4p-5q)^2$

or $(4p-5q)(4p-5q)$

$\frac{Hw^3}{28}$ (a) $x^3+27 = x^3+3^3$

$= (x+3)(x^2-3x+9)$ القانون الرابع

(b) $m^3-64n^3 = m^3-(4n)^3$

$= (m-4n)(m^2+4nm+16n^2)$ القانون الخامس

(c) $8q^6+125p^9$

$= (2q^2)^3+(5p^3)^3 = (q+5p^3)(q-10q^2p^3+25p^6)$ القانون الرابع

Page 29: (26) $16q^2-25 = (4q)^2-5^2 = (4q-5)(4q+5)$ القانون الأول

(27) $y^4-81 = (y^2)^2-9^2 = (y^2-9)(y^2+9) = (y-3)(y+3)(y^2+9)$

(24) $(2p+q)^2-10(2p+q)+25 \Rightarrow$ Let $x = 2p+q$

$= x^2-10x+25$

$= (x-5)(x-5)$

$= (x-5)^2 = (2p+q-5)^2$

التاريخ / / موضوع الدرس

Sec 1.4 Factoring Polynomials تحليل كثيرات الحدود

(A) By taking the greatest common factor

Page 29. (1) $15r - 27 = 3(5r - 9)$

(2) $9z^4 + 81z = 9z(z^3 + 9)$

(3) $5h^2j + hj = hj(5h + 1)$

(5) $28r^4s^2 + 7r^3s - 35r^4s^3 = 7r^3s(4rs + 1 - 5rs^2)$

(7) $(4z-5)(3z-2) - (3z-9)(3z-2) = (3z-2)(4z-5-3z+9)$

$= (3z-2)(z+4)$

(B) By grouping

Hw 25 (a) $\underbrace{mp^2 + 7m} + \underbrace{3p^2 + 21}$

$m(p^2+7) + 3(p^2+7) = (p^2+7)+(m+3)$

(b) $2y^2 + az - 2z - ay^2 = y^2(2-a) + z(a-2) = y^2(2-a) - z(2-a)$

$= (2-a)(y^2 - z)$

or $(a-2)(z-y^2)$

(c) $4x^3 + 2x^2 - 2x - 1 = 2x^2(2x+1) - (2x+1) = (2x+1)(2x^2-1)$

(C) Factoring Trinomial

Ex $x^2 - 5x - 6 = (x-6)(x+1)$

Page 29: (13) $8h^2 - 2h - 21 = (4h-7)(2h+3)$

ضرب البعيدين + ضرب القريبين = الحد الأوسط

(16) $36x^3 + 18x^2 - 4x = 2x(18x^2 + 9x - 2) = 2x(6x-1)(3x+2)$

Sec 1.3 Polynomials كثيرات الحدود موضوع الدرس / / التاريخ

Page 21: (7) $(-3m^4)(6m^2)(-4m^5) = 72m^{11}$

(15) $-(4m^3n^0)^2 = -16m^6$

(19) $\left(\frac{-4m^2}{7p^2}\right)^4 = \frac{256m^8}{7^4p^8}$

(31) $-7z^5-2z^3+1$ is trinomial Degree $= 5$

(41) $2(12y^2-8y+6)-4(3y^2-4y+2)$

$=24y^2-16y+12-12y^2+16y-8$

$=12y^2+4$

(48) $m^3(2m-\frac{1}{4})(3m+\frac{1}{2}) = m^3(6m^2+m-\frac{3}{4}m-\frac{1}{8})$

$= m^3(6m^2+\frac{1}{4}m-\frac{1}{8}) = 6m^5+\frac{1}{4}m^4-\frac{1}{8}m^3$

(67) $[(2p-3)+q]^2 = (2p-3)^2+2(2p-3)q+q^2$

$=(2p)^2-2(2p)(3)+9+2(2p-3)q+q^2$

$4p^2-12p+9+4pq-6q+q^2$

(73) $(y+2)^3 = (y+2)(y+2)^2 = (y+2)(y^2+4y+4)$

$= y^3+4y^2+4y+2y^2+8y+8 = y^3+6y^2+12y+8$

(87) $\frac{-4x^7-14x^6+10x^4-14x^2}{-2x^2} = 2x^5+7x^4-5x^2+7$

(89)

$$\begin{array}{r|l} & 4x^2+5x+10 \\ x-2 & 4x^3-3x^2+1 \\ & \underline{4x^3-8x^2} \\ & 5x^2+1 \\ & \underline{5x^2-10x} \\ & 10x+1 \\ & \underline{10x-20} \\ & 21 \end{array}$$

$= \frac{4x^3-3x^2+1}{x-2} = 4x^2+5x+10+\frac{21}{x-2}$

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Sec 1.3 Polynomials كثيرات الحدود موضوع الدرس التاريخ / /

Ex $2(x^2+5x+1)-3(2x^2-2x+7)$

$= 2x^2+10x+2-6x^2+6x-21$

$= -4x^2+16x-19$

Ex $(3x^2-x+1)(5x-4)$

$= 15x^3-12x^2-5x^2+4x+5x-4$

$= 15x^3-17x^2+9x-4$

special products

① $(x-y)(x+y) = x^2-y^2$

② $(x+y)^2 = x^2+2xy+y^2$

③ $(x-y)^2 = x^2-2xy+y^2$

Hw 4 / 20

ⓐ $(3p+11)(3p-11) = (3p)^2-(11)^2 = 9p^2-121$

ⓑ $(5m^3-3)(5m^3+3) = (5m^3)^2-3^2 = 25m^6-9$

ⓒ $(3x-7y^4)^2 = (3x)^2-2(3x)(7y^4)+(7y^4)^2$

$= 9x^2-42xy^4+49y^8$

5

$$\begin{array}{r|l} & 2m^2-3m+1 \\ \hline 2m-1 & 4m^3-8m^2+5m+6 \\ & -(4m^3-2m^2) \\ \hline & -6m^2+5m+6 \\ & -(-6m^2+3m) \\ \hline & 2m+6 \\ & -(2m-1) \\ \hline & 7 \end{array}$$

$7 \rightarrow$ Remainder

$$\frac{4m^3-8m^2+5m+6}{2m-1} = \underbrace{2m^2-3m+1}_{\text{Qoutient}} + \underbrace{\frac{7}{2m-1}}_{\text{Remainder}}$$

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Sec 1.3 Polynomials موضوع المحور كين نت الكورد

(4) polynomial is a finite sum of terms where the powers of variables are whole numbers

Ex $5xy^6 + 9xz - 8$ ⇒ is polynomial
deg = 7, deg = 2, deg =
Degree = 7   3 terms (Trinomial)

Ex $7xy - 6z^3$ is polynomial
Deg = 3   2 terms (Binomial)

Ex $3xy^5$ 1 term (monomial)

Ex $\sqrt{2}$ is a monomial   deg = 0

Ex $2\sqrt{x}$ is not a polynomial

Ex find the degree of $(3x^5(x+y^2)^7)$
Degree = 19

Ex $3x^2y(2z+t^4)$
Bionomial   Degree = 7

Hw2 / P.18   read from the book

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Sec 1.3 Poly nomials الموضوع / الدرس ... التاريخ / /

Exponent for Rules

① $a^n \cdot a^m = a^{n+m}$

② $\frac{a^n}{a^m} = a^{n-m}$

③ $(a^n)^{m} = a^{nm}$

④ $a^0 = 1$ where $a \neq 0$

⑤ 0^0 is undefined

⑥ $(ab)^n = a^n b^n$

⑦ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ $b \neq 0$

HW1 16

ⓐ $(5^3)^2 = 5^6$

ⓑ $(3^4 x^2)^3 = 3^{12} x^6$

ⓒ $\left(\frac{2^5}{b^4}\right)^3 = \frac{2^{15}}{b^{12}}$

ⓓ $\left(\frac{-2m^6}{t^2 z}\right)^5 = \frac{(-2)^5 m^{30}}{t^{10} z^5} = \frac{-32m^{30}}{t^{10} z^5}$

Ex: $3^0 = 1$, $-3^0 = -1$, $(-3)^0 = 1$

Ex ① Algebraic Expression

$2xy - 5z^2$, $\frac{-x^3y^2+1}{3t+\sqrt{x}}$, x

② like terms

$2xy^2$, $-5xy^2$

③ unlike terms

$3\sqrt{x}\,y$, $5xy$

$\underbrace{-2}_{\text{coefficient}}xyz$, xy

Sec 1.2 Real Numbers موضوع الدرس / /

Page 14

(25) $-2.5 + 12 \div 3 = -10 + 12 \div 3$
$= -10 + 4 = -6$

(63) $\frac{10}{11}(22\%) = 10(2\%) = 20\%$
قسمة

(88) $-|-12| = -12$

Read the table Page 10

Sec 1.2 Real Numbers الموضوع: / / التاريخ

Distribution ① $a(b+c) = ab + ac$

التوزيع ② $a(b-c) = ab - ac$

Hw 3 P.11

ⓐ $3(x+y) = 3x + 3y$

ⓑ $-(m-4n) = -m + 4n$

ⓒ $\frac{1}{3}\left(\frac{4}{5}m - \frac{3}{2}n - 27\right) = \left(\frac{4}{15}m - \frac{3}{6}n - 9\right) =$

$\frac{4}{15}m - \frac{1}{2}n - 9$

ⓓ $7p + 21 = 7(p+3)$

The Absolute Value

القيمة المطلقة

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Hw 4 12

ⓐ $|2x - 3y| = |2(-6) - 3(10)|$

$= |-12 - 30| = |-42| = 42$

ⓑ $\frac{2|x| - |3y|}{|xy|} = \frac{2|-6| - |3(10)|}{|(-6)(10)|} = \frac{12-30}{60} = \frac{-18}{60}$

$= \frac{-3}{10}$

The distance between a and b is $|a-b|$ or $|b-a|$

Ex Find the distance between -3 and 2

$d(-3, 2) = |-3-2| = |-5| = 5$

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Sec 1.2 Real Numbers

Note: $N \subseteq W \subseteq Z \subseteq Q \subseteq R$

Page 13: $A = \{-6, \frac{-12}{4}, \frac{-5}{8}, -\sqrt{3}, 0, \frac{1}{4}, 1, 2\pi, 3, \sqrt{12}\}$

(11) natural numbers are: $\{1, 3\}$
(12) whole numbers: $\{0, 1, 3\}$
(13) integer numbers: $\{-6, \frac{-12}{4}, 0, 1, 3\}$
(14) rational numbers: $\{-6, -\frac{12}{4}, \frac{-5}{8}, 0, 1, 3\}$
(15) irrational numbers: $\{-\sqrt{3}, \sqrt{12}, 2\pi\}$
(16) all are real numbers

Exponential Notation

$a^n = a \cdot a \cdot \ldots \cdot a$ n-times

Ex $a^2 = a \cdot a$, $a^3 = a \cdot a \cdot a$

order of operations

(1) fraction Bar
(2) Parantheses ( ), brackets [ ], braces { }
(3) Powers and roots
(4) multiplication and division (left to right)
(5) Addition and subtraction (left to right)

Hw 1 / 8

(a) $4^3 = 4 \cdot 4 \cdot 4 = 64$
(b) $(-6)^2 = (-6) \cdot (-6) = 36$
(c) $-6^2 = -36$
(d) $4 \cdot 3^2 = 4 \cdot 9 = 36$
(e) $(4 \cdot 3)^2 = 12^2 = 144$

Notes
1. $A \subseteq A$
2. $\emptyset \subseteq A$
3. $A \cap A' = \emptyset$
4. $A \cup A' = U$

Hw 2 P.4
$U = \{1, 2, 3, 4, 5, 6, 7\}$
$A = \{1, 3, 5, 7\}$ $B = \{3, 4, 6\}$
(a) $A' = \{2, 4, 6\}$ (b) $B' = \{1, 2, 5, 7\}$
(c) $\emptyset' = U$ (d) $U' = \emptyset$

Hw 3 P.5
(a) $\{1, 2, 3, 4, 5, 8, 9, 14\}$
(b) $\{1, 2, 3, 4, 5, 6, 7\}$
(c) $\{1, 2, \dots\} = N$

Page 6
(3) $= \{10, 11, 12, \dots\}$ infinite
(4) $= \{2, 4, 6, \dots\}$ infinite
(8) $= \{1, 2, 3, 4\}$
(9) $= 9 \in \{3, 2, 5, 9, 8\}$
(13) $= \{2\} \subseteq \{2, 4, 6, 8\}$
(14) $\emptyset \subseteq \emptyset$

Sec1.1 Sets موضوع الدرس المجموعات التاريخ / /

(3) $A \cup B = \{1,2,5,6,7\}$
(4) $A \cap B = \{2\}$
(5) $U \cap A = \{1,2,6\} \Longrightarrow A$
(6) $U' = \phi$
(7) $\phi' = U$
(8) $A'' = A$
(9) $(A \cap B)' = \{1,3,4,5,6,7\}$
(10) $A' \cap B' = \{3,4\}$
(11) $A \cap \phi = \phi$
(12) $A \cup \phi = A$

Sec1.1 Sets المجموعات موضوع الدرس / / التاريخ

(3) $A \cup B = \{1,2,5,6,7\}$

(4) $A \cap B = \{2\}$

(5) $U \cap A = \{1,2,6\} \Longrightarrow A$

(6) $U' = \phi$

(7) $\phi' = U$

(8) $A'' = A$

(9) $(A \cap B)' = \{1,3,4,5,6,7\}$

(10) $A' \cap B' = \{3,4\}$

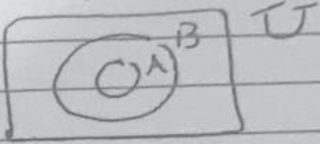
(11) $A \cap \phi = \phi$

(12) $A \cup \phi = A$

Defn ① The empty set or null set is denoted by ϕ or $\{\}$

شامل ② The universal set is denoted by U

جزئية ③ A set A is a subset of B denoted by $A \subset B$ if each element in A belongs to B

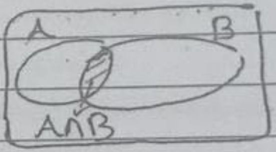


$A \subseteq B$

تقاطع

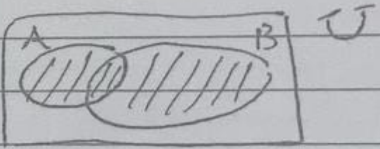
④ The intersection between A and B is given by

$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$



⑤ The union of A and B is given by

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$



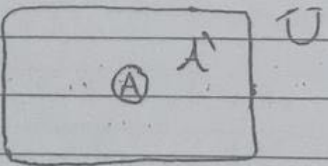
Ex $\sqrt{2}, \sqrt{3}, \sqrt{6}, \pi, e \in I$

Ex $\sqrt{4}, \sqrt{9}, 5, \frac{3}{9}, \frac{-6}{14}, 3.\bar{2} \in Q$

Note: $\phi \neq \{\phi\}$

⑥ The complement of A denoted by $A' = U \setminus A$

$= \{x \mid x \in U \text{ and } x \in A\}$



Ex $U = \{1, 2, 3, 4, 5, 6, 7\}$

$A = \{1, 2, 6\}$, $B = \{2, 5, 7\}$

① $A' = \{3, 4, 5, 7\}$ ② $B' = \{1, 3, 4, 6\}$

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Defn A set is a collection of objects. Each object is called an element or member in the set. We use braces $\{\}$ to denote a set. Ex $A = \{1,2,5\}$ $2 \in A$ belongs $3 \notin A$ doesn't belong

Sets

Finite

Ex $A = \{5,7,9,10\}$

Ex $B = \{x \mid x \text{ is natural number less than } 4\} = \{1,2,3\}$

Infinite

Ex ① The natural numbers $\mathbb{N} = \{1,2,3,\dots\}$

② The whole numbers $\mathbb{W} = \{0,1,2,3,\dots\}$

صحيحة ③ The integer numbers $\mathbb{Z} = \{\dots,-2,-1,0,1,2,\dots\}$

نسبية ④ The rational numbers $\mathbb{Q} = \{\frac{p}{q} \mid p,q \in \mathbb{Z}, q \neq 0\}$

⑤ The irrational numbers $I = \frac{R}{Q}$

⑥ The real numbers $\mathbb{R} = (-\infty, \infty)$

H.W. 1 P.3

ⓐ $\{x \mid x \text{ is natural number less than } 5\}$

$= \{1,2,3,4\}$

ⓑ $\{x \mid x \text{ is natural number greater than } 7 \text{ and less than } 14\}$ $= \{8,9,10,11,12,13\}$

Sec 1.1   sets المجموعات   موضوع الدرس   / /   التاريخ

Defn A set is a collection of objects. Each object is called an element or member in the set. We use braces { } to denote a set. Ex $A = \{1, 2, 5\}$ $2 \in A$ belongs $3 \notin A$ doesn't belong

Sets

Finite

Ex $A = \{5, 7, 9, 10\}$

Ex $B = \{x \mid x \text{ is natural number less than } 4\} = \{1, 2, 3\}$

Infinite

Ex ① The natural numbers

$\mathbb{N} = \{1, 2, 3, \dots\}$

② The whole numbers

$\mathbb{W} = \{0, 1, 2, 3, \dots\}$

صحيحة ③ The integer numbers

$\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$

نسبية ④ The rational numbers

$\mathbb{Q} = \left\{\frac{p}{q} \mid p, q \in \mathbb{Z}, q \neq 0\right\}$

⑤ The irrational numbers

$\mathbb{I} = \frac{\mathbb{R}}{\mathbb{Q}}$

⑥ The real numbers

$\mathbb{R} = (-\infty, \infty)$

$\frac{\text{H.w 1}}{\text{P3}}$ ⓐ $\{x \mid x \text{ is natural number less than } 5\}$

$= \{1, 2, 3, 4\}$

ⓑ $\{x \mid x \text{ is natural number greater than } 7 \text{ and less than } 14\} = \{8, 9, 10, 11, 12, 13\}$