KING SAUD UNIVERSITY
DEANSHIP OF THE FIRST YEAR COMMON
BASIC SCIENCES DEPARTMENT

MATH 101 HW # 1 /SUMMER SEMESTER 1442 Date: 3/06/2021

Question 1 **1 Mark**

A. Classify the following numbers into rational and irrationals.

$$2-\sqrt{5}\,,\ \frac{4.12}{6}\,,\ \sqrt[3]{125}\,,\ \frac{2\sqrt{3}}{\sqrt{12}}\,,\ \cos\frac{\pi}{6}\,,\ \frac{1}{3}\,,\ \tan\frac{\pi}{4}\,,\ \frac{(\sqrt{7}\sqrt[3]{27})}{\sqrt{28}}$$

$$\text{Rational} = \left\{ \frac{4.12}{6}\,,\ \sqrt[3]{125}\,,\ \frac{2\sqrt{3}}{\sqrt{12}}\ ,\ \frac{1}{3}\,,\ \tan\frac{\pi}{4}\,,\ \frac{\sqrt{7}\sqrt[2]{27}}{\sqrt{28}} \right\}$$

$$\text{Irrational} = \left\{ 2-\sqrt{5}, \cos\frac{\pi}{6}\ , \right\}$$

8 Marks

B. Solve the following inequalities and write the solution in interval notation.

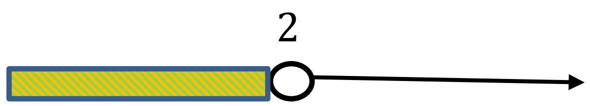
1. $5x-2<8$
2. $x^2-3x-4>0$
3. $\frac{|2x-5|}{|3x-2|}<1$
4. $|5x+2|\geq\frac{-3}{2}$

Answer:

1) $5x-2<8$

$$5x < 8+2 \Rightarrow 5x < 10$$

$$\frac{5x}{5} < \frac{10}{5} \Rightarrow x < 2$$



$$S.S:\ (-\infty\,,2)$$

2) $x^2 - 3x - 4 > 0$

$$x^2 - 3x - 4 > 0$$

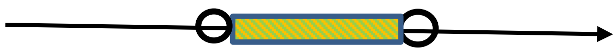
$$x^2 - 3x - 4 = 0$$

$$(x - 4)\ (x + 1) = 0$$

$$x - 4 = 0 \Rightarrow x = 4$$

$$x + 1 = 0 \Rightarrow x = -1$$

$+ + + \ -1 - - - - 4 + \ + \ + +$



S.S : $(-\infty\,, -1\ ) \cup (4\,, \infty)$

3) $\frac{|2x-5|}{|3x-2|} < 1$

$$3x - 2 \neq 0 \Rightarrow 3x \neq 2 \Rightarrow x \neq \frac{2}{3}$$

$$|2x - 5| < |3x - 2|$$

$$(\ |2x - 5|)^2 < \ (|3x - 2|)^2$$

$$(\ 2x - 5)^2 < \ (3x - 2)^2$$

$$(\ 2x - 5)^2 - \ (3x - 2)^2 < 0$$

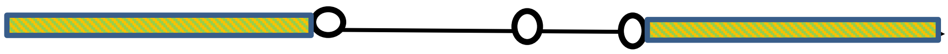
$$[(2x - 5) + (3x - 2)][(2x - 5) - (3x - 2)] < 0$$

$$\big((5x - 7)(-x - 3)\big) < 0$$

$5x - 7 = 0 \Rightarrow 5x = 7 \Rightarrow x = \frac{7}{5}$

$-x - 3 = 0 \Rightarrow x = -3$

$$------ -3 ++++ \frac{2}{3} ++ \frac{7}{5} -------$$



S.S : $\left(-\infty, -3\right) \cup \left(\frac{7}{5}, \infty\right)$

5) $|5x + 2| \geq \frac{-3}{2}$

S.S : $(-\infty, \infty) = R$

$|x| \geq 0 \;\; for\ any\ x \in R$

Question 2 **6 Marks**

A: Find the domain of the following functions.

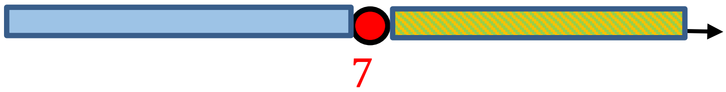
1. $f(x) = \sqrt{x-7} + \sqrt{7-x}$
2. $g(x) = \frac{2x-3}{|x-1|-4}$
3. $h(x) = \frac{\sqrt[3]{4-x}}{x}$

Answer:

1) $f(x) = \sqrt{x-7} + \sqrt{7-x}$

$$x - 7 \geq 0 \rightarrow x \geq 7$$

$$7 - x \geq 0 \rightarrow -x \geq -7 \rightarrow x \leq 7$$



$$D_f = (-\infty\,, 7] \cap [7\,, \infty) = \{7\}$$

2) $f(x) = \frac{2x-3}{|x-1|-4}$

$$|x-1| - 4 = 0 \rightarrow |x-1| = 4 \rightarrow x - 1 = \pm 4$$

$$\begin{cases} x - 1 = 4 \rightarrow x = 4 + 1 = 5 \\ x - 1 = -4 \rightarrow x = -4 + 1 = -3 \end{cases}$$

$$D_f = R - \{-3\,, 5\}$$

3) $f(x) = \frac{\sqrt[3]{4-x}}{x}$

$$x = 0$$

$$D_f = R - \{0\}$$

Question 3 **2 Marks**

$f(x) = \frac{x^2}{x}$ and $g(x) = x$

a): f, g are same function.

b): in which domain the two functions be same function.

Answer:

$$f(x) = \frac{x^2}{x}$$

$$D_f = R - \{0\}$$

$$g(x) = x$$

$$D_g = R$$

$$D_f \neq D_g$$

.f and g are not the same .

B) the domain f and g are the same is $R - \{0\}$.

$$f(x) = \frac{x^2}{x} = x = g(x)$$

Question 4 **6 Marks**

1. Find $f(x) + f(\frac{1}{x})$ if $f(x) = x^2 - \frac{1}{x^2}$, $x \neq 0$.
2. Let $f(x) = 9x^2 - 9$ and $g(x) = -3x$ find $\frac{f}{g}$, $f.g$ and their domains.

Answer:

1) $f(x) = x^2 - \frac{1}{x^2}$

$$f(x) + f\left(\frac{1}{x}\right) = x^2 - \frac{1}{x^2} + \left(\frac{1}{x}\right)^2 - \frac{1}{\left(\frac{1}{x}\right)^2}$$

$$= x^2 - \frac{1}{x^2} + \frac{1}{x^2} - x^2 = 0$$

2) $f(x) = 9x^2 - 9 \quad and \quad g(x) = -3x$

$$D_f = R \qquad , \qquad D_g = R$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

$$\left(\frac{f}{g}\right)(x) = \frac{9x^2 - 9}{-3x}$$

$$g(x) = 0 \rightarrow -3x = 0 \rightarrow x = 0$$

$$D_{\frac{f}{g}} = \left(D_f \cap D_g\right) - \{g(x) = 0\} = R - \{0\}$$

2)

$$(f.g)(x) = f(x).g(x)$$

$$(f.g)(x) = (9x^2 - 9).(-3x)$$

$$D_{f.g} = D_f \cap D_g = R$$

Question 5 **2 Marks**

Determine whether the function $f(x) = \frac{1}{x-2} + 2$ is a one-to-one or not. If yes then find f^{-1}.

Solution

Domaine $f(x) = \frac{1}{x-2} + 2$ $\quad x - 2 = 0 \rightarrow x = 2$

$$D_f = R - \{2\}$$

Let $x_1, x_2 \in D_f$ such that $f(x_1) = f(x_2)$ then

$$\frac{1}{x_1 - 2} + 2 = \frac{1}{x_2 - 2} + 2$$

$$-2 \qquad\qquad -2$$

$$\frac{1}{x_1-2} = \frac{1}{x_2-2}$$

$$x_2 - 2 = x_1 - 2$$

$$+2 \qquad +2$$

$$x_2 = x_1$$

$f(x)$ is a one -to – one

$$f(x) = \frac{1}{x-2} + 2$$

$$y = \frac{1}{x-2} + 2$$

$$y-2=\frac{1}{x-2}$$

$$(y-2)(x-2)=1$$

$$yx-2y-2x+4=1$$

$$yx-2x=1+2y-4$$

$$x(y-2)=2y-3$$

$$\frac{x(y-2)}{y-2}=\frac{2y-3}{y-2}$$

$$x=\frac{2y-3}{y-2}$$

$$f^{-1}(y)=\frac{2y-3}{y-2}$$

$$f^{-1}(x)=\frac{2x-3}{x-2}$$

$$D_{f^{-1}}=R-\{2\}$$

Question 6 **3 Marks**

Without using calculator find the values of:

1. $\cos(15^{\circ})$ 2. $\cos(\cos^{-1}(0.2))$ 3. $\sin(2\tan^{-1}2)$

1) $\cos(15°) = \cos(60° - 45°)$

$$= \cos 60° . \cos 45° + \sin 60° \sin 45°$$

$$= \frac{1}{2} \times \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} = \frac{\sqrt{2} + \sqrt{6}}{4}$$

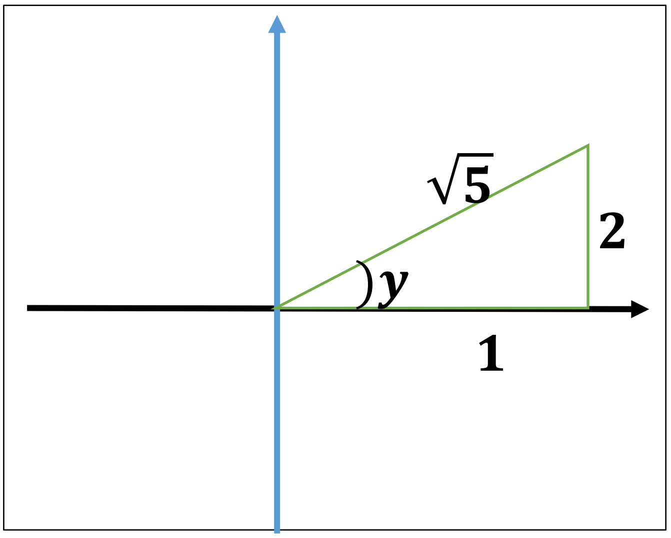
2) $\mathbf{\cos(\cos^{-1} 0.2)} = 0.2 \quad , \quad 0.2 \in [-1, 1]$

$$\mathbf{\cos(\cos^{-1} x)} = x \quad for\; x \in [-1, 1]$$

3) $\mathbf{\sin(2 \tan^{-1} 2)}$

Let : $y = \mathbf{\tan^{-1} 2} \rightarrow \tan y = \frac{2}{1}$ $\quad -\frac{\pi}{2} < y \leq \frac{\pi}{2}$

$$hyb = \sqrt{2^2 + 1^2} = \sqrt{5}$$



$$\sin y = \frac{2}{\sqrt{5}}$$

$$\cos y = \frac{1}{\sqrt{5}}$$

$$\sin\left(2\ \tan^{-1} 2\right) = \sin 2y$$

$$\sin 2y = 2 \sin y \cos y$$

$$\sin 2y = 2 \times \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{5}} = \frac{4}{5}$$

Question 7 **2 Marks**

1. Prove the identity $\sec x + \tan x = \frac{\cos x}{1 - \sin x}$.
2. use the definition of limit to show the following: $\lim_{x \to 8}(15 - 6x) = -33$.

$$\text{1) } \sec x + \tan x = \frac{\cos x}{1 - \sin x}$$

Solution

L.H.S= $\sec x + \tan x = \frac{1}{\cos x} + \frac{\sin x}{\cos x}$

$$= \frac{1 + \sin x}{\cos x} = \frac{1 + \sin x}{\cos x} \times \frac{1 - \sin x}{1 - \sin x}$$

$$= \frac{1 - \sin^2 x}{\cos x\,(1 - \sin x)} = \frac{\cos^2 x}{\cos x\,(1 - \sin x)}$$

$$= \frac{\cos x}{1 - \sin x} = R.H.S$$

$$\lim_{x\to 8}(15-6x)=-33$$

Solution

$$\lim_{x\to 8}(15-6x)=-33$$

Meanes that for any $\epsilon > 0$ there is $\delta > 0$ such that

If $0 < |x-8| < \delta$ then $|15-6x+33| < \epsilon$

$$|48-6x| < \epsilon$$

$$|-6x+48| < \epsilon$$

$$|-6(x+8)| < \epsilon$$

$$|-6||x+8| < \epsilon$$

$$6|x+8| < \epsilon$$

$$\frac{6|x+8|}{6} < \frac{\epsilon}{6}$$

$$|x+8| < \frac{\epsilon}{6}$$

We can take $\delta = \frac{\epsilon}{6}$