

MATH203 Calculus

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Power Series

Definition

If x is a variable, then an infinite series of the form

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + \cdots + a_n x^n + \cdots; \quad a_i \in \mathbb{R} \text{ is called a power series}$$

in x or. $\sum_{n=0}^{\infty} a_n (x - c)^n = a_0 + a_1 (x - c) + \cdots + a_n (x - c)^n + \cdots; \quad c \in \mathbb{R}$
is called a power series in $(x - c)$

Remarks:

- 1 We can check the convergence or divergence of a power series $\sum_{n=0}^{\infty} a_n x^n$ for different values of x .
- 2 Every power series in x converges if $x = 0$.
- 3 To find all other values of x for which $\sum_{n=0}^{\infty} a_n x^n$ is convergent, we often use **the absolute ratio test**.

Interval of convergence

After finding values of x which are convergent in the interval, say (a,b) , this is called the interval of convergence for the power series $\sum_{n=0}^{\infty} a_n x^n$.

Radius of convergence

Half of the length of interval of convergence is called the radius of convergence of the the power series $\sum_{n=0}^{\infty} a_n x^n$.

Theorem

Every power series $\sum_{n=0}^{\infty} a_n x^n$ satisfies one of the following:

- 1 The series converges only when $x = 0$ and this convergence is absolute.
- 2 The series converges for all x , and this convergence is absolute.
- 3 There is a number $R > 0$ such that the series converges absolutely when $x < R$ and diverges when $x > R$. Note that the series may converge or diverge depending on the particular series.

Examples

Find the interval of convergence and radius of convergence of the following series:

$$(1): \sum_{n=1}^{\infty} \frac{n}{3^n} x^n \quad (2): \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad (3): \sum_{n=0}^{\infty} (n!) x^n$$

$$(4): \sum_{n=0}^{\infty} (2x)^n \frac{1}{n} \quad (5): \sum_{n=0}^{\infty} x^n \frac{1}{\sqrt{n}} \quad (6): \sum_{n=0}^{\infty} (-1)^n \frac{1}{n+1} (x-3)^n$$

Solution:

Examples

Determine whether the series is absolute convergent, conditionally convergent or divergent

$$(1): \sum_{n=1}^{\infty} (-1)^n (1 + e^{-n})$$

$$(2): \sum_{n=1}^{\infty} (-1)^n \frac{n}{\ln n}$$

$$(3): \sum_{n=1}^{\infty} (-1)^n \frac{n^2 + 3}{(2n - 5)^2}$$

$$(4): \sum_{n=1}^{\infty} \frac{n!}{(-5)^n}$$

Solution:

Examples

Find the interval of convergence and radius of convergence of the following series:

$$(1): \sum_{n=0}^{\infty} \frac{1}{n+4} x^n \quad (2): \sum_{n=0}^{\infty} \frac{x^n n^2}{2^n}$$

$$(3): \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{\sqrt{n}} x^n \quad (4): \sum_{n=1}^{\infty} (-1)^n \frac{n+1}{10^n} (x-4)^n$$

Solution: