EXERCISES OF CHAPTER 2

8) The science club has challenged the math club to a friendly competition. Each club's team should be comprised of 2 boys and 3 girls. There are 20 boys and 15 girls in the science club and 25 boys and 30 girls in the math club. How many different teams are possible for each club?

Answer: The number of possible different teams for each club is:

For science club we have:

$$_{20}C_2 \times {_{15}C_3} = \frac{20!}{2!\ (20-2)!} \cdot \frac{15!}{3!\ (15-3)!} = 190 \times 455 = 86450 \text{ different teams}$$

For math club we have:

$$_{25}C_2 \times {_{30}C_3} = \frac{25!}{2!\ (25-2)!} \cdot \frac{30!}{3!\ (30-3)!} = 300 \times 4060 = 1218000 \text{ different teams}$$

9) A manager must choose five secretaries from among 12 applicants and assign them to different stations. How many different arrangements are possible?

Answer: The number of possible different arrangements is:

$$_{12}P_5 = \frac{12!}{(12-5)!} = \frac{12!}{7!} = \frac{12 \times 11 \times 10 \times 9 \times 8 \times (7!)}{7!} = 95040 \text{ different arrangements}$$

11) Nadia is a bit forgetful, and if she doesn't make a "to do" list, the probability that she forgets something she is supposed to do is .1. Tomorrow she intends to run three errands, and she fails to write them on her list.

a. What is the probability that Nadia forgets all three errands?

Answer: We suppose that:

A is the event that, Nadia forgets the first errand.

B is the event that, Nadia forgets the second errand.

C is the event that, Nadia forgets the third errand.

D is the event that Nadia forgets all three errands. Then, because the events A, B and C are independent, we get that:

$$P(D) = P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C) = (0.1)^3 = 0.001$$

b. What is the probability that Nadia remembers at least one of the three errands?

Answer: We suppose that E is the event that Nadia remembers at least one of the three errands. Then we get that:

$$P(E) = 1 - P(D) = 1 - 0.001 = 0.999$$

c. What is the probability that Nadia remembers the first errand but not the second or third?

Answer: We suppose that F is the event that Nadia remembers the first errand but not the second or third. Then we have $F = \bar{A} \cap B \cap C$. Then we get that:

$$P(F) = P(\bar{A} \cap B \cap C) = P(\bar{A}) \cdot P(B) \cdot P(C) = 0.9 \times 01 \times 0.1 = 0.009$$

16) Many fire stations handle emergency calls for medical assistance as well as calls requesting fire-fighting equipment. A particular station says that the probability that an incoming call is for medical assistance is .85.

a. What is the probability that a call is not for medical assistance?

Answer: We suppose that A the event that, the call is for medical assistance. Then the probability of being computed is:

$$P(\bar{A}) = 1 - 0.85 = 0.15.$$

b. Assuming that successive calls are independent, what is the probability that both of two successive calls will be for medical assistance?

Answer: We suppose that B and C are the events for two successive calls for medical assistance. Then the probability of being computed is:

$$P(B \cap C) = P(B) \cdot P(C) = 0.85 \times 0.85 = 0.7225.$$

c. What is the probability that three consecutive calls are not for medical assistance?

Answer: We suppose that D, E and F are the events for three successive calls for medical assistance. Then the probability of being computed is:

$$P(\bar{D} \cap \bar{E} \cap \bar{F}) = P(\bar{D}) \cdot P(\bar{E}) \cdot P(\bar{F}) = (1 - 0.85)^3 = 0.003375.$$

d. What is the probability that of the next 10 calls, at least one is for medical assistance?

Answer: We suppose that $A_1, A_2, \ldots and\ A_{10}$ are the events for 10 successive calls for medical assistance. Then the probability of being computed is:

$$P(\bigcup_{i=1}^{10} A_i) = 1 - P(\overline{\bigcup_{i=1}^{10} A_i}) = 1 - P(\bigcap_{i=1}^{10} \bar{A}_i) = 1 - \prod_{i=1}^{10} P(\bar{A}_i) = 1 - (0.15)^{10} = 0.9999$$

18) If you eat at Star Café there's a 40% chance that your food will be cold and a 30% chance your food will taste bad. We assume that these two events are independent. Then:

a. What is the probability that both will occur, your food is cold and it tastes bad?

Answer: We suppose that:

A is the event that, the food is cold. Then we have:

$$P(A) = 0.40$$

B is the event that, the food is taste bad. Then we have:

$$P(B) = 0.30$$

Then the probability of being computed is:

$$P(A \cap B) = P(A) \cdot P(B) = 0.40 \times 0.30 = 0.12$$

b. What is the probability that your food is cold or it tastes bad?

Answer: Taking into consideration previous assumptions, then the probability of being computed is:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A) + P(B) - P(A) \cdot P(B)$$
$$= 0.40 + 0.30 - 0.40 \times 0.30 = 0.58$$

20) A construction firm has bid on two different contracts. Let B_1 be the event that the first bid is successful and B_2, that the second bid is successful. Suppose that $P\ B_1\ = 0.4$, $P\ B_2\ = 0.6$ and that the bids are independent. What is the probability that:

a. Both bids are successful?

Answer: The probability of being computed is:

$$P(B_1 \cap B_2) = P(B_1) \cdot P(B_2) = 0.4 \times 0.6 = 0.24$$

b. Neither bid is successful?

Answer: The probability of being computed is:

$$P(\bar{B}_1 \cap \bar{B}_2) = P(\bar{B}_1) \cdot P(\bar{B}_2) = 0.6 \times 0.4 = 0.24$$

c. At least one of the bids is successful?

Answer: The probability of being computed is:

$$P(B_1 \cup B_2) = P(B_1) + P(B_2) - P(B_1 \cap B_2) \cdot$$
$$= P(B_1) + P(B_2) - P(B_1) \cdot P(B_2) = 0.4 + 0.6 - 0.6 \times 0.4 = 0.76$$

21) There are two traffic lights on the route used by Pikup Andropov to go from home to work. Let E denote the event that Pikup must stop at the first light and F in a similar manner for the second light. Suppose that $P\ E\ = 0.4$, $P\ F\ = 0.3$ and $P(E \cap F) = 0.15$. What is the probability that he:

a. Must stop for at least one light?

Answer: The probability of being computed is:

$$P(E \cup F) = P(E) + P(F) - P(E \cap F) = 0.4 + 0.3 - 0.15 = 0.55$$

b. Doesn't stop at either light?

Answer: The probability of being computed is:

$$P(\bar{E} \cap \bar{F}) = P(\overline{E \cup F}) = 1 - P(E \cup F) = 1 - 0.55 = 0.45$$

c. Must stop at exactly one light?

Answer: The probability of being computed is:

$$\begin{aligned} P\ (E \cap \bar{F}) \cup (\bar{E} \cap F) &= P(E \cap \bar{F}) + P(\bar{E} \cap F) \\ &= [P(E) - P(E \cap F)] + [P(F) - P(E \cap F)] \\ &= (0.4 - 0.15) + (0.3 - 0.15) = 0.25 + 0.15 = 0.40 \end{aligned}$$

d. Must stop just at the first light?

Answer: The probability of being computed is:

$$P(E \cap \bar{F}) = P(E) - P(E \cap F) = (0.4 - 0.15) = 0.25$$

22) There are 100 students enrolled in various AP (Advanced Placement) courses at American High School. There are 31 students in AP European History, 52 students in AP Calculus and 15 students in AP Spanish. Ten students study both AP European History and AP Calculus, five students study both AP European History and AP Spanish, eight students study both AP Calculus and AP Spanish, and three students study all three. What is the probability that a student takes an AP course other than these three?

Answer: We suppose that:

A is the event that, the student studies course European History. Then we have:

$$P(A) = 0.31$$

B is the event that, the student studies course Calculus. Then we have:

$$P(B) = 0.52$$

C is the event that, the student studies course Spanish. Then we have:

$$P(B) = 0.15$$

Then we have:

$P(A \cap B) = 0.10$, $P(A \cap C) = 0.05$, $P(B \cap C) = 0.08$ and $P(A \cap B \cap C) = 0.03$

Now we suppose that D is the student studies course other than the three above courses.

Then the probability of being computed is:

$$
\begin{aligned}
P(D) = P(\overline{A \cup B \cup C}) &= 1 - P(A \cup B \cup C) \\
&= 1 - \big[P(A) + P(B) + P(C) \\
&\quad - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)\big] \\
&= 1 - \big[0.31 + 0.52 + 0.15 - 0.10 - 0.05 - 0.08 + 0.03\big] \\
&= 1 - 0.78 = 0.22
\end{aligned}
$$

24) Suppose that 23% of adults, in a particular population, smoke cigarettes. It's known that 57% of smokers and 13% of non-smokers develop a certain lung condition by the age of 60. What is the probability that a randomly selected 60-year-old, of that population, has this lung condition?

Answer: We suppose that:

A is the event that, the randomly selected 60-year-old person is smoker. Then we have:

$$P(A) = 0.23$$

B is the event that, the randomly selected 60-year-old person has a certain lung condition. Then we have:

$$P(B) = ?$$

The probability that the randomly selected 60-year-old person has a certain lung condition, given that the person is smoker. Then we have:

$$P(B \mid A) = \frac{P(B \cap A)}{P(A)} = 0.57$$

$$\Rightarrow P(B \cap A) = 0.57\ P(A) = 0.57 \times 0.23 = 0.1311$$

The probability that the randomly selected 60-year-old person has a not certain lung condition, given that the person is non-smokers. Then we have:

$$P(\bar{B} \mid \bar{A}) = \frac{P(\bar{B} \cap \bar{A})}{P(\bar{A})} = 0.87$$

$$\Rightarrow P(\bar{B} \cap \bar{A}) = 0.87\ P(\bar{A}) = 0.87 \times 0.77 = 0.6699$$

$$\Rightarrow 0.6699 = P(\bar{B} \cap \bar{A}) = P(\overline{B \cup A}) = 1 - P(B \cup A)$$

$$\Rightarrow P(B \cup A) = 1 - 0.6699 = 0.3311$$

The probability of being computed is:

$$P(B) = P(B \cup A) - P(A) + P(B \cap A) = 0.3311 - 0.23 + 0.1311 = 0.2322$$