أسئلة مراجعة قبل الاختبار

هذه الأسئلة للمراجعة ليلة الاختبار .
هذه الأسئلة لا تغني عن الكتاب المقرر

Ex : **Reduce to the simplest form**

$$a)\ \frac{x+y}{x^2-y^2} \div \frac{x^2-xy}{x^2-2xy+y^2} \quad , \quad x \neq \pm y$$

$$b)\ \frac{4x}{x^2-y^2} + \frac{3}{x+y} - \frac{2}{x-y} \quad ; \ x \neq \pm y$$

solution:

$$a)\ \frac{x+y}{x^2-y^2} \div \frac{x^2-xy}{x^2-2xy+y^2} = \frac{x+y}{x^2-y^2} \cdot \frac{x^2-2xy+y^2}{x^2-xy}$$

تتحول القسمة الى ضرب و نقلب البسط و المقام

$$= \frac{x+y}{(x+y)(x-y)} \cdot \frac{(x-y)^2}{x(x-y)}$$

$$= \frac{1}{x}$$

$$b)\ \frac{4x}{x^2-y^2} + \frac{3}{x+y} - \frac{2}{x-y} = \frac{4x}{(x-y)(x+y)} + \frac{3(x-y)}{(x+y)(x-y)} - \frac{2(x+y)}{(x-y)(x+y)}$$

$$= \frac{4x+3(x-y)-2(x+y)}{(x-y)(x+y)}$$

$$= \frac{4x+3x-3y-2x-2y}{(x-y)(x+y)}$$

$$= \frac{5x-5y}{(x-y)(x+y)}$$

$$= \frac{5(x-y)}{(x-y)(x+y)}$$

$$= \frac{5}{x+y}$$

$$Ex.\ \frac{2x(1-3x)^3+9x^2(1-3x)^2}{(1-3x)^6} \quad ; \ x \neq \frac{1}{3}$$

solution:

نأخذ $x(1-3x)^2$ عامل مشترك من البسط

$$* \ \frac{2x(1-3x)^3+9x^2(1-3x)^2}{(1-3x)^6} = \frac{x(1-3x)^2\left(2(1-3x)+9x\right)}{(1-3x)^6}$$

$$= \frac{x\left(2-6x+9x\right)}{(1-3x)^4}$$

$$= \frac{x\left(3x+2\right)}{(1-3x)^4}$$

Ex : Find values of x and y such that

a) $2x - 4yi = 18 + \sqrt{-36}$

solution:

$$2x - 4yi = 18 + 6i$$

* Real part : $2x = 18 \rightarrow x = 9$

* Imaginary part : $-4y = 6 \rightarrow y = -\frac{6}{4} = -\frac{3}{2}$

b) $4x - 3yi = \frac{3}{4i^2}$

solution:

$$4x - 3yi = \frac{3}{4(-1)}$$

$$i^2 = -1$$

$$4x - 3yi = -\frac{3}{4} + 0i$$

* Real part : $4x = -\frac{3}{4} \longrightarrow x = -\frac{3}{16}$

* Imaginary part : $-3y = 0 \longrightarrow y = 0$

c) $-x + 2yi - 13 = \frac{2}{5i^3}$

solution:

$$-x + 2yi = 13 + \frac{2}{5i^3}$$

$$\frac{2}{5i^3} \cdot \frac{i}{i} = \frac{2i}{5i^4} = \frac{2i}{5(1)} = \frac{2}{5}i$$

$$-x + 2yi = 13 + \frac{2}{5}i$$

* Real Part : $-x = 13 \longrightarrow x = -13$

* Imaginary part : $2y = \frac{2}{5} \xrightarrow{\div 2} y = \frac{1}{5}$

Ex : Simplify i^{79} , i^{-33}

solution:

* $i^{79} = i^{76} \cdot i^3 = (i^4)^{16} \cdot i^3 = (1)^{16} \cdot -i = -i$

* $i^{-33} = \frac{1}{i^{33}} = \frac{1}{i^{32} \cdot i} = \frac{1}{(i^4)^8 \cdot i} = \frac{1}{i} \cdot \frac{i}{i} = \frac{i}{i^2} = -i$

Write in standard form

$Ex: \quad (2-\sqrt{-4})\cdot\left(3-\sqrt{-16}\right)$

solution:

$* \ (2-\sqrt{-4})\cdot\left(3-\sqrt{-16}\right) = (2-2i)(3-4i)$

$= 6-8i-6i-8$

$= -2-14i$

$Ex: \quad \left(-8i^2+\frac{3}{4}i\right)+\left(-7-\frac{2}{3}i^3\right)$

solution:

$* \ \left(-8i^2+\frac{3}{4}i\right)+\left(-7-\frac{2}{3}i^3\right) = \left(-8(-1)+\frac{3}{4}i\right)+\left(-7-\frac{2}{3}(-i)\right)$

$* \ i^3 = i^2 \cdot i = -i$

$= \left(8+\frac{3}{4}i\right)+\left(-7+\frac{2}{3}i\right)$

$= (8-7)+\left(\frac{3}{4}i+\frac{2}{3}i\right)$

$= 1+\frac{17}{12}i$

$Ex:$ Find the values of x and y such that

$$3x-2yi = \frac{26}{3-\sqrt{-4}}$$

solution:

standard form نضع الطرف الأيمن بالصيغة القياسية

$* \ \frac{26}{3-\sqrt{-4}} = \frac{26}{3-2i}$

$= \frac{26}{3-2i}\cdot\frac{3+2i}{3+2i}$

$= \frac{26(3+2i)}{(3)^2+(2)^2}$

$= \frac{26(3+2i)}{13} = 2(3+2i) = 6+4i$

$* \quad 3x-2yi = 6+4i$

Real part : $3x = 6 \longrightarrow x = 2$

Imaginary part : $-2y = 4 \longrightarrow y = -2$

0559108708 محمد ندا

Ex Let $z_1=4-3i$, $z_2=5-3i$, $z_3=-2i$, find:

$a)$ $\mathrm{Re}(z_1z_2)$ b) $z_1z_2^{-1}$ $c)$ $z_1\overline{z_2}$ $d)$ z_2^{3} $e)$ $\mathrm{Im}(3i^{34}-z_3^{3})$

solution:

$a)$ $\mathrm{Re}(z_1z_2)$

* $z_1z_2=(4-3i)(5-3i)=20-12i-15i-9=11-27i$

$\mathrm{Re}(z_1z_2)=11$

$b)$ $z_1z_2^{-1}$

* $z_1z_2^{-1}=\dfrac{z_1}{z_2}$

$=\dfrac{4-3i}{5-3i}\cdot\dfrac{5+3i}{5+3i}$

$=\dfrac{20+12i-15i+9}{25+9}$

$=\dfrac{29-3i}{34}$

$=\dfrac{29}{34}-\dfrac{3}{34}i$

$c)$ $z_1\overline{z_2}$

solution:

* $z_1\overline{z_2}=(4-3i)(5+3i)$

$=20+12i-15i+9=29-3i$

$d)$ z_2^{3}

$z_2^{2}=(5-3i)(5-3i)$

$=25-15i-15i-9=16-30i$

* $z_2^{3}=z_2^{2}\cdot z_2$

$=(16-30i)(5-3i)$

$=80-48i-150i-90$

$=-10-198i$

$e.$ $\mathrm{Im}(3i^{34}-z_3^{3})$

solution:

* $3i^{34}-z_3^{3}=3(i^{32})(i^{2})-(-2i)^{3}$

$=3(1)(-1)-(-8i^{3})$

$=-3-(-8\cdot -i)$

$=-3-8i$

$\mathrm{Im}(3i^{34}-z_3^{3})=-8$

Ex : Solve the equation $\frac{x}{2}+\frac{2x-1}{3}=\frac{3x+4}{4}$

solution

$\frac{3x+2(2x-1)}{6}=\frac{3x+4}{4}$ توحيد المقامات للطرف الأيسر

$\frac{3x+4x-2}{6}=\frac{3x+4}{4}$

$\frac{7x-2}{6}=\frac{3x+4}{4}$

$4(7x-2)=6(3x+4)$ حاصل ضرب الطرفين = ضرب الوسطين

$28x-8=18x+24$

$28x-18x=24+8$

$10x=32$

$x=\frac{32}{10}=\frac{16}{5}$

* Check

L.H.S: $\frac{\frac{16}{5}}{2}+\frac{2(\frac{16}{5})-1}{3}=\frac{16}{10}+\frac{9}{5}=\frac{17}{5}$

R.H.S: $\frac{3(\frac{16}{5})+4}{4}=\frac{17}{5}$

Ex : Given four consecutive even integers . the sum of the first three exceeds the fourth by 8 . Find these numbers

solution:

اربع اعداد صحيحة زوجية متتالية . مجموع أول ثلاثة أعداد تزيد عن الرابع بـ 8 . أوجد هذه الأعداد

Let the numbers are x , $x+2$, $x+4$, $x+6$

* The linear equation $(x)+(x+2)+(x+4)=(x+6)+8$

$3x+6=x+14$

$3x-x=14-6$

$2x=8$, $x=4$

The numbers are 4 , 6 , 8 and 10

محمد ندا 0559108708

Ex : A rectangular land has a perimeter 84 meters . If the length is 3 meters less than twice the width , find the dimentions of the rectangular (length and width)

solution:

أرض مستطيلة محيطها 84 . اذا كان طولها يقل عن ضعف العرض بـ 3 .أوجد أبعاد المستطيل (الطول و العرض)

Let the width is x and the length is $y = 2x - 3$

* Perimeter of rectangular $= 2(\text{length}) + 2(\text{width})$

$$2(2x - 3) + 2(x) = 84$$

$$4x - 6 + 2x = 84$$

$$6x = 84 + 6 = 90$$

$$x = \frac{90}{6} = 15$$

Then the width of rectangular is 15 and the length is $2(15) - 3 = 27$

Ex : Solve the inequality $\frac{y-3}{4} - 2 > \frac{y}{3} + 2$

solution: multiply all by $(3)(4)$

$$(3)(4)\frac{y-3}{4} - (3)(4)2 > (3)(4)\frac{y}{3} + (3)(4)2$$

$$3(y-3) - 24 > 4y + 24$$

$$3y - 9 - 24 > 4y + 24$$

$$3y - 4y > 24 + 33$$

$$-y > 57$$

$$y < -57$$

عند الضرب أو القسمة على عدد سالب نغير علامة المتباينة

The solution set is the interval $(-\infty, -57)$

Ex : $-3|x+5| + 6 = -15$

solution:

$$-3|x+5| = -15 - 6$$

$$-3|x+5| = -21 \qquad \text{divide by } (-3)$$

$$|x+5| = 7$$

The solutions are $x + 5 = \pm 7$

$$x + 5 = 7 \quad \text{or} \quad x + 5 = -7$$

$$x = 2 \quad \text{or} \quad x = -12$$

The solution set is $\{-12, 2\}$

Ex : Solve the inequality $\sqrt{(3-2x)^2} \le 4$

$\sqrt{(3-2x)^2} = |3-2x|$

solution:

$|3-2x| \le 4$

$|3-2x| = |2x-3|$

$|2x-3| \le 4$

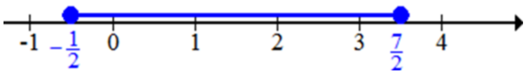
$-4 \le 2x-3 \le 4$

$-4+3 \le 2x \le 4+3$

$-1 \le 2x \le 7$

$-\frac{1}{2} \le x \le \frac{7}{2}$

The solution set is the interval $\left[-\frac{1}{2}, \frac{7}{2}\right]$



Ex : Solve the inequality $|7-3x| = 2x+5$

solution:

$|3x-7| = 2x+5$

$|3x-7| \ge 0$ then $2x+5 \ge 0$, $2x \ge -5$, $x \ge \frac{-5}{2}$

نبحث عن الفترة التي يقع بها الحل

* $|3x-7| = 2x+5$, $x \ge -\frac{5}{2}$

$3x-7 = \pm(2x+5)$

$3x-7 = 2x+5$ or $3x-7 = -(2x+5) = -2x-5$

$3x-2x = 5+7$ or $3x+2x = -5+7$

$x = 12$ or $5x = 2$, $x = \frac{2}{5}$

Remark: 12 and $\frac{2}{5} > -\frac{5}{2}$

The solution set is $\left\{\frac{2}{5}, 12\right\}$

Ex : Solve the equation $3x^2+12 = 0$

solution:

$3x^2 = -12$

$x^2 = -4$

$x = \pm\sqrt{-4} = \pm 2i$

The solution is $\{-2i, 2i\}$

Ex : Solve the equation $16x^2 + 9 = 24x$

solution: by completing square

$$16x^2 - 24x = -9 \qquad \text{divide all by (16)}$$

$$x^2 - \frac{3}{2}x = -\frac{9}{16}$$

$$x^2 - \frac{3}{2}x + \left(\frac{3}{4}\right)^2 = -\frac{9}{16} + \left(\frac{3}{4}\right)^2$$

$$\left(x - \frac{3}{4}\right)^2 = 0$$

$$x - \frac{3}{4} = 0$$

$$x = \frac{3}{4}$$

The solution set is $\left\{\frac{3}{4}\right\}$

Ex : Solve the equation $1 + \frac{8}{x^2} = \frac{4}{x}$, $x \neq 0$

solution: multiply all by x^2

$$x^2 \cdot 1 + x^2 \cdot \frac{8}{x^2} = x^2 \cdot \frac{4}{x}$$

$$x^2 + 8 = 4x$$

$$x^2 - 4x = -8 \qquad \text{(by completing square)}$$

$$x^2 - 4x + (2)^2 = (2)^2 - 8$$

$$(x - 2)^2 = -4$$

$$x - 2 = \pm\sqrt{-4}$$

$$x = 2 \pm 2i$$

The solution set is $\{2 - 2i , 2 + 2i\}$

Ex : The sum of a number and its reciprocal is $\frac{10}{3}$. Find the numbers.

مجموع عدد و مقلوبه يساوي $\frac{10}{3}$. أوجد العددين

solution:

Let the number is x and its reciprocal is $\frac{1}{x}$

* $x + \frac{1}{x} = \frac{10}{3}$

$$\frac{3x}{3x} \cdot \frac{x}{1} + \frac{3}{3} \cdot \frac{1}{x} = \frac{10}{3} \cdot \frac{x}{x}$$

$$\frac{3x^2}{3x} + \frac{3}{3x} = \frac{10x}{3x}$$

$$\frac{3x^2 + 3}{3x} = \frac{10x}{3x}$$

$$3x^2 + 3 = 10x$$

$$3x^2 - 10x + 3 = 0$$ (By factoring or any method)

$$(3x - 1)(x - 3) = 0$$

$3x - 1 = 0$ or $x - 3 = 0$

$3x = 1$ or $x = 3$

$x = \frac{1}{3}$ or $x = 3$

The numbers are $\frac{1}{3}$ and 3

Ex : The width of a rectangle is three centimeters less than the length . If the area of the rectangle is 54 cm^2 , find the dimensions of the rectangle.

solution:

عرض مستطيل أقل من طوله بـ 3 . اذا كان مساحة المستطيل 54 أوجد أبعاد المستطيل

Let the length $= x$, then the width $= x - 3$

* Area of rectangle = (length) · (width)

مساحة المستطيل = الطول × العرض

$$x(x - 3) = 54$$

$$x^2 - 3x - 54 = 0$$

$$(x - 9)(x + 6) = 0$$

$x = 9$ or $x = -6$ refused

The length of the rectangle is 9 and the width is $= 9 - 3 = 6$

Ex : Find the domain of $f(x)=\dfrac{\sqrt{x-1}}{x-3}$

نحصل على مجال الجذر و نحذف منها أصفار المقام

solution:

Domain of $\sqrt{x-1}$: $x-1\geq 0 \quad , \quad x\geq 1$

and non-zero of denominator: $x-3\neq 0 \quad , \quad x\neq 3$

Domain $D_f=\{x\in\mathbb{R}: x\geq 1 \text{ and } x\neq 3\}=[1,3)\cup(3,\infty)$

Ex : Find the domain of $f(x)=\dfrac{\sqrt{x}+4x}{x^3-x}$

نحصل على مجال الجذر و نحذف منها أصفار المقام

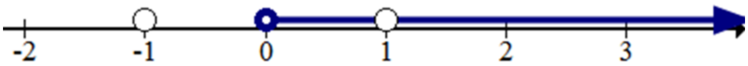
solution:

Domain of $\sqrt{x}$: $x\geq 0$

and non-zero of denominator: $x^3-x\neq 0 \quad , \quad x(x^2-1)\neq 0$

$x(x-1)(x+1)\neq 0$

$x\neq 0 \;,\; x\neq 1 \text{ and } x\neq -1$



Domain $D_f=\{x\in\mathbb{R}: x>0 \text{ and } x\neq 1\}=(0,1)\cup(1,\infty)$

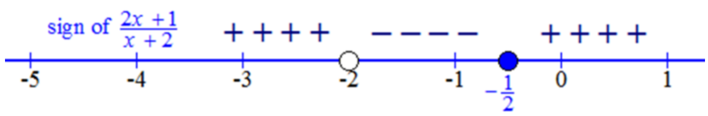
Ex : Find the domain of $f(x)=\sqrt{\dfrac{2x+1}{x+2}}$

نحصل أصفار البسط و المقام و نبحث اشارة الدالة و يكون
مجالها هي الفترات الموجبة
لاحظ أصفار البسط مقبولة في المجال و أصفار المقام مرفوضة

solution:

The domain of the function f is the solution of the inequality $\dfrac{2x+1}{x+2}\geq 0$

The zeros are $2x+1=0$ and $x+2=0 \Rightarrow x=-\dfrac{1}{2}$ and $x=-2$



Domain $D_f=\left\{x\in\mathbb{R}: x<-2 \text{ or } x\geq -\dfrac{1}{2}\right\}=(-\infty,-2)\cup\left[-\dfrac{1}{2},\infty\right)$

Ex : Find the domain of $f(x)=\dfrac{3x^2-x+4}{\sqrt{2x-4}-3}$

نحصل على مجال الجذر و نرفض أصفار المقام

solution:

The domain of $\sqrt{2x-4}$ is : $2x-4\geq 0$, $2x\geq 4$, $x\geq 2$

The zeros of denominator : $\sqrt{2x-4}-3=0$

$$\sqrt{2x-4}=3$$

$$2x-4=9$$

$$2x=13 \quad , \quad x=\frac{13}{2}$$

Domain D_f is : $\left[2,\frac{13}{2}\right)\cup\left(\frac{13}{2},\infty\right)$

Example , Use long division to find the quotient Q(x) and remainder R(x) of the each rational function $\dfrac{x^3+15x^2+49x-55}{x+7}$

solution:

$$\begin{array}{r|l} & x^2+8x-7 \\ \hline x+7 & x^3+15x^2+49x-55 \\ & \underline{-x^3-7x^2} \\ & \quad 8x^2+49x-55 \\ & \quad \underline{-8x^2-56x} \\ & \qquad -7x-55 \\ & \qquad \underline{+7x+49} \\ & \qquad\quad -6 \end{array}$$

$Q(x)=x^2+8x-7$, $R(x)=-6$

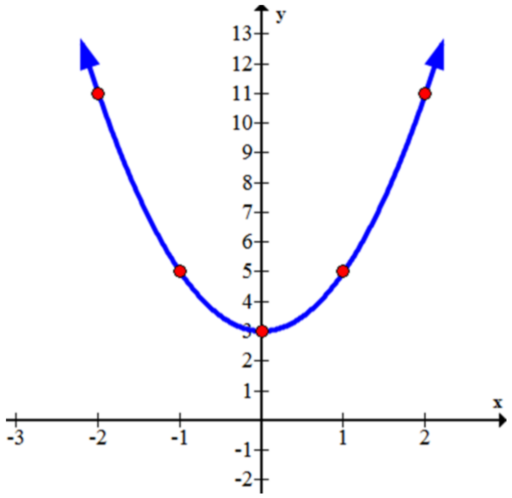
* $\dfrac{x^3+15x^2+49x-55}{x+7}=x^2+8x-7-\dfrac{6}{x+7}$

Ex : Graph the following function and find the domain $f(x) = 2x^2 + 3$

solution:

$y = 2x^2 + 3$ quadratic equation

x	-2	-1	0	1	2
y	11	5	3	5	11



* Domain : $(-\infty, \infty)$

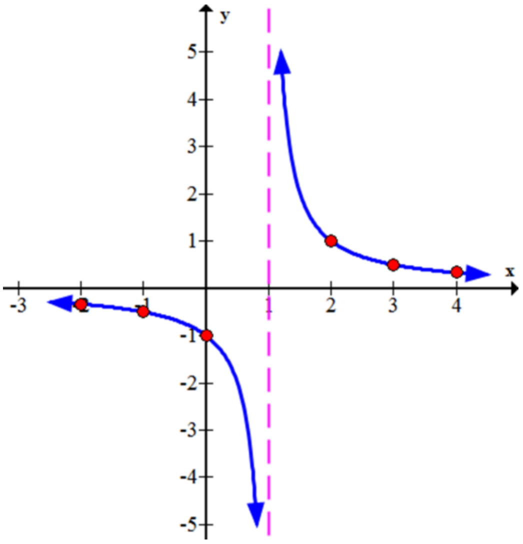
Ex : Graph the following function and find domain $f(x) = \dfrac{1}{x-1}$

solution:

The zero of denominator : $x - 1 = 0$, $x = 1$

x	-2	-1	0	1	2	3	4
y	≈ -0.33	-0.5	-1	undefined	1	0.5	≈ 0.33

* Domain of f is $(-\infty, 0) \cup (0, \infty)$

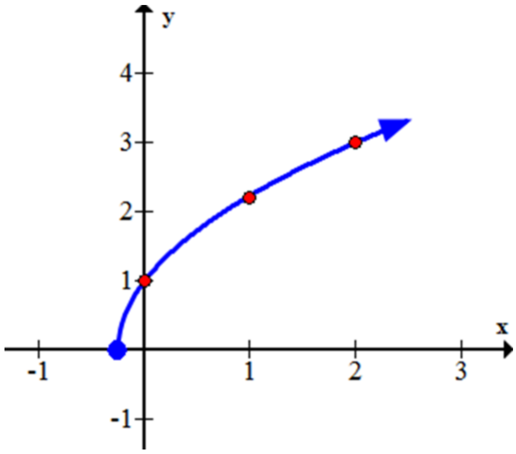


Ex : Graph the following function and find and domain $f(x)=\sqrt{4x+1}$

solution:

$4x+1\geq 0$, $4x\geq -1$, $x\geq -\frac{1}{4}$

x	$-\frac{1}{4}$	0	1	2
y	0	1	≈ 2.2	3



* Domain of f : $\left[-\frac{1}{4},\infty\right)$

Ex : Find the domain of the function $f(x)=\dfrac{5}{4-|2x-3|}$

solution:

مجال الدالة الكسرية : جميع الأعداد الحقيقية ما عدا أصفار المقام

$4-|2x-3|=0$

$-|2x-3|=-4$

$|2x-3|=4$

$2x-3=4$ or $2x-3=-4$

$2x=7$ or $2x=-1$

$x=\frac{7}{2}$ or $x=-\frac{1}{2}$

Domain of f is $\mathbb{R}-\left\{-\frac{1}{2},\frac{7}{2}\right\}$

Ex : Solve the equation $3(x-1)-4x=2x-5$

solution

$3x-3-4x=2x-5$

$3x-4x-2x=-5+3$

$-3x=-2$

$x=\frac{2}{3}$

The solution is $\left\{\frac{2}{3}\right\}$

Ex : Solve the equation $x^2 + \sqrt{(x-3)^2} = (x-1)^2$

solution:

$$x^2 + |x-3| = x^2 - 2x + 1$$

$$|x-3| = -2x + 1$$

$|x-3| \geq 0$ then $-2x + 1 \geq 0$, $-2x \geq -1$, $x \leq \frac{1}{2}$

* $|x-3| = -2x + 1$, $x \leq \frac{1}{2}$

$x - 3 = -2x + 1$ or $x - 3 = -(-2x + 1)$

$x + 2x = 1 + 3$ or $x - 3 = 2x - 1$

$3x = 4$ or $-x = 2$

$x = \frac{4}{3}$ or $x = -2$

* $-2 < \frac{1}{2}$ but $\frac{4}{3} > \frac{1}{2}$

The solution set is $\{-2\}$

Ex : Write in standard form : $\dfrac{2\sqrt{-16} - \sqrt{-25}}{1 + i^5}$

solution:

* $\dfrac{2\sqrt{-16} - \sqrt{-25}}{1 + i^5} = \dfrac{2(4i) - 5i}{1 + i^4 \cdot i}$

$$= \frac{8i - 5i}{1 + i}$$

$$= \frac{3i}{1 + i} \cdot \frac{1 - i}{1 - i}$$

$$= \frac{3i + 3}{1^2 + 1^2}$$

$$= \frac{3 + 3i}{2} = \frac{3}{2} + \frac{3}{2}\text{i}$$