Quadratic Functions

1. Definition and properties
2. How to convert from vertex form to standard and vise verse.
3. Find the equation from Given properties.
4. Solving quadratic inequalities

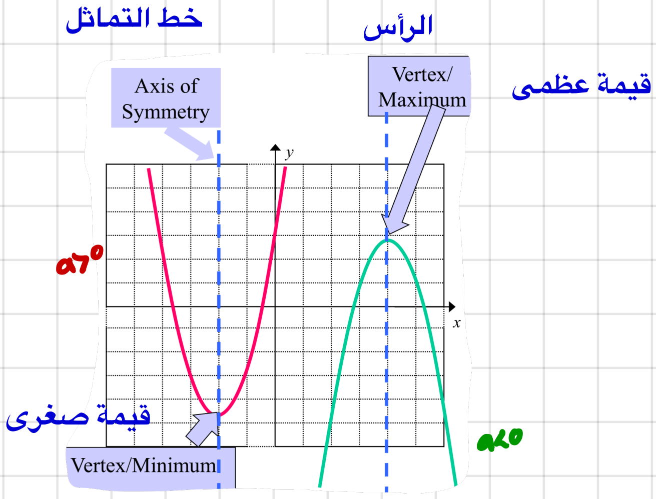
Quadratic Functions

Quadratic function: Any functions that contains an x^2 term.

Standard Form : $f(x) = ax^2 + bx + c \quad , \; a \neq 0$

Vertex Form : $f(x) = a(x-h)^2 + k \quad , \; a \neq 0$

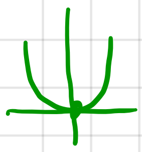
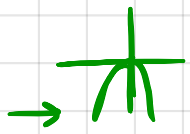
- The graph of a quadratic function is called Parabola. (U shape)



Type of Form / Properties	Vertex Form	General Form
	$f(x) = a(x-h)^2 + k$	$ax^2 + bx + c$
Vertex	(h, k)	$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$
Axis of Symmetry	$x = h$	$x = \frac{-b}{2a}$
Domain	$R = (-\infty, \infty)$	$R = (-\infty, \infty)$
Range	$(-\infty, k]$ if $a < 0$ $[k, \infty)$ if $a > 0$	$\left(-\infty, f\left(\frac{-b}{2a}\right)\right]$, $a < 0$ $\left[f\left(\frac{-b}{2a}\right), \infty\right)$, $a > 0$
Open (Up or down)	up , $a > 0$ down , $a < 0$	up , $a > 0$ down , $a < 0$
Max/Min	$a<0$ max $a>0$ min $\Big\} = k$	$a<0$ max $a>0$ min $\Big\} = f\left(\frac{-b}{2a}\right)$
Increasing and Decreasing Intervals	$(-\infty, h)$, (h, ∞)	$\left(-\infty, \frac{-b}{2a}\right)$ $\left(\frac{-b}{2a}, \infty\right)$

Quadratic Functions

Vertex Form

Properties \ Form	$f(x) = 2(x-2)^2 - 4$	$f(x) = -(x-2)^2 + 6$
Vertex	$(2, -4)$	$(2, 6)$
Domain	$\mathbb{R}$	$\mathbb{R}$
Range	$[-4, \infty)$	$(-\infty, 6]$
Axis of symmetry	$x = h = 2$	$x = 2$
Open (up or down)	up	Down
Max /Min Value	Min value: -4	max value: 6
Increasing or decreasing interval	Inc on $(2, \infty)$ Dec on $(-\infty, 2)$ 	Inc $[-\infty, 2]$ Dec on $[2, \infty)$ 

General Form

$$f(x) = 2x^2 - 8x + 4$$

vertex: $x = \frac{-b}{2a} = \frac{-(-8)}{2 \cdot 2} = \frac{8}{4} = 2$

$f\left(\frac{-b}{2a}\right) = f(2) = 2(2)^2 - 8(2) + 4$

$= 2 \cdot 4 - 16 + 4 = -4$

$\therefore$ vertex $(2, -4)$

- Domain: $\mathbb{R}$
- Range: $[-4, \infty)$
- Axis: $x = 2$
- open: up.
- Inc on $[2, \infty)$

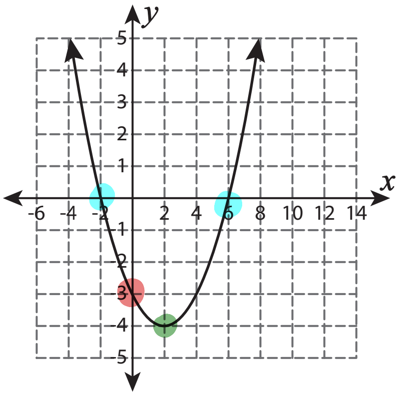
  dec on $(-\infty, 2]$

Properties of Quadratic Function

Sheet 1

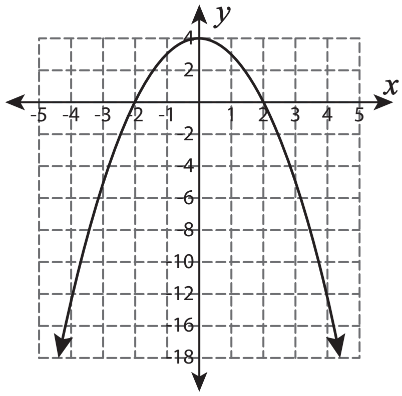
Find the properties of each quadratic function.

1)



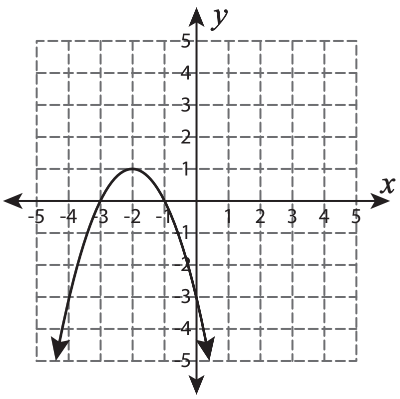
Domain	:	Real Numbers
Range	:	{ y is real : $y \geq -4$ }
x-intercepts	:	(–2, 0) and (6, 0)
y-intercept	:	(0, –3)
Vertex	:	(2, –4)
Minimum value	:	$y = -4$ or $k=-4$
Axis of symmetry	:	$x = 2$ or $h=2$
Open up or down	:	Up

2)



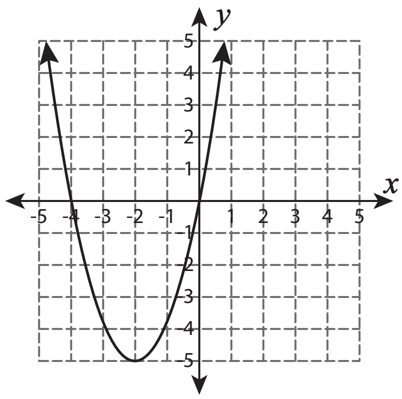
Domain	:	Real Numbers
Range	:	{ y is real : $y \leq 4$ }
x-intercepts	:	(–2, 0) and (2, 0)
y-intercept	:	(0, 4)
Vertex	:	(0, 4)
Maximum value	:	$y = 4$
Axis of symmetry	:	$x = 0$
Open up or down	:	Down

3)



Domain	:	Real Numbers
Range	:	{ y is real : $y \leq 1$ }
x-intercepts	:	(–3, 0) and (–1, 0)
y-intercept	:	(0, –3)
Vertex	:	(–2, 1)
Maximum value	:	$y = 1$
Axis of symmetry	:	$x = -2$
Open up or down	:	Down

4)



Domain	:	Real Numbers
Range	:	{ y is real : $y \geq -5$ }
x-intercepts	:	(–4, 0) and (0, 0)
y-intercept	:	(0, 0)
Vertex	:	(–2, –5)
Minimum value	:	$y = -5$
Axis of symmetry	:	$x = -2$
Open up or down	:	Up

2 How to convert from standard form to vertex form

Example : Convert the following quadratic equations from standard form.to vertex form

- $F(x) = 3x^2 - 18x + 5$

$a = 3$, $b = -18$

$$x = \frac{-b}{2a} = \frac{-(-18)}{2 \cdot 3} = \frac{18}{6} = 3$$

$$F(3) = 3(3)^2 - 18(3) + 5 = -22$$

$\therefore$ vertex : $(3, -22)$

$$f(x) = a(x-h)^2 + k$$

$$= 3(x-3)^2 - 22$$

How to convert from vertex form to standard form

Example : Convert the following quadratic equations from vertex form to standard form.

- $f(x) = (x-4)^2 - 1$

$$= (x^2 - 8x + 16) - 1 = x^2 - 8x + 15$$

- $F(x) = 2(x+3)^2 - 3$

$$= 2(x^2 + 6x + 9) - 3$$

$$= 2x^2 + 12x + 18 - 3$$

$$= 2x^2 + 12x + 15$$

Find the equation of a quadratic function that satisfy the given properties

Properties	**Equation**
• vertex : $(3, -2)$ • x intercept : 4 $(4, 0)$	$f(x) = a(x-3)^2 - 2$ x intercept 4 $\Rightarrow f(4) = 0$ $\Rightarrow a(4-3)^2 - 2 = 0$ $\Rightarrow a - 2 = 0 \Rightarrow a = 2$ $\therefore f(x) = 2(x-3)^2 - 2$
• vertex : $(4, -2)$ • y intercept : 2 $(0, 2)$	$f(x) = a(x-4)^2 - 2$ y intercept $f(0) = 2$ $\Rightarrow a(0-4)^2 - 2 = 2$ $\Rightarrow 16a = 2 + 2$ $\Rightarrow 16a = 4 \Rightarrow a = \frac{4}{16} = \frac{1}{4}$ $\therefore f(x) = \frac{1}{4}(x-4)^2 - 2$
• vertex : $(-3, -4)$ • addtional point $(1, 60)$	$f(x) = a(x+3)^2 - 4$ $(1, 60) \Rightarrow f(1) = 60$ $\Rightarrow a(1+3)^2 - 4 = 60$ $\Rightarrow 16a = 64$ $\Rightarrow a = 4$ $\therefore f(x) = 4(x+3)^2 - 4$

3 Solving Quadratic Inequalities

لحل أي معادلة من الدرجة الثانيه

$$\boxed{x = \frac{-b \pm \sqrt{4ac - b^2}}{2a}}$$

Solve: $x^2 - x > 12$

$x^2 - x - 12 > 0$

$x^2 - x - 12 = 0$

$(x - 4)(x + 3) = 0$

$\Rightarrow \quad x = 4 \quad \text{or} \quad x = -3$

$(-\infty, -3) \cup (4, \infty)$

Solve: $x^2 - 4x \geq 14$

$x^2 - 4x - 14 \geq 0$

$x^2 - 4x - 14 = 0$

$$x = \frac{-(-4) \pm \sqrt{4(1)(-14) - (-4)^2}}{2(1)}$$

$$= \frac{4 \pm \sqrt{72}}{2} = \frac{4 \pm 6\sqrt{2}}{2}$$

$= 2 + 3\sqrt{2}$ and $2 - 3\sqrt{2}$

$(-\infty, 2 - 3\sqrt{2}) \cup (2 + 3\sqrt{2}, \infty)$