Workshop Solutions to Section 3.3

1) If $f(x)=\begin{cases}2x+3; & x\geq -2\\ 2x+5; & x<-2\end{cases}$ then

$$\lim_{x\to(-2)^-} f(x)=$$

Solution:

$$\lim_{x\to(-2)^-} f(x)=\lim_{x\to(-2)^-}(2x+5)=2(-2)+5=-4+5$$
$$=1$$

2) If $f(x)=\begin{cases}2x+3; & x\geq -2\\ 2x+5; & x<-2\end{cases}$ then

$$\lim_{x\to(-2)^+} f(x)=$$

Solution:

$$\lim_{x\to(-2)^+} f(x)=\lim_{x\to(-2)^+}(2x+3)=2(-2)+3=-4+3$$
$$=-1$$

3) If $f(x)=\begin{cases}2x+3; & x\geq -2\\ 2x+5; & x<-2\end{cases}$ then

$$\lim_{x\to -2} f(x)=$$

Solution:

$\lim_{x\to -2} f(x)$ does not exist because

$$\lim_{x\to(-2)^-} f(x)\neq \lim_{x\to(-2)^+} f(x)$$

4) If $f(x)=\begin{cases}x^2-2x+3; & x\geq 3\\ x^3-3x-12; & x<3\end{cases}$ then

$$\lim_{x\to 3} f(x)=$$

Solution:

$$\lim_{x\to 3^-} f(x)=\lim_{x\to 3^-}(x^3-3x-12)=(3)^3-3(3)-12$$
$$=27-9-12=6$$
$$\lim_{x\to 3^+} f(x)=\lim_{x\to 3^+}(x^2-2x+3)=(3)^2-2(3)+3$$
$$=9-6+3=6$$
$$\therefore\ \lim_{x\to 3} f(x)=6$$

5) If $f(x)=\begin{cases}x^2-7x; & x<1\\ 5; & 1\leq x\leq 3\\ 3x+1; & x>3\end{cases}$ then

$$\lim_{x\to 1^-} f(x)=$$

Solution:

$$\lim_{x\to 1^-} f(x)=\lim_{x\to 1^-}(x^2-7x)=(1)^2-7(1)=1-7=-6$$

6) If $f(x)=\begin{cases}x^2-7x; & x<1\\ 5; & 1\leq x\leq 3\\ 3x+1; & x>3\end{cases}$ then

$$\lim_{x\to 1^+} f(x)=$$

Solution:

$$\lim_{x\to 1^+} f(x)=\lim_{x\to 1^+}(5)=5$$

7) If $f(x)=\begin{cases}x^2-7x; & x<1\\ 5; & 1\leq x\leq 3\\ 3x+1; & x>3\end{cases}$ then

$$\lim_{x\to 3^-} f(x)=$$

Solution:

$$\lim_{x\to 3^-} f(x)=\lim_{x\to 3^-}(5)=5$$

8) If $f(x)=\begin{cases}x^2-7x; & x<1\\ 5; & 1\leq x\leq 3\\ 3x+1; & x>3\end{cases}$ then

$$\lim_{x\to 3^+} f(x)=$$

Solution:

$$\lim_{x\to 3^+} f(x)=\lim_{x\to 3^+}(3x+1)=3(3)+1=9+1=10$$

9) If $f(x)=\begin{cases}\frac{x^2+x-6}{x^2-4}; & x^2-4>0\\ \frac{x^2+x-6}{4-x^2}; & x^2-4<0\end{cases}$ then

$$\lim_{x\to 2^+} f(x)=$$

Solution:

$$f(x)=\begin{cases}\dfrac{x^2+x-6}{x^2-4}; & x^2-4>0\\ \dfrac{x^2+x-6}{4-x^2}; & x^2-4<0\end{cases}$$
$$=\begin{cases}\dfrac{x^2+x-6}{x^2-4}; & x^2>4\\ \dfrac{x^2+x-6}{-(x^2-4)}; & x^2<4\end{cases}$$
$$=\begin{cases}\dfrac{(x+3)(x-2)}{(x-2)(x+2)}; & |x|>4\\ \dfrac{(x+3)(x-2)}{-(x-2)(x+2)}; & |x|<4\end{cases}$$
$$=\begin{cases}\dfrac{x+3}{x+2}; & x>2 \text{ or } x<-2\\ -\dfrac{x+3}{x+2}; & -2<x<2\end{cases}\quad \text{then}$$
$$\therefore\ \lim_{x\to 2^+} f(x)=\lim_{x\to 2^+}\left(\frac{x+3}{x+2}\right)=\frac{(2)+3}{(2)+2}=\frac{5}{4}$$

10) If $f(x)=\begin{cases}\frac{x^2+x-6}{x^2-4}; & x^2-4>0\\ \frac{x^2+x-6}{4-x^2}; & x^2-4<0\end{cases}$ then

$$\lim_{x\to 2^-} f(x)=$$

Solution:

$$f(x)=\begin{cases}\dfrac{x^2+x-6}{x^2-4}; & x^2-4>0\\ \dfrac{x^2+x-6}{4-x^2}; & x^2-4<0\end{cases}$$
$$=\begin{cases}\dfrac{x^2+x-6}{x^2-4}; & x^2>4\\ \dfrac{x^2+x-6}{-(x^2-4)}; & x^2<4\end{cases}$$
$$=\begin{cases}\dfrac{(x+3)(x-2)}{(x-2)(x+2)}; & |x|>4\\ \dfrac{(x+3)(x-2)}{-(x-2)(x+2)}; & |x|<4\end{cases}$$
$$=\begin{cases}\dfrac{x+3}{x+2}; & x>2 \text{ or } x<-2\\ -\dfrac{x+3}{x+2}; & -2<x<2\end{cases}\quad \text{then}$$
$$\therefore\ \lim_{x\to 2^-} f(x)=\lim_{x\to 2^-}\left(-\frac{x+3}{x+2}\right)=-\frac{(2)+3}{(2)+2}=-\frac{5}{4}$$

11)

$$\lim_{x\to a^-}\frac{|x-a|}{x-a}=$$

Solution:

$$f(x)=\frac{|x-a|}{x-a}=\begin{cases}\frac{x-a}{x-a} & ;\ x-a>0\\ \frac{-(x-a)}{x-a} & ;\ x-a<0\end{cases}=\begin{cases}1; & x>a\\ -1; & x<a\end{cases}$$

$$\therefore\quad \lim_{x\to a^-}\frac{|x-a|}{x-a}=\lim_{x\to a^-}\frac{-(x-a)}{x-a}=\lim_{x\to a^-}(-1)=-1$$

12)

$$\lim_{x\to a^+}\frac{|x-a|}{x-a}=$$

Solution:

$$f(x)=\frac{|x-a|}{x-a}=\begin{cases}\frac{x-a}{x-a} & ;\ x-a>0\\ \frac{-(x-a)}{x-a} & ;\ x-a<0\end{cases}=\begin{cases}1; & x>a\\ -1; & x<a\end{cases}$$

$$\therefore\quad \lim_{x\to a^+}\frac{|x-a|}{x-a}=\lim_{x\to a^+}\frac{(x-a)}{x-a}=\lim_{x\to a^+}(1)=1$$

13)

$$\lim_{x\to a}\frac{|x-a|}{x-a}=$$

Solution:

$\lim_{x\to a}\frac{|x-a|}{x-a}$ does not exist because

$$\lim_{x\to a^-}\frac{|x-a|}{x-a}\neq\lim_{x\to a^+}\frac{|x-a|}{x-a}$$

It is clearly obvious from questions (11) and (12) above.

14)

$$\lim_{x\to a^+}\frac{|a-x|}{x-a}=$$

Solution:

$$f(x)=\frac{|a-x|}{x-a}=\begin{cases}\frac{a-x}{x-a} & ;\ a-x>0\\ \frac{-(a-x)}{x-a} & ;\ a-x<0\end{cases}$$

$$=\begin{cases}\frac{-(x-a)}{x-a} & ;\ a>x\\ \frac{(x-a)}{x-a} & ;\ a<x\end{cases}=\begin{cases}-1; & x<a\\ 1; & x>a\end{cases}$$

$$\therefore\quad \lim_{x\to a^+}\frac{|a-x|}{x-a}=\lim_{x\to a^+}(1)=1$$

15)

$$\lim_{x\to a^-}\frac{|a-x|}{x-a}=$$

Solution:

$$f(x)=\frac{|a-x|}{x-a}=\begin{cases}\frac{a-x}{x-a} & ;\ a-x>0\\ \frac{-(a-x)}{x-a} & ;\ a-x<0\end{cases}$$

$$=\begin{cases}\frac{-(x-a)}{x-a} & ;\ a>x\\ \frac{(x-a)}{x-a} & ;\ a<x\end{cases}=\begin{cases}-1; & x<a\\ 1; & x>a\end{cases}$$

$$\therefore\quad \lim_{x\to a^-}\frac{|a-x|}{x-a}=\lim_{x\to a^-}(-1)=-1$$

16)

$$\lim_{x\to a}\frac{|a-x|}{x-a}=$$

Solution:

$\lim_{x\to a}\frac{|a-x|}{x-a}$ does not exist because

$$\lim_{x\to a^-}\frac{|a-x|}{x-a}\neq\lim_{x\to a^+}\frac{|a-x|}{x-a}$$

It is clearly obvious from questions (14) and (15) above.

17)

$$\lim_{x\to(-a)^-}\frac{|x+a|}{x+a}=$$

Solution:

$$f(x)=\frac{|x+a|}{x+a}=\begin{cases}\frac{x+a}{x+a} & ;\ x+a>0\\ \frac{-(x+a)}{x+a} & ;\ x+a<0\end{cases}=\begin{cases}1; & x>-a\\ -1; & x<-a\end{cases}$$

$$\therefore\quad \lim_{x\to(-a)^-}\frac{|x+a|}{x+a}=\lim_{x\to(-a)^-}(-1)=-1$$

18)

$$\lim_{x\to(-a)^+}\frac{|x+a|}{x+a}=$$

Solution:

$$f(x)=\frac{|x+a|}{x+a}=\begin{cases}\frac{x+a}{x+a} & ;\ x+a>0\\ \frac{-(x+a)}{x+a} & ;\ x+a<0\end{cases}=\begin{cases}1; & x>-a\\ -1; & x<-a\end{cases}$$

$$\therefore\quad \lim_{x\to(-a)^+}\frac{|x+a|}{x+a}=\lim_{x\to(-a)^+}(1)=1$$

19)

$$\lim_{x\to -a}\frac{|x+a|}{x+a}=$$

Solution:

$\lim_{x\to -a}\frac{|x+a|}{x+a}$ does not exist because

$$\lim_{x\to(-a)^-}\frac{|x+a|}{x+a}\neq\lim_{x\to(-a)^+}\frac{|x+a|}{x+a}$$

It is clearly obvious from questions (17) and (18) above.

20)

$$\lim_{x\to 0^+}\frac{2x-|x|}{x^2+|x|}=$$

Solution:

$$f(x)=\frac{2x-|x|}{x^2+|x|}=\begin{cases}\dfrac{2x-(x)}{x^2+(x)} & ;\ x>0\\ \dfrac{2x-(-x)}{x^2+(-x)} & ;\ x<0\end{cases}$$

$$=\begin{cases}\dfrac{2x-x}{x^2+x} & ;\ x>0\\ \dfrac{2x+x}{x^2-x} & ;\ x<0\end{cases}=\begin{cases}\dfrac{x}{x^2+x} & ;\ x>0\\ \dfrac{3x}{x^2-x} & ;\ x<0\end{cases}$$

$$=\begin{cases}\dfrac{x}{x(x+1)} & ;\ x>0\\ \dfrac{3x}{x(x-1)} & ;\ x<0\end{cases}$$

$$=\begin{cases}\dfrac{1}{x+1} & ;\ x>0\\ \dfrac{3}{x-1} & ;\ x<0\end{cases}$$

$$\therefore\quad \lim_{x\to 0^+}\frac{2x-|x|}{x^2+|x|}=\lim_{x\to 0^+}\frac{1}{x+1}=\frac{1}{0+1}=1$$

21)

$$\lim_{x\to 0^-}\frac{2x-|x|}{x^2+|x|}=$$

Solution:

$$f(x)=\frac{2x-|x|}{x^2+|x|}=\begin{cases}\dfrac{2x-(x)}{x^2+(x)} & ;\ x>0\\ \dfrac{2x-(-x)}{x^2+(-x)} & ;\ x<0\end{cases}$$

$$=\begin{cases}\dfrac{2x-x}{x^2+x} & ;\ x>0\\ \dfrac{2x+x}{x^2-x} & ;\ x<0\end{cases}=\begin{cases}\dfrac{x}{x^2+x} & ;\ x>0\\ \dfrac{3x}{x^2-x} & ;\ x<0\end{cases}$$

$$=\begin{cases}\dfrac{x}{x(x+1)} & ;\ x>0\\ \dfrac{3x}{x(x-1)} & ;\ x<0\end{cases}$$

$$=\begin{cases}\dfrac{1}{x+1} & ;\ x>0\\ \dfrac{3}{x-1} & ;\ x<0\end{cases}$$

$$\therefore\quad \lim_{x\to 0^-}\frac{2x-|x|}{x^2+|x|}=\lim_{x\to 0^-}\frac{3}{x-1}=\frac{3}{0-1}=-3$$

22)

$$\lim_{x\to 0}\frac{2x-|x|}{x^2+|x|}=$$

Solution:

$\lim_{x\to 0}\frac{2x-|x|}{x^2+|x|}$ does not exist because

$$\lim_{x\to 0^-}\frac{2x-|x|}{x^2+|x|}\neq\lim_{x\to 0^+}\frac{2x-|x|}{x^2+|x|}$$

It is clearly obvious from questions (20) and (21) above.

23)

$$\lim_{x\to\frac{\pi}{4}}\frac{\cos x-\sin x}{\cos^2 x-\sin^2 x}=$$

Solution:

$$\lim_{x\to\frac{\pi}{4}}\frac{\cos x-\sin x}{\cos^2 x-\sin^2 x}=\lim_{x\to\frac{\pi}{4}}\frac{\cos x-\sin x}{(\cos x-\sin x)(\cos x+\sin x)}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{1}{\cos x+\sin x}=\frac{1}{\cos\left(\frac{\pi}{4}\right)+\sin\left(\frac{\pi}{4}\right)}$$

$$=\frac{1}{\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}}=\frac{1}{\frac{2}{\sqrt{2}}}=\frac{\sqrt{2}}{2}$$

24)

$$\lim_{x\to 0}\frac{\cos^2 x+2\cos x-3}{2\cos^2 x-\cos x-1}=$$

Solution:

$$\lim_{x\to 0}\frac{\cos^2 x+2\cos x-3}{2\cos^2 x-\cos x-1}=\lim_{x\to 0}\frac{(\cos x+3)(\cos x-1)}{(2\cos x+1)(\cos x-1)}$$

$$=\lim_{x\to 0}\frac{\cos x+3}{2\cos x+1}=\frac{\cos(0)+3}{2\cos(0)+1}$$

$$=\frac{1+3}{2(1)+1}=\frac{4}{3}$$

25)

$$\lim_{x\to 0}(\sin^2 x+3\tan x-4)=$$

Solution:

$$\lim_{x\to 0}(\sin^2 x+3\tan x-4)=\sin^2(0)+3\tan(0)-4$$

$$=0+3(0)-4=-4$$

26) If $m\neq 0$, then

$$\lim_{x\to 0}\frac{\sin(nx)}{mx}=$$

Solution:

$$\lim_{x\to 0}\frac{\sin(nx)}{mx}=\frac{n}{m}\lim_{x\to 0}\frac{\sin(nx)}{nx}=\frac{n}{m}(1)=\frac{n}{m}$$

27) If $m\neq 0$, then

$$\lim_{x\to 0}\frac{\tan(nx)}{mx}=$$

Solution:

$$\lim_{x\to 0}\frac{\tan(nx)}{mx}=\frac{n}{m}\lim_{x\to 0}\frac{\tan(nx)}{nx}=\frac{n}{m}(1)=\frac{n}{m}$$

28) If $m\neq 0$, then

$$\lim_{x\to 0}\frac{nx}{\sin(mx)}=$$

Solution:

$$\lim_{x\to 0}\frac{nx}{\sin(mx)}=\frac{n}{m}\lim_{x\to 0}\frac{mx}{\sin(mx)}=\frac{n}{m}(1)=\frac{n}{m}$$

29) If $m\neq 0$, then

$$\lim_{x\to 0}\frac{nx}{\tan(mx)}=$$

Solution:

$$\lim_{x\to 0}\frac{nx}{\tan(mx)}=\frac{n}{m}\lim_{x\to 0}\frac{mx}{\tan(mx)}=\frac{n}{m}(1)=\frac{n}{m}$$

30) If $m \neq 0$, then

$$\lim_{x\to 0}\frac{\sin(nx)}{\sin(mx)} =$$

Solution:

$$\lim_{x\to 0}\frac{\sin(nx)}{\sin(mx)} = \frac{n}{m}\left(\lim_{x\to 0}\frac{\sin(nx)}{nx}\right)\left(\lim_{x\to 0}\frac{mx}{\sin(mx)}\right)$$

$$= \frac{n}{m}(1)(1) = \frac{n}{m}$$

31) If $m \neq 0$, then

$$\lim_{x\to 0}\frac{\sin(nx)}{\tan(mx)} =$$

Solution:

$$\lim_{x\to 0}\frac{\sin(nx)}{\tan(mx)} = \frac{n}{m}\left(\lim_{x\to 0}\frac{\sin(nx)}{nx}\right)\left(\lim_{x\to 0}\frac{mx}{\tan(mx)}\right)$$

$$= \frac{n}{m}(1)(1) = \frac{n}{m}$$

32) If $m \neq 0$, then

$$\lim_{x\to 0}\frac{\tan(nx)}{\tan(mx)} =$$

Solution:

$$\lim_{x\to 0}\frac{\tan(nx)}{\tan(mx)} = \frac{n}{m}\left(\lim_{x\to 0}\frac{\tan(nx)}{nx}\right)\left(\lim_{x\to 0}\frac{mx}{\tan(mx)}\right)$$

$$= \frac{n}{m}(1)(1) = \frac{n}{m}$$

33) If $m \neq 0$, then

$$\lim_{x\to 0}\frac{\tan(nx)}{\sin(mx)} =$$

Solution:

$$\lim_{x\to 0}\frac{\tan(nx)}{\sin(mx)} = \frac{n}{m}\left(\lim_{x\to 0}\frac{\tan(nx)}{nx}\right)\left(\lim_{x\to 0}\frac{mx}{\sin(mx)}\right)$$

$$= \frac{n}{m}(1)(1) = \frac{n}{m}$$

34)

$$\lim_{x\to 0}\frac{\sin(1-\cos x)}{1-\cos x} =$$

Solution:

$$\lim_{x\to 0}\frac{\sin(1-\cos x)}{1-\cos x} = 1$$

35)

$$\lim_{x\to 0}\frac{\sin(\sin(2x))}{\sin(2x)} =$$

Solution:

$$\lim_{x\to 0}\frac{\sin(\sin(2x))}{\sin(2x)} = 1$$

36)

$$\lim_{x\to 0}\frac{1-\cos(2x)}{x^2} =$$

Solution:

$$\lim_{x\to 0}\frac{1-\cos(2x)}{x^2} = \lim_{x\to 0}\frac{2\sin^2 x}{x^2} = 2\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2$$

$$= 2\left(\lim_{x\to 0}\frac{\sin x}{x}\right)^2 = 2(1)^2 = 2$$

37)

$$\lim_{x\to\infty}\sqrt{\frac{1}{x^2}-\frac{3}{x}+4} =$$

Solution:

$$\lim_{x\to\infty}\sqrt{\frac{1}{x^2}-\frac{3}{x}+4} = \sqrt{\lim_{x\to\infty}\left(\frac{1}{x^2}-\frac{3}{x}+4\right)} = \sqrt{0-0+4}$$

$$= 2$$

38)

$$\lim_{x\to\infty}\left(\frac{1}{x^{2/5}}+2\right) =$$

Solution:

$$\lim_{x\to -\infty}\left(\frac{1}{x^{2/5}}+2\right) = 0+2 = 2$$

39)

$$\lim_{x\to\infty}\frac{3x+15}{9x^2+4x-13} =$$

Solution:

$$\lim_{x\to\infty}\frac{3x+15}{9x^2+4x-13} = \lim_{x\to\infty}\frac{\frac{3x}{x^2}+\frac{15}{x^2}}{\frac{9x^2}{x^2}+\frac{4x}{x^2}-\frac{13}{x^2}}$$

$$= \lim_{x\to\infty}\frac{\frac{3}{x}+\frac{15}{x^2}}{9+\frac{4}{x}-\frac{13}{x^2}} = \frac{0+0}{9+0+0} = 0$$

40)

$$\lim_{x\to\infty}\frac{3x^2-8x+15}{9x^2+4x-13} =$$

Solution:

$$\lim_{x\to\infty}\frac{3x^2-8x+15}{9x^2+4x-13} = \lim_{x\to\infty}\frac{\frac{3x^2}{x^2}-\frac{8x}{x^2}+\frac{15}{x^2}}{\frac{9x^2}{x^2}+\frac{4x}{x^2}-\frac{13}{x^2}}$$

$$= \lim_{x\to\infty}\frac{3-\frac{8}{x}+\frac{15}{x^2}}{9+\frac{4}{x}-\frac{13}{x^2}} = \frac{3-0+0}{9+0+0} = \frac{1}{3}$$

41)

$$\lim_{x\to -\infty}\frac{3x^2-8x+15}{9x^2+4x-13} =$$

Solution:

$$\lim_{x\to -\infty}\frac{3x^2-8x+15}{9x^2+4x-13} = \lim_{x\to -\infty}\frac{\frac{3x^2}{-x^2}-\frac{8x}{-x^2}+\frac{15}{-x^2}}{\frac{9x^2}{-x^2}+\frac{4x}{-x^2}-\frac{13}{-x^2}}$$

$$= \lim_{x\to -\infty}\frac{-3+\frac{8}{x}-\frac{15}{x^2}}{-9-\frac{4}{x}+\frac{13}{x^2}} = \frac{-3+0-0}{-9-0+0} = \frac{1}{3}$$

42)

$$\lim_{x\to\infty}\frac{3x^5-8x+15}{9x^2+4x-13}=$$

Solution:

$$\lim_{x\to\infty}\frac{3x^5-8x+15}{9x^2+4x-13}=\lim_{x\to\infty}\frac{\frac{3x^5}{x^2}-\frac{8x}{x^2}+\frac{15}{x^2}}{\frac{9x^2}{x^2}+\frac{4x}{x^2}-\frac{13}{x^2}}$$

$$=\lim_{x\to\infty}\frac{3x^3-\frac{8}{x}+\frac{15}{x^2}}{9+\frac{4}{x}-\frac{13}{x^2}}=\frac{3(\infty)-0+0}{9+0+0}=\infty$$

43)

$$\lim_{x\to-\infty}\frac{3x^5-8x+15}{9x^2+4x-13}=$$

Solution:

$$\lim_{x\to-\infty}\frac{3x^5-8x+15}{9x^2+4x-13}=\lim_{x\to-\infty}\frac{\frac{3x^5}{-x^2}-\frac{8x}{-x^2}+\frac{15}{-x^2}}{\frac{9x^2}{-x^2}+\frac{4x}{-x^2}-\frac{13}{-x^2}}$$

$$=\lim_{x\to-\infty}\frac{-3x^3+\frac{8}{x}-\frac{15}{x^2}}{-9-\frac{4}{x}+\frac{13}{x^2}}=\frac{-3(-\infty)+0-0}{-9-0+0}=-\infty$$

44)

$$\lim_{x\to\infty}\left(\sqrt{x^2-3x+7}-x\right)=$$

Solution:

$$\lim_{x\to\infty}\left(\sqrt{x^2-3x+7}-x\right)$$

$$=\lim_{x\to\infty}\left[\left(\sqrt{x^2-3x+7}-x\right)\times\frac{\left(\sqrt{x^2-3x+7}+x\right)}{\left(\sqrt{x^2-3x+7}+x\right)}\right]$$

$$=\lim_{x\to\infty}\left(\frac{(x^2-3x+7)-x^2}{\sqrt{x^2-3x+7}+x}\right)=\lim_{x\to\infty}\left(\frac{-3x+7}{\sqrt{x^2-3x+7}+x}\right)$$

$$=\lim_{x\to\infty}\frac{\frac{-3x}{x}+\frac{7}{x}}{\frac{\sqrt{x^2-3x+7}}{x}+\frac{x}{x}}$$

$$=\lim_{x\to\infty}\frac{-3+\frac{7}{x}}{\sqrt{\frac{x^2}{x^2}-\frac{3x}{x^2}+\frac{7}{x^2}}+1}$$

$$=\lim_{x\to\infty}\frac{-3+\frac{7}{x}}{\sqrt{1-\frac{3}{x}+\frac{7}{x^2}}+1}$$

$$=\frac{-3+0}{\sqrt{1-0+0}+1}=\frac{-3}{1+1}=-\frac{3}{2}$$

45)

$$\lim_{x\to\infty}\left(\sqrt{x^2+x}-x\right)=$$

Solution:

$$\lim_{x\to\infty}\left(\sqrt{x^2+x}-x\right)$$

$$=\lim_{x\to\infty}\left[\left(\sqrt{x^2+x}-x\right)\times\frac{\sqrt{x^2+x}+x}{\sqrt{x^2+x}+x}\right]$$

$$=\lim_{x\to\infty}\left(\frac{(x^2+x)-x^2}{\sqrt{x^2+x}+x}\right)$$

$$=\lim_{x\to\infty}\left(\frac{x}{\sqrt{x^2+x}+x}\right)$$

$$=\lim_{x\to\infty}\frac{\frac{x}{x}}{\frac{\sqrt{x^2+x}}{x}+\frac{x}{x}}=\lim_{x\to\infty}\frac{1}{\sqrt{\frac{x^2}{x^2}+\frac{x}{x^2}}+1}$$

$$=\lim_{x\to\infty}\frac{1}{\sqrt{1+\frac{1}{x}}+1}=\frac{1}{\sqrt{1+0}+1}=\frac{1}{1+1}$$

$$=\frac{1}{2}$$

46)

$$\lim_{x\to\infty}(x^2-5x+4)=$$

Solution:

$$\lim_{x\to\infty}(x^2-5x+4)=\lim_{x\to\infty}x^2\left(\frac{x^2}{x^2}-\frac{5x}{x^2}+\frac{4}{x^2}\right)$$

$$=\lim_{x\to\infty}x^2\left(1-\frac{5}{x}+\frac{4}{x^2}\right)=(\infty)^2(1-0+0)=\infty$$

OR

$$\lim_{x\to\infty}(x^2-5x+4)=\lim_{x\to\infty}(x^2)=\infty$$

47)

$$\lim_{x\to-\infty}(x^4-2x^3+9)=$$

Solution:

$$\lim_{x\to-\infty}(x^4-2x^3+9)=\lim_{x\to-\infty}x^4\left(\frac{x^4}{x^4}-\frac{2x^3}{x^4}+\frac{9}{x^4}\right)$$

$$=\lim_{x\to-\infty}x^4\left(1-\frac{2}{x}+\frac{9}{x^4}\right)=(-\infty)^4(1-0+0)=\infty$$

OR

$$\lim_{x\to-\infty}(x^4-2x^3+9)=\lim_{x\to-\infty}(x^4)=\infty$$

48)

$$\lim_{x\to-\infty}\frac{\sqrt{3x^2-8}+2}{x+5}=$$

Solution:

$$\lim_{x\to-\infty}\frac{\sqrt{3x^2-8}+2}{x+5}=\lim_{x\to-\infty}\frac{\frac{\sqrt{3x^2-8}}{-x}+\frac{2}{-x}}{\frac{x}{-x}+\frac{5}{-x}}$$

$$=\lim_{x\to-\infty}\frac{\sqrt{\frac{3x^2-8}{x^2}}-\frac{2}{x}}{-1-\frac{5}{x}}=\lim_{x\to-\infty}\frac{\sqrt{\frac{3x^2}{x^2}-\frac{8}{x^2}}-\frac{2}{x}}{-1-\frac{5}{x}}$$

$$=\lim_{x\to-\infty}\frac{\sqrt{3-\frac{8}{x^2}}-\frac{2}{x}}{-1-\frac{5}{x}}=\frac{\sqrt{3-0}-0}{-1-0}=-\sqrt{3}$$

49)

$$\lim_{x\to\infty}\frac{\sqrt{3x^2-8}+2}{x+5}=$$

Solution:

$$\lim_{x\to\infty}\frac{\sqrt{3x^2-8}+2}{x+5}=\lim_{x\to\infty}\frac{\frac{\sqrt{3x^2-8}}{x}+\frac{2}{x}}{\frac{x}{x}+\frac{5}{x}}$$

$$=\lim_{x\to\infty}\frac{\sqrt{\frac{3x^2-8}{x^2}}+\frac{2}{x}}{1+\frac{5}{x}}=\lim_{x\to\infty}\frac{\sqrt{\frac{3x^2}{x^2}-\frac{8}{x^2}}+\frac{2}{x}}{1+\frac{5}{x}}$$

$$=\lim_{x\to\infty}\frac{\sqrt{3-\frac{8}{x^2}}+\frac{2}{x}}{1+\frac{5}{x}}=\frac{\sqrt{3-0}+0}{1+0}=\sqrt{3}$$

50) The horizontal asymptotes of

$$f(x)=\frac{\sqrt{3x^2-8}+2}{x+5}$$

Solution:

First, we have to find

$$\lim_{x\to\pm\infty}\frac{\sqrt{3x^2-8}+2}{x+5}$$

It is clear from the previous questions (48) and (49) that

$$\lim_{x\to\infty}\frac{\sqrt{3x^2-8}+2}{x+5}=\sqrt{3}$$

and

$$\lim_{x\to-\infty}\frac{\sqrt{3x^2-8}+2}{x+5}=-\sqrt{3}$$

Thus, the horizontal asymptotes are

$$y=\pm\sqrt{3}$$

51) The horizontal asymptote of

$$f(x)=\frac{1-x}{2x+1}$$

Solution:

First, we have to find

$$\lim_{x\to\pm\infty}\frac{1-x}{2x+1}$$

$$\lim_{x\to\infty}\frac{1-x}{2x+1}=\lim_{x\to\infty}\frac{\frac{1}{x}-\frac{x}{x}}{\frac{2x}{x}+\frac{1}{x}}=\lim_{x\to\infty}\frac{\frac{1}{x}-1}{2+\frac{1}{x}}=\frac{0-1}{2+0}=-\frac{1}{2}$$

$$\lim_{x\to-\infty}\frac{1-x}{2x+1}=\lim_{x\to-\infty}\frac{\frac{1}{-x}-\frac{x}{-x}}{\frac{2x}{-x}+\frac{1}{-x}}=\lim_{x\to-\infty}\frac{\frac{1}{-x}+1}{-2-\frac{1}{x}}=\frac{0+1}{-2-0}$$

$$=-\frac{1}{2}$$

Thus, the horizontal asymptote is

$$y=-\frac{1}{2}$$

52) The horizontal asymptote of

$$f(x)=\frac{7x^2+5}{3x^2+2}$$

Solution:

First, we have to find

$$\lim_{x\to\pm\infty}\frac{7x^2+5}{3x^2+2}$$

$$\lim_{x\to\infty}\frac{7x^2+5}{3x^2+2}=\lim_{x\to\infty}\frac{\frac{7x^2}{x^2}+\frac{5}{x^2}}{\frac{3x^2}{x^2}+\frac{2}{x^2}}=\lim_{x\to\infty}\frac{7+\frac{5}{x^2}}{3+\frac{2}{x^2}}=\frac{7+0}{3+0}=\frac{7}{3}$$

$$\lim_{x\to-\infty}\frac{7x^2+5}{3x^2+2}=\lim_{x\to-\infty}\frac{\frac{7x^2}{-x^2}+\frac{5}{-x^2}}{\frac{3x^2}{-x^2}+\frac{2}{-x^2}}$$

$$=\lim_{x\to-\infty}\frac{-7-\frac{5}{x^2}}{-3-\frac{2}{x^2}}=\frac{-7-0}{-3-0}=\frac{7}{3}$$

Thus, the horizontal asymptote is

$$y=\frac{7}{3}$$

53) The horizontal asymptote of

$$f(x)=\frac{\sqrt{x^2+2x-3}}{2x+7}$$

Solution:

First, we have to find

$$\lim_{x\to\pm\infty}\frac{\sqrt{x^2+2x-3}}{2x+7}$$

$$\lim_{x\to\infty}\frac{\sqrt{x^2+2x-3}}{2x+7}=\lim_{x\to\infty}\frac{\frac{\sqrt{x^2+2x-3}}{x}}{\frac{2x}{x}+\frac{7}{x}}$$

$$=\lim_{x\to\infty}\frac{\sqrt{\frac{x^2+2x-3}{x^2}}}{2+\frac{7}{x}}=\lim_{x\to\infty}\frac{\sqrt{\frac{x^2}{x^2}+\frac{2x}{x^2}-\frac{3}{x^2}}}{2+\frac{7}{x}}$$

$$=\lim_{x\to\infty}\frac{\sqrt{1+\frac{2}{x}-\frac{3}{x^2}}}{2+\frac{7}{x}}=\frac{\sqrt{1+0-0}}{2+0}=\frac{1}{2}$$

$$\lim_{x\to-\infty}\frac{\sqrt{x^2+2x-3}}{2x+7}=\lim_{x\to-\infty}\frac{\frac{\sqrt{x^2+2x-3}}{-x}}{\frac{2x}{-x}+\frac{7}{-x}}$$

$$=\lim_{x\to-\infty}\frac{\sqrt{\frac{x^2+2x-3}{x^2}}}{-2-\frac{7}{x}}=\lim_{x\to-\infty}\frac{\sqrt{\frac{x^2}{x^2}+\frac{2x}{x^2}-\frac{3}{x^2}}}{-2-\frac{7}{x}}$$

$$=\lim_{x\to-\infty}\frac{\sqrt{1+\frac{2}{x}-\frac{3}{x^2}}}{-2-\frac{7}{x}}=\frac{\sqrt{1+0-0}}{-2-0}=-\frac{1}{2}$$

Thus, the horizontal asymptotes are

$$y=\pm\frac{1}{2}$$

54) The horizontal asymptote of

$$f(x)=\frac{\sqrt{2x-3}}{2x^2+7x-1}$$

Solution:

First, we have to find

$$\lim_{x\to\pm\infty}\frac{\sqrt{2x-3}}{2x^2+7x-1}$$

$$\lim_{x\to\infty}\frac{\sqrt{2x-3}}{2x^2+7x-1}=\lim_{x\to\infty}\frac{\frac{\sqrt{2x-3}}{x^2}}{\frac{2x^2}{x^2}+\frac{7x}{x^2}-\frac{1}{x^2}}$$

$$=\lim_{x\to\infty}\frac{\sqrt{\frac{2x-3}{x^4}}}{2+\frac{7}{x}-\frac{1}{x^2}}=\lim_{x\to\infty}\frac{\sqrt{\frac{2x}{x^4}-\frac{3}{x^4}}}{2+\frac{7}{x}-\frac{1}{x^2}}$$

$$=\lim_{x\to\infty}\frac{\sqrt{\frac{2}{x^3}-\frac{3}{x^4}}}{2+\frac{7}{x}-\frac{1}{x^2}}=\frac{\sqrt{0-0}}{2+0-0}=\frac{0}{2}=0$$

$$\lim_{x\to-\infty}\frac{\sqrt{2x-3}}{2x^2+7x-1}=\lim_{x\to-\infty}\frac{\frac{\sqrt{2x-3}}{-x^2}}{\frac{2x^2}{-x^2}+\frac{7x}{-x^2}-\frac{1}{-x^2}}$$

$$=\lim_{x\to-\infty}\frac{\sqrt{\frac{2x-3}{x^4}}}{-2-\frac{7}{x}+\frac{1}{x^2}}=\lim_{x\to-\infty}\frac{\sqrt{\frac{2x}{x^4}-\frac{3}{x^4}}}{-2-\frac{7}{x}+\frac{1}{x^2}}$$

$$=\lim_{x\to-\infty}\frac{\sqrt{\frac{2}{x^3}-\frac{3}{x^4}}}{-2-\frac{7}{x}+\frac{1}{x^2}}=\frac{\sqrt{0-0}}{-2-0+0}=\frac{0}{-2}=0$$

Thus, the horizontal asymptote is

$$y=0$$

55)

$$\lim_{x\to-\infty}\frac{\sqrt{4x^2-8}+3}{x+1}=$$

Solution:

$$\lim_{x\to-\infty}\frac{\sqrt{4x^2-8}+3}{x+1}=\lim_{x\to-\infty}\frac{\frac{\sqrt{4x^2-8}}{-x}+\frac{3}{-x}}{\frac{x}{-x}+\frac{1}{-x}}$$

$$=\lim_{x\to-\infty}\frac{\sqrt{\frac{4x^2-8}{x^2}}-\frac{3}{x}}{-1-\frac{1}{x}}=\lim_{x\to-\infty}\frac{\sqrt{\frac{4x^2}{x^2}-\frac{8}{x^2}}-\frac{3}{x}}{-1-\frac{1}{x}}$$

$$=\lim_{x\to-\infty}\frac{\sqrt{4-\frac{8}{x^2}}-\frac{3}{x}}{-1-\frac{1}{x}}=\frac{\sqrt{4-0}-0}{-1-0}=-2$$

56)

$$\lim_{x\to\infty}\frac{\sqrt{4x^2-8}+3}{x+1}=$$

Solution:

$$\lim_{x\to\infty}\frac{\sqrt{4x^2-8}+3}{x+1}=\lim_{x\to\infty}\frac{\frac{\sqrt{4x^2-8}}{x}+\frac{3}{x}}{\frac{x}{x}+\frac{1}{x}}$$

$$=\lim_{x\to-\infty}\frac{\sqrt{\frac{4x^2-8}{x^2}}+\frac{3}{x}}{1+\frac{1}{x}}=\lim_{x\to\infty}\frac{\sqrt{\frac{4x^2}{x^2}-\frac{8}{x^2}}+\frac{3}{x}}{1+\frac{1}{x}}$$

$$=\lim_{x\to\infty}\frac{\sqrt{4-\frac{8}{x^2}}+\frac{3}{x}}{1+\frac{1}{x}}=\frac{\sqrt{4-0}+0}{1+0}=2$$