



Dr. George Karraz, Ph. D.

Motion tracking

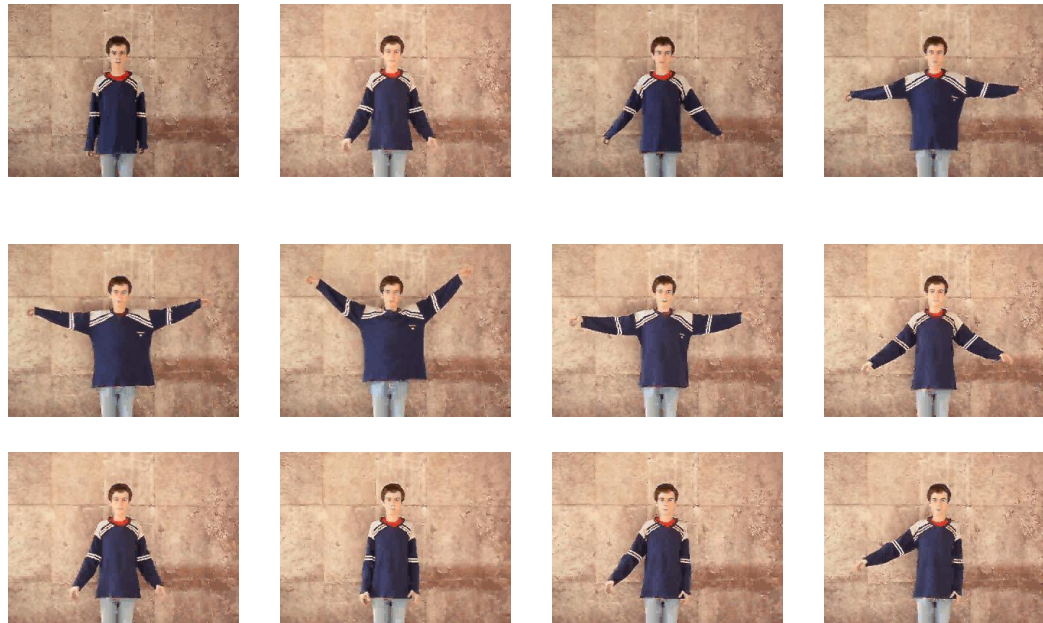
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Contents:

- 🔦 The Problem
- 🔦 Goals
- 🔦 Approaches
- 🔦 The Optical Flow Method
- 🔦 Algorithm

The Problem

- ✦ Given a set of images in time which are similar but not identical, derive a method for identifying the motion that has occurred (in 2d) between different images.



Goals

- ✦ Input:
 - an image sequence
 - captured with a fixed camera
 - containing one or more moving objects of interest
- ✦ Processing goals: determine the image regions where significant motion has occurred
- ✦ Output: an outline of the motion within the image sequence

Motion Detection and Estimation

🔦 Image differencing

- based on the thresholded difference of successive images
- difficult to reconstruct moving areas

🔦 Background subtraction

- foreground objects result by calculating the difference between an image in the sequence and the background image (previously obtained)
- remaining task: determine the movement of these foreground objects between successive frames

🔦 Block motion estimation

- Calculates the motion vector between frames for sub-blocks of the image
- mainly used in image compression
- too coarse

🔦 Optical Flow

What Is Optical Flow?

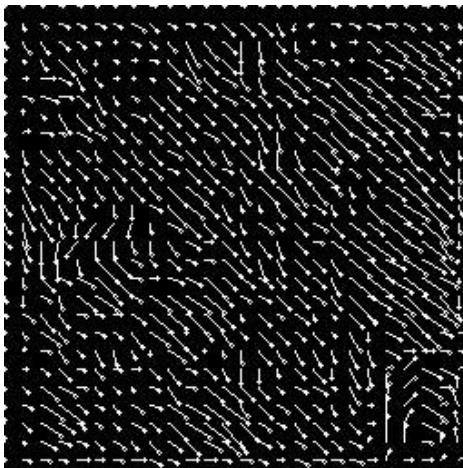
☀️ **Optical flow** is the displacement field for each of the pixels in an image sequence.

☀️ For every pixel, a velocity vector $\left(\frac{dx}{dt}, \frac{dy}{dt}\right)$ is found which says:

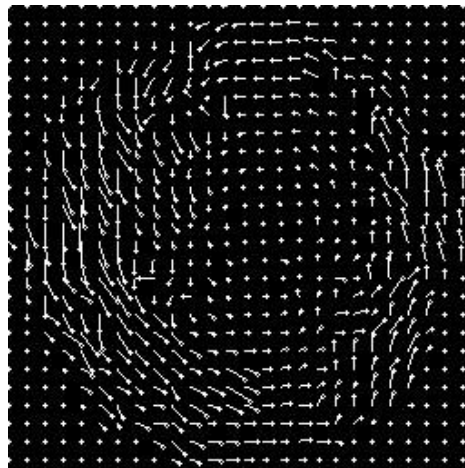
- how quickly a pixel is moving across the image
- the direction of its movement.

Optical Flow Examples

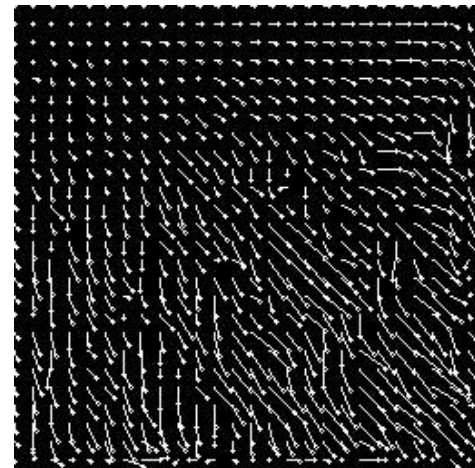
Translation



Rotation



Scaling



Algorithm

- ✦ Optical flow: maximum one pixel large movements
- ✦ Optical flow: larger movements
- ✦ Morphological filter
- ✦ Contour detection (demo purposes)

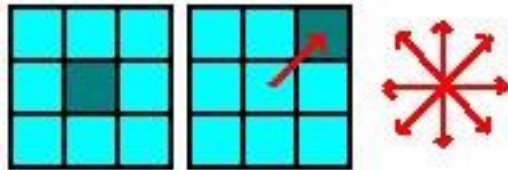
Optical Flow: maximum one pixel large movements

- ✨ The optical flow for a pixel (i, j) given 2 successive images k and $k+1$:

$$\overline{m_k(i, j)} = (x, y) \quad \text{so that}$$

$$|I_k(i, j) - I_{k+1}(i+x, j+y)| \quad (1)$$

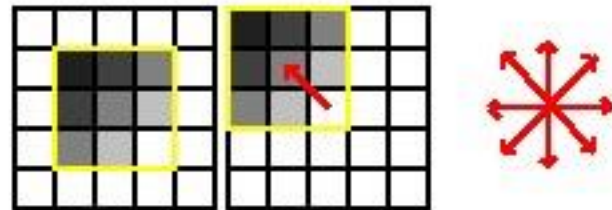
is minimum for $-1 \leq x \leq 1, -1 \leq y \leq 1$



k k+1

Optical Flow: maximum one pixel large movements (2)

- More precision: consider a 3×3 window around the pixel:



- Optical flow for pixel (i, j) becomes:

$\overline{m_k(i, j)} = (x, y)$ so that

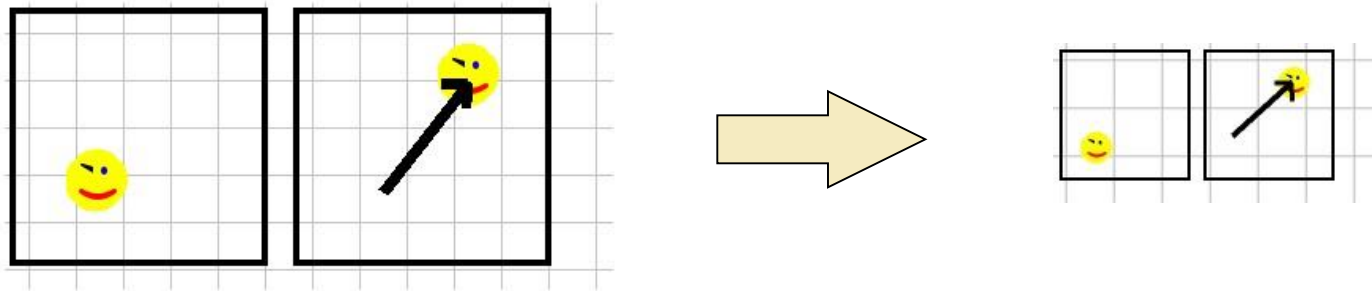
$$\left| \sum_{u=-1}^1 \sum_{v=-1}^1 I_k(i+u, j+v) - \sum_{u=-1}^1 \sum_{v=-1}^1 I_{k+1}(i+u+x, j+v+y) \right| \quad (2)$$

is minimum for $-1 \leq x \leq 1, -1 \leq y \leq 1$

Optical Flow: larger movements

🔦 Reduce the size of the image

=> reduced size of the movement

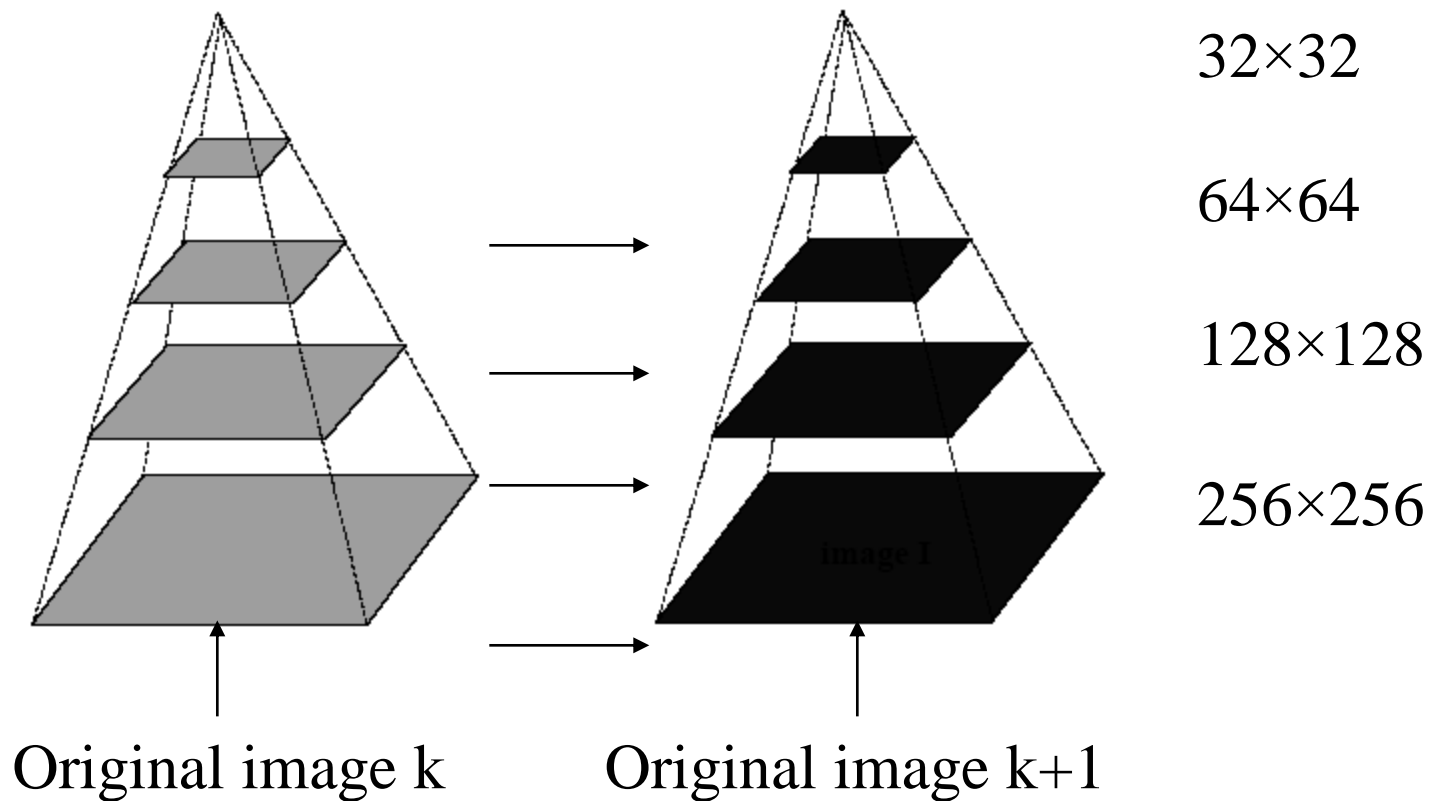


🔦 Solution: **multi-resolution analysis** of the images

🔦 Advantages: **computing efficiency, stability**

Multi-resolution Analysis

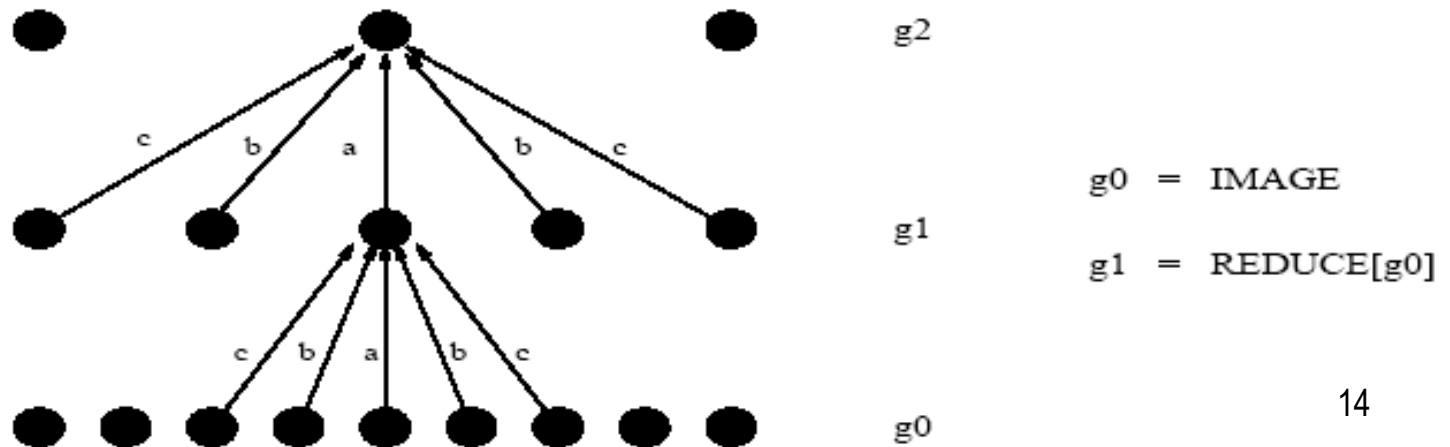
☀️ Coarse to fine optical flow estimation:



Gaussian Pyramid

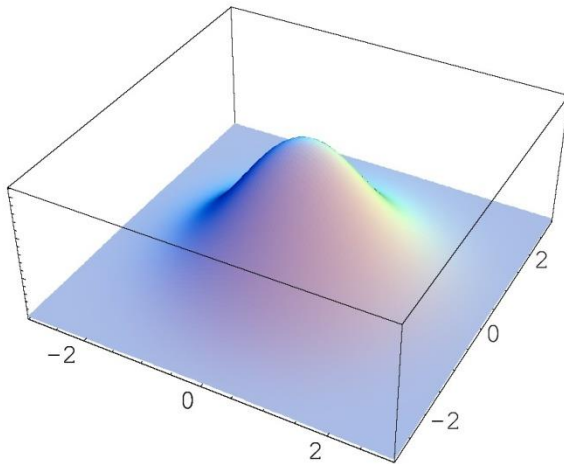
- ☀️ Lowest level g_0 - the original image
- ☀️ Level g_l - the weighed average of values in g_{l-1} in a 5×5 window:

$$g_l(i, j) = \sum_{m=-2}^2 \sum_{n=-2}^2 w(m, n) g_{l-1}(2i + m, 2j + n) \quad (3)$$



Gaussian Pyramid (2)

✦ The mask $G(m, n)$ is an approximation of the 2D Gaussian:



0.003	0.013	0.022	0.013	0.003
0.013	0.060	0.098	0.060	0.013
0.022	0.098	0.162	0.098	0.022
0.013	0.060	0.098	0.060	0.013
0.003	0.013	0.022	0.013	0.003

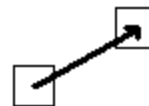
✦ The mask is symmetric and separable:

$$G(m, n) = G_r(m) * G_c(n) \quad (4)$$

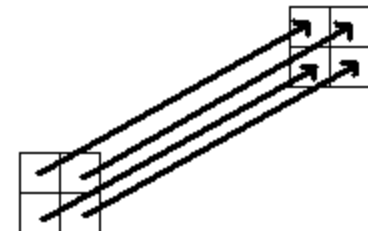
Optical Flow: Top-down Strategy

Algorithm (1/4 scale of resolution reduction):

- ✦ Step 1: compute optical flow vectors for the highest level of the pyramid l (smallest resolution)
- ✦ Step 2: double the values of the vectors
- ✦ Step 3: first approximation: optical flow vectors for the $(2i, 2j)$, $(2i+1, 2j)$, $(2i, 2j+1)$, $(2i+1, 2j+1)$ pixels in the $l-1$ level are assigned the value of the optical flow vector for the (i,j) pixel from the l level



Level l

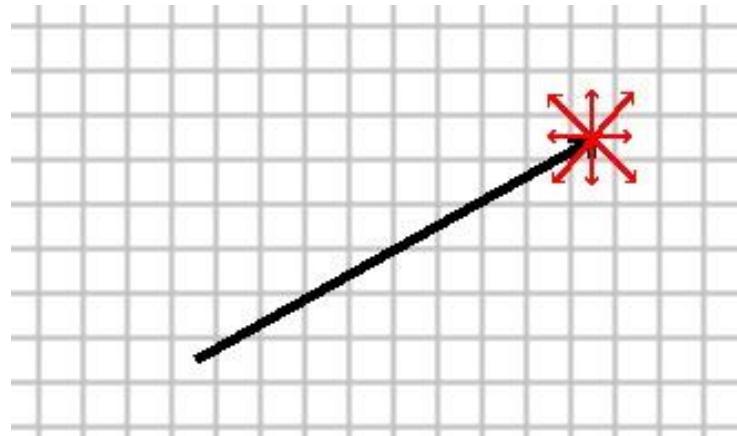


Level l-1

Optical Flow: Top-down Strategy (2)

☀ Step 4:

- adjustment of the vectors of the $l-1$ level in the pyramid
- method: detection of maximum one pixel displacements around the initially approximated position



☀ Step 5: smoothing of the optical flow field (Gaussian filter)

Filtering the Size of the Detected Regions

- ✦ Small isolated regions of motion detected by the optical flow method are classified as **noise** and are eliminated with the help of **morphological operations**:
- ✦ Step 1: Apply the **opening**:
- ✦ Step 2: Apply the **closing**:

Contour Detection

💡 For demonstration purposes, the **contours** of the moving regions detected are outlined

💡 Method: the **Sobel edge detector**:

➤ Compute the intensity gradient: $\nabla f(x, y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right) = (f_x, f_y)$ (5)

using the Sobel masks:

$$G_x = \frac{1}{4} \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}, \quad G_y = \frac{1}{4} \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \quad (6)$$

➤ Compute the magnitude of the gradient:

$$M(x, y) = \|\nabla f(x, y)\| = \sqrt{f_x^2 + f_y^2} \quad (7)$$

➤ **if** $M(x, y) > threshold$ **then** edge pixel
else non-edge pixel

A Block Diagram of the System

