



مدونة المناهج السعودية

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الموقع التعليمي لجميع المراحل الدراسية

في المملكة العربية السعودية

Q2

A) Write (T) for the true statement and write (F) for the false one: (1 mark for each)

1) The horizontal asymptote of $f(x) = \frac{x}{9-x^2}$, has an equation $y = 0$. \longleftrightarrow (T)

2) $\lim_{x \rightarrow 5^-} \left[\frac{3}{2x-10} \right] = \infty$. \longleftrightarrow (F)

3) Let $f(x) = \begin{cases} \frac{x^2-6x-7}{x-7}, & x \neq 7 \\ 2a, & x = 7 \end{cases}$, be continuous at $x = 7$ then $a = 4$. \longleftrightarrow (T)

4) $\frac{d}{dx} \left(\frac{x^2}{2} - \frac{1}{3x^3} \right) \Big|_{x=1} = -2$. \longleftrightarrow (F)

B) Evaluate each of the following limits:

(1) $\lim_{x \rightarrow 0} \left(\frac{x^3-2x^2}{2x} \right) = \frac{0}{0}$ (2 marks)

$$\lim_{x \rightarrow 0} \frac{x^2(x-2)}{2x} = \lim_{x \rightarrow 0} \frac{x(x-2)}{2} = \frac{0}{2} = 0$$

(3) $\lim_{x \rightarrow 5} \frac{x-5}{\sqrt{x+4}-3} = \frac{0}{0}$ (3 marks)

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(2) $\lim_{x \rightarrow 0} \left(\frac{5x+3\sin 3x}{2x} \right) =$ (2 marks)

$$\lim_{x \rightarrow 0} \frac{5x}{2x} + \frac{3 \sin 3x}{2x} = \frac{5}{2} + \frac{9}{2} = \frac{14}{2} = 7$$

C) Discuss the continuity to the function: $f(x) = \begin{cases} 2x-1, & x > 3 \\ x^2-4, & x \leq 3 \end{cases}$, at $x = 3$. (4 marks)

1) $f(3) = 5$

2) $\lim_{x \rightarrow 3^+} (2x-1) = 5$

$\lim_{x \rightarrow 3^-} (x^2-4) = 5$

$\lim_{x \rightarrow 3} f(x) = 5$

B) $f(3) = \lim_{x \rightarrow 3} f(x) = 5$

$f(x)$ is cont at $x = 3$

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Q1 Choose the correct answer (for ten) of the following: (one mark for each)					
(1)	If $\lim_{x \rightarrow a} f(x) = 2$, $\lim_{x \rightarrow a} g(x) = -4$ then $\lim_{x \rightarrow a} [f(x) + (g(x))^2] = \dots$	10			
	(a) 18	(b) -14	(c) -6	(d) 10	
(2)	The derivative of f at $x = a$,written $f'(a)$,is given by ...				
	(a) $\lim_{h \rightarrow 1} \left(\frac{f(a+h)-f(a)}{h} \right)$	(b) $\lim_{h \rightarrow 0} \left(\frac{f(a+h)+f(a)}{h} \right)$	(c) $\lim_{h \rightarrow 0} \left(\frac{f(a+h)-f(a)}{h} \right)$	(d) $\lim_{h \rightarrow 0} \left(\frac{f(h)-f(a)}{h} \right)$	
(3)	The vertical asymptote for the graph of $f(x) = \frac{x+2}{x^2-4}$ is equal to ...				
	(a) $x = -2$	(b) $x = \mp 2$	(c) $x = 2$	(d) $x = 0$	
(4)	The function $f(x) = 2 - x + 1 $ does not satisfy Rolle's theorem on $[-2, 0]$ because ...				
	(a) $-1 \notin \text{domain of } f$	(b) $f(-2) \neq f(0)$	(c) f is not continuous at $x = -1$	(d) f is not differentiable at $x = -1$	
(5)	For $f(x) = (x + 2)^{\frac{2}{3}}$, then $f''(-1) = \dots$				
	(a) $-\frac{2}{3}$	(b) $-\frac{2}{9}$	(c) $\frac{2}{9}$	(d) $\frac{2}{3}$	
(6)	The value of B so that $f(x) = 2x^2 - 3Bx$, has a critical number at $x = 3$ is ...				
	(a) 4	(b) 2	(c) 0	(d) -3	
(7)	Let $f(x)$ be a differentiable function, such that $f'(1) = 4$, then $\frac{d}{dx}(x^2 + f(x)) _{x=1} = \dots$				
	(a) 2	(b) 4	(c) 5	(d) 6	
(8)	The slope of the line tangent to the graph of $y = e^{-2x}$ at $x = 0$ is ...				
	(a) 2	(b) 1	(c) -2	(d) $-2e$	
	Given the graph of f'		<p>$x = -4$ min طالع المضي</p> <p>$x = 5$ max تازل المضي</p>		
(9)	(from the above figure) One of the critical numbers of $f(x)$ is ...	(a) $x = -5$	(b) $x = -2$	(c) $x = 5$	(d) $x = 4$
(10)	(from the above figure) The interval on which $f(x)$ is increasing only is ...	(a) $(-\infty, -2]$	(b) $[1, 5]$	(c) $[-4, 6]$	(d) $[5, \infty)$
(11)	(from the above figure) The x -coordinate on which $f(x)$ has a local minimum is ...	(a) $x = -4$	(b) $x = -3$	(c) $x = 1$	(d) $x = 5$
(12)	(from the above figure) The interval on which $f(x)$ is only concave downward is ...	(a) $(-\infty, -2.5)$	(b) $(-2.5, 4)$	(c) $(1, 4)$	(d) $(3.5, \infty)$

A) Find $\frac{dy}{dx}$ to each of the following functions:

(3 marks for each)

(1) $y = \frac{x}{x-1}$

$$\frac{x \cdot (-1) - (x-1) \cdot 1}{(x-1)^2}$$

(3) $y = \ln\left(\frac{x^4 \sin x}{\sqrt{x+3}}\right)$

$$4 \ln x + \ln \sin x - \frac{1}{2} \ln(x+3)$$

$$\frac{4}{x} + \frac{\cos x}{\sin x} - \frac{1}{2} \left(\frac{1}{x+3} \right)$$

$$\downarrow$$

$$\cot x$$

(2) $y = x^3 \cos x - 3x^2$

B) Find an equation of the tangent line to the curve: $x^2 - 4y^3 = 0$ at the point $(2,1)$. (4 marks)

$$2x - 12y^2 y' = 0$$

$$2x = 12y^2 y'$$

$$\frac{2x}{12y^2} = \frac{12y^2 y'}{12y^2}$$

$$m = \frac{4}{12}$$

$$m = \frac{1}{3}$$

$$y - 1 = \frac{1}{3}(x - 2)$$

C) Find the absolute extrema of $f(t) = t\sqrt{4-t^2}$ on the interval $[-1,2]$.

(2 marks)

$$f'(t) = 0$$

$$t \cdot \frac{-2t}{2\sqrt{4-t^2}} + \sqrt{4-t^2} = 0$$

$$\frac{-t^2}{\sqrt{4-t^2}} + \sqrt{4-t^2} = 0$$

$$\frac{-t^2 + 4 - t^2}{\sqrt{4-t^2}} = 0$$

$$f'(t) = 0$$

$$-2t^2 + 4 = 0$$

$$2t^2 = 4$$

$$t^2 = 2$$

$$t = \sqrt{2} \in (1, 2)$$

$$t = -\sqrt{2} \notin (1, 2)$$

$$f'(t) \text{ d.n.e}$$

$$\sqrt{4-t^2} = 0$$

$$4-t^2 = 0$$

$$4 = t^2$$

$$t = 2 \notin (1, 2)$$

$$t = -2 \notin (1, 2)$$

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$\sqrt{2}$	2	max
-1	$-\sqrt{3}$	min
2	0	min

Q4

A) Let $f(x) = x^3 - 6x^2 + 12x$, find the following:

(1) The interval(s) on which $f(x)$ is increasing and on which f is decreasing. (2 marks)

$$f'(x) = 0$$

$$3x^2 - 12x + 12 = 0$$

(2) Discuss the concavity. (2 marks)

$$f''(x) = 0$$

(3) The inflection point of $f(x)$. (one mark)

B) Show that $f(x) = x^2 - 3x + 1$, satisfies the condition of the mean value theorem on the interval $[1,3]$, and then find the number c , that satisfies the conclusion of the theorem. (3 marks)

1) $f(x)$ is cont on $[1,3]$ poly } $2x - 3 = 1$
 2) $f(x)$ is diff on $(1,3)$ poly }

$$f'(x) = \frac{f(3) - f(1)}{3 - 1}$$

$$2x - 3 = \frac{1 + 1}{3 - 1}$$

$$2x = 4$$

$$x = 2 \in (1,3)$$

$$c = 2$$

C) ~~If $f(x) = (x-5)^2$, then find the local extrema of f , using the second derivative test. (2 marks)~~

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