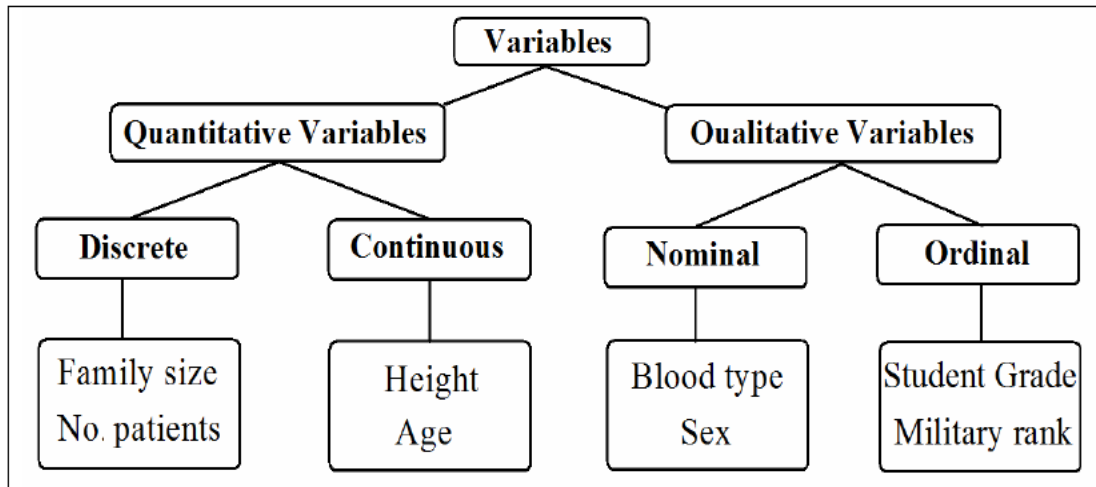
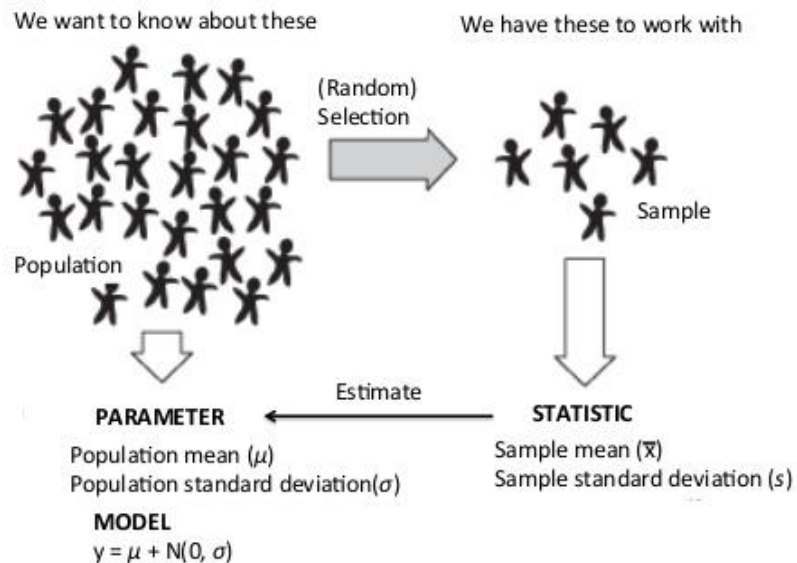


Introduction



Statistical Inference



Question 1:

1. Which of the following is an example of a statistic:

A)	the population variance	B)	<u>the sample median</u>	C)	the population mean	D)	the population mode
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2. Which of the following are examples of measures of dispersion:

A)	the median and the mode	B)	<u>the range and the variance</u>	C)	the parameter and the statistic	D)	the mean and the variance
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3. The number of students admitted in College of Medicine in King Saud University is a variable of type

(A) <u>Discrete</u>	(B) Qualitative	(C) Continuous	(D) nominal
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4. A mean of a population is called

(A) <u>Parameter</u>	(B) statistic	(C) Median	(D) Mode
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5. The measure that obtained from the population is called

(A) **parameter** (B) sample (C) population (D) statistic

6. The measure that obtained from the sample is called

(A) parameter (B) sample (C) population (D) **statistic**

7. A sample is defined as:

A) The entire population of values.

B) A measure of reliability of the population.

C) A subset of data selected from a population.

D) Inferential statistics.

Question 2:

From men with age more than 20 years living in Qaseem, we select 200 men. It was found that the average weight of the men was 76 kg.

1) The variable of interest is:

(A) Age	(B) weight	(C) 200 men	(D) 76 kg
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2) The sample size is:

(A) 76	(B) 20	(C) 200	(D) 1520
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Question 3 :

A study of 250 patients admitted to a hospital during the past year revealed that, on the average (mean), the patients lived 15 miles from the hospital.

1. The sample in the study is

(A) 250 patients	(B) 250 hospitals	(C) 250 houses	(D) 15 miles
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2. The population in this study is

(A) Some patients admitted to the hospital during the past year	(B) all patients admitted to the hospital during the past year	(C) 250 patients admitted to the hospital during the past year	(D) 500 patients admitted to the hospital during the past year
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3. The variable of interest is of type

(A) continuous	(B) discrete	(C) nominal	(D) Qualitative ordinal
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Measures of Central tendency and Dispersion

<i>Measures of central tendency</i>	مقاييس النزعة المركزية
$\bar{X} = \frac{\sum_{i=1}^n X_i}{n}$	Mean المتوسط
	Median الوسيط
تأخذ الأكثر تكرار في المجموعة . وقد لا يوجد لبعضها منوال وقد يوجد أكثر من منوال	Mode المنوال
<i>Measures of dispersions</i>	مقاييس التشتت
المدى = اكبر قيمة - اصغر قيمة	Range المدى
$S^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}$ $S^2 = \frac{\sum_{i=1}^n X_i^2 - n\bar{X}^2}{n-1}$	Variance التباين
$S = \sqrt{S^2}$	Standard deviation الانحراف المعياري
$C.V = \frac{S}{\bar{x}}$ الانحراف / المتوسط	C.V معامل الاختلاف

Question 1:

If the number of visits to the clinic made by 8 pregnant women in their pregnancy period is:

12 15 16 12 15 16 12 14

1. The type of the variable is:

A)	continuous	B)	ordinal	C)	nominal	D)	<u>discrete</u>
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2. The sample mean is:

$$\bar{X} = \frac{12+15+16+12+15+16+12+14}{8} = 14$$

A)	11	B)	<u>14</u>	C)	8	D)	15
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3. The sample standard deviation is:

$$S^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1} = \frac{(12-14)^2 + (15-14)^2 + (16-14)^2 + \dots + (14-14)^2}{8-1} = 3.14 \Rightarrow S = 1.77$$

A)	4.012	B)	-2.450	C)	<u>1.77</u>	D)	2.524
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4. The sample median is:

$$12 \ 12 \ 12 \ \boxed{14 \ 15} \ 15 \ 16 \ 16 \Rightarrow \frac{14+15}{2} = 14.5$$

A)	<u>14.5</u>	B)	15.5	C)	16.5	D)	15
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5. The coefficient of variation is:

$$C.V = \frac{s}{\bar{X}} = \frac{1.77}{14} = 0.1266$$

A)	70 %	B)	2.5 %	C)	28.25 %	D)	<u>12.66 %</u>
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6. The range is:

A)	11	B)	<u>4</u>	C)	6	D)	28
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Question 2:

Consider the following marks for a sample of students carried out on 10 quizzes:

6, 7, 6, 8, 5, 7, 6, 9, 10, 6

Q. 1. The mean mark is:

$$\bar{X} = \frac{6+7+6+8+5+7+6+9+10+6}{10} = 7$$

- A) 7 B) 10 C) 6 D) 7.5

Q. 2. The median mark is:

$$5 \ 6 \ 6 \ 6 \ \boxed{6 \ 7} \ 7 \ 8 \ 9 \ 10 \Rightarrow \frac{6+7}{2} = 6.5$$

- A) 6.5 B) 5.5 C) 7 D) 6

Q. 3. The mode for this data is:

- A) 7 B) 6 C) 0 D) 6 or 7

Q. 4. The range for this data is:

- A) 15 B) 10 C) 5 D) 0

Q. 5. The standard deviation for this data is:

$$S^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1} = \frac{(5-7)^2 + (6-7)^2 + \dots + (10-7)^2}{10-1} = 2.434 \Rightarrow S = 1.56$$

- A) 1.48 B) 2.44 C) 1 D) 1.56

Q. 6. The coefficient of variation for this data is:

$$C.V = \frac{s}{\bar{X}} = \frac{1.56}{7} = 0.223$$

- A) 44.9% B) 22.3% C) 19% D) 47.3%

Question 3:

The data for measurements of the left ischia tuberosity (in mm Hg) for the SCI and control groups are shown below.

Control	131	115	124	131	122
SCI	60	150	130	180	163

1. The mean for the control group is

$$\bar{X} = \frac{131+115+124+131+122}{5} = 124.60$$

(A) 125.10	(B) 128.10	(C) <u>124.60</u>	(D) 127.10
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2. The variance of the SCI group is

$$\bar{X} = \frac{60+150+130+180+163}{5} = 136.6$$

$$S^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1} = \frac{(60-136.6)^2 + (150-136.6)^2 + \dots + (163-136.6)^2}{5-1} = 2167.8$$

(A) 2025.10	(B) <u>2167.8</u>	(C) 2026.10	(D) 2037.10
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3. The unit of coefficient of variation for SCI group is

(A) mm Hg	(B) Hg	(C) mm	(D) <u>Unit-less</u>
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4. Which group has more variation

$$S^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1} = \frac{(131-124.6)^2 + (115-124.6)^2 + \dots + (12-124.6)^2}{5-1} = 45.3 \Rightarrow 6.7305$$

$$C.V_{Control} = \frac{S_{Control}}{\bar{X}_{Control}} = \frac{6.7305}{124.6} = 0.054$$

$$C.V_{SCI} = \frac{S_{SCI}}{\bar{X}_{SCI}} = \frac{\sqrt{2167.8}}{136.6} = 0.3408$$

(A) Control group	(B) <u>SCI group</u>	(C) Both groups have the same variation	(D) Cannot compare between their variations
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Question 4:

Temperature (in Faraheniet) recorded at 2 am in London on 8 days randomly chosen in a year were as follows:

40 -21 38 -9 26 -21 -49 44

1) The average temperature for the sample is:

(A) 248	(B) 1	(C) <u>6</u>	(D) 48
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2) The median temperature for the sample is:

(A) 17	(B) -21	(C) <u>8.5</u>	(D) -8.5
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3) The mode of temperature for the sample is:

(A) <u>-21</u>	(B) 44	(C) 2	(D) -49
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4) The standard deviation for the sample data is:

(A) <u>35.319</u>	(B) 30.904	(C) 1247.43	(D) 4
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5) The coefficient of variation for the sample is:

(A) 49%	(B) 17%	(C) 4%	(D) <u>588.7%</u>
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6) The range of the sample is:

(A) 4	(B) 8	(C) 40	(D) <u>93</u>
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Question 5:

Some families were selected and the number of children in each family were considered as follows: 5, 8, 0, 8, 3, 7, 8, 9

Then,

1) The sample size is:

(A) 9	(B) 6	(C) <u>8</u>	(D) 5
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2) The sample mode is:

(A) 9	(B) 0	(C) <u>8</u>	(D) No mode
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3) The sample mean is:

(A) 48	(B) <u>6</u>	(C) 8	(D) 0
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4) The sample variance is:

(A) 2.915	(B) 8.5	(C) <u>9.714</u>	(D) 3.117
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5) The sample median is:

(A) 5.5	(B) <u>7.5</u>	(C) 8	(D) 7
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6) The range of data is:

(A) 8	(B) 0	(C) 3	(D) <u>9</u>
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7) The sample coefficient of variation is:

(A) 5.5	(B) 8	(C) <u>0.52</u>	(D) 7
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Question 6:

Consider the following weights for a sample of 6 babies: 5, 3, 5, 2, 5, 4

[1] The sample mean is

A	<u>4</u>	B	5	C	3	D	6
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[2] The sample median is

A	4	B	5	C	<u>4.5</u>	D	3
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[3] The sample mode is

A	4	B	3	C	4.5	D	<u>5</u>
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[4] The sample standard deviation is

A	3.2649	B	8.2649	C	<u>1.2649</u>	D	2.2649
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[5] The coefficient of variation for this sample is

A	40.00%	B	<u>31.62%</u>	C	200%	D	12.50%
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Question 7:

1. Which of the following measures is not affected by the extreme values?

(A) <u>Median</u>	(B) Mean	(C) Variance	(D) Range
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2. Which of the following location (central tendency) measures is affected by extreme values?

(A) Range	(B) <u>Mean</u>	(C) Median	(D) Mode
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3. Which of the following measures can be used for the blood type in a given sample?

(A) Median	(B) Mean	(C) Variance	(D) <u>Mode</u>
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Question 8:

1.	The biggest advantage of the standard deviation over the variance is:							
	(A)	The standard deviation is always greater than the variance.						
	(B)	The standard deviation is calculated with the median instead of the mean.						
	(C)	The standard deviation is better for describing the qualitative data.						
	<u>(D)</u>	The standard deviation has the same units as the original data.						
2.	Parameters and statistics:							
	(A)	Describe the same group of individuals.						
	<u>(B)</u>	Describe the population and the sample, respectively.						
	(C)	Describe the sample and the population, respectively.						
	(D)	None of these.						
3.	Which of the following location (central tendency) measures is affected by extreme values?							
	(A)	Median						
	<u>(B)</u>	Mean						
	(C)	Variance						
	(D)	Range						
4.	Which of the following measures can be used for the blood type in a given sample?							
	<u>(A)</u>	Mode						
	(B)	Mean						
	(C)	Variance						
	(D)	Range						
5.	If x_1, x_2 and x_3 has mean $\bar{x} = 4$, then x_1, x_2, x_3 and $x_4 = 4$ has mean:							
	(A)	<u>equal 4</u>	(B)	less than 4	(C)	greater than 4	(D)	None of this

Question 9:

1. The sample mean is a measure of

- A) Relative position.
- B) Central tendency.
- C) Dispersion.
- D) all of the above.

2. The sample standard deviation is a measure of

- A) Relative position.
- B) Central tendency.
- C) Dispersion.
- D) all of the above.

Question 10:

The “life” of 40 similar car batteries recorded to the nearest tenth of a year. The batteries are guaranteed to last 3 years.

Class Interval	True class Interval	Midpoint	Frequency	Relative Frequency
1.5–1.9	1.45–1.95	1.72	2	0.050
2.0–2.4	1.95–2.45	2.2	D	0.025
2.5–2.9	2.45–2.95	C	4	F
A	2.95–3.45	3.2	15	0.375
3.5–3.9	B	3.7	E	0.250
4.0–4.4	3.95–4.45	4.2	5	0.125
4.5–4.9	4.45–4.95	4.7	3	0.075

1. The value of A:

A)	2.45–2.95	B)	3.5–4.9	C)	<u>3.0–3.4</u>	D)	3.55–3.95
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2. The value of B:

A)	40.5 – 50.5	B)	<u>3.45–3.95</u>	C)	54 - 64	D)	44.5 – 54.5
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3. The value of C: $C = \frac{2.45+2.95}{2} = 2.7$

A)	<u>2.7</u>	B)	28.5	C)	29	D)	59
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4. The value of D: $\frac{D}{40} = 0.025 \Rightarrow D = 40 \times 0.025 = 1$

A)	2	B)	4	C)	3	D)	<u>1</u>
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5. The value of E : $\frac{E}{40} = 0.25 \Rightarrow E = 40 \times 0.25 = 10$

A)	0	B)	<u>10</u>	C)	12	D)	11
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6. The value of F: $F = \frac{4}{40} = 0.10$

A)	<u>0.10</u>	B)	0.15	C)	0.35	D)	0.25
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Question 11:

Fill in the table given below. Answer the following questions.

<i>Class Interval</i>	<i>Frequency</i>	<i>Cumulative Frequency</i>	<i>Relative Frequency</i>	<i>Cumulative Relative Frequency</i>
5 - 9	8			
10 - 14	15		C	
15 - 19	11	B		D
20 - 24	A	40	0.15	

1) The value of A is: $A = 40 - (8 + 15 + 11) = 40 - 34 = 6$

(A) <u>6</u>	(B) 4	(C) 34	(D) 40
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2) The value of B is: $B = 8 + 15 + 11 = 34$

(A) 40	(B) <u>34</u>	(C) 0.85	(D) 0.275
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3) The value of C is: $C = \frac{15}{40} = 0.375$

(A) 23	(B) 0.575	(C) 0.275	(D) <u>0.375</u>
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4) The value of D is: $D = \frac{34}{40} = 0.85$

(A) 0.375	(B) 34	(C) 0.8	(D) <u>0.85</u>
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5) The true class interval for the first class is:

(A) 5 - 9	(B) 5 - 10	(C) <u>4.5 - 9.5</u>	(D) 5.5 - 9.5
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6) The number of observations less than 19.5 is:

$$8 + 15 + 11 = 34$$

(A) <u>34</u>	(B) 85	(C) 1	(D) 6
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Question 12:

The following table gives the age distribution for the number of deaths in New York State due to accidents for residents age 25 and older.

Age (Years)	Number of Deaths	Cumulative Frequency	True Class Interval	Relative Frequency	Cumulative Relative Frequency	Mid-Point
25 – 34	393	393	24.5 – 34.5	0.1188	0.1188	29.5
35 – 44	514	907	34.5 – 44.5	0.1554	0.2742	39.5
45 – 54	B	1367	-----	0.1104	0.3882	49.5
55 – 64	341	1708	54.5 – 64.5	0.1031	0.4913	59.5
65 – 74	A	2073	64.5 – 74.5	0.1391	C	69.5
75 – 84	616	2689	-----	0.1863	0.8167	79.5
85 – 94	618	3307	-----	0.1869	1.0000	89.5
Total	-----		-----	-----		

1. The value of A is

(A) <u>365</u>	(B) 341	(C) 514	(D) 616
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2. The value of B is

(A) <u>460</u>	(B) 441	(C) 414	(D) 406
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3. The true class interval of the highest frequency is

(A) 74.5 – 84.5	(B) 44.5 – 54.5	(C) 64.5 – 74.5	(D) <u>84.5 – 94.5</u>
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4. The value of the gap (jump) between (non-true) class intervals is

(A) No gaps	(B) 0.5	(C) 0	(D) <u>1</u>
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5. The true class interval that has the lowest relative frequency is

(A) 25 – 34	(B) 45 – 54	(C) 55 – 64	(D) <u>54.5 – 64.5</u>
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6. The value of C is

(A) 0.6101	(B) 0.6130	(C) <u>0.6304</u>	(D) 0.6011
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Question 13:

The table shows the weight loss (kg) of a sample of 40 healthy adults who fasted in Ramadan.

Class interval	Frequency	Cumulative Frequency
1.20 - 1.29	2	2
1.30 - 1.39	6	8
1.40 - 1.49	10	K
1.50 - 1.59	C	34
1.60 - 1.69	6	40

- 1) The value of the missing value K is
(a) 0 (b) 18 (c) 2 (d) 10 (e) None is correct
- 2) The value of the missing value C is
(a) 20 (b) 40 (c) 10 (d) 16 (e) None is correct

Question 14:

Consider the following frequency polygon of ages of 20 students in a certain school.



The frequency distribution of ages corresponding to above polygon is

(a)

True class limits	4.5- 6.5	6.5-8.5	8.5- 10.5	10.5 -12.5
frequency	2	5	8	5

(b)

True class limits	3.5- 5.5	5.5-7.5	7.5- 9.5	9.5 -11.5	11.5- 13.5
frequency	2	5	8	4	1

(c)

Class interval	5- 6	7-8	9- 10	11 -12
frequency	1	7	8	4

(d)

Class interval	5- 6	7-8	9- 10	11 -12
frequency	4	7	8	6

Question 15:

The following table gives the distribution of the ages of a sample of 50 patients who attend a dental clinic.

Age intervals (in years)	Frequency	Relative frequency	Less than	Cumulative Frequency
10 - 15	4	-	10	0
16 - 21	8	-	16	4
22 - 27	z	0.32	22	y
28 - 33	-	-	28	--
34 - 39	10	-	34	--
			40	x

1.	The class width is:							
	(A)	<u>6</u>	(B)	10	(C)	150	(D)	19
2.	The value of x is:							
	(A)	22	(B)	28	(C)	<u>50</u>	(D)	10
3.	The value of y is:							
	(A)	4	(B)	<u>12</u>	(C)	19	(D)	150
4.	The value of z is:							
	(A)	14	(B)	12	(C)	50	(D)	<u>16</u>
5.	Percent of the patients with age between 16 and 21 is:							
	(A)	<u>16%</u>	(B)	8%	(C)	20%	(D)	32%
6.	The 5 th interval midpoint is:							
	(A)	38	(B)	52	(C)	27	(D)	<u>36.5</u>

Question 16:

Consider the following Table showing a frequency distribution of weights in a sample of 20 cans of fruits:

<i>Class interval</i>	<i>True Class Limits</i>	<i>Midpoint</i>	<i>Frequency</i>	<i>Relative Frequency</i>	<i>Cumulative Frequency</i>
19.2 – 19.4			1		
19.5 – 19.7				0.10	
19.8 – 20.0			8		
			4		

1. The fifth class interval is:

- A) 20.2 - 20.4 B) 20.1-20.3 C) 21.0 - 21.2 **D) 20.4 - 20.6**

2. The second true class interval is

- A) 19.45 - 19.75** B) 19.5 – 19.7 C) 19.25 - 19.35 D) 20.2 - 20.4

3. The midpoint of the fourth class interval is:

- A) 20.5 **B) 20.2** C) 19.9 D) 20.1

4. The frequency of the second class interval is:

- A) 10 B) 4 **C) 2** D) 3

5. The relative frequency of the fourth class interval is:

- A) 0.20** B) 0.15 C) 0.13 D) 0.40

6. The cumulative frequency of the final class interval is:

- A) 13 B) 4 **C) 20** D) 100

Question 17:

Consider the following table showing a frequency distribution of blood test of 52 diabetes patients.

Class interval	Frequency	Cumulative frequency	Relative frequency	Cumulative relative frequency
101 – 120	--	--	0.4423	--
121 – 140	--	--	--	D
B	--	C	0.2115	--
161 – 180	--	--	0.0577	--
Total	A	--	1	--

[1] The value of A is

A	1	B	3	C	<u>52</u>	D	80
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[2] The class interval B is

A	122-140	B	161-180	C	131-140	D	<u>141-160</u>
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[3] The value of C is

A	<u>49</u>	B	15	C	34	D	52
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[4] The value of D is

A	0.5308	B	<u>0.7308</u>	C	0.4308	D	0.8308
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[5] The true class intervals are

A	100 – 120	B	99.5 – 119.5	C	100.5 – 120.5	D	100.5 – 120.5
	120.5 – 139.5		120.5 – 140.5		120.5 – 140.5		121.5 – 140.5
	141 – 160		140.5 – 159.5		140.5 – 160.5		141.5 – 160.5
	161 – 180		160.5 – 179.5		160.5 – 180.5		161.5 – 180.5

[6] The midpoint of the first class interval is

A	<u>110.5</u>	B	20	C	220	D	19
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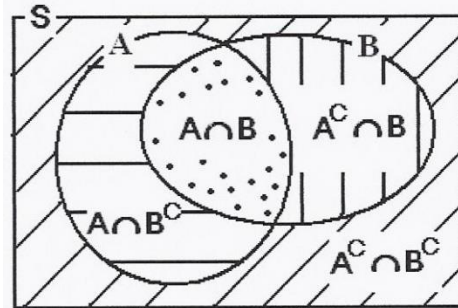
[7] Histogram of the frequency distribution is built based on

A	Frequency and cumulative relative frequency	B	Midpoints and cumulative relative frequency	C	True class interval and frequency	D	None of them
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Probability

Definitions and Theorems:

- * $0 \leq P(A) \leq 1$
- * $P(S) = 1$
- * $P(\emptyset) = 0$



- 1- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- 2- $P(A|B) = P(A \cap B)/P(B)$
- 3- $P(A \cap B) = P(A) \times P(B)$ (if A & B are independent.)
- 4- $P(A \cap B) = 0$ (if A & B are disjoint.)
- 5- $P(A^c) = 1 - P(A)$; $P(A^c) = P(\bar{A})$

Question 1:

Suppose that we have: $P(A) = 0.4$, $P(B) = 0.5$, $P(A \cap B) = 0.2$

1. The probability $P(A \cup B)$ is: $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.5 - 0.2 = 0.7$

A) <u>0.7</u>	B) 0.4	C) 0.5	D) 0
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2. The probability $P(A \cap B^c)$ is: $P(A \cap B^c) = P(A) - P(A \cap B) = 0.4 - 0.2 = 0.2$

A) 0.51	B) <u>0.20</u>	C) 0.40	D) 0.60
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3. The probability $P(A|B)$ is: $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.5} = 0.4$

A) 0.51	B) <u>0.40</u>	C) 0.20	D) 0.30
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4. The events A and B are: $P(A \cap B) \stackrel{?}{=} P(A) \times P(B) \Rightarrow 0.2 = 0.4 \times 0.5$

A) disjoint	B) dependent	C) equal	D) <u>Independent</u>
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Question 2:

If the events A, B we have: $P(A) = 0.2, P(B) = 0.5$ and $P(A \cap B) = 0.1$, then:

1) The events A, B are :

$$P(A \cap B) \stackrel{?}{=} P(A) \times P(B) \Rightarrow 0.1 = 0.2 \times 0.5$$

(a) Dependents (b) both are empties (c) Disjoints (d) Independents

2) The probability of A or B is:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.2 + 0.5 - 0.1 = 0.6$$

(a) 0.5 (b) 5.0 (c) 0.2 (d) 0.6 (e) None is correct

3) If $P(A) = 0.3, P(B) = 0.4$ and that A and B are disjoint, then $P(A \cup B) =$

$$P(A \cup B) = P(A) + P(B) - 0 = 0.3 + 0.4 - 0 = 0.7$$

(a) 0.7 (b) 0.12 (c) 0.6 (d) 0.1 (e) None

4) If $P(A) = 0.2$ and $P(B|A) = 0.4$, then $P(A \cap B) =$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \Rightarrow 0.4 = \frac{P(A \cap B)}{0.2} \Rightarrow P(A \cap B) = 0.2 \times 0.4 = 0.08$$

(a) 0.6 (b) 0.08 (c) 0.5 (d) 2.0 (e) None is correct

5) Suppose that the probability a patient smokes is 0.20. If the probability that the patient smokes and has a lung cancer is 0.15, then the probability that the patient has a lung cancer given that the patient smokes is

$$P(S) = 0.20 \quad P(S \cap C) = 0.15 \quad P(C|S) = ?$$
$$P(C|S) = \frac{P(S \cap C)}{P(S)} = \frac{0.15}{0.20} = 0.75$$

(a) 0.25 (b) 0.2 (c) 0.75 (d) 1.33 (e) None is correct

Question 3:

Suppose that we have two events A and B such that,

$$P(A) = 0.4, P(B) = 0.5, P(A \cap B) = 0.2.$$

[1] $P(A \cup B)$: $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.5 - 0.2 = 0.7$

A	0.1	B	0.9	C	0.7	D	0.8
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[2] $P(A^c \cap B)$: $P(A^c \cap B) = P(B) - P(A \cap B) = 0.5 - 0.2 = 0.3$

A	0.3	B	0.2	C	0.5	D	0.7
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[3] $P(A^c \cap B^c)$: $P(A^c \cap B^c) = 1 - P(A \cup B) = 1 - 0.7 = 0.3$

A	0.7	B	0.2	C	0.5	D	0.3
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[4] $P(A^c)$: $P(A^c) = 1 - P(A) = 1 - 0.4 = 0.6$

A	0.4	B	0.6	C	0.1	D	0.9
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[5] $P(A^c | B)$: $P(A^c | B) = \frac{P(A^c \cap B)}{P(B)} = \frac{0.3}{0.5} = 0.6$

A	0.4	B	0.6	C	0.1	D	0.9
----------	-----	----------	-----	----------	-----	----------	-----

[6] $P(B | A)$: $P(B | A) = \frac{P(A \cap B)}{P(A)} = \frac{0.2}{0.4} = 0.5$

A	0.4	B	0.6	C	0.3	D	0.5
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[7] The events A and B are ... $P(A \cap B) \stackrel{?}{=} P(A) \times P(B) \Rightarrow 0.2 = 0.4 \times 0.5$

A	Exhaustive	B	Disjoint	C	Independent	D	Dependent
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Question 4:

Following table shows 80 patients classified by sex and blood group.

Sex	Blood Group		
	A	B	O
Male (M)	25	17	15
Female (F)	11	9	3

1) The probability that a patient selected randomly is a male and has blood group A is

- (a) $25/36$ (b) $25/57$ (c) $25/80$ (d) $52/80$ (e) None

2) The probability that a patient selected randomly is a female is

- (a) $6/80$ (b) $40/80$ (c) $22/80$ (d) $23/80$ (e) None

3) In a certain population, 4% have cancer, 20% are smokers and 2% are both smokers and have cancer. If a person is chosen at random from the population, find the probability that the person chosen is a smoker or has cancer.

$P(C) = 0.04$ $P(S) = 0.20$ $P(S \cap C) = 0.02$ $P(S \cup C) = ?$ $P(S \cup C) = P(C) + P(S) - P(S \cap C)$ $P(S \cup C) = 0.04 + 0.20 - 0.02 = 0.22$
--

- (a) 0.02 (b) 0.24 (c) 0.2 (d) 0.22 (e) None

Question 5:

Gender	Diabetics (D)	Not Diabetic (D^c)	TOTAL
Male (M)	72	288	360
Female (F)	48	192	240
TOTAL	120	480	600

Consider the information given in the table above. A person is selected randomly

1. The probability that the person found is male and diabetic is:

$$P(M \cap D) = \frac{72}{600} = 0.12$$

(A) 72	(B) 0.12	(C) 0.60	(D) 0.67
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2. The probability that the person found is male or diabetic is:

$$P(M \cup D) = P(M) + P(D) - P(M \cap D) = \frac{360}{600} + \frac{120}{600} - \frac{72}{600} = \frac{408}{600}$$

(A) 0.12	(B) 0.68	(C) 0.60	(D) 0.97
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3. The probability that the person found is female is:

$$P(F) = \frac{240}{600} = 0.4$$

(A) 0.24	(B) 0.12	(C) 0.40	(D) 0.5
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4. Suppose we know the person found is a male, the probability that he is diabetic, is:

$$P(D|M) = \frac{P(M \cap D)}{P(M)} = \frac{72/600}{360/600} = \frac{72}{360} = 0.2$$

(A) 0.2	(B) 0.12	(C) 0.40	(D) 0.68
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5. The events M and D are:

$$P(M \cap D) = P(M) \times P(D) \Rightarrow \frac{72}{600} = \frac{360}{600} \times \frac{120}{600}$$

(A) Disjoint	(B) Independent	(C) mutually exclusive	(D) Dependent
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Question 6:

A group of people is classified by the amount of fruits eaten and the health status:

<i>Fruits Eaten</i> <i>Health Status</i>	<i>Few</i> <i>(F)</i>	<i>Some</i> <i>(S)</i>	<i>Many</i> <i>(M)</i>	<i>Total</i>
<i>Poor (B)</i>	80	35	20	135
<i>Good (G)</i>	25	110	45	180
<i>Excellent (E)</i>	15	95	75	185
<i>Total</i>	120	240	140	500

If one of these people is randomly chosen give:

5. The event “(eats few fruits) and (has good health)“, is defined as.

A)	$F \cup G^c$	B)	$F \cap G$	C)	$F \cup E$	D)	$S \cup E$
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6. $P(B \cup M) =$

A)	<u>0.51</u>	B)	0.028	C)	0.27	D)	0.04
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7. $P(G \cap S) =$

A)	0.48	B)	0.36	C)	<u>0.22</u>	D)	0.62
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8. $P(E^c) =$

A)	<u>0.63</u>	B)	0.37	C)	0.50	D)	1
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9. $P(G | S) =$

A)	0.6111	B)	0.2200	C)	<u>0.4583</u>	D)	0.36
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10. $P(M | E) =$

A)	0.6111	B)	0.2200	C)	<u>0.405</u>	D)	0.36
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Question 7:

The following table classifies a sample of individuals according to gender and period (in years) attendance in the college:

College Attended	Gender		
	Male	Female	Total
None	12	41	53
Two Years	14	63	77
Three Years	9	49	58
Four Years	7	50	57
Total	42	203	245

Suppose we select an individual at random, then:

1.	The probability that the individual is male is:							
	(A)	0.8286	(B)	<u>0.1714</u>	(C)	0.0490	(D)	0.2857
2.	The probability that the individual did not attend college (None) and female is:							
	(A)	0.0241	(B)	0.0490	(C)	<u>0.1673</u>	(D)	0.2163
3.	The probability that the individual has three year or two year college attendance is:							
	(A)	<u>0.551</u>	(B)	0.0939	(C)	0.4571	(D)	0
4.	If we pick an individual at random and found that he had three year college attendance, the probability that the individual is male is:							
	(A)	0.0367	(B)	0.2143	(C)	<u>0.1552</u>	(D)	0.1714
5.	The probability that the individual is not a four year college attendance is:							
	(A)	<u>0.7673</u>	(B)	0.2327	(C)	0.0286	(D)	0.1429
6.	The probability that the individual is a two year college attendance or male is:							
	(A)	0.0571	(B)	0.8858	(C)	0.2571	(D)	<u>0.4286</u>
7.	The events: the individual is a four year college attendance and male are:							
	(A)	Mutually exclusive	(B)	Independent	(C)	<u>Dependent</u>	(D)	None of these

Question 8:

	<i>Blood pressure</i>		
	<i>Low (L)</i>	<i>Medium (M)</i>	<i>High (H)</i>
<i>Has obesity (B)</i>	50	150	300
<i>Does not have obesity (\bar{B})</i>	250	240	110

If an individual is selected at random from this group, then the probability that he/she

1. has obesity or has medium blood pressure is equal to

- A) 0.442 B) 0.50 C) 0.725 **D) 0.673**

2. has low blood pressure given that he/she has obesity is equal to

- A) 0.90 **B) 0.1** C) 0.66 D) 0.44

Bayes' Theorem, Screening Tests, Sensitivity, Specificity, and Predictive Value Positive and Negative

Test Result	Disease		Total
	Present (D)	Absent (\bar{D})	
Positive (T)	a	b	a + b = n(T)
Negative (\bar{T})	c	d	c + d = n(\bar{T})
Total	a + c = n(D)	b + d = n(\bar{D})	n

1. The probability of false positive result:

$$P(T | \bar{D}) = \frac{n(T \cap \bar{D})}{n(\bar{D})} = \frac{b}{b+d}$$

2. The probability of false negative result:

$$P(\bar{T} | D) = \frac{n(\bar{T} \cap D)}{n(D)} = \frac{c}{a+c}$$

3. The sensitivity of the screening test:

$$P(T | D) = \frac{n(T \cap D)}{n(D)} = \frac{a}{a+c}$$

4. The specificity of the screening test:

$$P(\bar{T} | \bar{D}) = \frac{n(\bar{T} \cap \bar{D})}{n(\bar{D})} = \frac{d}{b+d}$$

• **The predictive value positive:**

$$\begin{aligned} P(D|T) &= \frac{P(D \cap T)}{P(T)} = \frac{P(T|D)P(D)}{P(T|D)P(D) + P(T|\bar{D})P(\bar{D})} \\ &= \frac{(\text{sensitivity}) P(D_{\text{given}})}{(\text{sensitivity}) P(D_{\text{given}}) + (\text{false + ve})P(\bar{D}_{\text{given}})} \end{aligned}$$

• **The predictive value negative:**

$$\begin{aligned} P(\bar{D}|\bar{T}) &= \frac{P(\bar{T} \cap \bar{D})}{P(\bar{T})} = \frac{P(\bar{T} \cap \bar{D})P(\bar{D})}{P(\bar{T} \cap \bar{D})P(\bar{D}) + P(\bar{T}|D)P(D)} \\ &= \frac{(\text{specificity}) P(\bar{D}_{\text{given}})}{(\text{specificity}) P(\bar{D}_{\text{given}}) + (\text{false - ve})P(D_{\text{given}})} \end{aligned}$$

Question 1:

The following table shows the results of a screening test:

	Disease confirmed (D)	Disease not confirmed (\bar{D})
Positive test (T)	38	10
Negative test (\bar{T})	5	18

1. The probability of false positive of the test is: $\frac{10}{28} = 0.3571$

A)	<u>0.3571</u>	B)	0.2083	C)	0.7916	D)	0.2173
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2. The probability of false negative of the test is: $\frac{5}{43} = 0.1163$

A)	0.3571	B)	0.7826	C)	0.2173	D)	<u>0.1163</u>
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3. The sensitivity value of the test is: $\frac{38}{43} = 0.8837$

A)	0.2173	B)	<u>0.8837</u>	C)	0.6429	D)	0.3571
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4. The specificity value of the test is: $\frac{18}{28} = 0.6429$

A)	<u>0.6429</u>	B)	0.3571	C)	0.2173	D)	0.2535
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Suppose it is known that the rate of the disease is 0.113,

5. The predictive value positive of a symptom is: $1 - 0.113 = 0.887$

$$= \frac{(\text{sensitivity}) P(D_{\text{given}})}{(\text{sensitivity}) P(D_{\text{given}}) + (\text{false +ve}) P(\bar{D}_{\text{given}})} = \frac{0.8837 \times 0.113}{0.8837 \times 0.113 + 0.3571 \times 0.887} = 0.2397$$

A)	0.9797	B)	0.5714	C)	<u>0.2397</u>	D)	0.34591
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6. The predictive value negative of a symptom is:

$$= \frac{(\text{specificity}) P(\bar{D}_{\text{given}})}{(\text{specificity}) P(\bar{D}_{\text{given}}) + (\text{false -ve}) P(D_{\text{given}})} = \frac{0.6429 \times 0.887}{0.6429 \times 0.887 + 0.1163 \times 0.113} = 0.9772$$

A)	<u>0.9775</u>	B)	0.5714	C)	0.2397	D)	0.34591
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Question 2:

It is known that 40% of the population is diabetic. 330 persons who were diabetics went through a test where the test confirmed the disease for 288 persons. Among 270 healthy persons, test showed high sugar level for 22 persons. The information obtained is given in the table below.

Test	Diabetics (D)	Not Diabetic (D ^c)	TOTAL
Positive (T)	288	72	360
Negative (\bar{T})	42	198	240
TOTAL	330	270	600

1. The sensitivity of the test is: $\frac{288}{330} = 0.873$

- (A) **0.873** (B) 0.480 (C) 0.733 (D) 0.33

2. The specificity of the test is: $\frac{198}{270} = 0.733$

- (A) 0.873 (B) 0.330 (C) 0.48 (D) **0.733**

3. The probability of false positive is: $\frac{72}{270} = 0.267$

- (A) 0.1549 (B) 0.127 (C) 0.713 (D) **0.267**

4. The predictive probability positive for the disease is:

$$= \frac{(\text{sensitivity}) P(D_{\text{given}})}{(\text{sensitivity}) P(D_{\text{given}}) + (\text{false +ve}) P(\bar{D}_{\text{given}})} = \frac{0.873 \times 0.40}{0.873 \times 0.40 + 0.267 \times 0.60} = 0.686$$

- (A) **0.686** (B) 0.800 (C) 0.480 (D) 0.873

Question 3:

The following table shows the results of a screening test evaluation in which a random sample of 700 subjects with the disease and an independent random sample of 1300 subjects without the disease participated:

<i>Disease</i>	<i>Present</i>	<i>Absent</i>
<i>Test result</i>		
<i>Positive</i>	500	100
<i>Negative</i>	200	1200

1) *The sensitivity value of the test is:* $\frac{500}{700} = 0.7143$

- (A) 0.2649 (B) **0.7143** (C) 0.7538 (D) 0.923

2) *The specificity value of the test is:* $\frac{1200}{1300} = 0.923$

- (A) 0.1 (B) 0.7143 (C) 0.9943 (D) **0.923**

3) *The probability of false positive of the test is:* $\frac{100}{1300} = 0.0769$

- (A) 0.0583 (B) 0.2462 (C) **0.0769** (D) 0.2649

4) *If the rate of the disease in the general population is 0.002, then the predictive value positive of the test is:*

$$\begin{aligned} &= \frac{(\text{sensitivity}) P(D_{\text{given}})}{(\text{sensitivity}) P(D_{\text{given}}) + (\text{false +ve}) P(\bar{D}_{\text{given}})} \\ &= \frac{0.7143 \times 0.002}{0.7143 \times 0.002 + 0.0769 \times 0.998} = 0.01827 \end{aligned}$$

- (A) 0.9748 (B) **0.01827** (C) 0.002 (D) 0.0252

Question 4:

In a study of high blood pressure, 188 persons found positive, of a sample of 200 persons with the disease subjected to a screening test. While, 27 persons found positive, of an independent sample of 300 persons without the disease subjected to the same screening test. That is,

	High Blood Pressure		
Test Result	Yes D	No \bar{D}	Total
Positive T	188	27	215
Negative \bar{T}	12	273	285
Total	200	300	500

[1] Given that a person has the disease, the probability of a positive test result, that is, the "sensitivity" of this test is:

A	0.49	B	0.94	C	0.35	D	0.55
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[2] Given that a person does not have the disease, the probability of a negative test result, that is, the "specificity" of this test is:

A	0.91	B	0.75	C	0.63	D	0.49
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[3] The "false negative" results when a test indicates a negative status given that the true status is positive is:

A	0.01	B	0.15	C	0.21	D	0.06
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[4] The "false positive" results when a test indicates a positive status given that the true status is negative is:

A	0.16	B	0.31	C	0.09	D	0.02
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Assuming that 15% of the population under study is known to be with high blood pressure.

[5] Given a positive screening test, what is the probability that the person has the disease? That is, the "predictive value positive" is:

A	0.22	B	0.65	C	0.93	D	0.70
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[6] Given a negative screening test result, what is the probability that the person does not have the disease? That is, the "predictive value negative" is:

A	0.258	B	0.778	C	0.988	D	0.338
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Question 5:

Suppose that the ministry of health intends to check the reliability of the central Diabetic Lab in Riyadh. A sample person with Diabetic disease (D) and another without the disease (\bar{D}) had the Lab tests and the results are given below:

	Present (D)	Absence (\bar{D})
Positive (T)	950	40
Negative (\bar{T})	25	640

Then:

1.	The probability of false negative result is:							
	(A)	0.0256	(B)	0.9412	(C)	0.9744	(D)	0.0588
2.	The probability of false positive result is:							
	(A)	0.0256	(B)	0.9412	(C)	0.9744	(D)	0.0588
3.	The sensitivity of the test is:							
	(A)	0.0256	(B)	0.9412	(C)	0.9744	(D)	0.0588
4.	The specificity of the test is:							
	(A)	0.0256	(B)	0.9412	(C)	0.9744	(D)	0.0588

Assume that the true percentage of Diabetic patients in Riyadh is 25%.

Then

5.	The predictive value positive of the test is:							
	(A)	0.847	(B)	0.924	(C)	0.991	(D)	0.695
6.	The predictive value negative of the test is:							
	(A)	0.195	(B)	0.982	(C)	0.847	(D)	0.991

Question 6:

A Fecal Occult Blood Screen Outcome Test is applied for 875 patients with bowel cancer. The same test was applied for another sample of 925 without bowel cancer. Obtained results are shown in the following table:

	<i>Present Disease (D)</i>	<i>Absent Disease (\bar{D})</i>
<i>Test Positive (T)</i>	850	10
<i>Test Negative (\bar{T})</i>	25	915

1. *The sensitivity of the test is*

- A) 0.85 **B) 0.971** C) 0.915 D) 0.988

2. *The specificity of the test is*

- A) 0.850 B) 0.250 C) 0.915 **D) 0.989**

3. *The probability of false positive is*

- A) 0.989 **B) 0.011** C) 0.250 D) 0.915

4. *The probability of false negative is*

- A) 0.250 B) 0.971 **C) 0.029** D) 0.10

5. *If the rate of the disease in the general population is equal to 15% then the predictive value positive of the test is*

- A) 0.941** B) 0.995 C) 0.674 D) 0.150

More Exercises

Question 1:

Givens:

$$P(A) = 0.5, \quad P(B) = 0.4, \quad P(C \cap A^c) = 0.6, \\ P(C \cap A) = 0.2, \quad P(A \cup B) = 0.9$$

(a) *What is the probability of $P(C)$:*

$$P(C) = P(C \cap A^c) + P(C \cap A) = 0.6 + 0.2 = 0.8$$

(b) *What is the probability of $P(A \cap B)$:*

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \\ \Rightarrow 0.9 = 0.5 + 0.4 - P(A \cap B) \\ P(A \cap B) = 0$$

(c) *What is the probability of $P(C | A)$:*

$$P(C | A) = \frac{P(C \cap A)}{P(A)} = \frac{0.2}{0.5} = 0.4$$

(d) *What is the probability of $P(B^c \cap A^c)$:*

$$P(B^c \cap A^c) = 1 - P(B \cup A) = 1 - 0.9 = 0.1$$

Question 2:

Givens:

$$P(B) = 0.3, \quad P(A | B) = 0.4$$

Then find $P(A \cap B) = ?$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \\ \Rightarrow 0.4 = \frac{P(A \cap B)}{0.3} \\ \Rightarrow P(A \cap B) = 0.4 \times 0.3 = 0.12$$

Question 3:

Givens:

$$P(A) = 0.3, \quad P(B) = 0.4, \quad P(A \cap B \cap C) = 0.03, \quad P(\overline{A \cap B}) = 0.88$$

(1) Are the event A and b independent?

$$P(A \cap B) = 1 - P(\overline{A \cap B}) = 1 - 0.88 = 0.12$$

$$P(A) \times P(B) = 0.3 \times 0.4 = 0.12$$

$$\Rightarrow P(A \cap B) = P(A) \times P(B)$$

Therefore, A and B are independent.

(2) What is the probability of $P(C | A \cap B)$:

$$P(C | A \cap B) = \frac{P(A \cap B \cap C)}{P(A \cap B)} = \frac{0.03}{0.12} = 0.25$$

Question 4:

Givens:

$$P(A_1) = 0.4, \quad P(A_1 \cap A_2) = 0.2, \quad P(A_3 | A_1 \cap A_2) = 0.75$$

(1) Find the $P(A_2|A_1)$:

$$P(A_2|A_1) = \frac{P(A_1 \cap A_2)}{P(A_1)} = \frac{0.2}{0.4} = 0.5$$

(2) Find the $P(A_1 \cap A_2 \cap A_3)$:

$$P(A_3 | A_1 \cap A_2) = \frac{P(A_1 \cap A_2 \cap A_3)}{P(A_1 \cap A_2)}$$

$$0.75 = \frac{P(A_1 \cap A_2 \cap A_3)}{0.2}$$

$$P(A_1 \cap A_2 \cap A_3) = 0.75 \times 0.2 = 0.15$$

Random Variables:

- $0 \leq P(X = x) \leq 1$
- $\sum P(X = x) = 1$
- $E(X) = \sum x P(X = x)$

Question 1:

Given the following discrete distribution:

x	-1	0	1	2	3	4
$P(X=x)$	0.15	0.30	M	0.15	0.10	0.10

1. The value of M is equal to

$$M = 1 - (0.15 + 0.30 + 0.15 + 0.10 + 0.10) = 1 - 0.80 = 0.20$$

(A) <u>0.20</u>	(B) 0.0	(C) 0.10	(D) 0.25
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2. $P(X \leq 0.5) =$

$$0.15 + 0.30 = 0.45$$

(A) 0.0	(B) 0.50	(C) <u>0.45</u>	(D) 1.0
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3. $P(X=0) =$

(A) 0	(B) <u>0.30</u>	(C) 0.80	(D) 1.0
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4. The expected (mean) value $E[X]$ is equal to

$$E(X) = (-1 \times 0.15) + (0 \times 0.30) + (1 \times 0.20) + (2 \times 0.15) + (3 \times 0.10) + (4 \times 0.10) = 1.05$$

(A) 0.0	(B) 1.35	(C) <u>1.05</u>	(D) 1.20
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Question 2:

The average length of stay in a hospital is useful for planning purposes. Suppose that the following is the probability distribution of the length of stay (X) in a hospital after a minor operation:

Length of stay (days)	3	4	5	6
Probability	0.4	0.2	0.1	k

(1) The value of k is

$$k = 1 - (0.4 + 0.2 + 0.1) = 1 - 0.7 = 0.3$$

A	0.0	B	1	C	0.3	D	6
---	-----	---	---	---	-----	---	---

(2) $P(X < 0) =$

A	0.0	B	0.5	C	1	D	0.75
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(3) $P(0 < X \leq 5) =$

$$0.4 + 0.2 + 0.1 = 0.7$$

A	0.32	B	0.5	C	0.7	D	0.1
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(4) $P(X \leq 5.5) =$

$$0.4 + 0.2 + 0.1 = 0.7$$

A	0.7	B	0.6	C	0	D	0.1
---	-----	---	-----	---	---	---	-----

(5) The probability that the patient will stay at most 4 days in a hospital after a minor operation is equal to

$$0.4 + 0.2 = 0.6$$

A	0.4	B	0.1	C	0.2	D	0.6
---	-----	---	-----	---	-----	---	-----

(6) The average length of stay in a hospital is

$$E(X) = (3 \times 0.4) + (4 \times 0.2) + (5 \times 0.1) + (6 \times 0.3) = 4.3$$

A	2.3	B	0.7	C	1	D	4.3
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Binomial Distribution:

$$P(X = x) = \binom{n}{x} p^x q^{n-x} ; \quad x = 0, 1, \dots, n$$

$$* E(X) = np \quad * Var(X) = npq$$

$$q = 1 - p$$

Question 1:

Suppose that 25% of the people in a certain large population have high blood pressure. A Sample of 7 people is selected at random from this population. Let X be the number of people in the sample who have high blood pressure, follows a binomial distribution then

1) The values of the parameters of the distribution are:

$$p = 0.25 \quad q = 0.75 \quad n = 7$$

(A) 7, 0.75	(B) <u>7, 0.25</u>	(C) 0.25, 0.75	(D) 25, 7
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2) The probability that we find exactly one person with high blood pressure, is:

$$P(X = 1) = \binom{7}{1} (0.25)^1 (0.75)^6 = 0.31146$$

(A) <u>0.31146</u>	(B) 0.143	(C) 0.125	(D) 0.25
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3) The probability that there will be at most one person with high blood pressure, is:

$$P(X \leq 1) = \binom{7}{0} (0.25)^0 (0.75)^7 + \binom{7}{1} (0.25)^1 (0.75)^6 = 0.4449$$

(A) 0.311	(B) 0.25	(C) <u>0.4449</u>	(D) 0.5551
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4) The probability that we find more than one person with high blood pressure, is:

$$P(X > 1) = 1 - P(X \leq 1) = 1 - 0.4449 = 0.5551$$

(A) 0.689	(B) 0.857	(C) 0.4449	(D) <u>0.5551</u>
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Question 2:

In some population it was found that the percentage of adults who have hypertension is 24 percent. Suppose we select a simple random sample of five adults from this population. Then the probability that the number of people who have hypertension in this sample, will be:

$$p = 0.24 \quad q = 0.76 \quad n = 5$$

1. Zero:

$$P(X = 0) = \binom{5}{0} (0.24)^0 (0.76)^5 = 0.2536$$

2. Exactly one

$$P(X = 1) = \binom{5}{1} (0.24)^1 (0.76)^4 = 0.4003$$

3. Between one and three, inclusive

$$P(1 \leq X \leq 3) = \binom{5}{1} (0.24)^1 (0.76)^4 + \binom{5}{2} (0.24)^2 (0.76)^3 + \binom{5}{3} (0.24)^3 (0.76)^2 \\ = 0.7330$$

4. Two or fewer (at most two):

$$P(X \leq 2) = \binom{5}{0} (0.24)^0 (0.76)^5 + \binom{5}{1} (0.24)^1 (0.76)^4 + \binom{5}{2} (0.24)^2 (0.76)^3 \\ = 0.9067$$

5. Five:

$$P(X = 5) = \binom{5}{5} (0.24)^5 (0.76)^0 = 0.0008$$

6. The mean of the number of people who have hypertension is equal to:

$$E(X) = np = 5 \times 0.24 = 1.2$$

7. The variance of the number of people who have hypertension is equal to:

$$\text{Var}(X) = npq = 5 \times 0.24 \times 0.76 = 0.912$$

Poisson distribution:

$$P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!} ; x = 0,1,2, \dots$$

$$E(X) = \text{Var}(X) = \lambda$$

Question 1:

The number of serious cases coming to a hospital during a night follows a Poisson distribution with an average of 10 persons per night, then:

1) The probability that 12 serious cases coming in the next night, is:

$\lambda_{\text{one night}} = 10$
$P(X = 12) = \frac{e^{-10} 10^{12}}{12!} = 0.09478$

(A) <u>0.09478</u>	(B) 0.3456	(C) 12	(D) 0.5
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2) The average number of serious cases in a two nights period is:

$\lambda_{\text{two nights}} = 20$

(A) 10.5	(B) <u>20</u>	(C) 0.2065	(D) 0.0867
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3) The probability that 20 serious cases coming in next two nights is:

$\lambda_{\text{two nights}} = 20$
$P(X = 20) = \frac{e^{-20} 20^{20}}{20!} = 0.0888$

(A) 10.5	(B) 0.7694	(C) 0.20	(D) <u>0.0888</u>
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Question 2:

Given the mean number of serious accidents per year in a large factory is five. If the number of accidents follows a Poisson distribution, then the probability that in the next year there will be:

1. Exactly seven accidents:

$$\lambda_{\text{one year}} = 5$$

$$P(X = 7) = \frac{e^{-5} 5^7}{7!} = 0.1044$$

2. No accidents

$$P(X = 0) = \frac{e^{-5} 5^0}{0!} = 0.0067$$

3. one or more accidents

$$\begin{aligned} P(X \geq 1) &= 1 - P(X < 1) \\ &= 1 - P(X = 0) \\ &= 1 - 0.0067 = 0.9933 \end{aligned}$$

4. The expected number (mean) of serious accidents in the next two years is equal to

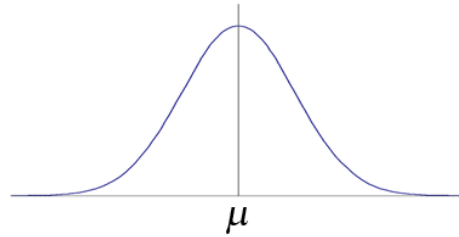
$$\lambda_{\text{two years}} = 10$$

5. The probability that in the next two years there will be three accidents

$$\lambda_{\text{two years}} = 10$$

$$P(X = 3) = \frac{e^{-10} 10^3}{3!} = 0.0076$$

The Normal Distribution:



Normal distribution $X \sim N(\mu, \sigma^2)$

Standard normal $Z \sim N(0, 1)$

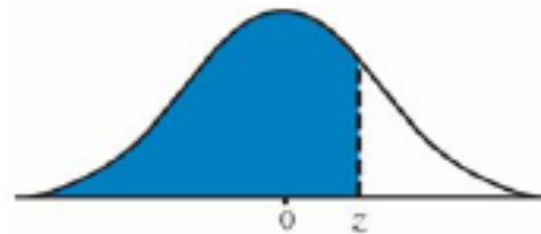
Question 1:

Given the standard normal distribution, $Z \sim N(0, 1)$, find:

1. $P(Z < 1.43) = 0.92364$

z	0.00	0.01	0.02	0.03	0.04
0.00	0.50000	0.50399	0.50798	0.51197	0.51595
0.10	0.53983	0.54380	0.54776	0.55172	0.55567
0.20	0.57926	0.58317	0.58706	0.59095	0.59483
0.30	0.61791	0.62172	0.62552	0.62930	0.63307
0.40	0.65542	0.65910	0.66276	0.66640	0.67003
0.50	0.69146	0.69497	0.69847	0.70194	0.70540
0.60	0.72575	0.72907	0.73237	0.73565	0.73891
0.70	0.75804	0.76115	0.76424	0.76730	0.77035
0.80	0.78814	0.79103	0.79389	0.79673	0.79955
0.90	0.81594	0.81859	0.82121	0.82381	0.82639
1.00	0.84134	0.84375	0.84614	0.84849	0.85083
1.10	0.86433	0.86650	0.86864	0.87076	0.87286
1.20	0.88493	0.88686	0.88877	0.89065	0.89251
1.30	0.90320	0.90490	0.90658	0.90824	0.90988
1.40	0.91924	0.92073	0.92220	0.92364	0.92507
1.50	0.93319	0.93448	0.93574	0.93699	0.93822
1.60	0.94520	0.94630	0.94738	0.94845	0.94950
1.70	0.95543	0.95637	0.95728	0.95818	0.95907
1.80	0.96407	0.96485	0.96562	0.96638	0.96712

$$P\left(Z < \begin{array}{|c|} \hline \text{اطراف} \\ \hline \text{الجدول} \\ \hline \end{array}\right) = \begin{array}{|c|} \hline \text{داخل} \\ \hline \text{الجدول} \\ \hline \end{array}$$



2. $P(Z > 1.67) = 1 - P(Z < 1.67) = 1 - 0.9525 = 0.0475$

3.

$$\begin{aligned} &P(-2.16 < Z < -0.65) \\ &= P(Z < -0.65) - P(Z < -2.16) \\ &= 0.2578 - 0.0154 = 0.2424 \end{aligned}$$

Question 2:

Given the standard normal distribution, $Z \sim N(0,1)$, find:

1. $P(Z > 2.71) = 1 - P(Z < 2.71) = 1 - 0.9966 = 0.0034$

(A) 0.7088	(B) <u>0.0034</u>	(C) 0.3645	(D) 0.1875
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2. $P(-1.96 < Z < 1.96)$

$$= P(Z < 1.96) - P(Z < -1.96)$$

$$= 0.9750 - 0.0250 = 0.9500$$

(A) 0.0746	(B) 0.9950	(C) <u>0.9500</u>	(D) 0.9750
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3. If $P(Z < a) = 0.9929$, then the value of $a =$

(A) -2.54	(B) 0	(C) 1.64	(D) <u>2.45</u>
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4. $P(Z=1.33)=$

(A) 0.1220	(B) 0.1660	(C) 0.1550	(D) <u>0.0</u>
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5. If $P(-k < Z < k) = 0.8132$, then the value of $k =$



$$\Rightarrow 2 \times P(0 < Z < k) = 0.8132$$

$$\Rightarrow P(0 < Z < k) = 0.4066$$

$$\Rightarrow P(Z < k) - P(Z < 0) = 0.4066$$

$$\Rightarrow P(Z < k) - 0.5 = 0.4066$$

$$\Rightarrow P(Z < k) = 0.9066$$

(A) 2.54	(B) 2.31	(C) <u>1.32</u>	(D) 0.5
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Question 3:

Given the standard normal distribution, then:

1) $P(-1.1 \leq z \leq 1.1)$ is:

$$\Rightarrow P(Z < 1.1) - P(Z < -1.1)$$

$$0.86433 - 0.13567 = 0.74731$$

(A) 0.3254	(B) 0.8691	(C) <u>0.7473</u>	(D) 0.1475
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2) $P(z > -0.15)$ is:

$$P(Z > -0.15) = 1 - P(Z < -0.15) = 1 - 0.44038 = 0.55962$$

(A) <u>0.5596</u>	(B) 0.9394	(C) 0.0606	(D) 0.4404
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3) The z value that has an area of 0.883 to its right, is:

$$P(Z > z) = 0.883$$

$$1 - P(Z < z) = 0.883$$

$$P(Z < z) = 0.117$$

(A) -0.811	(B) 1.19	(C) 0.811	(D) <u>-1.19</u>
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Question:

Q2. The finished inside diameter of a piston ring is normally distributed with a mean of 12 centimeters and a standard deviation of 0.03 centimeter. Then,

1) the proportion of rings that will have inside diameter less than 12.05 centimeters is:

(A) 0.0475 (B) 0.9525 (C) 0.7257 (D) 0.8413

2) the proportion of rings that will have inside diameter exceeding 11.97 centimeters is:

(A) 0.0475 (B) 0.8413 (C) 0.1587 (D) 0.4514

3) the probability that a piston ring will have an inside diameter between 11.95 and 12.05 centimeters is:

(A) 0.905 (B) -0.905 (C) 0.4514 (D) 0.7257

Solution:

$$X \sim N(12, 0.03^2)$$

(1):

$$\begin{aligned} P(X < 12.05) &= P\left(Z < \frac{12.05 - \mu}{\sigma}\right) \\ &= P\left(Z < \frac{12.05 - 12}{0.03}\right) = P(Z < 1.67) = 0.9525 \end{aligned}$$

(2):

$$\begin{aligned} P(X > 11.97) &= P\left(Z > \frac{11.97 - \mu}{\sigma}\right) \\ &= P\left(Z > \frac{11.97 - 12}{0.03}\right) = P(Z > -1) \\ &= 1 - P(Z < -1) = 1 - 0.1587 = 0.8413 \end{aligned}$$

(3):

$$\begin{aligned} P(11.95 < X < 12.05) &= P\left(\frac{11.95 - 12}{0.03} < Z < \frac{12.05 - 12}{0.03}\right) \\ &= P(-1.67 < Z < 1.67) \\ &= P(Z < 1.67) - P(Z < -1.67) \\ &= 0.9525 - 0.0475 = 0.905 \end{aligned}$$

Question:

Q6. The weight of a large number of fat persons is nicely modeled with a normal distribution with mean of 128 kg and a standard deviation of 9 kg.

- (1) The percentage of fat persons with weights at most 110 kg is
(A) 0.09 % (B) 90.3 % (C) 99.82 % (D) 2.28 %
- (2) The percentage of fat persons with weights more than 149 kg is
(A) 0.09 % (B) 0.99 % (C) 9.7 % (D) 99.82 %
- (3) The weight x above which 86% of those persons will be
(A) 118.28 (B) 128.28 (C) 154.82 (D) 81.28
- (4) The weight x below which 50% of those persons will be
(A) 101.18 (B) 128 (C) 154.82 (D) 81

Solution:

$$X \sim N(128, 9^2)$$

(1):

$$P(X \leq 110) = P\left(Z < \frac{110 - 128}{9}\right) = P(Z < -2) = 0.0228$$

(2):

$$\begin{aligned} P(X > 149) &= P\left(Z > \frac{149 - 128}{9}\right) \\ &= 1 - P(Z < 2.33) = 1 - 0.9901 = 0.0099 \end{aligned}$$

(3):

$$P(X > x) = 0.86 \Rightarrow P(X < x) = 0.14 \Rightarrow P\left(Z < \frac{x - 128}{9}\right) = 0.14$$

by searching inside the table for 0.14, and transforming X to Z , we got:

$$\frac{x - 128}{9} = -1.08 \Rightarrow x = 118.28$$

(4):

$P(X < x) = 0.5$, by searching inside the table for 0.5, and transforming X to Z

$$\frac{x - 128}{9} = 0 \Rightarrow x = 128$$

Q8. If the random variable X has a normal distribution with the mean μ and the variance σ^2 , then $P(X < \mu + 2\sigma)$ equals to

- (A) 0.8772 (B) 0.4772 (C) 0.5772 (D) 0.7772 (E) 0.9772

Q9. If the random variable X has a normal distribution with the mean μ and the variance 1, and if $P(X < 3) = 0.877$, then μ equals to

- (A) 3.84 (B) 2.84 (C) 1.84 (D) 4.84 (E) 8.84

Q10. Suppose that the marks of the students in a certain course are distributed according to a normal distribution with the mean 70 and the variance 25. If it is known that 33% of the student failed the exam, then the passing mark x is

- (A) 67.8 (B) 60.8 (C) 57.8 (D) 50.8 (E) 70.8

Solution of Q8

$$P(X < \mu + 2\sigma) = P\left(Z < \frac{(\mu + 2\sigma) - \mu}{\sigma}\right) = P(Z < 2) = 0.9772$$

Solution of Q9

Given that $\sigma = 1$

$$P(X < 3) = 0.877 \Rightarrow P\left(Z < \frac{3 - \mu}{1}\right) = 0.877$$

$$3 - \mu = 1.16 \Rightarrow \mu = 1.84$$

Solution of Q10

$X \sim N(70, 25)$

$$P(X < x) = 0.33 \Rightarrow P\left(Z < \frac{x - 70}{5}\right) = 0.33$$

by searching inside the table for 0.33, and transforming X to Z , we got:

$$\frac{x - 70}{5} = -0.44 \Rightarrow x = 67.8$$

Question:

A nurse supervisor has found that staff nurses, on the average, complete a certain task in 10 minutes. If the times required to complete the task are approximately normally distributed with a standard deviation of 3 minutes, then:

- 1) *The probability that a nurse will complete the task in less than 8 minutes is:*

(A) 0.3221	(B) <u>0.2514</u>	(C) 0.5288	(D) 0.1565
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- 2) *The probability that a nurse will complete the task in more than 4 minutes is:*

(A) 0.5461	(B) 0.7558	(C) <u>0.9772</u>	(D) 0.8712
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- 3) *If eight nurses were assigned the task, the expected number of them who will complete it within 8 minutes is approximately equal to:*

(A) 4	(B) 1	(C) 5	(D) <u>2</u>
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- 4) *If a certain nurse completes the task within k minutes with probability 0.6293; then k equals approximately:*

(A) 15	(B) 11	(C) 7	(D) 21
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Question:

Given the normally distributed random variable X with mean 491 and standard deviation 119,

1) If $P(X < k) = 0.9082$, the value of k is equal to

(A) <u>649.27</u>	(B) 390.58	(C) 128.90	(D) 132.65
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2) If $P(292 < X < M) = 0.8607$, the value of M is equal to

(A) 766	(B) <u>649</u>	(C) 108	(D) 136
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Question:

The IQ (Intelligent Quotient) of individuals admitted to a state school for the mentally retarded are approximately normally distributed with a mean of 60 and a standard deviation of 10, then:

1) The probability that an individual picked at random will have an IQ greater than 75 is:

(A) 0.9332	(B) 0.8691	(C) 0.7286	(D) <u>0.0668</u>
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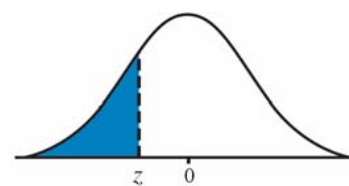
2) The probability that an individual picked at random will have an IQ between 55 and 75 is:

(A) 0.3085	(B) 0.6915	(C) <u>0.6247</u>	(D) 0.9332
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3) If the probability that an individual picked at random will have an IQ less than k is 0.1587. Then the value of k

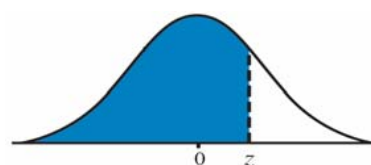
(A) <u>50</u>	(B) 45	(C) 51	(D) 40
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Standard Normal Table
Areas Under the Standard Normal Curve



z	-0.09	-0.08	-0.07	-0.06	-0.05	-0.04	-0.03	-0.02	-0.01	-0.00	z
-3.50	0.00017	0.00017	0.00018	0.00019	0.00019	0.00020	0.00021	0.00022	0.00022	0.00023	-3.50
-3.40	0.00024	0.00025	0.00026	0.00027	0.00028	0.00029	0.00030	0.00031	0.00032	0.00034	-3.40
-3.30	0.00035	0.00036	0.00038	0.00039	0.00040	0.00042	0.00043	0.00045	0.00047	0.00048	-3.30
-3.20	0.00050	0.00052	0.00054	0.00056	0.00058	0.00060	0.00062	0.00064	0.00066	0.00069	-3.20
-3.10	0.00071	0.00074	0.00076	0.00079	0.00082	0.00084	0.00087	0.00090	0.00094	0.00097	-3.10
-3.00	0.00100	0.00104	0.00107	0.00111	0.00114	0.00118	0.00122	0.00126	0.00131	0.00135	-3.00
-2.90	0.00139	0.00144	0.00149	0.00154	0.00159	0.00164	0.00169	0.00175	0.00181	0.00187	-2.90
-2.80	0.00193	0.00199	0.00205	0.00212	0.00219	0.00226	0.00233	0.00240	0.00248	0.00256	-2.80
-2.70	0.00264	0.00272	0.00280	0.00289	0.00298	0.00307	0.00317	0.00326	0.00336	0.00347	-2.70
-2.60	0.00357	0.00368	0.00379	0.00391	0.00402	0.00415	0.00427	0.00440	0.00453	0.00466	-2.60
-2.50	0.00480	0.00494	0.00508	0.00523	0.00539	0.00554	0.00570	0.00587	0.00604	0.00621	-2.50
-2.40	0.00639	0.00657	0.00676	0.00695	0.00714	0.00734	0.00755	0.00776	0.00798	0.00820	-2.40
-2.30	0.00842	0.00866	0.00889	0.00914	0.00939	0.00964	0.00990	0.01017	0.01044	0.01072	-2.30
-2.20	0.01101	0.01130	0.01160	0.01191	0.01222	0.01255	0.01287	0.01321	0.01355	0.01390	-2.20
-2.10	0.01426	0.01463	0.01500	0.01539	0.01578	0.01618	0.01659	0.01700	0.01743	0.01786	-2.10
-2.00	0.01831	0.01876	0.01923	0.01970	0.02018	0.02068	0.02118	0.02169	0.02222	0.02275	-2.00
-1.90	0.02330	0.02385	0.02442	0.02500	0.02559	0.02619	0.02680	0.02743	0.02807	0.02872	-1.90
-1.80	0.02938	0.03005	0.03074	0.03144	0.03216	0.03288	0.03362	0.03438	0.03515	0.03593	-1.80
-1.70	0.03673	0.03754	0.03836	0.03920	0.04006	0.04093	0.04182	0.04272	0.04363	0.04457	-1.70
-1.60	0.04551	0.04648	0.04746	0.04846	0.04947	0.05050	0.05155	0.05262	0.05370	0.05480	-1.60
-1.50	0.05592	0.05705	0.05821	0.05938	0.06057	0.06178	0.06301	0.06426	0.06552	0.06681	-1.50
-1.40	0.06811	0.06944	0.07078	0.07215	0.07353	0.07493	0.07636	0.07780	0.07927	0.08076	-1.40
-1.30	0.08226	0.08379	0.08534	0.08691	0.08851	0.09012	0.09176	0.09342	0.09510	0.09680	-1.30
-1.20	0.09853	0.10027	0.10204	0.10383	0.10565	0.10749	0.10935	0.11123	0.11314	0.11507	-1.20
-1.10	0.11702	0.11900	0.12100	0.12302	0.12507	0.12714	0.12924	0.13136	0.13350	0.13567	-1.10
-1.00	0.13786	0.14007	0.14231	0.14457	0.14686	0.14917	0.15151	0.15386	0.15625	0.15866	-1.00
-0.90	0.16109	0.16354	0.16602	0.16853	0.17106	0.17361	0.17619	0.17879	0.18141	0.18406	-0.90
-0.80	0.18673	0.18943	0.19215	0.19489	0.19766	0.20045	0.20327	0.20611	0.20897	0.21186	-0.80
-0.70	0.21476	0.21770	0.22065	0.22363	0.22663	0.22965	0.23270	0.23576	0.23885	0.24196	-0.70
-0.60	0.24510	0.24825	0.25143	0.25463	0.25785	0.26109	0.26435	0.26763	0.27093	0.27425	-0.60
-0.50	0.27760	0.28096	0.28434	0.28774	0.29116	0.29460	0.29806	0.30153	0.30503	0.30854	-0.50
-0.40	0.31207	0.31561	0.31918	0.32276	0.32636	0.32997	0.33360	0.33724	0.3409	0.34458	-0.40
-0.30	0.34827	0.35197	0.35569	0.35942	0.36317	0.36693	0.37070	0.37448	0.37828	0.38209	-0.30
-0.20	0.38591	0.38974	0.39358	0.39743	0.40129	0.40517	0.40905	0.41294	0.41683	0.42074	-0.20
-0.10	0.42465	0.42858	0.43251	0.43644	0.44038	0.44433	0.44828	0.45224	0.45620	0.46017	-0.10
-0.00	0.46414	0.46812	0.47210	0.47608	0.48006	0.48405	0.48803	0.49202	0.49601	0.50000	-0.00

Standard Normal Table (continued)
 Areas Under the Standard Normal Curve



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	z
0.00	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586	0.00
0.10	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535	0.10
0.20	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409	0.20
0.30	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173	0.30
0.40	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793	0.40
0.50	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240	0.50
0.60	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490	0.60
0.70	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524	0.70
0.80	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327	0.80
0.90	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891	0.90
1.00	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214	1.00
1.10	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298	1.10
1.20	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147	1.20
1.30	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774	1.30
1.40	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189	1.40
1.50	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408	1.50
1.60	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449	1.60
1.70	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327	1.70
1.80	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062	1.80
1.90	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670	1.90
2.00	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169	2.00
2.10	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574	2.10
2.20	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899	2.20
2.30	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158	2.30
2.40	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361	2.40
2.50	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520	2.50
2.60	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643	2.60
2.70	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736	2.70
2.80	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807	2.80
2.90	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861	2.90
3.00	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900	3.00
3.10	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929	3.10
3.20	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950	3.20
3.30	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965	3.30
3.40	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976	3.40
3.50	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983	3.50

Sampling Distribution

Sampling Distribution: Single Mean

$$* \bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

$$* E(\bar{X}) = \bar{X} = \mu \quad * \text{Var}(\bar{X}) = \frac{\sigma^2}{n}$$

Sampling Distribution: Two Means

$$* \bar{X}_1 - \bar{X}_2 \sim N\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$$

$$* E(\bar{X}_1 - \bar{X}_2) = \mu_1 - \mu_2 \quad * \text{Var}(\bar{X}_1 - \bar{X}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

Sampling Distribution: Single Proportion

$$* \hat{p} \sim N\left(p, \frac{pq}{n}\right)$$

$$* E(\hat{p}) = p \quad * \text{Var}(\hat{p}) = \frac{pq}{n}$$

Sampling Distribution: Two Proportions

$$* \hat{p}_1 - \hat{p}_2 \sim N\left(p_1 - p_2, \frac{p_1q_1}{n_1} + \frac{p_2q_2}{n_2}\right)$$

$$* E(\hat{p}_1 - \hat{p}_2) = p_1 - p_2 \quad * \text{Var}(\hat{p}_1 - \hat{p}_2) = \frac{p_1q_1}{n_1} + \frac{p_2q_2}{n_2}$$

Q2. The average life of a certain battery is 5 years, with a standard deviation of 1 year. Assume that the live of the battery approximately follows a normal distribution.

1) The sample mean \bar{X} of a random sample of 5 batteries selected from this product has a mean

$E(\bar{X}) = \mu_{\bar{X}}$ equal to:

- (A) 0.2 (B) 5 (C) 3 (D) None of these

2) The variance $Var(\bar{X}) = \sigma_{\bar{X}}^2$ of the sample mean \bar{X} of a random sample of 5 batteries selected from this product is equal to:

- (A) 0.2 (B) 5 (C) 3 (D) None of these

3) The probability that the average life of a random sample of size 16 of such batteries will be between 4.5 and 5.4 years is:

- (A) 0.1039 (B) 0.2135 (C) 0.7865 (D) 0.9224

4) The probability that the average life of a random sample of size 16 of such batteries will be less than 5.5 years is:

- (A) 0.9772 (B) 0.0228 (C) 0.9223 (D) None of these

5) The probability that the average life of a random sample of size 16 of such batteries will be more than 4.75 years is:

- (A) 0.8413 (B) 0.1587 (C) 0.9452 (D) None of these

6) If $P(\bar{X} > a) = 0.1492$ where \bar{X} represents the sample mean for a random sample of size 9 of such batteries, then the numerical value of a is:

- (A) 4.653 (B) 6.5 (C) 5.347 (D) None of these

Solution:

$$\mu = 5 ; \sigma = 1 ; n = 5$$

(1): $E(\bar{X}) = \mu = 5$

(2): $Var(\bar{X}) = \frac{\sigma^2}{n} = \frac{1}{5} = 0.2$

(3): $n = 16 \rightarrow \frac{\sigma}{\sqrt{n}} = \frac{1}{4}$

$$\begin{aligned} P(4.5 < \bar{X} < 5.4) &= P\left(\frac{4.5 - \mu}{\frac{\sigma}{\sqrt{n}}} < Z < \frac{5.4 - \mu}{\frac{\sigma}{\sqrt{n}}}\right) = P\left(\frac{4.5 - 5}{\frac{1}{4}} < Z < \frac{5.4 - 5}{\frac{1}{4}}\right) \\ &= P(-2 < Z < 1.6) \\ &= P(Z < 1.6) - P(Z < -2) \\ &= 0.9452 - 0.0228 = 0.9224 \end{aligned}$$

(4): $P(\bar{X} < 5.5) = P\left(Z < \frac{5.5 - \mu}{\frac{\sigma}{\sqrt{n}}}\right) = P\left(Z < \frac{5.5 - 5}{1/4}\right) = P(Z < 2) = 0.9772$

$$(5): P(\bar{X} > 4.75) = P\left(Z > \frac{4.75 - \mu}{\frac{\sigma}{\sqrt{n}}}\right) = P\left(Z > \frac{4.75 - 5}{\frac{1}{4}}\right) = P(Z > -1) \\ = 1 - P(Z < -1) = 1 - 0.1587 = 0.841$$

$$(6): P(\bar{X} > a) = 0.1492 ; n = 9$$

$$P\left(Z > \frac{a - \mu}{\frac{\sigma}{\sqrt{n}}}\right) = 0.1492 \\ \Rightarrow 1 - P\left(Z < \frac{a - 5}{\frac{1}{3}}\right) = 0.1492 \\ \Rightarrow P\left(Z < \frac{a - 5}{\frac{1}{3}}\right) = 0.8508 \\ \frac{a - 5}{\frac{1}{3}} = 1.04 \\ a = 5 + \frac{1.04}{3} = 5.347$$

Q4. Suppose that you take a random sample of size $n=64$ from a distribution with mean $\mu=55$ and standard deviation $\sigma=10$. Let $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$ be the sample mean.

- (a) What is the approximated sampling distribution of \bar{X} ?
- (b) What is the mean of \bar{X} ?
- (c) What is the standard error (standard deviation) of \bar{X} ?

(d) Find the probability that the sample mean \bar{X} exceeds 52.

Solution

$$\mu = 55 ; \sigma = 10 ; n = 64$$

$$(a) \bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right) = \bar{X} \sim N\left(55, \frac{100}{64}\right)$$

$$(b) E(\bar{X}) = \mu = 55$$

$$(c) S.D(\bar{X}) = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{64}} = \frac{10}{8}$$

$$(d) P(\bar{X} > 52) = P\left(Z > \frac{52 - 55}{\frac{10}{8}}\right) \\ = P(Z > -2.4) \\ = 1 - P(Z < -2.4) \\ = 1 - 0.0082 = 0.9918$$

Old exams:

Question: Suppose that the hemoglobin levels (in g/dl) of healthy Saudi females are approximately normally distributed with mean of 13.5 and a standard deviation of 0.7. If 15 healthy adult Saudi female is randomly chosen, then:

- (1) The mean of \bar{x} ($E(\bar{x})$ or $\mu_{\bar{x}}$) is:
 (A) 0.7 (B) 13.5 (C) 15 (D) 3.48
- (2) The standard error of \bar{x} ($\sigma_{\bar{x}}$) is:
 (A) 0.181 (B) .0327 (C) 0.7 (D) 13.5
- (3) $P(\bar{X} < 14) =$
 (A) 0.99720 (B) 0.99440 (C) 0.76115 (D) 0.9971
- (4) $P(\bar{X} > 13.5) =$
 (A) 0.99 (B) 0.50 (C) 0.761 (D) 0.622
- (5) $P(13 < \bar{X} < 14) =$
 (A) 0.9972 (B) 0.9942 (C) 0.7615 (D) 0.5231

Question: If the uric acid value in normal adult males is approximately normally distributed with a mean and standard derivation of 5.7 and 1 mg percent, respectively, find the probability that a sample of size 9 will yield a mean:

(1) Greater than 6 is

(A)	0.2109	(B)	<u>0.1841</u>	(C)	0.8001	(D)	0.8159
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(2) At most 5.2 is

(A)	0.6915	(B)	0.9331	(C)	0.8251	(D)	<u>0.0668</u>
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(3) Between 5 and 6 is

(A)	0.1662	(B)	<u>0.7981</u>	(C)	0.8791	(D)	0.9812
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Sampling Distribution: Two Means:

$$* \bar{X}_1 - \bar{X}_2 \sim N\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$$

$$* E(\bar{X}_1 - \bar{X}_2) = \mu_1 - \mu_2 \quad * \text{Var}(\bar{X}_1 - \bar{X}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

Q1. A random sample of size $n_1 = 36$ is taken from a normal population with a mean $\mu_1 = 70$ and a standard deviation $\sigma_1 = 4$. A second independent random sample of size $n_2 = 49$ is taken from a normal population with a mean $\mu_2 = 85$ and a standard deviation $\sigma_2 = 5$. Let \bar{X}_1 and \bar{X}_2 be the averages of the first and second samples, respectively.

b) Find $E(\bar{X}_1 - \bar{X}_2)$ and $\text{Var}(\bar{X}_1 - \bar{X}_2)$.

d) Find $P(\bar{X}_1 - \bar{X}_2 > -16)$.

Solution

$$n_1 = 36, \mu_1 = 70, \sigma_1 = 4$$

$$n_2 = 49, \mu_2 = 85, \sigma_2 = 5$$

$$(b): E(\bar{X}_1 - \bar{X}_2) = \mu_1 - \mu_2 = 70 - 85 = -15$$

$$\text{Var}(\bar{X}_1 - \bar{X}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} = \frac{16}{36} + \frac{25}{49} = 0.955$$

$$(d): P(\bar{X}_1 - \bar{X}_2 > -16) = P\left(Z > \frac{-16 - (-15)}{\sqrt{0.955}}\right) = 1 - P\left(Z < \frac{-16 - (-15)}{\sqrt{0.955}}\right) \\ = 1 - P(Z < -1.02) = 0.8461$$

Q2. A random sample of size 25 is taken from a normal population (first population) having a mean of 100 and a standard deviation of 6. A second random sample of size 36 is taken from a different normal population (second population) having a mean of 97 and a standard deviation of 5. Assume that these two samples are independent.

- (1) the probability that the sample mean of the first sample will exceed the sample mean of the second sample by at least 6 is
 (A) 0.0013 (B) 0.9147 (C) 0.0202 (D) 0.9832
- (2) the probability that the difference between the two sample means will be less than 2 is
 (A) 0.099 (B) 0.2480 (C) 0.8499 (D) 0.9499

Solution

$$n_1 = 25, \mu_1 = 100, \sigma_1 = 6$$

$$n_2 = 36, \mu_2 = 97, \sigma_2 = 5$$

$$E(\bar{X}_1 - \bar{X}_2) = \mu_1 - \mu_2 = 100 - 97 = 3$$

$$Var(\bar{X}_1 - \bar{X}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} = \frac{36}{25} + \frac{25}{36} = 2.35$$

$$\begin{aligned} (1) P(\bar{X}_1 > \bar{X}_2 + 6) &= P(\bar{X}_1 - \bar{X}_2 > 6) \\ &= P\left(Z > \frac{6-(3)}{\sqrt{2.35}}\right) = P(Z > 2.05) \\ &= 1 - P(Z < 2.05) \\ &= 1 - 0.9798 = 0.0202 \end{aligned}$$

$$\begin{aligned} (2) P(\bar{X}_1 - \bar{X}_2 < 2) &= P\left(Z < \frac{2-(3)}{\sqrt{2.35}}\right) \\ &= P(Z < -0.68) = 0.2483 \end{aligned}$$

Question: Given two normally distributed populations with equal means and variances of

$\sigma_1^2 = 100$, $\sigma_2^2 = 350$. Two random samples of sizes $n_1 = 40$, $n_2 = 35$ are drawn and the sample means \bar{X}_1 , \bar{X}_2 are calculated, respectively, then

(1) $P(\bar{X}_1 - \bar{X}_2 > 12)$ is

(A)	0.1499	(B)	0.8501	(C)	0.9997	(D)	<u>0.0003</u>
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(2) $P(5 < \bar{X}_1 - \bar{X}_2 < 12)$ is

<u>(A)</u>	<u>0.0783</u>	(B)	0.9217	(C)	0.8002	(D)	None of these
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Sampling Distribution: Single Proportion

$$* \hat{p} \sim N\left(p, \frac{pq}{n}\right)$$

$$* E(\hat{p}) = p \quad * Var(\hat{p}) = \frac{pq}{n}$$

Q1. Suppose that 20% of the students in a certain university smoke cigarettes. A random sample of 5 students is taken from this university. Let \hat{p} be the proportion of smokers in the sample.

- (1) Find $E(\hat{p}) = \mu_{\hat{p}}$, the mean \hat{p} .
- (2) Find $Var(\hat{p}) = \sigma_{\hat{p}}^2$, the variance of \hat{p} .
- (3) Find an approximate distribution of \hat{p} .
- (4) Find $P(\hat{p} > 0.25)$.

Solution

$$p = 0.2 \ ; \ n = 5 \ ; \ q = 1 - p = 0.8$$

(1): $E(\hat{p}) = p = 0.2$

(2): $Var(\hat{p}) = \frac{pq}{n} = \frac{0.2 \times 0.8}{5} = 0.032$

(3): $\hat{p} \sim N(0.2, 0.032)$

(4): $P(\hat{p} > 0.25) = P\left(Z > \frac{0.25 - 0.2}{\sqrt{0.032}}\right) = P(Z > 0.28)$
 $= 1 - P(Z < 0.28) = 1 - 0.6103 = 0.3897$

Question: A random sample of 35 students in a certain university resulted in the sample proportion of smokers $\hat{p} = 0.15$. Then:

1. The point estimate of p is:

(A) 0.35	(B) 0.85	(C) 0.15	(D) 0.80
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2. The standard deviation of \hat{p} is:

(A) 0.3214	(B) .0036	(C) 0.1275	(D) 0.0604
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Question: In a study, it was found that 31% of the adult population in a certain city has a diabetic disease. 100 people are randomly sampled from the population. Then

(6) The mean for the sample proportion ($\mu_{\hat{p}}$ or $E(\hat{p})$) is:

- (A) 0.4 (B) 0.31 (C) 0.69 (D) 0.1

(7) $P(\hat{p} > 0.4) =$

- (A) 0.02619 (B) 0.02442 (C) 0.0256 (D) 0.7054

Sampling Distribution: Two Proportions:

$$* \hat{p}_1 - \hat{p}_2 \sim N\left(p_1 - p_2, \frac{p_1q_1}{n_1} + \frac{p_2q_2}{n_2}\right)$$

$$* E(\hat{p}_1 - \hat{p}_2) = p_1 - p_2 \qquad * Var(\hat{p}_1 - \hat{p}_2) = \frac{p_1q_1}{n_1} + \frac{p_2q_2}{n_2}$$

Q1. Suppose that 25% of the male students and 20% of the female students in a certain university smoke cigarettes. A random sample of 5 male students is taken. Another random sample of 10 female students is independently taken from this university. Let \hat{p}_1 and \hat{p}_2 be the proportions of smokers in the two samples, respectively.

- (1) Find $E(\hat{p}_1 - \hat{p}_2) = \mu_{\hat{p}_1 - \hat{p}_2}$, the mean of $\hat{p}_1 - \hat{p}_2$.
- (2) Find $Var(\hat{p}_1 - \hat{p}_2) = \sigma_{\hat{p}_1 - \hat{p}_2}^2$, the variance of $\hat{p}_1 - \hat{p}_2$.
- (3) Find an approximate distribution of $\hat{p}_1 - \hat{p}_2$.
- (4) Find $P(0.10 < \hat{p}_1 - \hat{p}_2 < 0.20)$.

Solution

$$p_1 = 0.25 \quad ; \quad n_1 = 5$$

$$p_2 = 0.2 \quad ; \quad n_2 = 10$$

$$(1): E(\hat{p}_1 - \hat{p}_2) = p_1 - p_2 = 0.25 - 0.2 = 0.05$$

$$(2): Var(\hat{p}_1 - \hat{p}_2) = \frac{p_1q_1}{n_1} + \frac{p_2q_2}{n_2} = \frac{0.25 \times 0.75}{5} + \frac{0.2 \times 0.8}{10} = 0.054$$

$$(3): \hat{p}_1 - \hat{p}_2 \sim N(0.05, 0.054)$$

$$(4): P(0.1 < \hat{p}_1 - \hat{p}_2 < 0.2) = P\left(\frac{0.1 - 0.05}{\sqrt{0.054}} < Z < \frac{0.2 - 0.05}{\sqrt{0.054}}\right)$$

$$= P(0.22 < Z < 0.65)$$

$$= P(Z < 0.65) - P(Z < 0.22)$$

$$= 0.7422 - 0.5871 = 0.1551$$

Question: Suppose that 7 % of the pieces from a production process A are defective while that proportion of defective for another production process B is 5 %. A random sample of size 400 pieces is taken from the production process A while the sample size taken from the production process B is 300 pieces. If \hat{P}_1 and \hat{P}_2 be the proportions of defective pieces in the two samples, respectively, then:

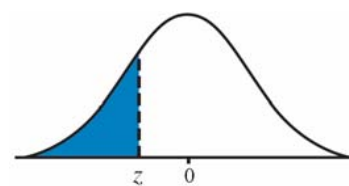
3. The sampling distribution of $\hat{P}_1 - \hat{P}_2$ is:

(A) $N(0, 1)$	(B) Normal	(C) T	(D) unknown
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4. The value of the standard error of the difference ($\hat{P}_1 - \hat{P}_2$) is:

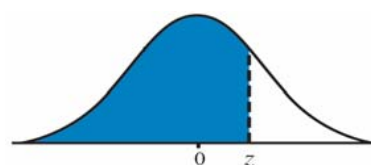
(A) 0.02	(B) 0.10	(C) 0	(D) 0.22
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Standard Normal Table
Areas Under the Standard Normal Curve



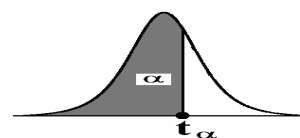
z	-0.09	-0.08	-0.07	-0.06	-0.05	-0.04	-0.03	-0.02	-0.01	-0.00	z
-3.50	0.00017	0.00017	0.00018	0.00019	0.00019	0.00020	0.00021	0.00022	0.00022	0.00023	-3.50
-3.40	0.00024	0.00025	0.00026	0.00027	0.00028	0.00029	0.00030	0.00031	0.00032	0.00034	-3.40
-3.30	0.00035	0.00036	0.00038	0.00039	0.00040	0.00042	0.00043	0.00045	0.00047	0.00048	-3.30
-3.20	0.00050	0.00052	0.00054	0.00056	0.00058	0.00060	0.00062	0.00064	0.00066	0.00069	-3.20
-3.10	0.00071	0.00074	0.00076	0.00079	0.00082	0.00084	0.00087	0.00090	0.00094	0.00097	-3.10
-3.00	0.00100	0.00104	0.00107	0.00111	0.00114	0.00118	0.00122	0.00126	0.00131	0.00135	-3.00
-2.90	0.00139	0.00144	0.00149	0.00154	0.00159	0.00164	0.00169	0.00175	0.00181	0.00187	-2.90
-2.80	0.00193	0.00199	0.00205	0.00212	0.00219	0.00226	0.00233	0.00240	0.00248	0.00256	-2.80
-2.70	0.00264	0.00272	0.00280	0.00289	0.00298	0.00307	0.00317	0.00326	0.00336	0.00347	-2.70
-2.60	0.00357	0.00368	0.00379	0.00391	0.00402	0.00415	0.00427	0.00440	0.00453	0.00466	-2.60
-2.50	0.00480	0.00494	0.00508	0.00523	0.00539	0.00554	0.00570	0.00587	0.00604	0.00621	-2.50
-2.40	0.00639	0.00657	0.00676	0.00695	0.00714	0.00734	0.00755	0.00776	0.00798	0.00820	-2.40
-2.30	0.00842	0.00866	0.00889	0.00914	0.00939	0.00964	0.00990	0.01017	0.01044	0.01072	-2.30
-2.20	0.01101	0.01130	0.01160	0.01191	0.01222	0.01255	0.01287	0.01321	0.01355	0.01390	-2.20
-2.10	0.01426	0.01463	0.01500	0.01539	0.01578	0.01618	0.01659	0.01700	0.01743	0.01786	-2.10
-2.00	0.01831	0.01876	0.01923	0.01970	0.02018	0.02068	0.02118	0.02169	0.02222	0.02275	-2.00
-1.90	0.02330	0.02385	0.02442	0.02500	0.02559	0.02619	0.02680	0.02743	0.02807	0.02872	-1.90
-1.80	0.02938	0.03005	0.03074	0.03144	0.03216	0.03288	0.03362	0.03438	0.03515	0.03593	-1.80
-1.70	0.03673	0.03754	0.03836	0.03920	0.04006	0.04093	0.04182	0.04272	0.04363	0.04457	-1.70
-1.60	0.04551	0.04648	0.04746	0.04846	0.04947	0.05050	0.05155	0.05262	0.05370	0.05480	-1.60
-1.50	0.05592	0.05705	0.05821	0.05938	0.06057	0.06178	0.06301	0.06426	0.06552	0.06681	-1.50
-1.40	0.06811	0.06944	0.07078	0.07215	0.07353	0.07493	0.07636	0.07780	0.07927	0.08076	-1.40
-1.30	0.08226	0.08379	0.08534	0.08691	0.08851	0.09012	0.09176	0.09342	0.09510	0.09680	-1.30
-1.20	0.09853	0.10027	0.10204	0.10383	0.10565	0.10749	0.10935	0.11123	0.11314	0.11507	-1.20
-1.10	0.11702	0.11900	0.12100	0.12302	0.12507	0.12714	0.12924	0.13136	0.13350	0.13567	-1.10
-1.00	0.13786	0.14007	0.14231	0.14457	0.14686	0.14917	0.15151	0.15386	0.15625	0.15866	-1.00
-0.90	0.16109	0.16354	0.16602	0.16853	0.17106	0.17361	0.17619	0.17879	0.18141	0.18406	-0.90
-0.80	0.18673	0.18943	0.19215	0.19489	0.19766	0.20045	0.20327	0.20611	0.20897	0.21186	-0.80
-0.70	0.21476	0.21770	0.22065	0.22363	0.22663	0.22965	0.23270	0.23576	0.23885	0.24196	-0.70
-0.60	0.24510	0.24825	0.25143	0.25463	0.25785	0.26109	0.26435	0.26763	0.27093	0.27425	-0.60
-0.50	0.27760	0.28096	0.28434	0.28774	0.29116	0.29460	0.29806	0.30153	0.30503	0.30854	-0.50
-0.40	0.31207	0.31561	0.31918	0.32276	0.32636	0.32997	0.33360	0.33724	0.3409	0.34458	-0.40
-0.30	0.34827	0.35197	0.35569	0.35942	0.36317	0.36693	0.37070	0.37448	0.37828	0.38209	-0.30
-0.20	0.38591	0.38974	0.39358	0.39743	0.40129	0.40517	0.40905	0.41294	0.41683	0.42074	-0.20
-0.10	0.42465	0.42858	0.43251	0.43644	0.44038	0.44433	0.44828	0.45224	0.45620	0.46017	-0.10
-0.00	0.46414	0.46812	0.47210	0.47608	0.48006	0.48405	0.48803	0.49202	0.49601	0.50000	-0.00

Standard Normal Table (continued)
 Areas Under the Standard Normal Curve



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	z
0.00	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586	0.00
0.10	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535	0.10
0.20	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409	0.20
0.30	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173	0.30
0.40	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793	0.40
0.50	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240	0.50
0.60	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490	0.60
0.70	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524	0.70
0.80	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327	0.80
0.90	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891	0.90
1.00	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214	1.00
1.10	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298	1.10
1.20	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147	1.20
1.30	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774	1.30
1.40	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189	1.40
1.50	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408	1.50
1.60	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449	1.60
1.70	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327	1.70
1.80	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062	1.80
1.90	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670	1.90
2.00	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169	2.00
2.10	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574	2.10
2.20	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899	2.20
2.30	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158	2.30
2.40	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361	2.40
2.50	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520	2.50
2.60	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643	2.60
2.70	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736	2.70
2.80	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807	2.80
2.90	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861	2.90
3.00	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900	3.00
3.10	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929	3.10
3.20	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950	3.20
3.30	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965	3.30
3.40	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976	3.40
3.50	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983	3.50

Critical Values of the t-distribution (t_α)



v=df	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$	$t_{0.995}$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
35	1.3062	1.6896	2.0301	2.4377	2.7238
40	1.3030	1.6840	2.0210	2.4230	2.7040
45	1.3006	1.6794	2.0141	2.4121	2.6896
50	1.2987	1.6759	2.0086	2.4033	2.6778
60	1.2958	1.6706	2.0003	2.3901	2.6603
70	1.2938	1.6669	1.9944	2.3808	2.6479
80	1.2922	1.6641	1.9901	2.3739	2.6387
90	1.2910	1.6620	1.9867	2.3685	2.6316
100	1.2901	1.6602	1.9840	2.3642	2.6259
120	1.2886	1.6577	1.9799	2.3578	2.6174
140	1.2876	1.6558	1.9771	2.3533	2.6114
160	1.2869	1.6544	1.9749	2.3499	2.6069
180	1.2863	1.6534	1.9732	2.3472	2.6034
200	1.2858	1.6525	1.9719	2.3451	2.6006
∞	1.282	1.645	1.960	2.326	2.576

Estimation and Confidence Interval

Estimation and Confidence Interval: Single Mean:

To find the confidence intervals for a single mean:

$$1- \bar{X} \pm \left(Z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \right) \quad \sigma \text{ known}$$

$$2- \bar{X} \pm \left(t_{1-\frac{\alpha}{2}, n-1} \frac{S}{\sqrt{n}} \right) \quad \sigma \text{ unknown}$$

Estimation and Confidence Interval: Two Means

To find the confidence intervals for two means:

$$1- (\bar{X}_1 - \bar{X}_2) \pm \left(Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \right) \quad \sigma_1 \text{ and } \sigma_2 \text{ known}$$

$$2- (\bar{X}_1 - \bar{X}_2) \pm \left(t_{1-\frac{\alpha}{2}, n_1+n_2-2} Sp \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \right) \quad \sigma_1 \text{ and } \sigma_2 \text{ unknown}$$

$$Sp^2 = \frac{S_1^2(n_1 - 1) + S_2^2(n_2 - 1)}{n_1 + n_2 - 2}$$

Estimation and Confidence Interval: Single Proportion

* Point estimate for P is: $\frac{x}{n}$

$$* \text{ Interval estimate for P is: } \hat{p} \pm \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}\hat{q}}{n}} \right)$$

Estimation and Confidence Interval: Two Proportions

* Point estimate for $P_1 - P_2 = \hat{p}_1 - \hat{p}_2 = \frac{x_1}{n_1} - \frac{x_2}{n_2}$

$$* \text{ Interval estimate for } P_1 - P_2 \text{ is: } (\hat{p}_1 - \hat{p}_2) \pm \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} \right)$$

Q2. Suppose that we are interested in making some statistical inferences about the mean, μ , of a normal population with standard deviation $\sigma=2.0$. Suppose that a random sample of size $n=49$ from this population gave a sample mean $\bar{X}=4.5$.

a. Find the upper limit of 95% of the confident interval for μ

$$\sigma = 2 \quad , \quad \bar{X} = 4.5 \quad , \quad n = 49$$

$$95\% \rightarrow \alpha = 0.05 \quad \quad Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$$

$$\bar{X} + \left(Z_{1-\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}} \right) = 4.5 + \left(1.96 \times \frac{2}{7} \right) = 5.06$$

b. Find the lower limit of 95% of the confident interval for μ

$$\bar{X} - \left(Z_{1-\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}} \right) = 4.5 - \left(1.96 \times \frac{2}{7} \right) = 3.94$$

Q4. A researcher wants to estimate the mean lifespan of a certain light bulbs. Suppose that the distribution is normal with standard deviation of 5 hours.

Suppose that the researcher selected a random sample of 49 bulbs and found that the sample mean is 390 hours.

$$\sigma = 5 \quad , \quad \bar{X} = 390 \quad , \quad n = 49$$

a. find $Z_{0.975}$:

$$Z_{0.975} = 1.96$$

b. find a point estimate for μ

$$\hat{\mu} = \bar{X} = 390$$

c. Find the upper limit of 95% of the confident interval for μ

$$\bar{X} + \left(Z_{1-\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}} \right) = 390 + \left(1.96 \times \frac{5}{\sqrt{49}} \right) = 391.3$$

d. Find the lower limit of 95% of the confident interval for μ

$$\bar{X} - \left(Z_{1-\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}} \right) = 390 - \left(1.96 \times \frac{5}{\sqrt{49}} \right) = 388.6$$

➤ A sample of 16 college students were asked about time they spent doing their homework. It was found that the average to be 4.5 hours. Assuming normal population with standard deviation 0.5 hours.

(1) The point estimate for μ is:

- (A) 0 hours (B) 10 hours (C) 0.5 hours (D) 4.5 hours

(2) The standard error of \bar{x} is:

- (A) 0.125 hours (B) 0.266 hours (C) 0.206 hours (D) 0.245 hours

(3) The correct formula for calculating **100(1 - α)%** confidence interval for μ is:

- (A) $\bar{x} \pm t_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$ (B) $\bar{x} \pm z_{1-\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$
 (C) $\bar{x} \pm z_{1-\frac{\alpha}{2}} \frac{\sigma^2}{n}$ (D) $\bar{x} \pm t_{1-\frac{\alpha}{2}} \frac{\sigma^2}{n}$

(4) The upper limit of 95% confidence interval for μ is:

- (A) 4.745 (B) 4.531 (C) 4.832 (D) 4.891

(5) The lower limit of 95% confidence interval for μ is:

- (A) 5.531 (B) 7.469 (C) 3.632 (D) 4.255

(6) The length of the 95% confidence interval for μ is:

- (A) 4.74 (B) 0.49 (C) 0.83 (D) 0.89

Estimation and Confidence Interval: Two Means

To find the confidence intervals for two means:

$$1- (\bar{X}_1 - \bar{X}_2) \pm \left(Z_{1-\frac{\alpha}{2}} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \right)$$

$$2- (\bar{X}_1 - \bar{X}_2) \pm \left(t_{1-\frac{\alpha}{2}, n_1+n_2-2} Sp \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \right)$$

$$Sp^2 = \frac{S_1^2(n_1 - 1) + S_2^2(n_2 - 1)}{n_1 + n_2 - 2}$$

Q1.(I) The tensile strength of type I thread is approximately normally distributed with standard deviation of 6.8 kilograms. A sample of 20 pieces of the thread has an average tensile strength of 72.8

Q1.(II). The tensile strength of type II thread is approximately normally distributed with standard deviation of 6.8 kilograms. A sample of 25 pieces of the thread has an average tensile strength of 64.4 kilograms. Then for the 98% confidence interval of the difference in tensile strength means between type I and type II , we have:

$$\text{Thread 1 : } n_1 = 20, \bar{X}_1 = 72.8, \sigma_1 = 6.8$$

$$\text{Thread 2 : } n_2 = 25, \bar{X}_2 = 64.4, \sigma_2 = 6.8$$

$$98\% \rightarrow \alpha = 0.02 \rightarrow Z_{1-\frac{\alpha}{2}} = Z_{0.99} = 2.33$$

$$(\bar{X}_1 - \bar{X}_2) \pm \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \right)$$

$$(72.8 - 64.4) \pm \left(2.33 \times \sqrt{\frac{6.8^2}{20} + \frac{6.8^2}{25}} \right)$$

$$(3.65, 13.15)$$

(1): The lower limit = 3.65

(2): The upper limit = 13.15

	<i>First sample</i>	<i>Second sample</i>
<i>Sample size (n)</i>	12	14
<i>Sample mean (\bar{X})</i>	10.5	10
<i>Sample variance (S^2)</i>	4	5

(1) Estimate the difference $\mu_1 - \mu_2$:

$$E(\bar{X}_1 - \bar{X}_2) = \bar{X}_1 - \bar{X}_2 = 10.5 - 10 = 0.5$$

(2) Find the pooled estimator S_p :

$$S_p^2 = \frac{S_1^2(n_1-1) + S_2^2(n_2-1)}{n_1+n_2-2} = \frac{4(11) + 5(13)}{24} = 4.54 \Rightarrow S_p = 2.13$$

(3) The upper limit of 95% confidence interval for μ is:

$$95\% \rightarrow \alpha = 0.05 \rightarrow t_{1-\frac{\alpha}{2}, n_1+n_2-2} = t_{0.975, 24} = 2.064,$$

$$(\bar{X}_1 - \bar{X}_2) + \left(t_{1-\frac{\alpha}{2}, n_1+n_2-2} \times S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \right)$$

$$(0.5) + \left(2.064 \times 2.13 \sqrt{\frac{1}{12} + \frac{1}{14}} \right) = 2.23$$

(4) The lower limit of 95% confidence interval for μ is:

$$(\bar{X}_1 - \bar{X}_2) - \left(t_{1-\frac{\alpha}{2}, n_1+n_2-2} \times S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \right)$$

$$(0.5) - \left(2.064 \times 2.13 \sqrt{\frac{1}{12} + \frac{1}{14}} \right) = -1.23$$

➤ A researcher was interested in comparing the mean score of female students μ_1 , with the mean score of male students μ_2 in a certain test. Assume the populations of score are normal with equal variances. Two independent samples gave the following results:

	Female	Male
Sample size	$n_1 = 5$	$n_2 = 7$
Mean	$\bar{x}_1 = 82.63$	$\bar{x}_2 = 80.04$
Variance	$s_1^2 = 15.05$	$s_2^2 = 20.79$

(1) The point estimate of $\mu_1 - \mu_2$ is:

- (A) 2.63 (B) -2.37 (C) 2.59 (D) 0.59

(2) The estimate of the pooled variance (s_p^2) is:

- (A) 17.994 (B) 18.494 (C) 17.794 (D) 18.094

(3) The upper limit of the 95% confidence interval for $\mu_1 - \mu_2$ is :

- (A) 26.717 (B) 7.525 (C) 7.153 (D) 8.2

(4) The lower limit of the 95% confidence interval for $\mu_1 - \mu_2$ is :

- (A) -21.54 (B) -2.345 (C) -3.02 (D) -1.973

Estimation and Confidence Interval: Single Proportion

* Point estimate for P is: $\frac{x}{n}$

* Interval estimate for P is: $\hat{p} \pm \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}\hat{q}}{n}} \right)$

Q1. A random sample of 200 students from a certain school showed that 15 students smoke. Let p be the proportion of smokers in the school.

1. Find a point Estimate for p.
2. Find 95% confidence interval for p.

Solution

$$n = 200 \text{ \& \ } x = 15$$

(1):

$$\hat{p} = \frac{x}{n} = \frac{15}{200} = 0.075 \rightarrow \hat{q} = 0.925$$

(2):

$$95\% \rightarrow \alpha = 0.05 \rightarrow Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$$

$$\hat{p} \pm \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}\hat{q}}{n}} \right) = 0.075 \pm \left(1.96 \times \sqrt{\frac{0.075 \times 0.925}{200}} \right)$$

The 95% confidence interval is: (0.038, 0.112)

➤ ***A researchers group has perfected a new treatment of a disease which they claim is very efficient. As evidence, they say that they have used the new treatment on 50 patients with the disease and cured 25 of them. To calculate a 95% confidence interval for the proportion of the cured.***

1. The point estimate of p is equal to:

- (A) 0.25 (B) 0.5 (C) 0.01 (D) 0.33

2. The reliability coefficient ($Z_{1-\frac{\alpha}{2}}$) is equal is :

- (A) 1.96 (B) 1.645 (C) 2.02 (D) 1.35

3. The 95% confidence interval is equal to:

- (A) (0.1114, 0.3886) (B) (0.3837, 0.6163) (C) (0.1614, 0.6386) (D) (0.3614, 0.6386)

Estimation and Confidence Interval: Two Proportions

* Point estimate for $P_1 - P_2 = \hat{p}_1 - \hat{p}_2 = \frac{x_1}{n_1} - \frac{x_2}{n_2}$

* Interval estimate for $P_1 - P_2$ is: $(\hat{p}_1 - \hat{p}_2) \pm \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} \right)$

Q3. A random sample of 100 students from school "A" showed that 15 students smoke. Another independent random sample of 200 students from school "B" showed that 20 students smoke. Let p_1 be the proportion of smokers in school "A" and p_2 is the proportion of smokers in school "B".

- (1) Find a point Estimate for $p_1 - p_2$.
- (2) Find 95% confidence interval for $p_1 - p_2$.

Solution

$$n_1 = 100 \quad x_1 = 15 \quad \rightarrow \quad \hat{p}_1 = \frac{15}{100} = \boxed{0.15} \quad \Rightarrow \quad \hat{q}_1 = 1 - 0.15 = \boxed{0.85}$$

$$n_2 = 200 \quad x_2 = 20 \quad \rightarrow \quad \hat{p}_2 = \frac{20}{200} = \boxed{0.10} \quad \Rightarrow \quad \hat{q}_2 = 1 - 0.10 = \boxed{0.90}$$

(1)

$$\hat{p}_1 - \hat{p}_2 = 0.15 - 0.1 = 0.05$$

(2)

$$95\% \rightarrow \alpha = 0.05 \quad \rightarrow \quad Z_{1-\frac{\alpha}{2}} = Z_{0.975} = 1.96$$

$$(\hat{p}_1 - \hat{p}_2) \pm \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} \right)$$

$$= (0.05) \pm \left(1.96 \times \sqrt{\frac{(0.15)(0.85)}{100} + \frac{(0.1)(0.9)}{200}} \right)$$

$$= 0.05 \pm (1.96 \times \sqrt{0.001725})$$

The 95% confidence interval is: (-0.031, 0.131)

➤ In a first sample of 100 store customers, 43 used a MasterCard. In a second sample of 100 store customers, 58 used a Visa card. To find the 95% confidence interval for difference in the proportion ($\hat{p}_1 - \hat{p}_2$) of people who use each type of credit card?

1. The value of α is :

- (A) 0.95 (B) 0.5 (C) 0.05 (D) 0.025

2. The upper limit of 95% confidence interval for the proportion difference is:

$$n_1 = 100 \quad x_1 = 43 \rightarrow \hat{p}_1 = \frac{43}{100} = \boxed{0.43} \Rightarrow \hat{q}_1 = 1 - 0.43 = \boxed{0.57}$$

$$n_2 = 100 \quad x_2 = 58 \rightarrow \hat{p}_2 = \frac{58}{100} = \boxed{0.58} \Rightarrow \hat{q}_2 = 1 - 0.58 = \boxed{0.42}$$

$$(\hat{p}_1 - \hat{p}_2) + \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}} \right)$$

$$= (0.43 - 0.58) + \left(1.96 \times \sqrt{\frac{(0.43)(0.57)}{100} + \frac{(0.58)(0.42)}{100}} \right)$$

- (A) 0.137 (B) -0.013 (C) 0.518 (D) 0.150

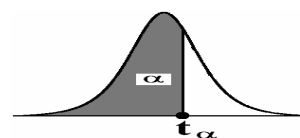
3. The lower limit of 95% confidence interval for the proportion difference is:

$$(\hat{p}_1 - \hat{p}_2) - \left(Z_{1-\frac{\alpha}{2}} \times \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}} \right)$$

$$= (0.05) - \left(1.96 \times \sqrt{\frac{(0.15)(0.85)}{100} + \frac{(0.1)(0.9)}{200}} \right)$$

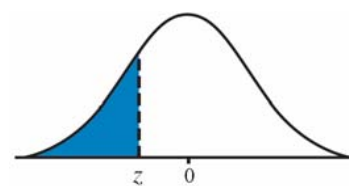
- (A) -0.278 (B) 1.547 (C) 0.421 (D) -0.129

Critical Values of the t-distribution (t_α)



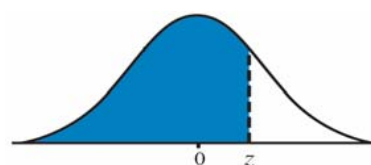
v=df	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$	$t_{0.995}$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
35	1.3062	1.6896	2.0301	2.4377	2.7238
40	1.3030	1.6840	2.0210	2.4230	2.7040
45	1.3006	1.6794	2.0141	2.4121	2.6896
50	1.2987	1.6759	2.0086	2.4033	2.6778
60	1.2958	1.6706	2.0003	2.3901	2.6603
70	1.2938	1.6669	1.9944	2.3808	2.6479
80	1.2922	1.6641	1.9901	2.3739	2.6387
90	1.2910	1.6620	1.9867	2.3685	2.6316
100	1.2901	1.6602	1.9840	2.3642	2.6259
120	1.2886	1.6577	1.9799	2.3578	2.6174
140	1.2876	1.6558	1.9771	2.3533	2.6114
160	1.2869	1.6544	1.9749	2.3499	2.6069
180	1.2863	1.6534	1.9732	2.3472	2.6034
200	1.2858	1.6525	1.9719	2.3451	2.6006
∞	1.282	1.645	1.960	2.326	2.576

Standard Normal Table
Areas Under the Standard Normal Curve



z	-0.09	-0.08	-0.07	-0.06	-0.05	-0.04	-0.03	-0.02	-0.01	-0.00	z
-3.50	0.00017	0.00017	0.00018	0.00019	0.00019	0.00020	0.00021	0.00022	0.00022	0.00023	-3.50
-3.40	0.00024	0.00025	0.00026	0.00027	0.00028	0.00029	0.00030	0.00031	0.00032	0.00034	-3.40
-3.30	0.00035	0.00036	0.00038	0.00039	0.00040	0.00042	0.00043	0.00045	0.00047	0.00048	-3.30
-3.20	0.00050	0.00052	0.00054	0.00056	0.00058	0.00060	0.00062	0.00064	0.00066	0.00069	-3.20
-3.10	0.00071	0.00074	0.00076	0.00079	0.00082	0.00084	0.00087	0.00090	0.00094	0.00097	-3.10
-3.00	0.00100	0.00104	0.00107	0.00111	0.00114	0.00118	0.00122	0.00126	0.00131	0.00135	-3.00
-2.90	0.00139	0.00144	0.00149	0.00154	0.00159	0.00164	0.00169	0.00175	0.00181	0.00187	-2.90
-2.80	0.00193	0.00199	0.00205	0.00212	0.00219	0.00226	0.00233	0.00240	0.00248	0.00256	-2.80
-2.70	0.00264	0.00272	0.00280	0.00289	0.00298	0.00307	0.00317	0.00326	0.00336	0.00347	-2.70
-2.60	0.00357	0.00368	0.00379	0.00391	0.00402	0.00415	0.00427	0.00440	0.00453	0.00466	-2.60
-2.50	0.00480	0.00494	0.00508	0.00523	0.00539	0.00554	0.00570	0.00587	0.00604	0.00621	-2.50
-2.40	0.00639	0.00657	0.00676	0.00695	0.00714	0.00734	0.00755	0.00776	0.00798	0.00820	-2.40
-2.30	0.00842	0.00866	0.00889	0.00914	0.00939	0.00964	0.00990	0.01017	0.01044	0.01072	-2.30
-2.20	0.01101	0.01130	0.01160	0.01191	0.01222	0.01255	0.01287	0.01321	0.01355	0.01390	-2.20
-2.10	0.01426	0.01463	0.01500	0.01539	0.01578	0.01618	0.01659	0.01700	0.01743	0.01786	-2.10
-2.00	0.01831	0.01876	0.01923	0.01970	0.02018	0.02068	0.02118	0.02169	0.02222	0.02275	-2.00
-1.90	0.02330	0.02385	0.02442	0.02500	0.02559	0.02619	0.02680	0.02743	0.02807	0.02872	-1.90
-1.80	0.02938	0.03005	0.03074	0.03144	0.03216	0.03288	0.03362	0.03438	0.03515	0.03593	-1.80
-1.70	0.03673	0.03754	0.03836	0.03920	0.04006	0.04093	0.04182	0.04272	0.04363	0.04457	-1.70
-1.60	0.04551	0.04648	0.04746	0.04846	0.04947	0.05050	0.05155	0.05262	0.05370	0.05480	-1.60
-1.50	0.05592	0.05705	0.05821	0.05938	0.06057	0.06178	0.06301	0.06426	0.06552	0.06681	-1.50
-1.40	0.06811	0.06944	0.07078	0.07215	0.07353	0.07493	0.07636	0.07780	0.07927	0.08076	-1.40
-1.30	0.08226	0.08379	0.08534	0.08691	0.08851	0.09012	0.09176	0.09342	0.09510	0.09680	-1.30
-1.20	0.09853	0.10027	0.10204	0.10383	0.10565	0.10749	0.10935	0.11123	0.11314	0.11507	-1.20
-1.10	0.11702	0.11900	0.12100	0.12302	0.12507	0.12714	0.12924	0.13136	0.13350	0.13567	-1.10
-1.00	0.13786	0.14007	0.14231	0.14457	0.14686	0.14917	0.15151	0.15386	0.15625	0.15866	-1.00
-0.90	0.16109	0.16354	0.16602	0.16853	0.17106	0.17361	0.17619	0.17879	0.18141	0.18406	-0.90
-0.80	0.18673	0.18943	0.19215	0.19489	0.19766	0.20045	0.20327	0.20611	0.20897	0.21186	-0.80
-0.70	0.21476	0.21770	0.22065	0.22363	0.22663	0.22965	0.23270	0.23576	0.23885	0.24196	-0.70
-0.60	0.24510	0.24825	0.25143	0.25463	0.25785	0.26109	0.26435	0.26763	0.27093	0.27425	-0.60
-0.50	0.27760	0.28096	0.28434	0.28774	0.29116	0.29460	0.29806	0.30153	0.30503	0.30854	-0.50
-0.40	0.31207	0.31561	0.31918	0.32276	0.32636	0.32997	0.33360	0.33724	0.3409	0.34458	-0.40
-0.30	0.34827	0.35197	0.35569	0.35942	0.36317	0.36693	0.37070	0.37448	0.37828	0.38209	-0.30
-0.20	0.38591	0.38974	0.39358	0.39743	0.40129	0.40517	0.40905	0.41294	0.41683	0.42074	-0.20
-0.10	0.42465	0.42858	0.43251	0.43644	0.44038	0.44433	0.44828	0.45224	0.45620	0.46017	-0.10
-0.00	0.46414	0.46812	0.47210	0.47608	0.48006	0.48405	0.48803	0.49202	0.49601	0.50000	-0.00

Standard Normal Table (continued)
 Areas Under the Standard Normal Curve

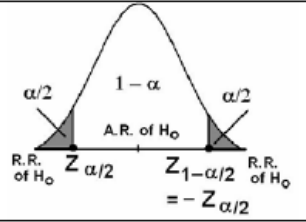
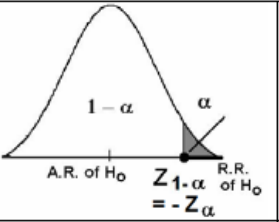
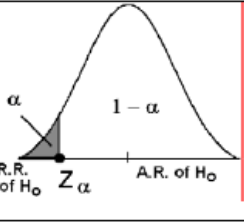


z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	z
0.00	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586	0.00
0.10	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535	0.10
0.20	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409	0.20
0.30	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173	0.30
0.40	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793	0.40
0.50	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240	0.50
0.60	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490	0.60
0.70	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524	0.70
0.80	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327	0.80
0.90	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891	0.90
1.00	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214	1.00
1.10	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298	1.10
1.20	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147	1.20
1.30	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774	1.30
1.40	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189	1.40
1.50	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408	1.50
1.60	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449	1.60
1.70	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327	1.70
1.80	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062	1.80
1.90	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670	1.90
2.00	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169	2.00
2.10	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574	2.10
2.20	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899	2.20
2.30	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158	2.30
2.40	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361	2.40
2.50	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520	2.50
2.60	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643	2.60
2.70	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736	2.70
2.80	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807	2.80
2.90	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861	2.90
3.00	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900	3.00
3.10	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929	3.10
3.20	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950	3.20
3.30	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965	3.30
3.40	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976	3.40
3.50	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983	3.50

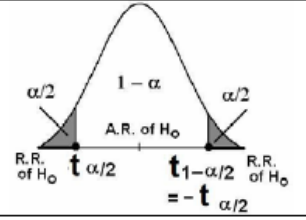
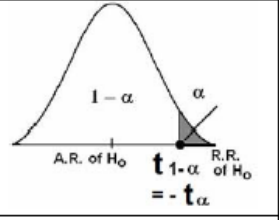
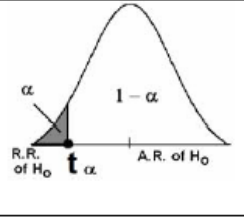
Hypotheses Testing

1-Single Mean

(if σ known):

Hypotheses	$H_0: \mu = \mu_0$ $H_A: \mu \neq \mu_0$	$H_0: \mu \leq \mu_0$ $H_A: \mu > \mu_0$	$H_0: \mu \geq \mu_0$ $H_A: \mu < \mu_0$
Test Statistic (T.S.)	Calculate the value of: $Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$		
R.R. & A.R. of H_0			
Critical value (s)	$Z_{\alpha/2}$ and $-Z_{\alpha/2}$	$Z_{1-\alpha} = -Z_{\alpha}$	Z_{α}
Decision:	We reject H_0 (and accept H_A) at the significance level α if:		
	$Z < Z_{\alpha/2}$ or $Z > Z_{1-\alpha/2} = -Z_{\alpha/2}$ Two-Sided Test	$Z > Z_{1-\alpha} = -Z_{\alpha}$ One-Sided Test	$Z < Z_{\alpha}$ One-Sided Test

(if σ unknown):

Hypotheses	$H_0: \mu = \mu_0$ $H_A: \mu \neq \mu_0$	$H_0: \mu \leq \mu_0$ $H_A: \mu > \mu_0$	$H_0: \mu \geq \mu_0$ $H_A: \mu < \mu_0$
Test Statistic (T.S.)	Calculate the value of: $t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} \sim t(n-1)$ (df = v = n-1)		
R.R. & A.R. of H_0			
Critical value (s)	$t_{\alpha/2}$ and $-t_{\alpha/2}$	$t_{1-\alpha} = -t_{\alpha}$	t_{α}
Decision:	We reject H_0 (and accept H_A) at the significance level α if:		
	$t < t_{\alpha/2}$ or $t > t_{1-\alpha/2} = -t_{\alpha/2}$ Two-Sided Test	$t > t_{1-\alpha} = -t_{\alpha}$ One-Sided Test	$t < t_{\alpha}$ One-Sided Test

Question 1:

Suppose that we are interested in estimating the true average time in seconds it takes an adult to open a new type of tamper-resistant aspirin bottle. It is known that the population standard deviation is $\sigma = 5.71$ seconds. A random sample of 40 adults gave a mean of 20.6 seconds. Let μ be the population mean, then, to test if the mean μ is 21 seconds at level of significance 0.05 ($H_0: \mu = 21$ vs $H_A: \mu \neq 21$) then:

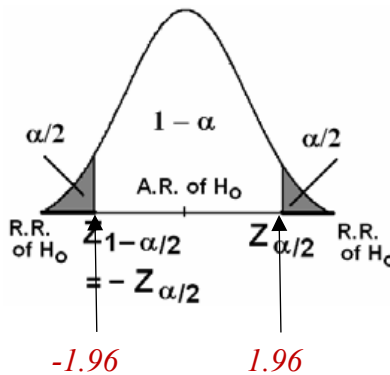
(1) The value of the test statistic is:

$$\sigma = 5.71 \quad n = 40 \quad \bar{X} = 20.6$$

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} = \frac{20.6 - 21}{5.71/\sqrt{40}} = -0.443$$

- (A) 0.443 (B) - 0.012 (C) -0.443 (D) 0.012

(2) The acceptance area is:



$$Z_{\frac{\alpha}{2}} = Z_{\frac{0.05}{2}} = Z_{0.025} = 1.96$$

- (A) (-1.96, 1.96) (B) (1.96, ∞) (C) ($-\infty$, 1.96) (D) ($-\infty$, 1.645)

$P - value = 2 \times P(Z < -0.443) = 2 \times (0.33) = 0.66$

(3) The decision is:

- (A) Reject H_0 (B) Accept H_0 (C) no decision (D) None of these

Question 2:

If the hemoglobin level of pregnant women (امراه حامل) is normally distributed, and if the mean and standard deviation of a sample of 25 pregnant women were $\bar{X} = 13$ (g/dl), $s = 2$ (g/dl). Using $\alpha = 0.05$, to test if the average hemoglobin level for the pregnant women is greater than 10 (g/dl) [$H_0 : \mu \leq 10$, $H_A : \mu > 10$].

(1) The test statistic is:

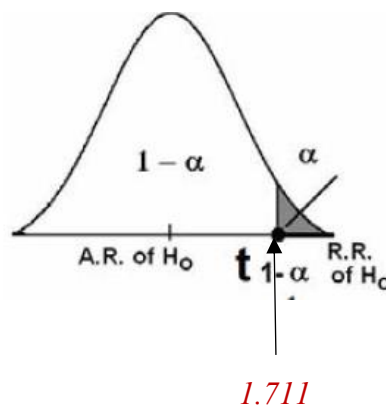
(A) $Z = \frac{\bar{X}-10}{\sigma/\sqrt{n}}$ (B) $Z = \frac{\bar{X}-10}{S/\sqrt{n}}$ (C) $t = \frac{\bar{X}-10}{\sigma/\sqrt{n}}$ (D) $t = \frac{\bar{X}-10}{S/\sqrt{n}}$

(2) The value of the test statistic is:

$$s = 2 \quad n = 25 \quad \bar{X} = 13$$
$$t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = \frac{13 - 10}{2/\sqrt{25}} = 7.5$$

- (A) 10 (B) 1.5 (C) 7.5 (D) 37.5

(3) The rejection of H_0 is :



$$t_{1-\alpha, n-1} = t_{0.95, 24} = 1.711$$

- (A) $Z < -1.645$ (B) $z > 1.645$ (C) $t < -1.711$ (D) $t > 1.711$

(4) The decision is:

- (A) Reject H_0 (B) Do not reject (Accept) H_0
(C) Accept both H_0 and H_A (D) Reject both H_0 and H_A

2-Two Means:

Hypotheses	$H_0: \mu_1 - \mu_2 = 0$ $H_A: \mu_1 - \mu_2 \neq 0$	$H_0: \mu_1 - \mu_2 \leq 0$ $H_A: \mu_1 - \mu_2 > 0$	$H_0: \mu_1 - \mu_2 \geq 0$ $H_A: \mu_1 - \mu_2 < 0$
Test Statistic For the First Case:	$Z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0,1)$ {if σ_1^2 and σ_2^2 are known}		
R.R. and A.R. of H_0 (For the First Case)			
Test Statistic For the Second Case:	$T = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}} \sim t(n_1+n_2-2)$ {if $\sigma_1^2 = \sigma_2^2 = \sigma^2$ is unknown}		
R.R. and A.R. of H_0 (For the Second Case)			
Decision:	Reject H_0 (and accept H_A) at the significance level α if:		
	T.S. \in R.R. Two-Sided Test	T.S. \in R.R. One-Sided Test	T.S. \in R.R. One-Sided Test

Question 1:

A standardized chemistry test was given to 50 girls and 75 boys. The girls made an average of 84, while the boys made an average grade of 82. Assume the population standard deviations are 6 and 8 for girls and boys respectively. To test the null hypothesis $H_0: \mu_1 - \mu_2 \leq 0$ against the alternative hypothesis $H_A: \mu_1 - \mu_2 > 0$ at 0.05 level of significance:

(1) The standard error of $(\bar{X}_1 - \bar{X}_2)$ is:

$$\begin{aligned} \text{girls: } n_1 &= 50, \bar{X}_1 = 84, \sigma_1 = 6 \\ \text{boys: } n_2 &= 75, \bar{X}_2 = 82, \sigma_2 = 8 \end{aligned}$$

$$S.E(\bar{X}_1 - \bar{X}_2) = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{6^2}{50} + \frac{8^2}{75}} = 1.2543$$

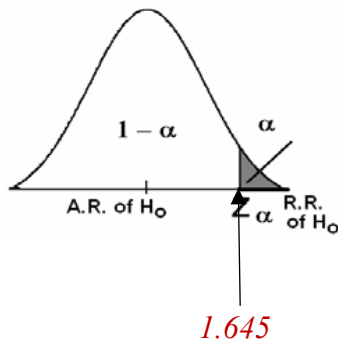
- (A) 0.2266 (B) 2 (C) 1.5733 (D) 1.2543

(2) The value of the test statistic is:

$$Z = \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(84 - 82)}{\sqrt{\frac{6^2}{50} + \frac{8^2}{75}}} = \frac{2}{1.2543} = 1.5945$$

- (A) -1.59 (B) 1.59 (C) 1.25 (D) 4.21

(3) The rejection region (RR) of H_0 is:



$$Z_{1-\alpha} = Z_{1-0.05} = Z_{0.95} = 1.645$$

- (A) (1.645, ∞) (B) $(-\infty, -1.645)$ (C) (1.96, ∞) (D) $(-\infty, -1.96)$

$$P\text{-value} = P(Z > 1.5945) = 1 - P(Z < 1.59) = 1 - 0.9441 = 0.0559$$

(4) The decision is:

- (A) Reject H_0 (B) Do not reject (Accept) H_0
(C) Accept both H_0 and H_A (D) Reject both H_0 and H_A

Question 2:

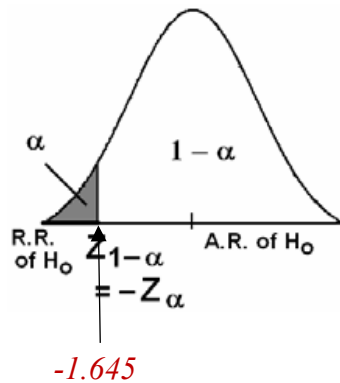
Cortisol level determinations were made on two samples of women at childbirth. Group 1 subjects underwent emergency cesarean section following induced labor. Group 2 subjects natural childbirth route following spontaneous labor. The sample sizes, mean cortisol levels, and standard deviations were $(n_1 = 40, \bar{x}_1 = 575, \sigma_1 = 70)$, $(n_2 = 44, \bar{x}_2 = 610, \sigma_2 = 80)$. If we are interested to test if the mean Cortisol level of group 1 (μ_1) is less than that of group 2 (μ_2) at level 0.05 (or $H_0: \mu_1 \geq \mu_2$ vs $H_1: \mu_1 < \mu_2$), then:

(1) The value of the test statistic is:

$$Z = \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(575 - 610)}{\sqrt{\frac{70^2}{40} + \frac{80^2}{44}}} = -2.138$$

- (A) -1.326 (B) -2.138 (C) -2.576 (D) -1.432

(2) Reject H_0 if :



$$Z_{1-\alpha} = Z_{0.95} = 1.645$$

- (A) $Z > 1.645$ (B) $T > 1.98$ (C) $Z < -1.645$ (D) $T < -1.98$

$$P\text{-value} = P(Z < -2.138) = P(Z < -2.14) = 0.0162$$

(3) The decision is:

- (A) Reject H_0 (B) Accept H_0 (C) no decision (D) none of these

Question 3:

An experiment was conducted to compare time length (duration time in minutes) of two types of surgeries (A) and (B). 10 surgeries of type (A) and 8 surgeries of type (B) were performed. The data for both samples is shown below.

Surgery type	A	B
Sample size	10	8
Sample mean	14.2	12.8
Sample standard deviation	1.6	2.5

Assume that the two random samples were independently selected from two normal populations with equal variances. If μ_A and μ_B are the population means of the time length of surgeries of type (A) and type (B), then, to test if μ_A is greater than μ_B at level of significant 0.05 ($H_0: \mu_A \leq \mu_B$ vs $H_A: \mu_A > \mu_B$) then:

(4) The value of the test statistic is:

$$Sp^2 = \frac{S_1^2(n_1 - 1) + S_2^2(n_2 - 1)}{n_1 + n_2 - 2} = \frac{1.6^2(10 - 1) + 2.5^2(8 - 1)}{10 + 8 - 2} = 4.174$$

$$t = \frac{(\bar{X}_1 - \bar{X}_2)}{Sp \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{(14.2 - 12.8)}{\sqrt{4.174} \sqrt{\frac{1}{10} + \frac{1}{8}}} = 1.44$$

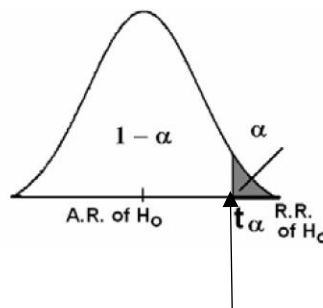
(A) -1.44

(B) 1.44

(C) - 0.685

(D) 0.685

(5) Reject H_0 if :



1.746

$$t_{1-\alpha, n_1+n_2-2} = t_{1-0.05, 10+8-2} = t_{0.95, 16} = 1.746$$

(A) $Z > 1.645$

(B) $Z < -1.645$

(C) $T > 1.746$

(D) $T < -1.746$

(6) The decision is:

(A) Reject H_0

(B) Accept H_0

(C) no decision

(D) none of these

Question 4:

A researcher was interested in comparing the mean score of female students μ_1 , with the mean score of male students μ_2 in a certain test. Assume the populations of score are normal with equal variances. Two independent samples gave the following results:

	Female	male
Sample size	$n_1 = 5$	$n_2 = 7$
Mean	$\bar{x}_1 = 82.63$	$\bar{x}_2 = 80.04$
Variance	$s_1^2 = 15.05$	$s_2^2 = 20.79$

Test that is there is a difference between the mean score of female students and the mean score of male students.

(1) The hypotheses are:

- (A) $H_0: \mu_1 = \mu_2$ (B) $H_0: \mu_1 = \mu_2$ (C) $H_0: \mu_1 < \mu_2$ (D) $H_0: \mu_1 \leq \mu_2$
 $H_A: \mu_1 \neq \mu_2$ $H_A: \mu_1 < \mu_2$ $H_A: \mu_1 > \mu_2$ $H_A: \mu_1 > \mu_2$

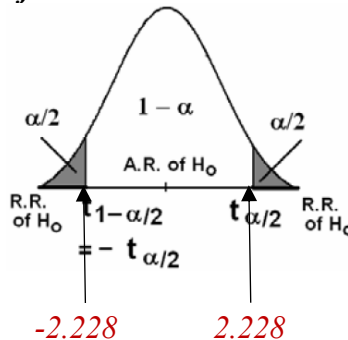
(2) The value of the test statistic is:

$$Sp^2 = \frac{S_1^2(n_1 - 1) + S_2^2(n_2 - 1)}{n_1 + n_2 - 2} = \frac{15.05(4) + 20.79(6)}{5 + 7 - 2} = 18.494$$

$$t = \frac{(\bar{X}_1 - \bar{X}_2)}{Sp \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{82.63 - 80.04}{\sqrt{18.494} \sqrt{\frac{1}{5} + \frac{1}{7}}} = 1.029$$

- (A) 1.3 (B) 1.029 (C) 0.46 (D) 0.93

(4) The acceptance region (AR) of H_0 is:



$$t_{1-\frac{\alpha}{2}, n_1+n_2-2} = t_{1-\frac{0.05}{2}, 5+7-2} = t_{0.975, 10} = 2.228$$

- (A) $(2.2281, \infty)$ (B) $(-\infty, -2.2281)$
(C) $(-2.228, 2.228)$ (D) $(-1.96, 1.96)$

Question 5:

A nurse researcher wished to know if graduates of baccalaureate nursing program and graduate of associate degree nursing program differ with respect to mean scores on personality inventory at $\alpha = 0.02$. A sample of 50 associate degree graduates (sample A) and a sample of 60 baccalaureate graduates (sample B) yielded the following means and standard deviations:

$$\begin{aligned}\bar{X}_A &= 88.12, S_A = 10.5, n_A = 50 \\ \bar{X}_B &= 83.25, S_B = 11.2, n_B = 60\end{aligned}$$

1) **The hypothesis is:**

- A) $H_0: \mu_1 \leq \mu_2$ vs $H_1: \mu_1 > \mu_2$ B) $H_0: \mu_1 \geq \mu_2$ vs $H_1: \mu_1 < \mu_2$
 C) $H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2$ D) None of the above.

2) **The test statistic is:**

- A) Z B) t C) F D) None of the above.

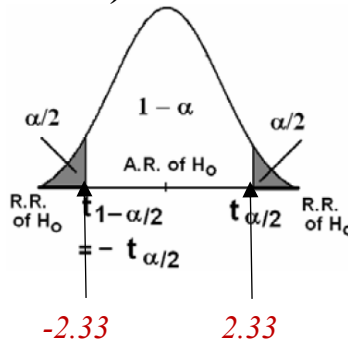
3) **The computed value of the test statistic is:**

$$Sp^2 = \frac{S_1^2(n_1 - 1) + S_2^2(n_2 - 1)}{n_1 + n_2 - 2} = \frac{10.5^2(50 - 1) + 11.2^2(60 - 1)}{50 + 60 - 2} = 118.55$$

$$t = \frac{(\bar{X}_1 - \bar{X}_2)}{Sp \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{88.12 - 83.25}{\sqrt{118.55} \sqrt{\frac{1}{50} + \frac{1}{60}}} = 48.19$$

- A) 2.72 B) 1.50 C) 1.86 D) 2.33

4) **The critical region (rejection area) is:**



$$t_{1-\frac{\alpha}{2}, n_1+n_2-2} = t_{1-\frac{0.02}{2}, 50+60-2} = t_{0.99, 108} = \mathbf{2.33}$$

- A) 2.60 Or -2.60 B) 2.06 Or -2.06 C) 2.33 Or - 2.33 D) 2.58

5) **Your decision is:**

- A) Accept & reject H_0 B) accept H_0 C) reject H_0 D) no decision.

Single proportion:

Hypotheses	$H_0: p = p_0$ $H_A: p \neq p_0$	$H_0: p \leq p_0$ $H_A: p > p_0$	$H_0: p \geq p_0$ $H_A: p < p_0$
Test Statistic (T.S.)	$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \sim N(0,1)$		
R.R. & A.R. of H_0			
Decision:	Reject H_0 (and accept H_A) at the significance level α if:		
	$Z < Z_{\alpha/2}$ or $Z > Z_{1-\alpha/2} = -Z_{\alpha/2}$ Two-Sided Test	$Z > Z_{1-\alpha} = -Z_{\alpha}$ One-Sided Test	$Z < Z_{\alpha}$ One-Sided Test

Question 1:

Toothpaste (معجون الأسنان) company claims that more than 75% of the dentists recommend their product to the patients. Suppose that 161 out of 200 dental patients reported receiving a recommendation for this toothpaste from their dentist. Do you suspect that the proportion is actually more than 75%. If we use 0.05 level of significance to test $H_0: P \leq 0.75$, $H_A: P > 0.75$, then:

(1) The sample proportion \hat{p} is:

$$n = 200, \hat{p} = \frac{161}{200} = 0.8050$$

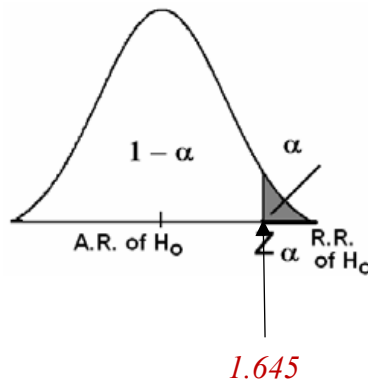
- (A) 0.75 (B) 0.195 (C) 0.805 (D) 0.25

(2) The value of the test statistic is:

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.805 - 0.75}{\sqrt{\frac{(0.75)(0.25)}{200}}} = 1.7963$$

- (A) 1.963 (B) 1.796 (C) -1.796 (D) -1.963

(3) The decision is:



$$\alpha = 0.05 \rightarrow Z_{1-\alpha} = Z_{0.95} = 1.645$$

- (A) Reject H_0 (B) Do not reject (Accept) H_0
(C) Accept both H_0 and H_A (D) Reject both H_0 and H_A

$P - \text{value} = P(Z > 1.7963) = P(Z > 1.80) = 1 - P(Z < 1.80) = 1 - 0.9641 = 0.0359$
--

Question 2:

A researcher was interested in studying the obesity (السمنة) disease in a certain population. A random sample of 400 people was taken from this population. It was found that 152 people in this sample have the obesity disease. If p is the population proportion of people who are obese. Then, to test if p is greater than 0.34 at level 0.05 ($H_0: p \leq 0.34$ vs $H_A: p > 0.34$) then:

(1) The value of the test statistic is:

$$n = 400, \hat{p} = \frac{152}{400} = 0.38$$

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.38 - 0.34}{\sqrt{\frac{0.34 \times 0.66}{400}}} = 1.69$$

- (A) 0.023 (B) 1.96 (C) 2.50 (D) 1.69

(2) The P-value is

$$P - \text{value} = P(Z > 1.96) = 1 - P(Z < 1.96) = 1 - 0.9545 = 0.0455$$

- (A) 0.9545 (B) 0.0910 (C) 0.0455 (D) 1.909

(3) The decision is:

$$P - \text{value} = 0.0455 < 0.05$$

- (A) Reject H_0 (B) Accept H_0 (C) no decision (D) none of these

Two proportions:

Hypotheses	$H_0: p_1 - p_2 = 0$ $H_A: p_1 - p_2 \neq 0$	$H_0: p_1 - p_2 \leq 0$ $H_A: p_1 - p_2 > 0$	$H_0: p_1 - p_2 \geq 0$ $H_A: p_1 - p_2 < 0$
Test Statistic (T.S.)	$Z = \frac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\frac{\bar{p}(1-\bar{p})}{n_1} + \frac{\bar{p}(1-\bar{p})}{n_2}}} \sim N(0,1)$		
R.R. and A.R. of H_0			
Decision:	Reject H_0 (and accept H_1) at the significance level α if $Z \in R.R.$:		
Critical Values	$Z > Z_{\alpha/2}$ or $Z < -Z_{\alpha/2}$ Two-Sided Test	$Z > Z_{\alpha}$ One-Sided Test	$Z < -Z_{\alpha}$ One-Sided Test

Question 1:

In a first sample of 200 men, 130 said they used seat belts and a second sample of 300 women, 150 said they used seat belts. To test the claim that men are more safety-conscious than women ($H_0: p_1 - p_2 \leq 0, H_1: p_1 - p_2 > 0$), at 0.05 level of significant:

(1) The value of the test statistic is:

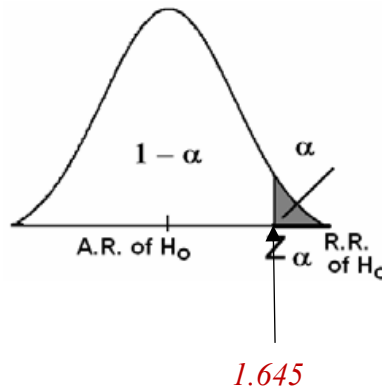
$$n_1 = 200, \hat{p}_1 = \frac{130}{200} = 0.65 \quad n_2 = 300, \hat{p}_2 = \frac{150}{300} = 0.5$$

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{130 + 150}{200 + 300} = 0.56$$

$$Z = \frac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(0.65 - 0.5)}{\sqrt{(0.56)(0.44)\left(\frac{1}{200} + \frac{1}{300}\right)}} = 3.31$$

- (A) -3.31 (B) 5.96 (C) 1.15 (D) 3.31

(2) The decision is:



$$Z_{1-\alpha} = Z_{1-0.05} = Z_{0.95} = 1.645$$

- (A) Reject H_0 (B) Do not reject (Accept) H_0
(C) Accept both H_0 and H_A (D) Reject both H_0 and H_A

$P - \text{value} = P(Z > 3.31) = 1 - P(Z < 3.31) = 1 - 0.9995 = 0.0005$
--

Question 2:

In a study of diabetes, the following results were obtained from samples of males and females between the ages of 20 and 75. Male sample size is 300 of whom 129 are diabetes patients, and female sample size is 200 of whom 50 are diabetes patients. If P_M, P_F are the diabetes proportions in both populations and \hat{p}_M, \hat{p}_F are the sample proportions, then:
A researcher claims that the Proportion of diabetes patients is found to be more in males than in female ($H_0: P_M - P_F \leq 0$ vs $H_A: P_M - P_F > 0$). Do you agree with his claim, take $\alpha = 0.10$

(1) The pooled proportion is:

$$\hat{p} = \frac{x_m + x_f}{n_m + n_f} = \frac{129 + 50}{300 + 200} = 0.358$$

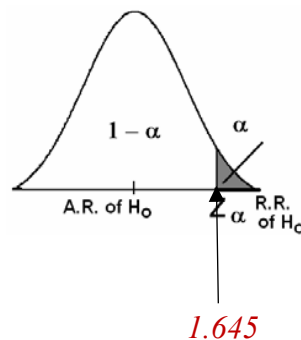
- (A) 0.43 (B) 0.18 (C) 0.358 (D) 0.68

(2) The value of the test statistic is:

$$Z = \frac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(0.43 - 0.25)}{\sqrt{(0.358)(1 - 0.358)\left(\frac{1}{300} + \frac{1}{200}\right)}} = 0.411$$

- (A) -4.74 (B) 4.74 (C) 4.11 (D) -4.11

(3) The decision is:

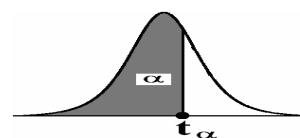


$$Z_{1-\alpha} = Z_{1-0.05} = Z_{0.95} = 1.645$$

- (A) Agree with the claim (B) do not agree with the claim (C) Can't say

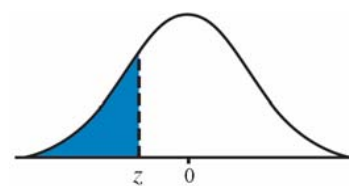
$P - value = P(Z > 0.41) = 1 - P(Z < 0.41) = 1 - 0.6591 = 0.3409$

Critical Values of the t-distribution (t_α)



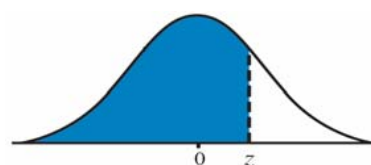
v=df	$t_{0.90}$	$t_{0.95}$	$t_{0.975}$	$t_{0.99}$	$t_{0.995}$
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
35	1.3062	1.6896	2.0301	2.4377	2.7238
40	1.3030	1.6840	2.0210	2.4230	2.7040
45	1.3006	1.6794	2.0141	2.4121	2.6896
50	1.2987	1.6759	2.0086	2.4033	2.6778
60	1.2958	1.6706	2.0003	2.3901	2.6603
70	1.2938	1.6669	1.9944	2.3808	2.6479
80	1.2922	1.6641	1.9901	2.3739	2.6387
90	1.2910	1.6620	1.9867	2.3685	2.6316
100	1.2901	1.6602	1.9840	2.3642	2.6259
120	1.2886	1.6577	1.9799	2.3578	2.6174
140	1.2876	1.6558	1.9771	2.3533	2.6114
160	1.2869	1.6544	1.9749	2.3499	2.6069
180	1.2863	1.6534	1.9732	2.3472	2.6034
200	1.2858	1.6525	1.9719	2.3451	2.6006
∞	1.282	1.645	1.960	2.326	2.576

Standard Normal Table
Areas Under the Standard Normal Curve



z	-0.09	-0.08	-0.07	-0.06	-0.05	-0.04	-0.03	-0.02	-0.01	-0.00	z
-3.50	0.00017	0.00017	0.00018	0.00019	0.00019	0.00020	0.00021	0.00022	0.00022	0.00023	-3.50
-3.40	0.00024	0.00025	0.00026	0.00027	0.00028	0.00029	0.00030	0.00031	0.00032	0.00034	-3.40
-3.30	0.00035	0.00036	0.00038	0.00039	0.00040	0.00042	0.00043	0.00045	0.00047	0.00048	-3.30
-3.20	0.00050	0.00052	0.00054	0.00056	0.00058	0.00060	0.00062	0.00064	0.00066	0.00069	-3.20
-3.10	0.00071	0.00074	0.00076	0.00079	0.00082	0.00084	0.00087	0.00090	0.00094	0.00097	-3.10
-3.00	0.00100	0.00104	0.00107	0.00111	0.00114	0.00118	0.00122	0.00126	0.00131	0.00135	-3.00
-2.90	0.00139	0.00144	0.00149	0.00154	0.00159	0.00164	0.00169	0.00175	0.00181	0.00187	-2.90
-2.80	0.00193	0.00199	0.00205	0.00212	0.00219	0.00226	0.00233	0.00240	0.00248	0.00256	-2.80
-2.70	0.00264	0.00272	0.00280	0.00289	0.00298	0.00307	0.00317	0.00326	0.00336	0.00347	-2.70
-2.60	0.00357	0.00368	0.00379	0.00391	0.00402	0.00415	0.00427	0.00440	0.00453	0.00466	-2.60
-2.50	0.00480	0.00494	0.00508	0.00523	0.00539	0.00554	0.00570	0.00587	0.00604	0.00621	-2.50
-2.40	0.00639	0.00657	0.00676	0.00695	0.00714	0.00734	0.00755	0.00776	0.00798	0.00820	-2.40
-2.30	0.00842	0.00866	0.00889	0.00914	0.00939	0.00964	0.00990	0.01017	0.01044	0.01072	-2.30
-2.20	0.01101	0.01130	0.01160	0.01191	0.01222	0.01255	0.01287	0.01321	0.01355	0.01390	-2.20
-2.10	0.01426	0.01463	0.01500	0.01539	0.01578	0.01618	0.01659	0.01700	0.01743	0.01786	-2.10
-2.00	0.01831	0.01876	0.01923	0.01970	0.02018	0.02068	0.02118	0.02169	0.02222	0.02275	-2.00
-1.90	0.02330	0.02385	0.02442	0.02500	0.02559	0.02619	0.02680	0.02743	0.02807	0.02872	-1.90
-1.80	0.02938	0.03005	0.03074	0.03144	0.03216	0.03288	0.03362	0.03438	0.03515	0.03593	-1.80
-1.70	0.03673	0.03754	0.03836	0.03920	0.04006	0.04093	0.04182	0.04272	0.04363	0.04457	-1.70
-1.60	0.04551	0.04648	0.04746	0.04846	0.04947	0.05050	0.05155	0.05262	0.05370	0.05480	-1.60
-1.50	0.05592	0.05705	0.05821	0.05938	0.06057	0.06178	0.06301	0.06426	0.06552	0.06681	-1.50
-1.40	0.06811	0.06944	0.07078	0.07215	0.07353	0.07493	0.07636	0.07780	0.07927	0.08076	-1.40
-1.30	0.08226	0.08379	0.08534	0.08691	0.08851	0.09012	0.09176	0.09342	0.09510	0.09680	-1.30
-1.20	0.09853	0.10027	0.10204	0.10383	0.10565	0.10749	0.10935	0.11123	0.11314	0.11507	-1.20
-1.10	0.11702	0.11900	0.12100	0.12302	0.12507	0.12714	0.12924	0.13136	0.13350	0.13567	-1.10
-1.00	0.13786	0.14007	0.14231	0.14457	0.14686	0.14917	0.15151	0.15386	0.15625	0.15866	-1.00
-0.90	0.16109	0.16354	0.16602	0.16853	0.17106	0.17361	0.17619	0.17879	0.18141	0.18406	-0.90
-0.80	0.18673	0.18943	0.19215	0.19489	0.19766	0.20045	0.20327	0.20611	0.20897	0.21186	-0.80
-0.70	0.21476	0.21770	0.22065	0.22363	0.22663	0.22965	0.23270	0.23576	0.23885	0.24196	-0.70
-0.60	0.24510	0.24825	0.25143	0.25463	0.25785	0.26109	0.26435	0.26763	0.27093	0.27425	-0.60
-0.50	0.27760	0.28096	0.28434	0.28774	0.29116	0.29460	0.29806	0.30153	0.30503	0.30854	-0.50
-0.40	0.31207	0.31561	0.31918	0.32276	0.32636	0.32997	0.33360	0.33724	0.3409	0.34458	-0.40
-0.30	0.34827	0.35197	0.35569	0.35942	0.36317	0.36693	0.37070	0.37448	0.37828	0.38209	-0.30
-0.20	0.38591	0.38974	0.39358	0.39743	0.40129	0.40517	0.40905	0.41294	0.41683	0.42074	-0.20
-0.10	0.42465	0.42858	0.43251	0.43644	0.44038	0.44433	0.44828	0.45224	0.45620	0.46017	-0.10
-0.00	0.46414	0.46812	0.47210	0.47608	0.48006	0.48405	0.48803	0.49202	0.49601	0.50000	-0.00

Standard Normal Table (continued)
 Areas Under the Standard Normal Curve



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	z
0.00	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586	0.00
0.10	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535	0.10
0.20	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409	0.20
0.30	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173	0.30
0.40	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793	0.40
0.50	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240	0.50
0.60	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490	0.60
0.70	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524	0.70
0.80	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327	0.80
0.90	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891	0.90
1.00	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214	1.00
1.10	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298	1.10
1.20	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147	1.20
1.30	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774	1.30
1.40	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189	1.40
1.50	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408	1.50
1.60	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449	1.60
1.70	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327	1.70
1.80	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062	1.80
1.90	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670	1.90
2.00	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169	2.00
2.10	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574	2.10
2.20	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899	2.20
2.30	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158	2.30
2.40	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361	2.40
2.50	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520	2.50
2.60	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643	2.60
2.70	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736	2.70
2.80	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807	2.80
2.90	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861	2.90
3.00	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900	3.00
3.10	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929	3.10
3.20	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950	3.20
3.30	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965	3.30
3.40	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976	3.40
3.50	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983	3.50

- **Two Samples Test for Paired Observation**

Q1. The following contains the calcium levels of eleven test subjects at zero hours and three hours after taking a multi-vitamin containing calcium.

Pair	0 hour (X_i)	3 hours (Y_i)	Difference $D_i = X_i - Y_i$
1	17.0	17.0	0.0
2	13.2	12.9	0.3
3	35.3	35.4	-0.1
4	13.6	13.2	0.4
5	32.7	32.5	0.2
6	18.4	18.1	0.3
7	22.5	22.5	0.0
8	26.8	26.7	0.1
9	15.1	15.0	0.1

The sample mean and sample standard deviation of the differences D are 0.144 and 0.167, respectively. To test whether the data provide sufficient evidence to indicate a difference in mean calcium levels ($H_0: \mu_1 = \mu_2$ against $H_1: \mu_1 \neq \mu_2$) with $\alpha = 0.10$ we have: $\bar{D} = 0.144$, $S_d = 0.167$, $n = 9$

[1]. the reliability coefficient (the tabulated value) is:

$$t_{1-\frac{\alpha}{2}, n-1} = t_{1-\frac{0.1}{2}, 9-1} = t_{0.95, 8} = 1.860$$

[2]. the value of the test statistic is:

$$\begin{matrix} H_0: \mu_1 = \mu_2 \\ H_1: \mu_1 \neq \mu_2 \end{matrix} \Rightarrow \begin{matrix} H_0: \mu_1 - \mu_2 = 0 \\ H_1: \mu_1 - \mu_2 \neq 0 \end{matrix} \Rightarrow \begin{matrix} H_0: \mu_D = 0 \\ H_1: \mu_D \neq 0 \end{matrix}$$

$$T = \frac{\bar{D} - \mu_D}{S_d/\sqrt{n}} = \frac{0.144 - 0}{0.167/\sqrt{9}} = 2.5868$$

[3]. the decision is:

$$T = 2.5868 \notin AR: (-1.86, 1.86), \text{ then we Reject } H_0$$

Q2. Scientists and engineers frequently wish to compare two different techniques for measuring or determining the value of a variable. Reports the accompanying data on amount of milk ingested by each of 14 randomly selected infants.

Pair	DD method (X_i)	TW method (Y_i)	Difference $D_i = X_i - Y_i$
1	1509	1498	11
2	1418	1254	164
3	1561	1336	225
4	1556	1565	-9
5	2169	2000	169
6	1760	1318	442
7	1098	1410	-312
8	1198	1129	69
9	1479	1342	137
10	1281	1124	157
11	1414	1468	-54
12	1954	1604	350
13	2174	1722	452
14	2058	1518	540

1. The sample mean of the differences \bar{D} is:

$$\bar{D} = \frac{11+164+225-9+169+442-312+\dots+540}{14} = 167.21$$

(A) 167.21	(B) 0.71	(C) 0.61	(D) 0.31
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2. The sample standard deviation of the differences S_D is:

$$S_D = \sqrt{\frac{(D_i - \bar{D})^2}{n-1}} = 228.21$$

(A) 3.15	(B) -0.71	(C) 71.53	(D) 228.21
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3. The reliability coefficient to construct 90% confidence interval for the true average difference between intake values measured by the two methods μ_D is:

$$\text{The reliability coefficient} = t_{1-\frac{\alpha}{2}, n-1} = t_{0.95, 13} = 1.771$$

(A) 1.96	(B) 1.771	(C) 2.58	(D) 1.372
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4. The 90% lower limit for μ_D is:

$$\begin{aligned} &= \bar{D} - \left(t_{1-\frac{\alpha}{2}, n-1} \times \frac{S_D}{\sqrt{n}} \right) \\ &= 167.21 - \left(1.771 \times \frac{228.12}{\sqrt{14}} \right) = 59.19 \end{aligned}$$

(A) 24.92	(B) 22.55	(C) 59.19	(D) 44.96
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5. The 90% upper limit for μ_D is:

$$\begin{aligned} &= \bar{D} + \left(t_{1-\frac{\alpha}{2}, n-1} \times \frac{S_D}{\sqrt{n}} \right) \\ &= 167.21 + \left(1.771 \times \frac{228.12}{\sqrt{14}} \right) = 275.23 \end{aligned}$$

(A) 224.92	(B) 322.55	(C) 275.23	(D) 24.96
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To test $H_0: \mu_D = 0$ versus $H_A: \mu_D \neq 0$, $\alpha = 0.10$ as a level of significance we have:

6. The value of the test statistic is:

$$T = \frac{\bar{D} - \mu_D}{S_d / \sqrt{n}} = \frac{167.21 - 0}{228.12 / \sqrt{14}} = 2.74$$

(A) 2.74	(B) -0.7135	(C) -7.1530	(D) -0.3157
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7. The decision is:

$$2.74 \notin AR: (-1.771, 1.771)$$

- (A) Reject H_0 (B) Not reject H_0
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Q3. In a study of a surgical procedure used to decrease the amount of food that person can eat. A sample of 10 persons measures their weights before and after one year of the surgery, we obtain the following data:

Before surgery (X)	148	154	107	119	102	137	122	140	140	117
After surgery (Y)	78	133	80	70	70	63	81	60	85	120
$D_i = X_i - Y_i$	70	21	27	49	32	74	41	80	55	-3

We assume that the data comes from normal distribution.

➤ For 90% confidence interval for μ_D , where μ_D is the difference in the average weight before and after surgery.

1. The sample mean of the differences \bar{D} is:

$$\bar{D} = \frac{70+21+27+\dots+55-3}{10} = 44.6$$

2. The sample standard deviation of the differences S_D is:

$$S_D = \sqrt{\frac{(D_i - \bar{D})^2}{n-1}} = 26.2$$

3. The 90% upper limit of the confidence interval for μ_D is:

$$\begin{aligned} t_{1-\frac{\alpha}{2}, n-1} &= t_{0.95, 9} = 1.833 \\ &= \bar{D} + \left(t_{1-\frac{\alpha}{2}, n-1} \times \frac{S_D}{\sqrt{n}} \right) \\ &= 44.6 + \left(1.833 \times \frac{26.2}{\sqrt{10}} \right) = 59.38 \end{aligned}$$

➤ To test $H_0: \mu_D \geq 43$ versus $H_A: \mu_D < 43$, with $\alpha = 0.10$ as a level of significance, we have:

4. The value of the test statistic is:

$$T = \frac{\bar{D} - \mu_D}{S_d / \sqrt{n}} = \frac{44.6 - 43}{26.2 / \sqrt{10}} = \boxed{0.19}$$

5. The decision is:

$$-t_{1-\alpha, n-1} = -t_{0.90, 9} = -1.383 \Rightarrow \boxed{0.19 \notin RR: (-\infty, -1.383)}$$

(A) Reject H_0

(B) Do not reject H_0

(C) None of them