- We consider a tossing four identical fair coins at the same time. Now, let X be a random variable observe the number of heads that appears. Then:
  a. Determine the space of elementary events of this random experiment.
  b. Compare this result with the example 3.2.5. What do you notice?
  c. Determine the set $\mathbf{X}$ of possible values of the random variable X.
  d. Compare the set $\mathbf{X}$ for this random variable with the values set of X in the example 3.2.5. what do you notice?
  e. Determine the probability mass function $P(X = \bullet)$.

4 3 2 1 0

a) $\Omega = \{HHHH, HHHT, HHTT, HTTT, TTTT\}$ $|\Omega| = 5$

b) the elementry event not the same

c) $X = \{0, 1, 2, 3, 4\}$

H

3) the same

e)

X	0	1	2	3	4
$P(X=\cdot)$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

$\sum P(X=\cdot) = 1$ so mass

2- Consider rolling two fair different dice one time only (and at the same time), and let X be a disc... variable observe the minimum of two numbers that appears. Then:

a. Determine the space of elementary events of this random experiment.

b. Determine the set **X** of possible values of the random variable X.

c. Determine is the probability mass function $P(X = \bullet)$.

d. Represent the random variable X tabular and graphical.

a) $\Omega = \{$ (1,1) (1,2) (1,3) (1,4) (1,5) (1,6)

(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)

(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)

(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)

(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)

(6,1) (6,2) (6,3) (6,4) (6,5) (6,6) $\}$ $|\Omega| = 36$

b) $X = \{1, 2, 3, 4, 5, 6\}$

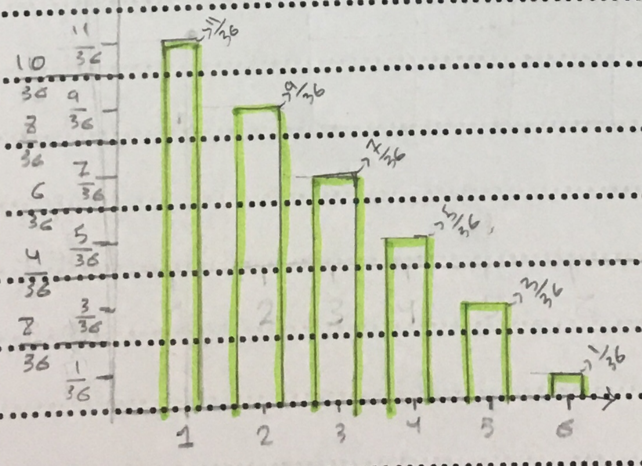
c)

X	1	2	3	4	5	6
P(X=·)	$\frac{11}{36}$	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{5}{36}$	$\frac{3}{36}$	$\frac{1}{36}$

Mass

$\sum P(X=\cdot) = 1$

d) $P_x (P_x(x=\cdot) \geq 0$



3- We consider the experiment of rolling two identical balanced dice at the same time and for one time only, and let X be a random variable observe the sum of the two appear numbers. Then:

a. Determine the space of elementary events of this random experiment.

b. Determine the set **X** of possible values of the random variable X.

c. Determine is the probability mass function $P(X=\bullet)$.

d. Calculate $\mathbf{E}(X)$, $\mathbf{E}(X^2)$, $\mathbf{var}(X)$ and the standard deviation of X.

A $\Omega = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$

$(2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$

$(3,3)\ (3,4)\ (3,5)\ (3,6)$

$(4,4)\ (4,5)\ (4,6)$ $\qquad |\Omega| = 21$

$(5,5)\ (5,6)$

$(6,6)\}$

B) $X = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

C)

x^2	4	9	16	25	36	49	64	81	100	121	144
x	2	3	4	5	6	7	8	9	10	11	12
$P(x=\cdot)$	$\frac{1}{21}$	$\frac{1}{21}$	$\frac{2}{21}$	$\frac{2}{21}$	$\frac{3}{21}$	$\frac{3}{21}$	$\frac{3}{21}$	$\frac{2}{21}$	$\frac{2}{21}$	$\frac{1}{21}$	$\frac{1}{21}$

$\sum (P=\cdot) = 1$

$P_X\ P(x=\cdot) \geq 0$

D) $E(x) = \sum x \cdot P(x=\cdot) = (2\cdot\frac{1}{21}) + (3\cdot\frac{1}{21}) + (4\cdot\frac{2}{21}) + (5\cdot\frac{2}{21}) + (6\cdot\frac{3}{21}) + (7\cdot\frac{3}{21}) + (8\cdot\frac{3}{21})$
$+ (9\cdot\frac{2}{21}) + (10\cdot\frac{2}{21}) + (11\cdot\frac{1}{21}) + (12\cdot\frac{2}{21}) = 7$

$E(x^2) = \sum x^2 \cdot P(x=\cdot) = (4\cdot\frac{1}{21}) + (9\cdot\frac{1}{21}) + (16\cdot\frac{2}{21}) + (25\cdot\frac{2}{21}) + (36\cdot\frac{3}{21}) + (49\cdot\frac{3}{21})$
$+ (64\cdot\frac{3}{21}) + (81\cdot\frac{2}{21}) + (100\times\frac{2}{21}) + (121\times\frac{1}{21}) + (144\times\frac{1}{21}) = 55.67$

$Var(x) = E(x^2) - [E(x)]^2 = 55.67 - (7)^2 = 6.67$

$\sigma = \sqrt{Var(x)} = \sqrt{6.67} = 2.58$

4- Let X be a random variable with probability mass function:

$$P(X=k)=\frac{c}{k} \quad ;k=2,4,6,8$$

Then:

a. Determine the value of the constant c that make $P(X=\bullet)$ probability density function.

b. Determine the distribution function of X. $F(x)$

c. Calculate the mathematical expectation $E(x)$ and variance $Var(x)$ of X and $X-8$.

A) if it density function $\Sigma = 1$ so $\frac{c}{2}+\frac{c}{4}+\frac{c}{6}+\frac{c}{8}=1$

$= \frac{c}{2}+\frac{c}{4}+\frac{c}{6}+\frac{c}{8}=1 = \frac{12c}{24}+\frac{6c}{24}+\frac{4c}{24}+\frac{3c}{24}=1$

$= 24 \cdot \frac{25c}{24} = 1 \cdot 24 = c = \frac{24}{25} = 0.96$

B)

x^2	4	16	36	64
X	2	4	6	8
$P(x=\circ)$	$\frac{0.96}{2}=\frac{12}{25}$	$\frac{6}{25}$	$\frac{4}{25}$	$\frac{3}{25}$

$F(x)$ $\Sigma P(x=\circ)=1$

$P = P(x=\circ) \geq 0$

B) $F(x) = \begin{cases} 0 & x<2 \\ \frac{12}{25} & 2\leq x<4 \\ \frac{18}{25} & 4\leq x<6 \\ \frac{22}{25} & 6\leq x<8 \\ \frac{25}{25}=1 & x\geq 8 \end{cases}$

$E(x-8) = E(x)-E(8) = 3.84-8 = -4.16$

C) $= E(x) = \Sigma x \, P(x=\circ) = (2 \cdot \frac{12}{25})+(4 \cdot \frac{6}{25})+(6 \cdot \frac{4}{25})+(8 \cdot \frac{3}{25}) = 3.84$

$- E(x^2) = \Sigma x^2 P(x=\circ) = (4 \cdot \frac{12}{25})+(16 \cdot \frac{6}{25})+(36 \cdot \frac{4}{25})+(64 \cdot \frac{3}{25}) = 19.2$

$- Var(x) = E(x^2) - (E(x))^2 = 19.2 - (3.84)^2 = 4.4544$

$Var(x-8) = Var(x) - Var(8) = 4.4544 - 0 = 4.4544$

5- We consider a discreet random variable X given by the following table:

x	-1	1	3	4	7	12
$P_x = P(X = x)$	0.20	0.15	0.10	0.10	0.20	0.25

a. Determine **X** the values set of X.

b. Determine is the probability mass function $P(X = \bullet)$.

c. Represent the random variable X graphical.

d. Determine the distribution function F_X and sketch it.

a) $X = \{-1, 1, 3, 4, 7, 12\}$

b)

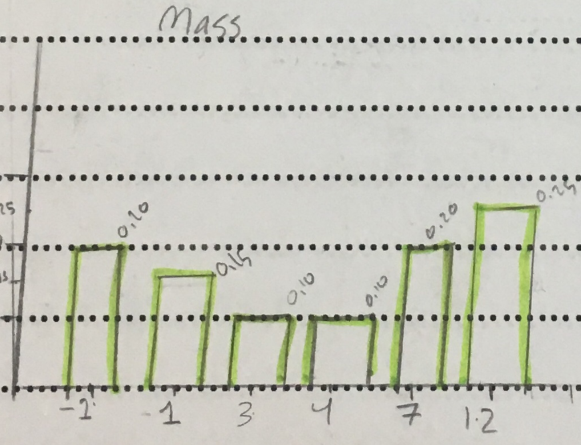
X	-1	1	3	4	7	12
P(x=x)	0.20	0.15	0.10	0.10	0.20	0.25

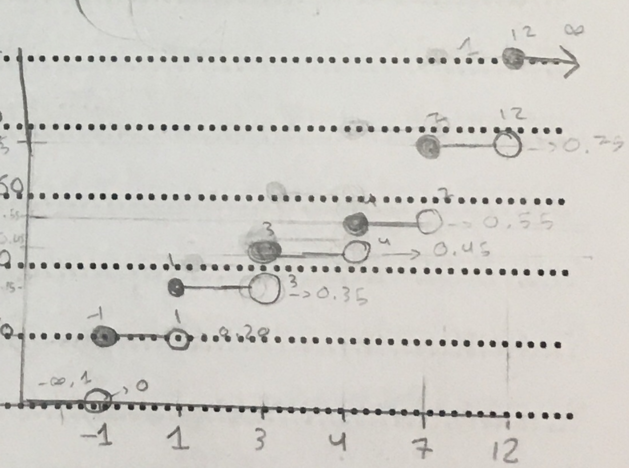
B) $P_x = P(X=x) \geq 0 = \Sigma P(X=x) = 1$

$0.20 + 0.15 + 0.10 + 0.10 + 0.20 + 0.25 = 1$

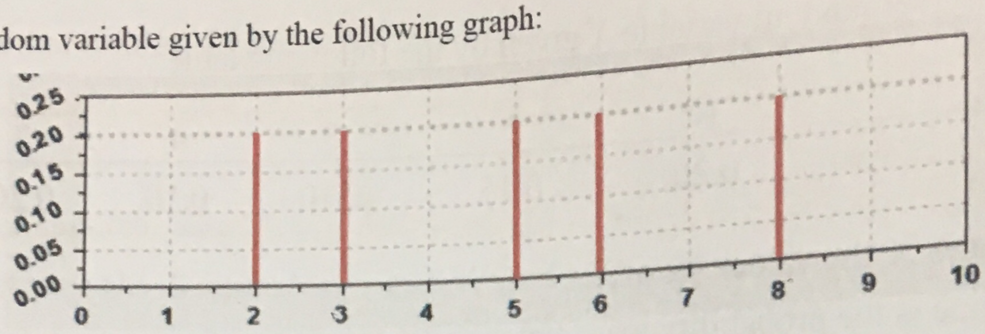
$$F(x) = \begin{cases} 0 & x < -1 \\ 0.20 & -1 \leq x < 1 \\ 0.35 & 1 \leq x < 3 \\ 0.45 & 3 \leq x < 4 \\ 0.55 & 4 \leq x < 7 \\ 0.75 & 7 \leq x < 12 \\ 1 & x \geq 12 \end{cases}$$

Mass





6- Let X be a random variable given by the following graph:



Then:

a. Represent the given random variable X tabular.

b. Determine is the probability mass function $P(X = \bullet)$.

c. As studied in this course. Is this random variable of famous random variables (has a special name)? If yes, what is it?

d. Determine the distribution function F_X and sketch it.

e. Calculate $\mathbf{E}(X)$, $\mathbf{E}(X^2)$, $\mathrm{var}(X)$ and the standard deviation of X.

a)

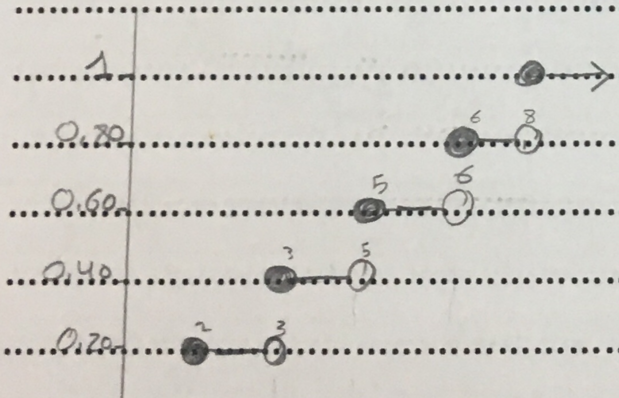
x^2	4	9	25	36	64
X	2	3	5	6	8
$P(X=\cdot)$	0.20	0.20	0.20	0.20	0.20

b) $P|P(X=\cdot)| \geq 0$; $\sum P(X=\cdot) = 1 = 0.20 + 0.20 + 0.20 + 0.20 + 0.20 = 1$

c) Yeas Uniform

d)

$$F(x) = \begin{cases} 0 & X < 2 \quad (-\infty, 2) \\ 0.20 & 2 \leq X < 3 \\ 0.20+0.20 = 0.40 & 3 \leq X < 5 \\ 0.60 & 5 \leq X < 6 \\ 0.80 & 6 \leq X < 8 \\ 1 & X \geq 8 \end{cases}$$



e) $E(X) = \sum x . P(X=\cdot) =$

$= 2(0.20) + 3(0.20) + 5(0.20) + 6(0.20) + 8(0.20)$

$= 4.8$

$E(x^2) = 4(0.20) + 9(0.20) + 25(0.20) + 36(0.20)$

$+ 64(0.20) = 27.6$

$Var(x) = E(x^2) - (E(x))^2 = 27.6 - (4.8)^2 = 4.56$

$\sigma = \sqrt{Var(x)} = \sqrt{4.56} = 2.135$

1- Let X be a given continuous random variable. Can we accept that the following function as probability density function for X, and why?

$$f_X(x) = \begin{cases} x-1 & \text{for } 0 \le x < 4 \\ 0 & \text{otherwise} \end{cases}$$

Probaility density function has to be $\int_{-\infty}^{+\infty} f_{(x)}\, \partial x = 1$

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$\int_{-\infty}^{0} f_{(x)} \partial x + \int_{0}^{4} f_{(x)} \partial x + \int_{4}^{\infty} f_{(x)} \partial x = 1$

$\int_{-\infty}^{0} 0 \, \partial x + \int_{0}^{4} (x-1) \partial x + \int_{4}^{\infty} f_{(x)} \partial x$

$= 0 + \left.\frac{x^2}{2} - x\right]_0^4 + 0$

$= \left(\frac{4^2}{2} - 4\right) - \left(\frac{0^2}{2} - 0\right) = 4$ So it is not density function

$\neq 1$

8- Let the time for a mechanical to finish the car repair (in hours) is a continuous random variable X with probability density function:

$$f_X(x) = \begin{cases} \frac{2}{9}(x-1) & \text{for } 1 \le x < 4 \\ 0 & \text{otherwise} \end{cases}$$

Where c is a constant. Then:

a. Determine the distribution function F_X.

b. What is the probability that he finishing the car repair in the interval $[1.5, 2]$?

c. Calculate the average time for the car repair and the standard deviation for the car repair.

a) $F(x) = \int f_X(x)\,dx \quad P(x<1) = \int_{-\infty} 0 = 0$

$P(1 \le x \le 4) = \int_{-\infty}^{1} f_X(x)\,dx + \int_{1}^{x} f_X(x)\,dx = \int_{1}^{x} \frac{2x-2}{9} = \frac{x^2-2x}{9}\Big|_1^x = \frac{x^2-2x}{9} - \frac{1-2}{9}$

$= \frac{x^2-2x}{9} - \left(\frac{-1}{9}\right) = \frac{x^2-2x+1}{9} = \frac{(x-1)^2}{9}$

$F(x) = \begin{cases} 0 & x < 1 \\ \frac{(x-1)^2}{9} & 1 \le x < 4 \\ 1 & x \ge 4 \end{cases}$ $\quad P(x \ge 4) = \int f(x)dx + \int f(x)dx + \int f(x)dx = 0 + \frac{x^2-2x}{9}\Big|_1^4 + 0 = \frac{4^2-2(4)}{9} - \frac{1-2}{9} = 1$

B) $P(1.5 \le x \le 2) = \int_{1.5}^{2} f(x)\,dx = \int_{1.5}^{2} \frac{2x}{9} - \frac{2}{9} = \frac{x^2}{9} - \frac{2x}{9}\Big|_{1.5}^{2} = \left(\frac{2^2}{9} - \frac{2(2)}{9}\right) - \left(\frac{1.5^2}{9} - \frac{2(1.5)}{9}\right)$

$= 0 - \left(-\frac{1}{12}\right) = \frac{1}{12}$

C) $\mu = E(x) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{1}^{4} x \cdot \left(\frac{2x}{9} - \frac{2}{9}\right) = \int_{1}^{4} \frac{2x^2}{9} - \frac{2x}{9}\,dx$

$E(x) = \frac{2x^3}{27} - \frac{x^2}{9}\Big|_1^4 = \left(\frac{2(4^3)}{27} - \frac{4^2}{9}\right) - \left(\frac{2(1^3)}{27} - \frac{1}{9}\right) = \frac{80}{27} - \frac{-1}{27} = 3$

$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_{1}^{4} x^2 \left(\frac{2x}{9} - \frac{2}{9}\right) dx = \int_{1}^{4} \frac{2x^3}{9} - \frac{2x^2}{9} = \frac{x^4}{18} - \frac{2x^3}{27}\Big|_1^4$

$= \left(\frac{(4)^4}{18} - \frac{2(4^3)}{27}\right) - \left(\frac{1^4}{18} - \frac{2(1)}{27}\right) = \frac{256}{27} - \left(\frac{-1}{54}\right) = 9.5$

$Var(x) = E(x^2) - (E(x))^2 = 9.5 - (3)^2 = 0.5 \quad \sigma = \sqrt{var(x)} = \sqrt{0.5} = \frac{\sqrt{2}}{2} = 0.7$

9- Let X be a random variable with density function f_X given by the following relation:

$$f_X(x)=\begin{cases}\alpha x & \text{for } 0\le x\le 1\\ 0 & \text{otherwise}\end{cases}$$

Where α is a constant. Then:

a) Determine the value of α and draw the graph of f_X.

c) Determine the distribution function F_X.

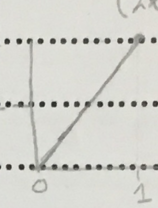
d) Determine the distribution function F_X and sketch it.

e) Calculate the following probabilities:

c-1) $P(-0.25 < X \le 0.25)$ c-2) $P(X > 1.75)$ c-3) $P(X = 0.5)$

f) Calculate $\mathbf{E}(X)$, $\mathbf{E}(X^2)$, $\mathbf{var}(X)$ and the standard deviation of X.

a) $\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx = 1$ a) 2 (2x)

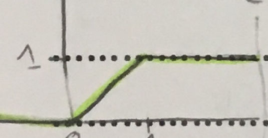


So $\int_{-\infty}^{0} 0\,dx + \int_{0}^{1} \alpha x\,dx + \int_{1}^{\infty} 0\,dx = 1$

$0 + \frac{\alpha x^2}{2}\Big|_0^1 + 0 = 1 = \frac{\alpha(1)^2}{2} - \frac{\alpha(0)^2}{2} = 1 \therefore \frac{\alpha}{2} = 1 \therefore \alpha = 2$

b)

$$F(x)=\begin{cases} 0 & x\le 0 \rightarrow P(x\le 0) = \int_{-\infty}^{0} 0 = 0 \\ x^2 & 0\le x\le 1 \rightarrow \int_0^x 2x = x^2\Big|_0^x = x^2 - 0 = x^2 \\ 1 & x\ge 1 \rightarrow P(x\ge 1) = \int_{-\infty}^{0} 0 + \int_0^1 2x + \int_1^\infty 0 = x^2\Big|_0^1 = 1 \end{cases}$$



e) $P(-0.25 < x \le 0.25) = \int_{-0.25}^{0} 0\,dx + \int_{0}^{0.25} 2x\,dx = 0 + x^2\Big|_0^{0.25} = (0.25)^2 - 0^2 = \frac{1}{16}$

$P(x \ge 1.75) = 1 - \int_{-\infty}^{0} f(x)\,dx + \int_0^1 f(x)\,dx + \int_1^x f(x)\,dx = 1 - 1 = 0$

$E(x) = \int_{-\infty}^{\infty} x\, f(x)\,dx = \int_0^1 x \cdot 2x\,dx = \int_0^1 2x^2\,dx = \frac{2x^3}{3}\Big|_0^1 = \frac{2}{3} - 0 = \frac{2}{3}$

$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx = \int_0^1 x^2\, 2x = \int_0^1 2x^3\,dx = \frac{x^4}{2}\Big|_0^1 = \frac{1}{2} - 0 = \frac{1}{2}$

$Var(x) = E(x^2) - (E(x))^2 = \frac{1}{2} - \left(\frac{2}{3}\right)^2 = \frac{1}{18}$ $\sigma = \sqrt{Var(x)} = \sqrt{\frac{1}{18}} = \frac{\sqrt{2}}{6} = 0.24$

$P(x = 0.5) = \int_{0.5}^{0.5} 2x\,dx = x^2\Big|_{0.5}^{0.5} = 0.5^2 - 0.5^2 = 0$