1) a. $_nP_r = \dfrac{n!}{(n-r)!}$

$$_7P_4 = \frac{7!}{(7-4)!} = \frac{7\times6\times5\times4\times\cancel{3!}}{\cancel{3!}} = 840$$

b. $_nC_r = \dfrac{n!}{r!(n-r)!}$

$$_{52}C_5 = \frac{52!}{5!(52-5)!} = \frac{52\times51\times50\times49\times48\times\cancel{47!}}{(5\times4\times3\times2\times1)(\cancel{47!})} = 2598960$$

c. $26\times26\times26\times10\times10\times10 = 26^3\times10^3 = 17576000$

2) a. $nCr = \dfrac{n!}{r!(n-r)!}$

$$_{12}C_4 = \frac{12!}{4!(12-4)!} = \frac{12\times11\times10\times9\times\cancel{8!}}{(4\times3\times2\times1)(\cancel{8!})} = 495$$

$$_4C_2 = \frac{4!}{2!(4-2)!} = \frac{4\times3\times\cancel{2!}}{(2\times1)(\cancel{2!})} = 6$$

$\therefore 6\cdot495 = 2970$

b.

$P(A) = \dfrac{n(A)}{N}$

$$= \frac{_{12}C_4\cdot{}_4C_2}{_{16}C_6} = \frac{2970}{8008} = 0.370$$

3) a. $P(A\setminus B) = P(A) - P(A\cap B) \rightarrow 0.15 = P(A) - 0.10 \rightarrow P(A) = 0.25$

$P(A\setminus C) = P(A) - P(A\cap C) \rightarrow 0.25 - 0.15 \rightarrow P(A\setminus C) = 0.10$

$P(B\setminus A) = P(B) - P(B\cap A) \rightarrow 0.30 = P(B) - 0.10 \rightarrow P(B) = 0.40$

$P(C\setminus A) = P(C) - P(A\cap C) \rightarrow 0.35 = P(C) - 0.15 \rightarrow P(C) = 0.50$

$P(B\setminus C) = P(B) - P(B\cap C) \rightarrow = 0.4 - 0.2 \rightarrow P(B\setminus C) = 0.2$

$P(C\setminus B) = P(C) - P(C\cap B) \rightarrow = 0.5 - 0.2 \rightarrow P(C\setminus B) = 0.30$

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{0.10}{0.40} = 0.25$$

$$P(A|B\cap C) = \frac{P(A\cap B\cap C)}{P(B\cap C)} = \frac{0.05}{0.20} = 0.25$$

3)

b. $P(A\cup B\cup C) = 0.25 + 0.40 + 0.5 - 0.10 - 0.15 - 0.20 + 0.05$

$= 0.75$

$P(\bar{A}\cap\bar{B}\cap\bar{C}) = P(\overline{A\cup B\cup C}) = 1 - P(A\cup B\cup C) = 1 - 0.75 = 0.25$

$P(A|B\cap C) = \dfrac{P(A\cap B\cap C)}{P(B\cap C)} = \dfrac{0.05}{0.20} = 0.25$

c. $P(A\cap B\cap C) = P(A)\cdot P(B)\cdot P(C)$

$0.05 = (0.25)\cdot(0.40)\cdot(0.5)$

$0.05 = 0.05$

$\therefore$ The events are independent

4) a. $P(E_1) = 0.15$
$P(E_2) = 0.2$
$P(E_3) = 0.25$
$P(E_4) = 0.4$

$P(B|E_1) = 0.01$
$P(B|E_2) = 0.02$
$P(B|E_3) = 0.03$
$P(B|E_4) = 0.04$

$P(B) = P(E_1)\cdot P(B|E_1) + P(E_2)\cdot P(B|E_2) + P(E_3)\cdot P(B|E_3) + P(E_4)\cdot P(B|E_4)$

$= 0.15\cdot 0.01 + 0.2\cdot 0.02 + 0.25\cdot 0.03 + 0.4\cdot 0.04$

$= 0.029$

$P(\bar{B}) = 1 - P(B) \to 1 - 0.029 \to = 0.971$

b. $P(E_4|B) = \dfrac{P(B|E_4)\cdot P(E_4)}{P(B)}$

$= \dfrac{0.04\cdot 0.4}{0.029} = 0.55$

5) a. $P(E \cup F) = P(E) + P(F) - P(E \cap F)$
$= 0.40 + 0.30 - 0.15$
$= 0.55$

b. $P(\overline{E} \cap \overline{F}) = P(\overline{E \cup F}) = 1 - P(E \cup F) \Rightarrow 1 - 0.55 = 0.45$

c. $P(E \cap \overline{F}) \cup P(\overline{E} \cap F) = (P(E) - P(E \cap F)) + (P(F) - P(E \cap F))$
$= (0.40 - 0.15) + (0.30 - 0.15)$

$= 0.25 + 0.15$
$= 0.40$

d. $P(E \cap \overline{F}) = P(E) - P(E \cap F)$
$= 0.40 - 0.15$
$= 0.25$

6) $P(A) = 0.23$
$P(\overline{A}) = 1 - P(A) \Rightarrow 1 - 0.23 = 0.77$
$P(B|A) = 0.57$
$P(B|\overline{A}) = 0.13$

$P(B \cap A) + P(B \cap \overline{A}) = ??$

$P(B|A) = \frac{P(B \cap A)}{P(A)} \Rightarrow 0.57 = \frac{P(B \cap A)}{0.23} \Rightarrow P(B \cap A) = 0.57(0.23) \Rightarrow = 0.1311$

$P(B|\overline{A}) = \frac{P(B \cap \overline{A})}{P(\overline{A})} \Rightarrow 0.13 = \frac{P(B \cap \overline{A})}{0.77} \Rightarrow P(B \cap \overline{A}) = 0.13(0.77) \Rightarrow = 0.1001$

$P(B \cap A) + P(B \cap \overline{A}) = 0.1311 + 0.1001 \Rightarrow = 0.2312$

OR

$P(B) = P(B|A) \cdot P(A) + P(B|\overline{A}) \cdot P(\overline{A})$

$= 0.57 \cdot 0.23 + 0.13 \cdot 0.77$

$= 0.2312$