HOW TO WORK WORD PROBLEMS IN ALGEBRA: PART II

Word Problem Examples:

1. Age:

Abigail is 6 years older than Jonathan. Six years ago she was twice as old as he. How old is each now?

Solution:

Let x = Jonathan's age now (smaller number)
x + 6 = Abigail's age now
x - 6 = Jonathan's age 6 years ago
(x + 6) - 6 = Abigail's age 6 years ago

Six years ago she was twice as old as he.

Equation:

$$(x + 6) - 6 = 2(x - 6)$$
$$x = 2x - 12$$
$$-x = -12$$

Answers: x = 12 (Jonathan's age now)
x + 6 = 18 (Abigail's age now)

2. Numbers:

There are two numbers whose sum is 72. One number is twice the other. What are the numbers?

Steps:

1. Read the problem. It is about numbers.

2. The question at the end asks, "What are the numbers?" So we start out "let x = smaller number." Be sure you always start with x (that is, 1x). Never start off with "let 2x = something," because it is meaningless unless you know what x stands for.

3. Now let's put it all together and solve.

Solution:

Let x = smaller number
2x = larger number

Then $x + 2x = 72$
$3x = 72$

Answers: x = 24 (smaller number)
2x = 48 (larger number)

Check: The sum of the numbers is 72. Thus 24 + 48 = 72.

3. Rate, Time, and Distance:

A freight train starts from Los Angeles and heads for Chicago at 40 mph. Two hours later a passenger train leaves the same station for Chicago traveling 60 mph. How long before the passenger train overtakes the freight train?

Steps:

1. Read the problem through carefully.
2. The question at the end of the problem asks "how long?" (which means time) for the passenger train. **Time** is your unknown.

Solution:

Draw sketch of movement.

	Rate	**Time**	**Distance**
Freight			
Passenger			

Now read the problem again from the beginning for those individual steps. The freight train traveled 40 mph, and "Two hours later a passenger train... traveling 60 mph":

	Rate	**Time**	**Distance**
Freight	40		
Passenger	60		

But so far you haven't any unknown. The question asked is, "**How long** before the passenger train overtakes the freight train?"

Let x = time in hours for the passenger train.

The passenger train started 2 hours after the freight train, so the freight train took 2 hours longer. You can represent the time for the freight train by x + 2:

	Rate	**Time**	**Distance**
Freight	40	x + 2	
Passenger	60	x	

Now, rate times time equals distance ($r \times t = d$), so multiply what you have in the rate box times what you have in the time box, and put the result in the distance box:

	Rate	**Time**	**Distance**
Freight	40	x + 2	40(x + 2)
Passenger	60	x	60x

Set these two distances equal for your equation:

$$40(x + 2) = 60x$$
$$40x + 80 = 60x$$
$$-20x = -80$$

Answer: x = 4

Check: $40(4 + 2) = 60(4)$
$240 = 240$

4. Mixtures:

A mixture containing 6% boric acid is to be mixed with 2 quarts of a mixture which is 15% boric acid in order to obtain a solution which is 12% boric acid. How much of the 6% solution must be used?

Steps:

1. Draw three diagrams to represent the three solutions.

1st Mixture		2nd Mixture		Total Mixture

2. Find out what the percents are in each mixture and the amount and put it in the proper place:

1st		2nd 2 qt		Total
6%	+	15%	=	12%

3. The question asks, "How much of the 6% solution must be used?" Therefore, let x represent the amount of 6% solution used. The total solution will have **x** quarts **plus 2** quarts (or the total of the other two solutions).

   Let x = quarts of 6% solution

x qt		2 qt		(x + 2) qt
6%	+	15%	=	12%

   You now are ready to solve the problem. If you multiply the amount of solution by the percent of acid in the solution, you will find the amount of **pure** boric acid in each solution. The amount of pure acid in the final solution is **equal** to the sum of the amounts of pure acid in the two original solutions.

0.06x	2(0.15)	0.12(x + 2)
Pure acid	Pure acid	Pure acid

   Equation: $0.06x + 2\,(0.15) = 0.12(x + 2)$

   Eliminate parentheses first: $0.06x + 0.30 = 0.12x + 0.24$

   To clear decimals, multiply by 100 which moves all decimal points two places to the right:

$$6x + 30 = 12x + 24$$
$$-6x = -6$$

Answer: $x = 1$

Check: $0.06x + 0.30 = 0.12x + 0.24$
$0.06(1) + 0.30 = 0.12(1) + 0.24$
$0.36 = 0.36$

5. Coins:

Michael has some coins in his pocket consisting of dimes, nickels, and pennies. He has two more nickels than dimes, and three times as many pennies as nickels. How many of each kind of coins does he have if the total value is 52 cents?

Steps:

1. Determine which of the coin he has the **fewest**. This is often a good way to find what x represent. Here he has fewer dimes than nickels or pennies.

2. The question asks **how many** of each kind of coin does he have. (Not how much they are worth!). That is, what **number** of each kind of coin does he have? So, let x = number of dimes.

3. Look at one fact at a time. He has two more nickels than dimes. Let

   x + 2 = number of nickels

4. Next fact, he has "three times as many pennies as nickels."

   3(x + 2) = number of pennies

Number of Coins	Value in Cents
x = number of dimes	10x = number of cents in dimes
x + 2 = number of nickels	5(x + 2) = number of cents in nickels
3(x + 2) = number of pennies	3(x + 2) = number of cents in pennies

Now add the **amounts of money**. If you make it all pennies, there are no decimals.

$$10x + 5(x + 2) + 3(x + 2) = 52$$
$$10x + 5x + 10 + 3x + 6 = 52$$
$$8x = 36$$

Answers: x = 2 (number of dimes)
x + 2 = 4 (number of nickels)
3(x + 2) = 12 (number of pennies)

Check: 2(10) = 20 cents in dimes
4(5) = 20 cents in nickels
12(1) = **12** cents in pennies

Total = 52 cents

FACTS TO REMEMBER ABOUT SOLVING AN EQUATION

These are facts you should have already learned about procedures in problem solutions.

1. Remove parentheses first.

   subtraction -(3x + 2) = -3x - 2
   multiplication (distributive law) 3(x + 2) = 3x + 6

2. Remove fractions by multiplying by the **l**owest **c**ommon **d**enominator.

   3/x + 4 = 5/2x + 3

   The LCD is 2x. Multiplying both sides of the equation by 2x:
   6 + 8x = 5 + 6x

3. Decimals **MAY** be removed from an equation before solving. Multiply by a power of 10 large enough to make all decimal numbers whole numbers.

   Example:
   0.03x + 201.2 - x = 85

   Multiply both sides of the equation by 100.

   3x + 20120 - 100x = 8500

Word Problem Practice:

1. Think of a number. Double the number. Subtract 6 from the result and divide the answer by 2. The quotient will be 20. What is the number?

2. There are three consecutive even numbers such that twice the first is 20 more than the second. Find the numbers.

3. Jay's father is twice as old as Jay. In 20 years Jay will be two-thirds as old as his father. How old is each now?

4. Wolfgang and Heinrich worked as electricians at $14 and $12 per hour respectively. One month Wolfgang worked 10 hours more than Heinrich. If their total income for the month was $3520, how many hours did each work during the month?

5. Three-fifths of the men in chemistry class have beards and two-thirds of the women have long hair. If there are 120 men in the class and 46 are not in the above groups, how many men and how many women are there in the class?

6. A service station checks Mr. Gittleboro's radiator and finds it contains only 30% antifreeze. If the radiator holds 10 quarts and is full, how much must be drained off and replaced with pure antifreeze in order to bring it up to a required 50% antifreeze?

7. Tickets for the baseball games were $2.50 for general admission and 50 cents for kids. If there were six times as many general admissions sold as there were kids' tickets, and total receipts were $7750, how many of each type of ticket were sold?

8. Bob has a coin collection made up of pennies and nickels. If he has three times as many pennies as nickels and the total face value of the coins is $416, how many coins of each kind are in the collection?

9. The Allisons are on a cross-country trip traveling with the Jensons. One day they get separated and the Jensons are 20 miles a head of the Allisons on the same road. If the Jensons average 50 mph and the Allisons travel at 60 mph, how long will it be before the Allisons catch up with the Jensons?

10. A reservoir can be filled by an inlet pipe in 24 hours and emptied by an outlet pipe in 28 hours. The foreman starts to fill the reservoir, but he forgets to close the outlet pipe. Six hours later he remembers and closes the outlet. How long does it take altogether to fill the reservoir

Answers:

1. 23
2. 22, 24, 26
3. 20, 40
4. 130, 140
5. 90 men, 30 women, 29 1/7 hours
6. 2 6/7 quarts
7. 6500, 3000
8. 200 nickels, 600 pennies
9. 2 hours
10. 29 1/7 hours

Source: Johnson, Mildred. *How to Solve Word Problems in Algebra: A Solved Problem Approach.* McGraw-Hill, Inc. 1976.
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